Elias Firisa
20220773
Theoretical Neuroscience (BCS304)
HW #1 Report

Dear TAs/Professor: This file might seem to have many pages but each page has just simple figure and brief explanation

Prob 1.a

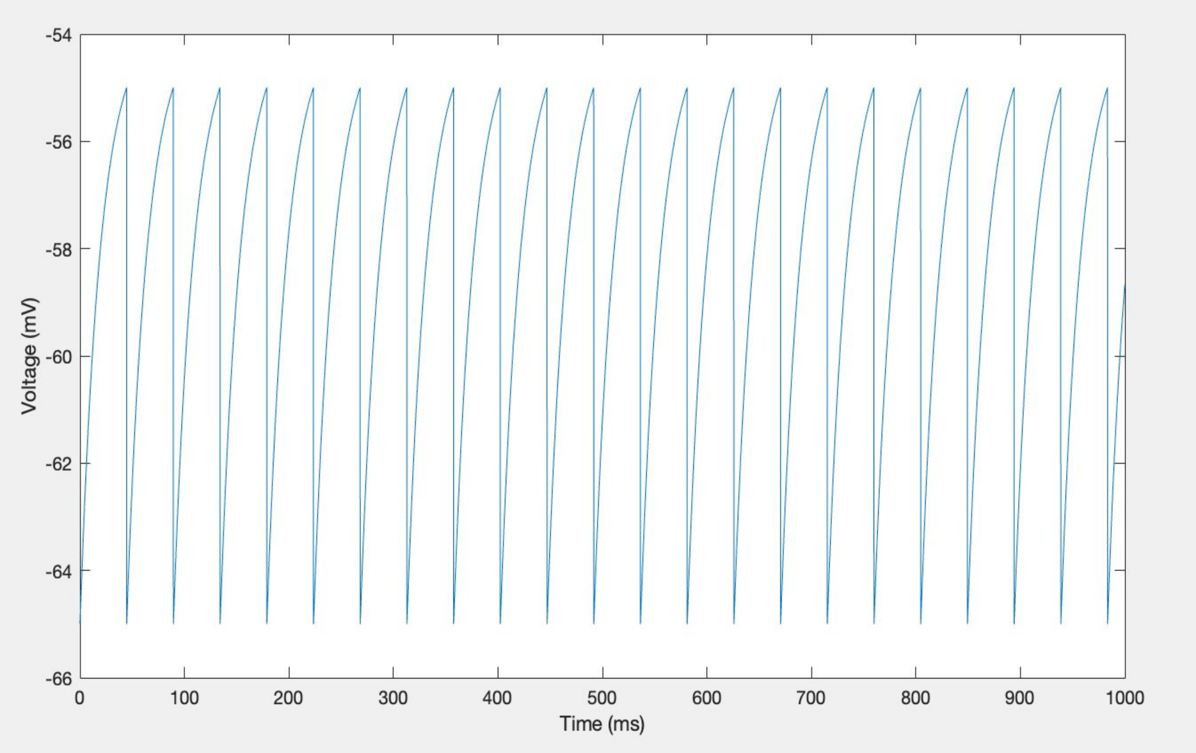


Fig 1a.1 **Membrane Potential of LIF Neuron Over 1 Second**
This plot shows the voltage trace of a leaky integrate-and-fire (LIF) neuron receiving a constant current injection (**1.12 mA**). The neuron fires repeatedly, resetting to its resting potential (Erest) each time the membrane potential crosses the threshold (Eth), resulting in an average firing rate of approximately **22 Hz**

