BCS304 Theoretical Neuroscience Spring 2025

Homework set #4: Due 5/30(Fri), 6:00 PM

Please prepare your report as outlined below and send it to: **bcs304ta@gmail.com**

- Summarize your results, including discussions and comments for each question, in a PDF file. Name the file as: **"HW#_answer.pdf"**.
- Zip all the MATLAB® codes and text files into one compressed folder, named as: **"HW#_YourSID_YourName.zip"**.
- For each problem, clearly indicate which file to run by naming them as: **"Prob1a.m"**, **"Prob1b.m"**, etc. You may also include as many sub-function files as necessary.
- Failure to follow these instructions may result in a penalty.

Comments: In this problem set, detailed instructions are not provided, as it mirrors your own research. You'll need to use your reasoning and hypothesis to determine the necessary steps. Be as specific as possible in your explanations. Successfully demonstrating this may earn you extra points, regardless of the outcome.

Problem 1

The entropy of spike trains in the firing rate coding model can be estimated as:

$$S = -\sum_n P(n)\log_2 P(n)$$

where p(n) is the probability of observing exactly n spikes within a time window T.

- **a.** Assume that a neuron's firing follows a Poisson distribution, where the probability of observing n spikes is given by $P(n) = \frac{\lambda^n}{n!}e^{-\lambda}$. Here, λ represents the expected number of spikes. Implement a model neuron with an average firing rate of r = 15Hz, and generate spike counts over T = 1sec, repeated across N = 100 trials. Plot a histogram of the observed spike counts n to empirically reconstruct the distribution of P(n).
- **b.** Numerically estimate the entropy S using the data from part **a**. Compare this result with the analytical estimation of entropy for a Poisson distribution $S = \frac{1}{2}(\log_2 n + \log_2 2\pi + \log_2 e)$. Discuss the discrepancy and explain under what conditions (i.e., values of r, T, and N) would the numerical estimate better approximate the analytical entropy.
- **c.** Choose appropriate values of r, T, and N based on your hypothesis in part **b**. Re-estimate the entropy numerically under these conditions and compare it with the analytical estimation.
- **d.** Now suppose that a neuron's firing follows an exponential distribution, i.e. the number of spikes is sampled from the probability density function $P(n) = \frac{e^{-\frac{n}{\lambda}}}{\lambda}$, where λ represents the average number of spikes. Using the values of r, T, and N in part **a**, generate spike counts and numerically estimate the entropy S. Discuss the difference between the numerical and analytical entropy values $S = \log_2(1+n) + n\log_2\left(1+\frac{1}{n}\right)$.
- **e.** When both models from parts **a** and **d** have the same average number of spikes, discuss the advantages of each in terms of information processing, particularly when the average firing rate is low versus when it is high.