

FYS 5429, FEB 15, 2023

CNN

ODE

$$x_{t+1} = x_t + \Delta t \times \overset{\uparrow}{\frac{dx}{dt}} \Big|_{x=x_t} \text{ ODE method}$$

$$\frac{dx}{dt} = f(x, t)$$

$$t \rightarrow t = \{t_0, t_1, \dots, t_n\}$$
$$\Delta t = \frac{t_n - t_0}{n}$$

$$x \rightarrow x = \{x_0, x_1, \dots, x_n\}$$

Euler's method

$$x_{i+1} = x(t + \Delta t) \approx$$

$$x_i + \Delta t f(x_i, t_i)$$

$$f(x_i, t_i) = \frac{dx}{dt} \Big|_{t=t_i}$$

$$t_i = t_0 + i \Delta t$$

$$i = \{0, 1, 2, \dots, n\}$$

## Basics of CNNs

I

a	b	c	d
e	f	g	h
i	j	k	l

3x4

W

w	x
y	z

2x2

stride = 1

$aw + bx$ $ey + fz$	$bw + cx$ $+ fg + gz$	$cw + dx$ $+ gy + hz$
$ew + fx$ $+ ig + jz$	$fw + gx$ $+ jy + kz$	$sw + hx$ $+ ky + lz$

2x3 matrix

output matrix assume a symmetric  
 $N = 7$

input 7x7

assume a filter of dim 3x3

$$F = 3$$

output size

$$(N - F) / \text{stride} + 1$$

$$N = 7, F = 3 \quad \text{hyperparameter.} \quad S = \text{stride} = 1$$

$$(7 - 3) / 1 + 1 = 5 \Rightarrow 5 \times 5 \text{ output.}$$

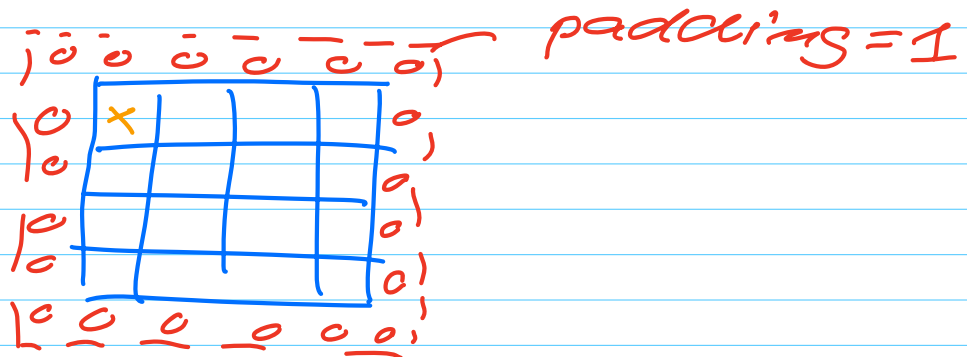
$$N = 7, F = 3, \text{stride} = 2$$

$$(7 - 3) / 2 + 1 = 3$$

$$S = 3$$

$$(7 - 3) / 3 + 1 = 2.33 \text{ not possible}$$

Additional hyperparameter is padding  $P$



Convolution as another way of doing matrix-vector multiplication,

Example

$$\begin{bmatrix} \lambda'_{00} & \lambda'_{01} & \lambda'_{02} \\ \lambda'_{10} & \lambda'_{11} & \lambda'_{12} \\ \lambda'_{20} & \lambda'_{21} & \lambda'_{22} \end{bmatrix} \otimes \begin{bmatrix} w_{00} & w_{01} \\ w_{10} & w_{11} \end{bmatrix}$$

$3 \times 3 \qquad \qquad \qquad 2 \times 2$

$$S = 1 \quad p = 0$$

$$= \begin{bmatrix} \lambda'_{00} w_{00} + \lambda'_{01} w_{01} & \lambda'_{01} w_{00} + \lambda'_{02} w_{01} \\ \lambda'_{10} w_{10} + \lambda'_{11} w_{11} & \lambda'_{11} w_{10} + \lambda'_{12} w_{11} \\ \lambda'_{20} w_{00} + \lambda'_{21} w_{01} & \lambda'_{21} w_{00} + \lambda'_{22} w_{01} \\ \lambda'_{20} w_{10} + \lambda'_{21} w_{11} & \lambda'_{21} w_{10} + \lambda'_{22} w_{11} \end{bmatrix}$$

$$\underline{I}' = \begin{bmatrix} \lambda'_{00} & \lambda'_{01} & \lambda'_{02} & \lambda'_{10} & \lambda'_{11} & \lambda'_{12} & \lambda'_{20} & \lambda'_{21} \\ & & & & & & & \lambda'_{22} \end{bmatrix}$$

$\in \mathbb{R}^9$

$$\underline{w}' = \begin{bmatrix} w_{00} & w_{01} & 0 & w_{10} & w_{11} & 0 & 0 & 0 \\ 0 & w_{00} & w_{01} & 0 & w_{10} & w_{11} & 0 & 0 \\ 0 & 0 & 0 & w_{00} & w_{01} & 0 & w_{10} & w_{11} \\ 0 & 0 & 0 & 0 & w_{00} & w_{10} & 0 & 0 \end{bmatrix}$$

$$w' \in \mathbb{R}^{4 \times 9}$$

$$w' * I' = 0' \in \mathbb{R}^4$$