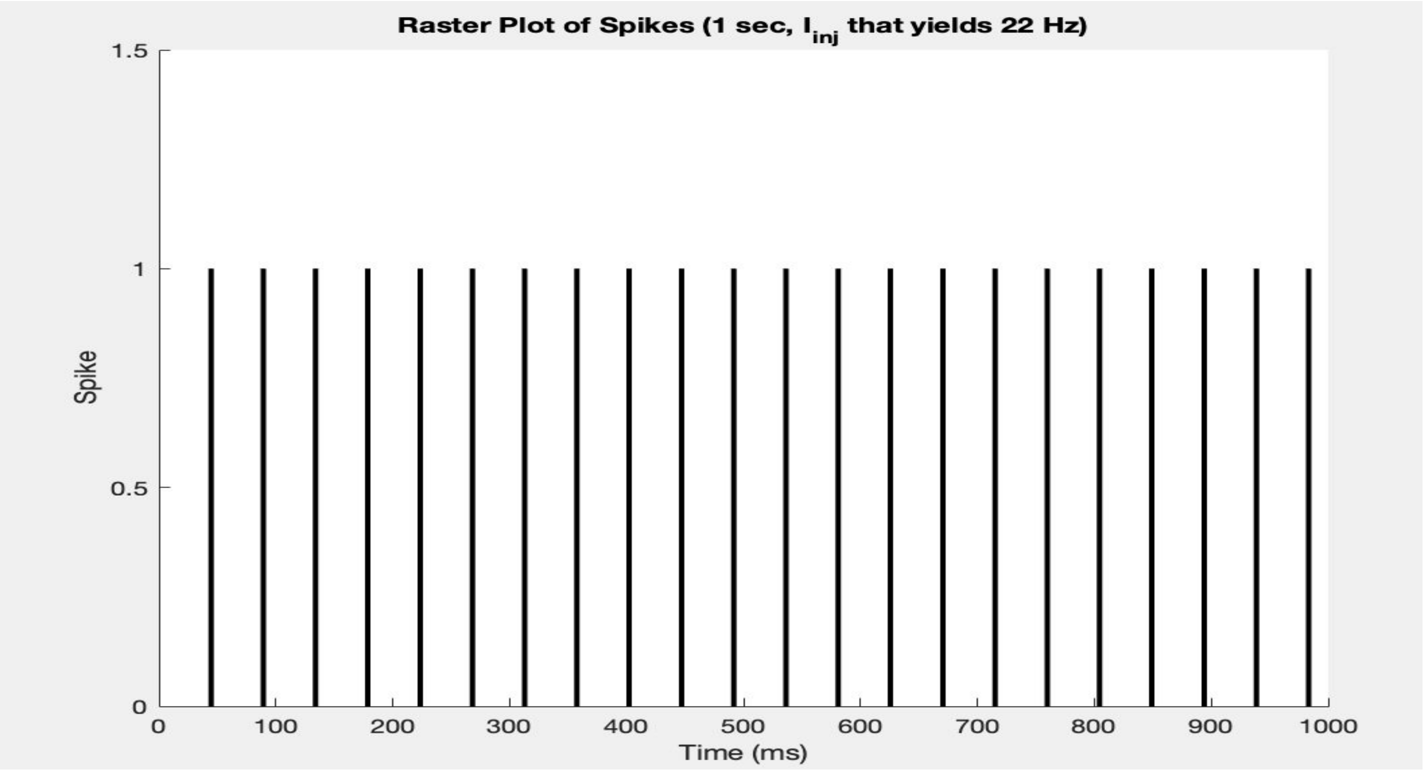


Fig. 1a.2 ***Raster Plot of Spikes (1 sec, I_inj that yields 22 Hz)***

This raster plot shows the spike times of a leaky integrate-and-fire neuron during a 1-second simulation under constant current injection (~**1.12mA**). Each vertical line indicates a spike event, and the injected current was chosen so that the neuron fires at an average rate of ~**22 Hz**.

How I (precisely and efficiently) estimated *I_c (I_inj)*

To **precisely** and **efficiently** find the constant current that yields an average firing rate of 22 Hz, I used a **two-stage** approach:

1. **Initial Parameter Sweep**
   I first tested a few coarse values of ***I_c*** (e.g., 1 mA, 2 mA, 5 mA, 10 mA) to see how the firing rate changed. This quickly showed that 1 mA gave 0 Hz (no spikes), while 2 mA produced a much higher rate (71 Hz). Thus, the target 22 Hz must lie somewhere between 1 mA and 2 mA.

2. **Refined Bisection Method(motivated by binary search algorithm)**

   - I bracketed the target rate between ***I_low***=1 mA and ***I_high***=2 mA.
   - I then **halved** the interval: ***I_mid***=(***I_low***+***I_high***)/2.
   - After simulating the neuron for 1 second at ***I_mid*** and measuring the firing rate, I updated either ***I_low or I_high*** depending on whether the firing rate was below or above 22 Hz.
   - Repeating this process **rapidly converged** on a value that produced the desired 22 Hz, without exhaustively testing every current value

Ultimately, this **binary search** approach led to an ***I_inj*** of approximately **1.12 mA**, which yielded a stable firing rate of about 22 Hz. The bisection method is both efficient and precise because the firing rate in a leaky integrate-and-fire model typically changes **monotonically** with injected current, ensuring each halving of the interval quickly narrows in on the correct value

Problem 1b: finding *I_inj that produces 2Hz firing rate (within a 1-sec duration)*

A 2 Hz firing rate means the neuron fires only twice per second, so the injected current is barely enough to push the membrane potential from its reset (hyperpolarized) state to threshold. In this region, the neuron is on the cusp between not firing anything (0 Hz) and occasional firing (2 Hz). Consequently, even extremely small changes in ***I_inj*** can cause large differences in the firing rate. For example, my experiments showed that:

- ***I_inj***=1.01 mA produced 10 Hz,
- ***I_inj***=1.001 mA produced 7 Hz,
- ***I_inj***=1.0000 mA produced 4 Hz,
- ***I_inj***=1.0000001 mA produced 3 Hz,
- ***I_inj***=1.00000001 mA produced 2 Hz.

This demonstrates that near the threshold for low firing rates, the system is extremely sensitive to minute changes in the injected current. As a result, achieving a precise 2 Hz rate requires high numerical precision and possibly longer simulation times to accurately capture this delicate balance

