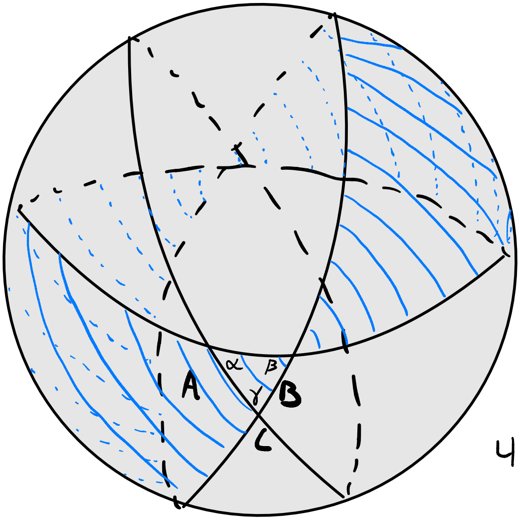
Th: Area of triangle is $\alpha+\beta+\gamma-\pi$

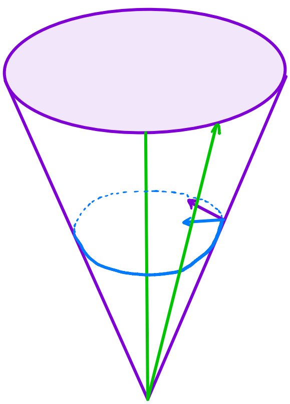


Blue wedge has area 4B

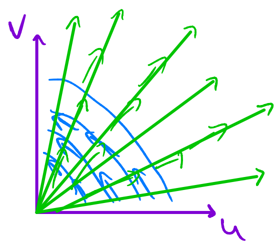
+ same for α and γ

$$4(\alpha+\beta+\gamma-\pi) = 4(\text{Area of triangle})$$

Tranformation of Parameterization



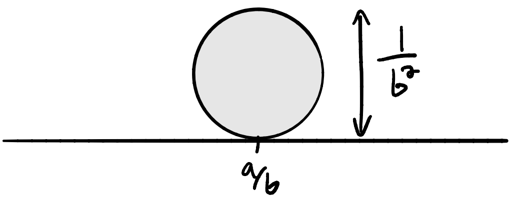
$E' = 2$	$L' = 0$
$F' = 0$	$M' = 0$
$G' = u'^2$	$N' = u'/\sqrt{2}$



Def. a/c & b/d are Farey neighbors if $ad - bc = \pm 1$ $\quad b, d \geq 1$

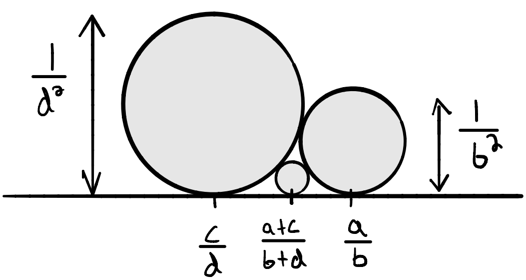
we write $\frac{a}{b} \heartsuit \frac{c}{d}$

Def The Ford circle atop the reduced fraction a/b is the circle centered at $\left(\frac{a}{b}, \frac{1}{2b^2}\right)$ of radius $\frac{1}{2b^2}$

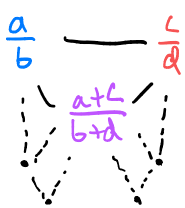


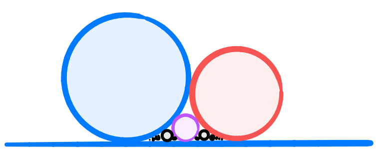
Thm 3.15 Let a/b, c/d be reduced.

