BCS304 Theoretical Neuroscience　　　　　　　　　　　Spring 2025

Final Exam: Due 6/13(Fri), 6:00PM

Please prepare your report as outlined below and send it to: **bcs304ta@gmail.com**

- Summarize your results, including discussions and comments for each question, in a PDF file. Name the file as: **“Final_answer.pdf”**.
- Zip all the MATLAB® codes and text files into one compressed folder, named as: **“Final_YourSID_Your Name.zip”**.
- For each problem, clearly indicate which file to run by naming them as: **“Prob1a.m”**, **“Prob1b.m”**, etc. You may also include as many sub-function files as necessary.
- Failure to follow these instructions may result in a penalty.

Problem 1 [50pts]

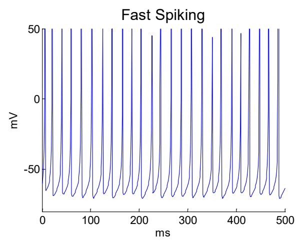
The Izhikevich neuron is modeled as

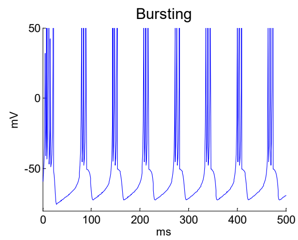
$$\frac{dV(t)}{dt} = 0.04V^2 + 5V + 140 - u + I(t)$$

$$\frac{du(t)}{dt} = a(bV - u)$$

with the reset condition

$$V(V > 30) = c,\ u(V > 30) = u + d$$



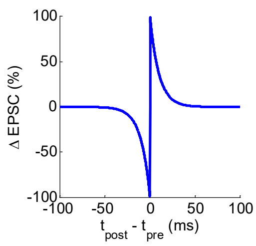


To compute $V(t)$ and $u(t)$, use the Euler method with a time step dt/τ = 1 and temporal resolution $t_n - t_{n-1}$= 1ms.

a. For $I(t)$= 10mA, find a set of parameters (*a, b, c, d*) such that the neuron fires at approximately 50Hz in the “fast spiking” mode as shown. Plot V(t) and ISI (inter-spike-interval) histogram for 500ms.

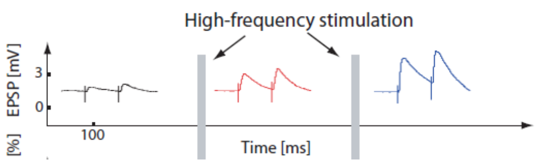
b. For the same input $I(t)$= 10mA, find a set of parameters (*a, b, c, d*) such that the neuron exibits “bursting” at approximately 50Hz. Plot V(t) and ISI histogram over 500ms.

c. Assume this neuron provides presynaptic input to a target neuron modeled as an Integrate-and-Fire neuron. Using the open-and-decay model of EPSC (Excitatory Post-Synaptic Current), find a condition of the model synapse such that this target neuron induces a spike with the input in **b**, but does not fire with the input in **a**. Plot the pre- and post-synaptic neuron firings for the two conditions, respectively.



d. Model the synapse using STDP (Spike Timing Dependent Plasticity) described by an exponential function of Δt= t_{post} - t_{pre} as shown; ΔEPSC (Δt=10ms) ≈ 1/e *ΔEPSC (Δt=0ms). Implement this STDP using appropriate equations and parameters. Show your design of the STDP and plot the ΔEPSC (%) as a function of Δt, for Δt = [-100, 100] ms as shown.

e. Based on your STDP model, estimate the change in EPSC amplitude (ΔEPSC) after a single “bursting” input from **b**. Plot the EPSCs before and after the bursting input for visual comparison. Discuss qualitatively how high-frequency stimulation can lead to an increase in EPSC amplitude.

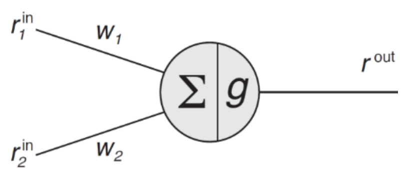


Problem 2 [20pts]

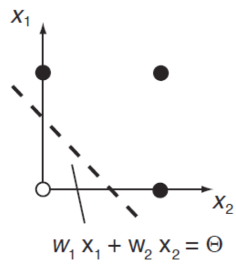
The spike pattern of a neuron can be represented as a binary string (e.g. [0 1 0 0 0 0 1 0 0 1]). Use Δt=10ms time bin for a sample neuron with the refractory period of 10ms.

- **a.** The neuron is observed to generate exactly 3 spikes in a 100ms window. Estimate analytically the total number of possible spike patterns.
- **b.** Assume that these spike patterns are equally likely. Generate N= 1,000 patterns randomly (Hint: Use "randperm"), and plot the probability distribution of them.
- **c.** Based on the result from **b**, compute the entropy S_1[bits] numerically.
- **d.** Now assume that spike generation follows a Poisson point process with an average firing rate of 30Hz (i.e. 3 spikes in 100ms). Generate and display 5 example spike trains.
- **e.** Estimate analytically the number of possible spike patterns. Compare and discuss the difference between the results in **a** and **e**.
- **f.** Under this condition, generate N=1,000 spike patterns randomly. Plot the probability distribution of them.
- **g.** Calculate the entropy S_2[bits] numerically from **f**. Compare S_1 and S_2 and discuss the reason for the difference in entropy values.

Problem 3 [20pts]



(Trappenberg)



You are given a linear perceptron model where the binary inputs and output are described as r_1^{in}, r_2^{in} and $r^{out} = g\left(w_1 r_1^{in} + w_2 r_2^{in}\right)$, where $g(x) = \begin{cases} 1 & x > \Theta \\ 0 & x \le \Theta \end{cases}$. You can implement some Boolean functions by choosing proper values of w_1, w_2 and Θ.

- **a.** Manually pick the values of w_1, w_2, and Θ so that your perceptron can perform a Boolean "AND" function. Calculate r^{out} for all pairs of r_1^{in}, r_2^{in} to show your choice of parameters works.
- **b.** The problem can be simplified by matching the equation of a line $y = ax + b$, that has only two free parameters, to $w_1 r_1^{in} + w_2 r_2^{in} = \Theta$. Explain the relationship between (a, b) and (w_1, w_2, Θ) under this simplification and describe how to decide r^{out}.
- **c.** Try a couple of randomly chosen values of (a, b) to estimate an error (cost) ***E*** for each case. In other word, calculate r^{out} for all pairs of r_1^{in}, r_2^{in} and estimate the "total difference" between the observed and expected r^{out}.
- **d.** In a 2D space of parameters *a* (x-axis) and *b* (y-axis), estimate the error $\boldsymbol{E}(a, b)$ in all locations within a proper boundary. Visualize the profile of $\boldsymbol{E}(a, b)$ using a 2D heat map. Discuss the condition of (a, b) for the system to perform a Boolean "AND" function.

Problem 4 (Questions for questions) [10pts]

Choose one question from a previous homework assignment (e.g., HW2, Problem 1b). Your task is to improve the question by rewriting it in a way that helps students better understand the key scientific concept or idea behind it. Write the revised version of the question, provide a sample answer, and explain the educational goal you intended to achieve with the new version.