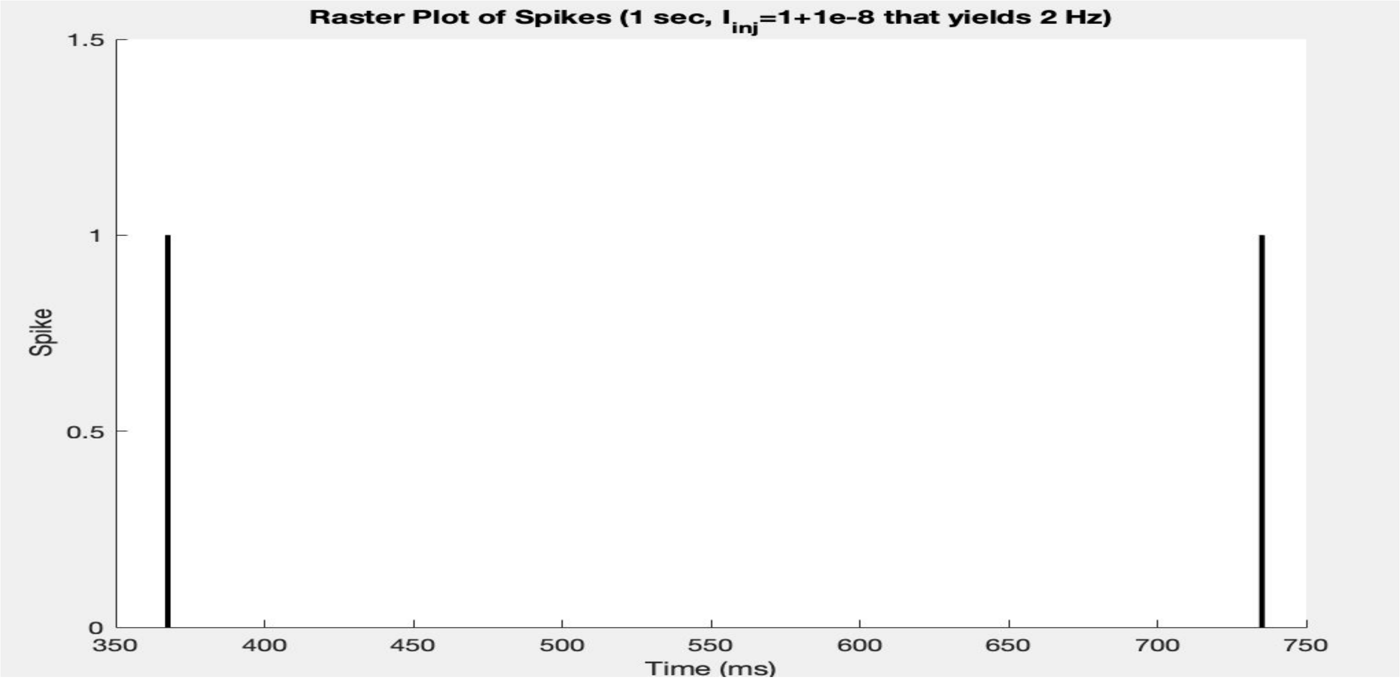


Fig. 1b.1 ***Raster Plot of Spikes (1 sec, I_inj that yields 2 Hz)***

This raster plot shows the spike times of a leaky integrate-and-fire neuron during a 1-second simulation under constant current injection (**1.00000001mA**). Each vertical line indicates a spike event, and the injected current was chosen so that the neuron fires at an average rate of ~**2 Hz**.

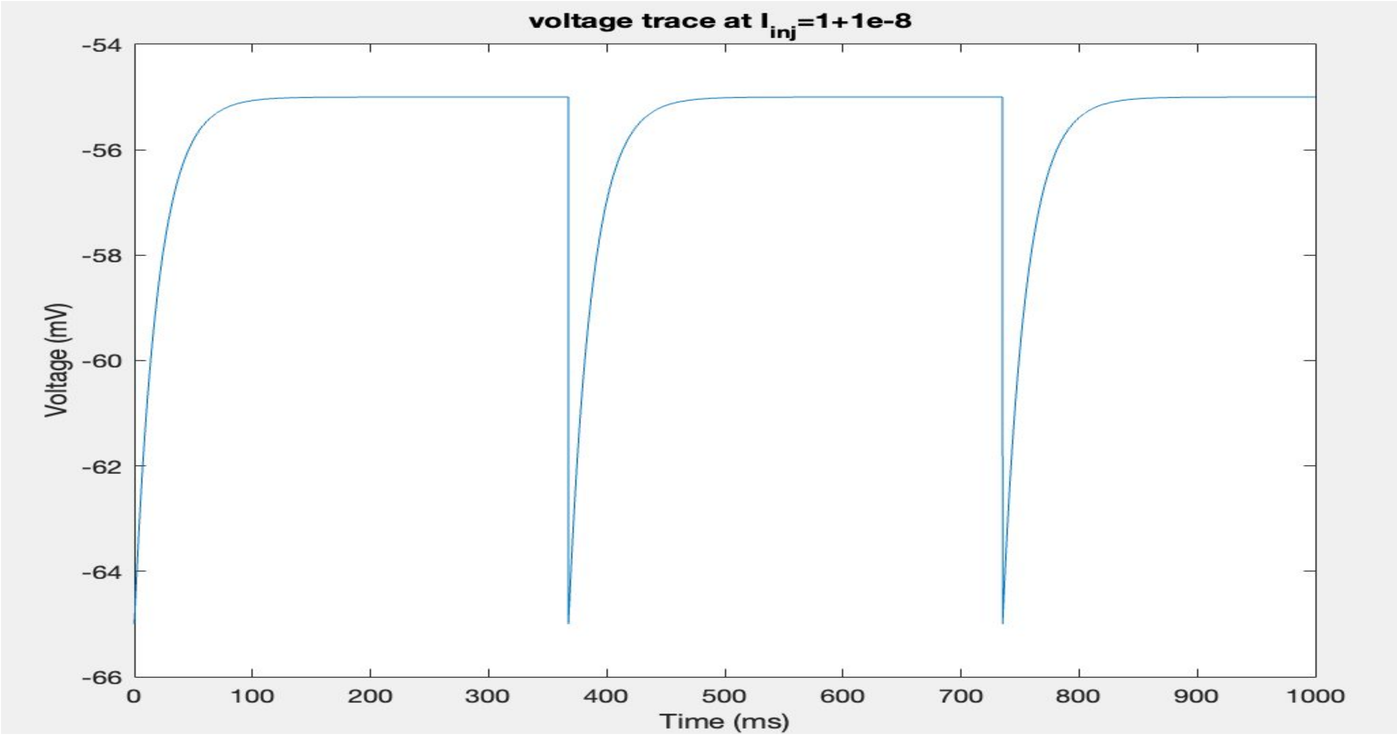


Fig. 1b.2 Membrane potential of LIF neuron at very small I_inj value

The figure shows only two spikes in a period of 1 second (firing rate of 2Hz). This happens when the injected current is barely enough to push the membrane potential from its reset (hyperpolarized after each spike) state to threshold.

Problem 1c

Estimating the smallest I_c at which the neuron begins to fire (recording at least one spike) within a 1-sec duration