$\frac{a}{b} \heartsuit \frac{c}{d} \iff$ The Ford circles atop $\frac{a}{b}$ & $\frac{c}{d}$ are tangent

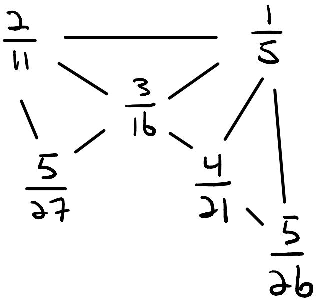


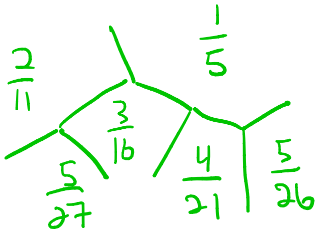
How a Farey Tree relates to Ford Circles

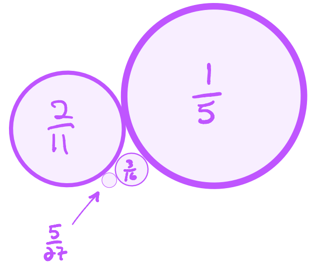




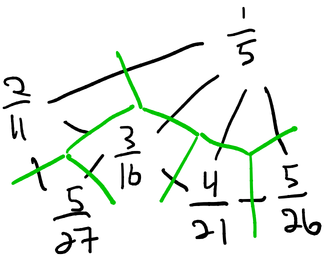
- Farey Tree
- -Topograph
- Ford Cirles

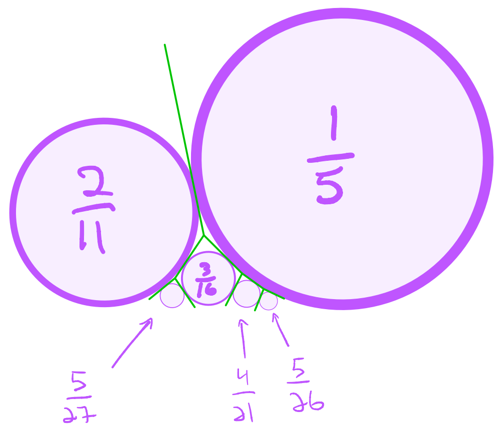




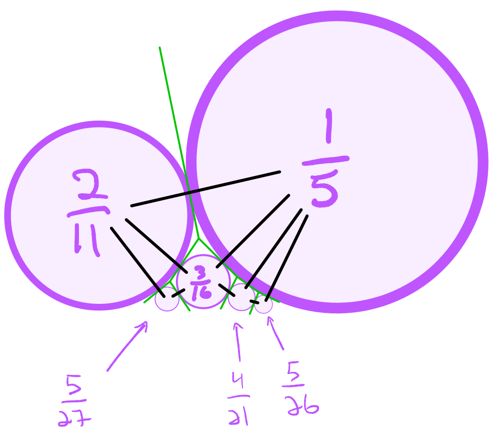


We can combine them to see how they relate





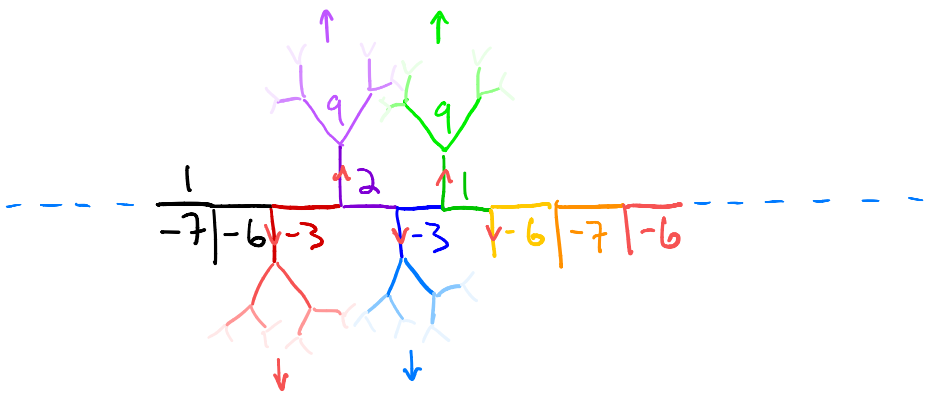
Hard to see, but lines in Topograph are tangent to Ford Circles



All Three represenations are inter related!

Pell Equation $x^2 - ny^2 \quad n > 0$

We can draw the Farey Tree as follows:



There is only one "river" and it is infinitely long
'Per Topograph $x^2 - ny^2$

The river is Periodic: There are finitely many river segments of a given discriminant. Hence, segments repeat. This proves periodicity.

Diagram Explaining Span in Linear Algebra