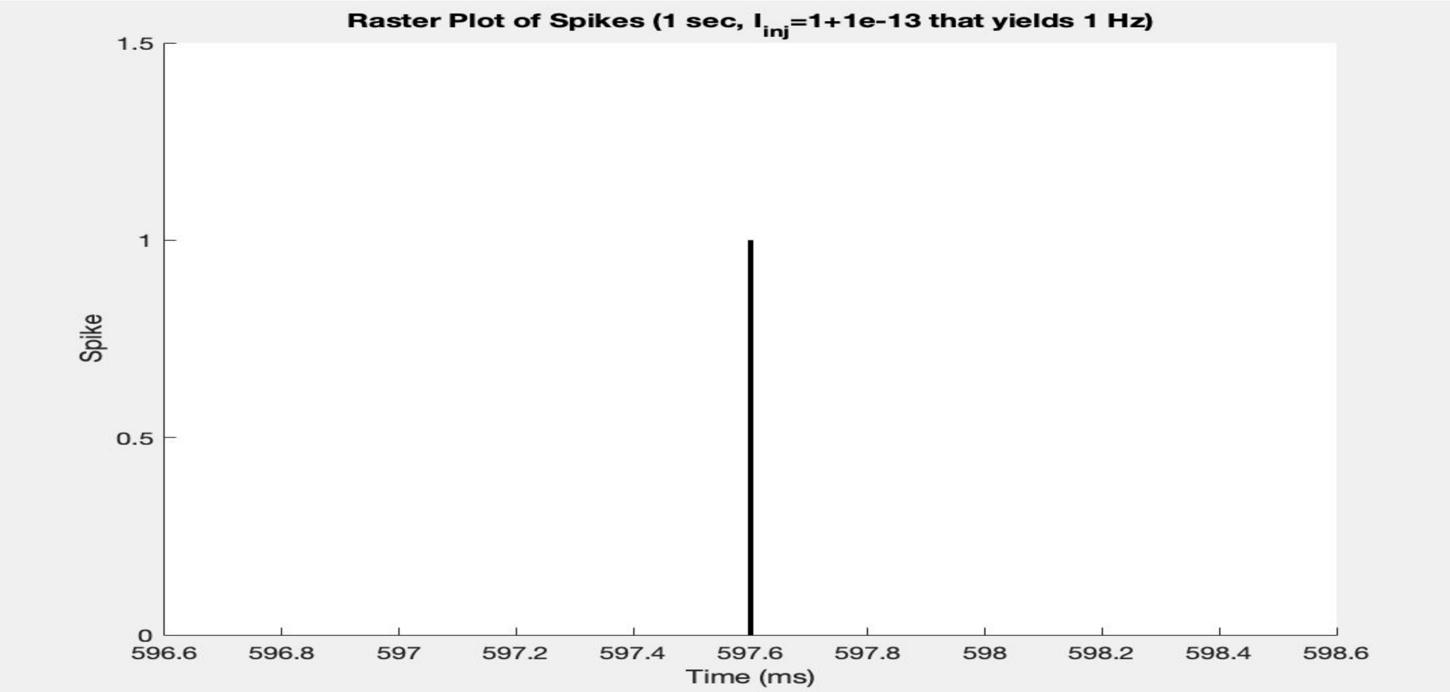


Fig. 1c.1 Raster Plot of Spikes (1 sec, I_inj that yields 1 Hz)

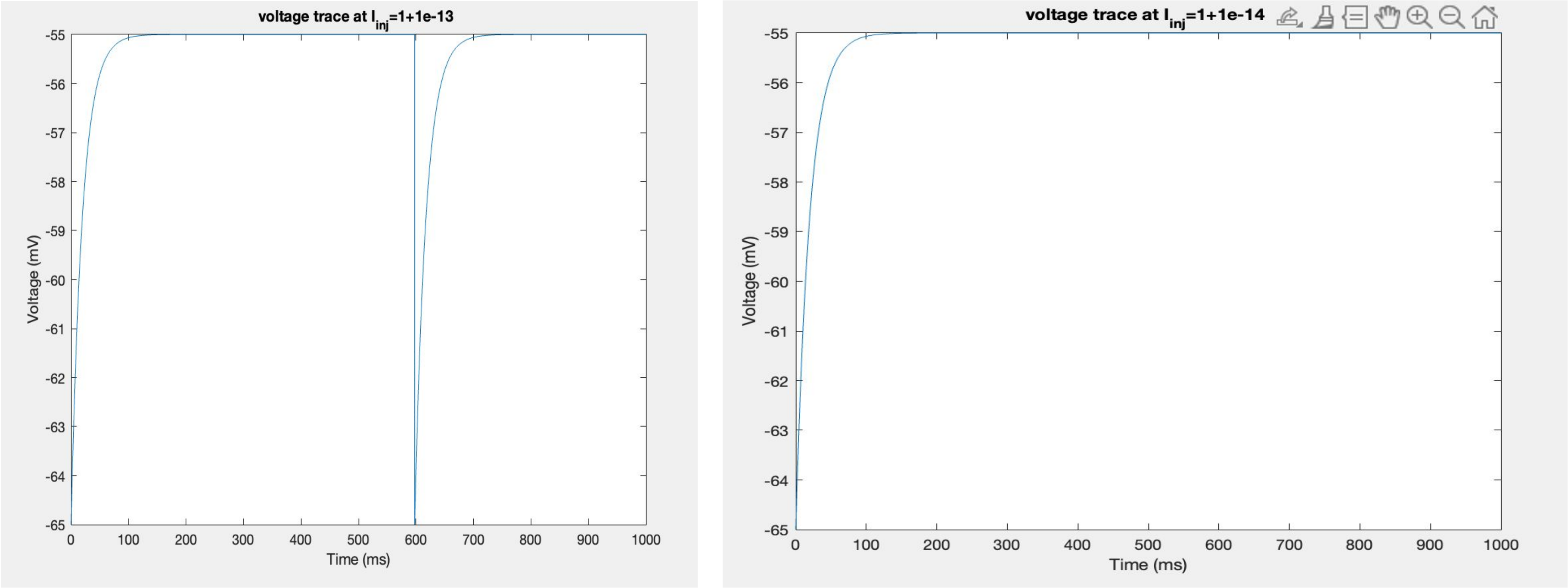


Fig. 1c.2 **Membrane potential of LIF neuron at very small I_inj value**

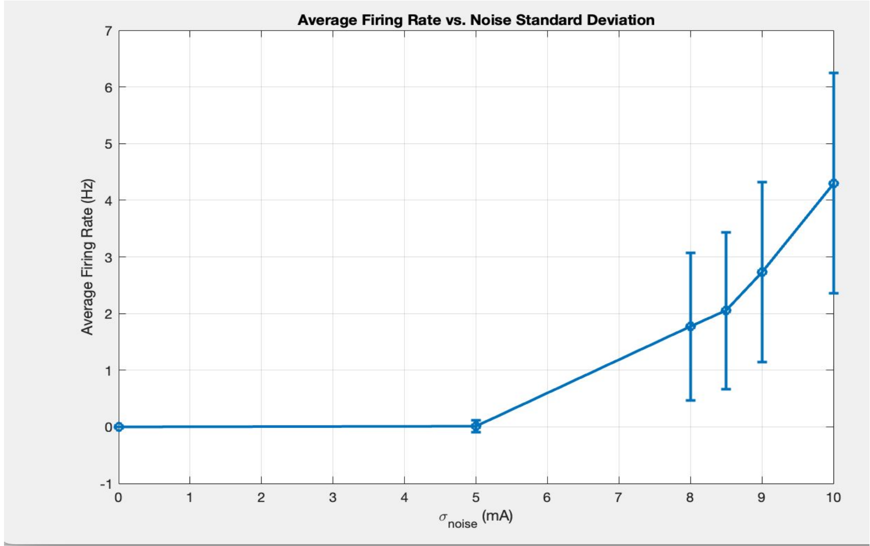
The neuron remains subthreshold for ~600 ms, fires exactly one spike, and then resets. A slightly lower current yields zero spikes (since I_inj≈1+1×10−14 mA produces no spike), indicating that I_inj≈1+1×10−13 mA is near the threshold for non-zero firing under these conditions

So, how would I measure this threshold as accurately as possible?

From earlier questions 1a and 1b, I have established 1+1e-8 and 1 to be a current values that produce 2Hz and 0Hz firing rates respectively under 1-second duration.(this experiment is done under 1-second duration, which means 1+1e-8 can produce more spikes if we grow time to -for example- 8 seconds). Given those values, now it's clear that I_c that produces 1Hz lies between 1 and 1+1e-8. I first did a course search by checking for values such as 1+1e-10, 1+1e-12,1+1e-14. The firing rate didn't change from 2Hz until I tried with 1+1e-14, where there's no spike. Now, using the bisection approach, I_c obviously lies between 1+1e-12 and 1+1e-14. So , I tried the middle value 1+1e-13 which resulted in 1Hz firing rate. Therefore, we can say that-with very god precision- I_c=1+1e-13 is the threshold current value at which non-zero firing begins.

Again, this experiment is done under 1-second duration to compare result with those of problems 1a and 1b.

Problem 1d: simulating with random noise




```
For sigma_noise = 0, Average firing rate = 0.00 Hz
For sigma_noise = 5, Average firing rate = 0.01 Hz
For sigma_noise = 8, Average firing rate = 1.77 Hz
For sigma_noise = 8.5, Average firing rate = 2.05 Hz
For sigma_noise = 9, Average firing rate = 2.73 Hz
For sigma_noise = 10, Average firing rate = 4.30 Hz
```

Fig. 1d.1 Average Firing Rate vs. Noise Standard Deviation

This plot shows the mean firing rate (± standard deviation) of a leaky integrate-and-fire neuron across 100 trials for different noise standard deviations (σnoise). No constant current (Iinj=0) is injected, so all spikes arise solely from random fluctuations. At low σnoise, the neuron rarely spikes, while higher noise amplitudes significantly increase the firing rate. σnoise=8.5 produces approximately~2Hz firing rate from this experiment.

Discussion

From fig. 1d.1 each data point represents the mean firing rate across 100 independent trials, with vertical error bars indicating the standard deviation. At low noise levels (σnoise≤5), the neuron rarely or never fires, as random fluctuations are insufficient to push the membrane potential above threshold. Once σnoise surpasses a critical level (around 8 mA), the firing rate rises sharply, indicating that noise-driven threshold crossings become more frequent. This transition reflects the neuron's high sensitivity near threshold—small increases in noise can cause large changes in firing rate. The standard deviations show that, even at a fixed σnoise, there is trial-to-trial variability in the spiking behavior due to the stochastic nature of the input. Overall, this analysis demonstrates that purely noise-driven spiking emerges once the noise amplitude is sufficient to overcome the leak and reset dynamics of the LIF model

Prob 1d: My attempt to accurately measure the optimal sigma

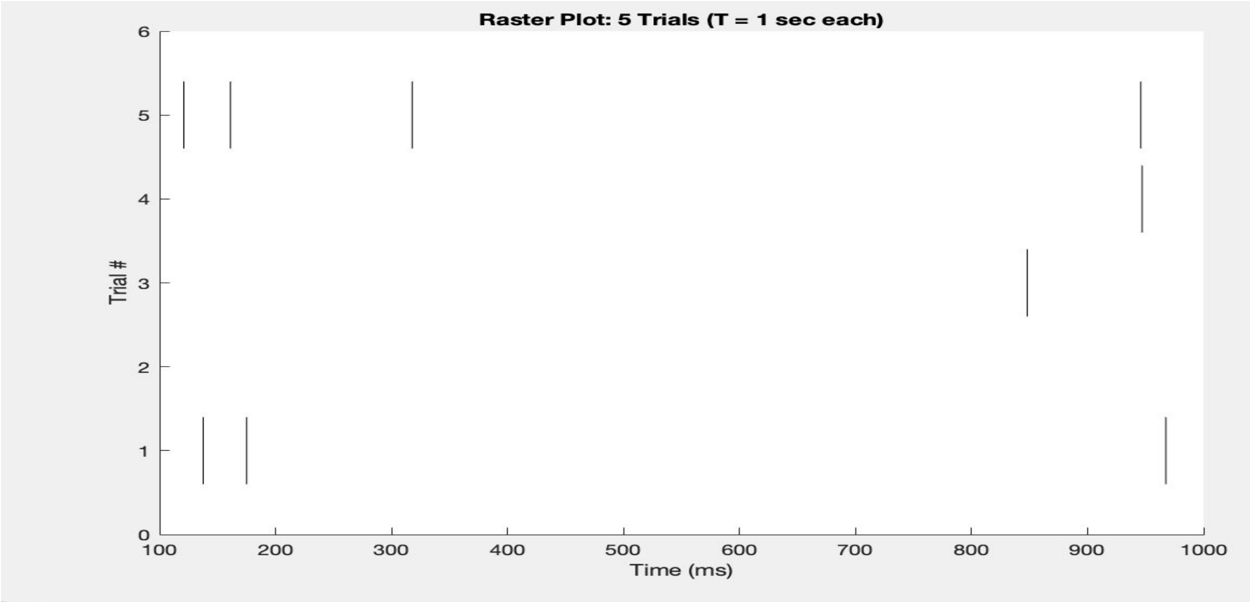
To **measure** the σnoise value that yields a spontaneous firing rate of approximately 2 Hz, I combined:

- **Longer Simulations & Multiple Trials:** Running the neuron for a few seconds (**1 s** in the figure, though 5–10 s can further reduce variability) and averaging over multiple trials (with different random seeds) mitigates the inherent stochastic fluctuations of the noise.

- **Coarse-to-Fine Parameter Search:** Starting with a coarse sweep (σnoise=1 mA and 10 mA) quickly brackets the region of interest. I then used intermediate values (5,8, 8.5, 9 mA) to home in on a noise level near 2 Hz. This approach resembles a bisection method, narrowing down σnoise in a structured way.

- **Averaging Firing Rates:** At each noise level, I recorded the firing rate across **100** independent trials. By averaging these rates, I obtained a stable estimate of the neuron's mean firing behavior, despite variability in individual runs

Problem 1e

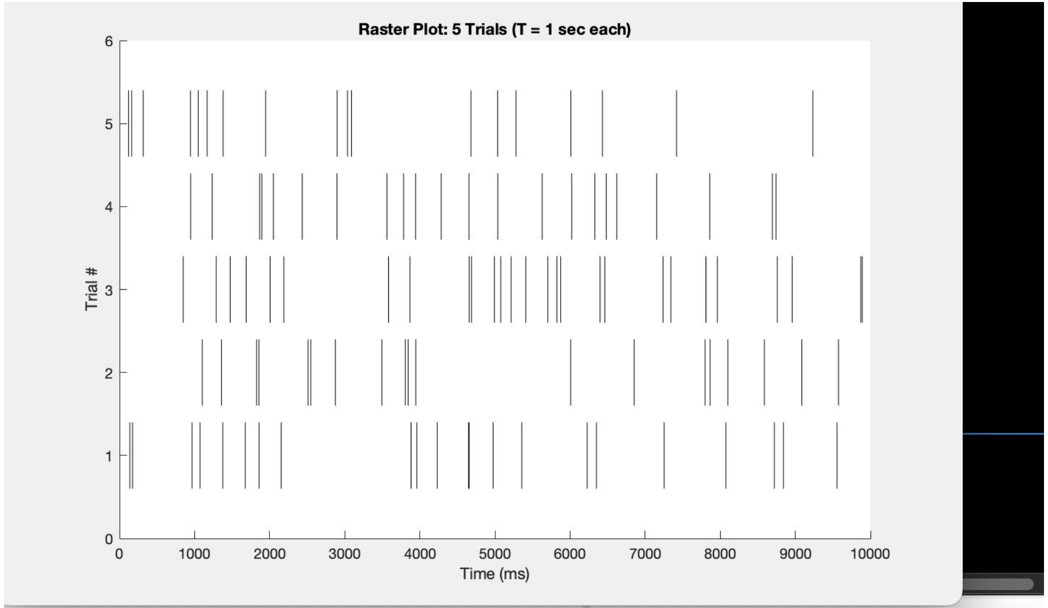



```
>> problem1e
Firing rates for each trial: 3   0   1   1   4
Mean firing rate: 1.80 Hz
Variance of firing rates: 2.70 (Hz^2)
```

Fig. 1e.1 Raster Plot of 5 Trials (T = 1 sec each, σnoise≈8.5)

This figure shows the spike times for five independent 1-second trials of a leaky integrate-and-fire (LIF) neuron receiving only Gaussian noise (I_inj=0). Each row corresponds to one trial, and each vertical line indicates a spike. Across these trials, the firing rates were 3, 0, 1, 1, and 4 Hz, yielding an average firing rate of 1.80 Hz and a variance of 2.70 (Hz2^22). The neuron operates near threshold, so even small random differences in the noise produce large variability in spiking behavior

Problem 1f



Command Window

```
>> problem1f
Firing rates for each trial: 2.2        1.9        2.7        2.2
Mean firing rate: 2.16 Hz
Variance of firing rates: 0.12 (Hz^2)
```

Fig. 1f.1 Raster Plot of 5 Trials (T = 1 0sec each, σnoise≈8.5)

This raster plot shows the spike times for five independent 10-second trials of a leaky integrate-and-fire neuron receiving only Gaussian noise (I_inj=0). Each row corresponds to one trial, and each vertical line indicates a spike event. With a longer observation period, the neuron's firing rate is measured more reliably, as evidenced by a lower variance across trials

Problem 1f: Relationship between mean firing rate and length of observation time for accurate estimation

To investigate the impact of simulation time on accuracy of estimated mean firing rate, I repeated a procedure from part(e) but increased simulation time from 1-sec to 10-secs. I run 5 independent trials each with same sigma_noise($\sigma_{noise} \approx 8.5$) that approximately produced 2Hz in part (d), recorded spike trains and then computed firing rate for each trial. As can be seen in fig. 1f.1, the firing rates across trials are now closer to each other compared to the 1-sec case. The mean 2.16 Hz stays closer to the target 2Hz and variance 0.12Hz^2 is much lower than that of shorter 1-sec case. When the neuron is near threshold, spikes are infrequent, and short simulations can produce large trial-to-trial differences. However, longer simulations ensure that each trial captures the neuron's "typical" spiking behavior more consistently, thus reducing the measurement variance. From my observation, increasing the observation length from 1 second to 10 seconds substantially improves the reliability of the firing rate measurement at low firing rates. The results confirm that the average firing rate revolves around 2 Hz, and the variance across trials drops significantly when more spikes are collected, making the estimate more accurate.

Problem 1g

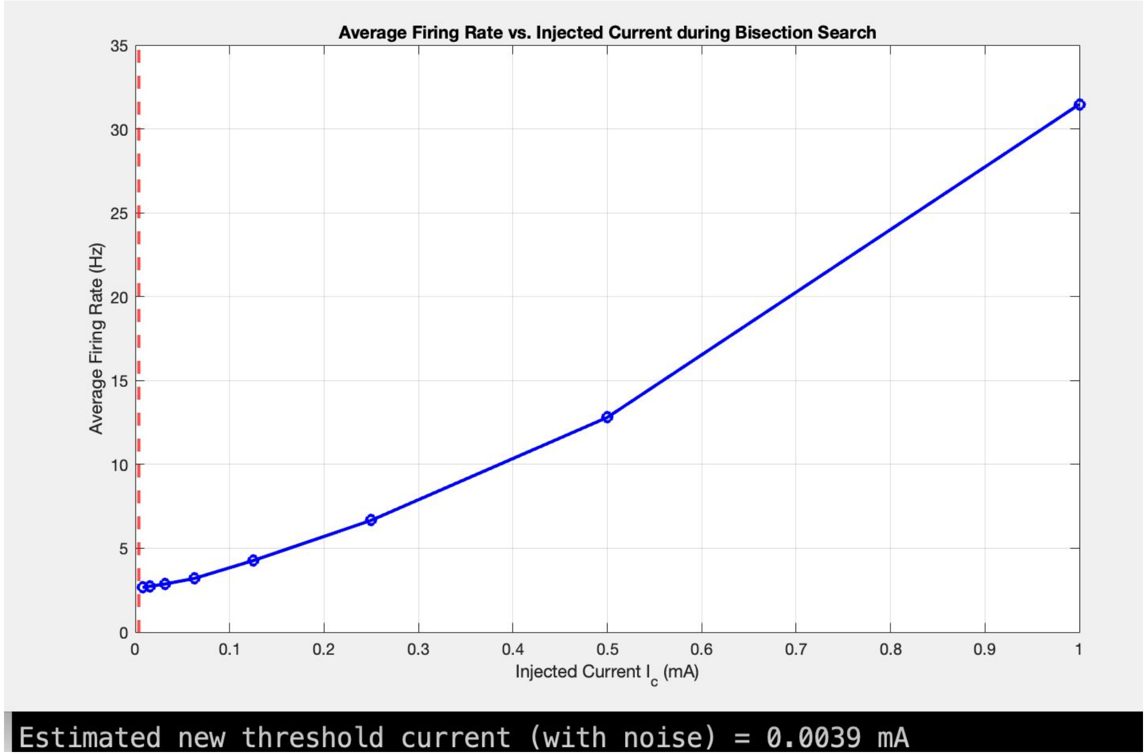


Fig. 1g.1 Average Firing Rate vs. Injected Current (I_c) in the Presence of Background Noise

This plot shows how the neuron's firing rate increases with the injected current (I_c) when background noise (σnoise=8.5) is present. Even at I_c=0, the neuron fires at a non-zero rate due to noise alone, and the firing rate grows steadily with small increases in I_c. The data illustrate that background noise significantly lowers the effective threshold current, causing spontaneous spiking at near-zero I_c.

The approach I used to determine new threshold current

I chose an initial lower bound, I_low, (where I expected no-spiking) and upper bound, I_high, where I knew(from the previous parts) the neuron would fire robustly. For each candidate I_c I run multiple independent trials and computed mean firing rate. I used bisection method to narrow down the range and computed firing rate at the midpoint of the bounds. This update is done based on whether rate exceeded a threshold (min non-zero spiking rate defined in code). This process was repeated until the difference (I_high-I_low) fell below a small tolerance value (defined in code). Because the neuron is already spiking at Ic=0, the **bisection search** likely converged to a very small positive Ic (~0.0039 mA) as the "new threshold."

Problem 1h: comparing response function curves

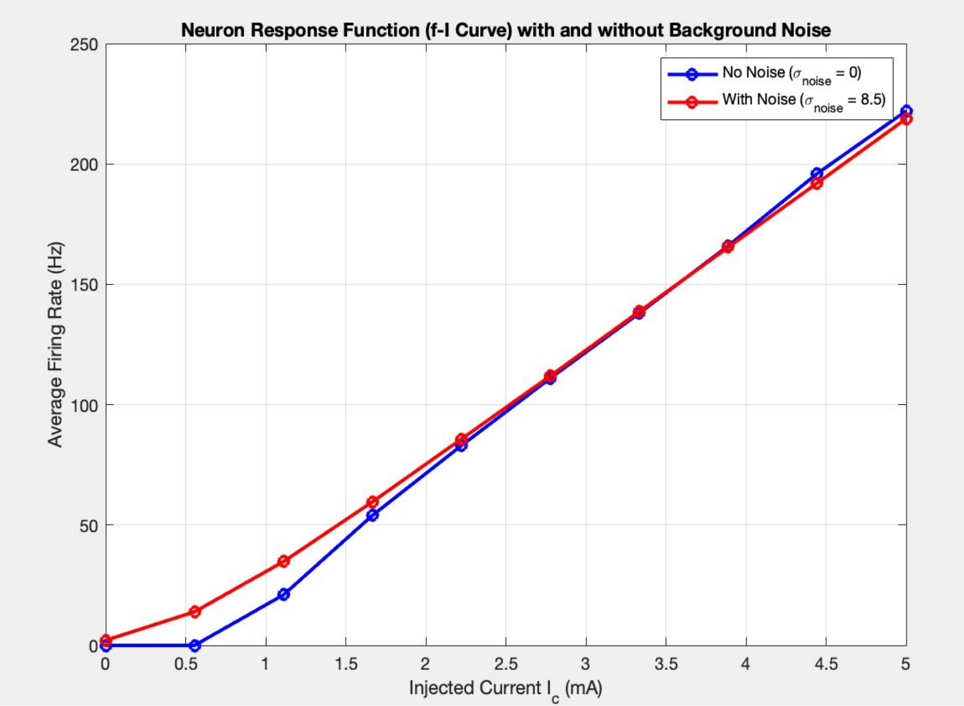


Fig. 1h.1 Neuron Response Function (f–I Curves) with and without Background Noise (for 1-sec, 50 trials)

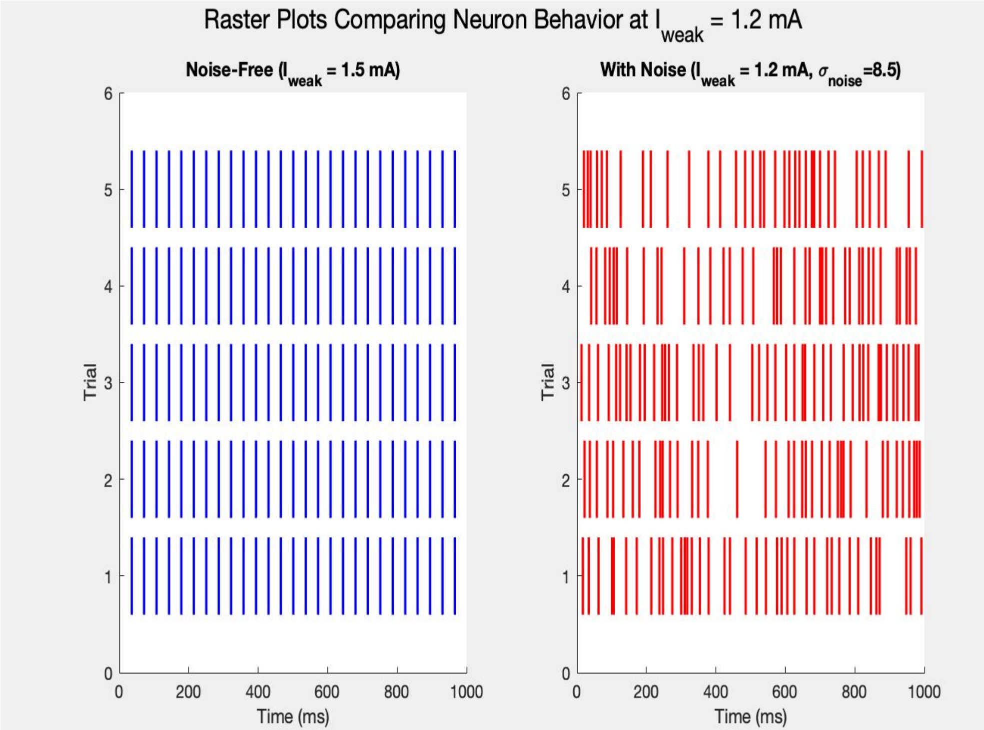
This plot shows the average firing rate as a function of the injected current (I_c) for a leaky integrate-and-fire neuron under two conditions: noise-free (σnoise=0) and noise-driven (σnoise=8.5). Each curve is based on 10 values of I_c from 0 to 2 mA, with multiple trials averaged at each point. The noise-free curve (blue) exhibits a sharp threshold, remaining at 0 Hz until I_c≈0.8mA, while the noise-driven curve (red) demonstrates spontaneous firing at I_c=0 and generally higher firing rates, indicating that background noise significantly reduces the effective threshold current

Problem 1h: Discussion

Choice of input currents: From Part (a) (Noise-Free Case) I discovered that in the absence of noise, and 2 mA yielded a very high firing rate (70+ Hz). And **from Part (d) (With Noise) I** found that background noise alone could drive the neuron to spike at I_c = 0. Therefore, starting at 0 mA naturally covers the case of "noise-only" spiking and captures how even small additional currents (<1 mA) boost the firing rate in the noise-driven regime. Then I used ***linspace(0, 2, 10)*** to ensure that I sample both low-current and high-current regions.

Insight from the curves: Both curves become **steeper** at higher I_c, but the noise-driven curve is consistently **above** the noise-free curve for most I _c. This is because noise adds random depolarizations that, on average, boost the firing rate. The **noise-free curve** remains flat (0 Hz) until the classical threshold current is exceeded, then ramps up. The **noise-present curve** transitions more smoothly from low to high firing rates, lacking the abrupt "hard" threshold

Problem 1i




```
>> problem1i
Noise-free condition: Mean firing rate = 27.00 Hz
With noise: Mean firing rate = 39.60 Hz
```

Fig. 1i.1

Fig. 1i.1 ***Raster Plots Comparing Neuron Behavior at I-weak=1.2 mA With and Without Noise***

These raster plots show the spike times of a leaky integrate-and-fire neuron across multiple trials at a fixed input current of 1.2 mA (a value for which even the noise-free neuron starts firing). The left subplot (noise-free condition) reveals a deterministic and regular spiking pattern at an average rate of 27 Hz, while the right subplot (with background noise, σnoise=8.5) exhibits a higher average firing rate (39.6 Hz) and greater variability in spike timing. This comparison depicts how background noise both increases the neuron's firing rate and introduces stochastic fluctuations in spike occurrence.

Problem 1i: Discussion of Neuron Behavior at I_weak=1.2 mA

Noise-free setting (σnoise=0):

- The neuron fires at a **mean firing rate of 27 Hz**, producing a **regular** spiking pattern across trials.
- Since there is no stochastic input, the spike timing is **deterministic**, and each trial closely resembles the next, with minimal variation in interspike intervals.

With background noise (σnoise=8.5):

- The neuron's mean firing rate **increases** to about 39.6 Hz.
- The presence of noise adds **random depolarizations** that push the membrane potential above threshold more frequently, leading to a higher overall firing rate.
- Spike timing becomes **less regular**, as noise introduces variability from trial to trial. Small random fluctuations can accelerate or delay individual spikes, making the raster plot appear denser and less periodic than in the noise-free condition