FYS5429 lecture, January 23, 2024

1 hidden layer

m - input nodes

$W \in \mathbb{R}^{m \times n}$ $\qquad b \in \mathbb{R}^{n}$

$$W = \begin{bmatrix} | & | & & | \\ W_0 & W_1 & \cdots & W_{n-1} \\ | & | & & | \end{bmatrix}$$

The input x is modulated to give an input to the various nodes

$$z(x) = W^T x + b$$

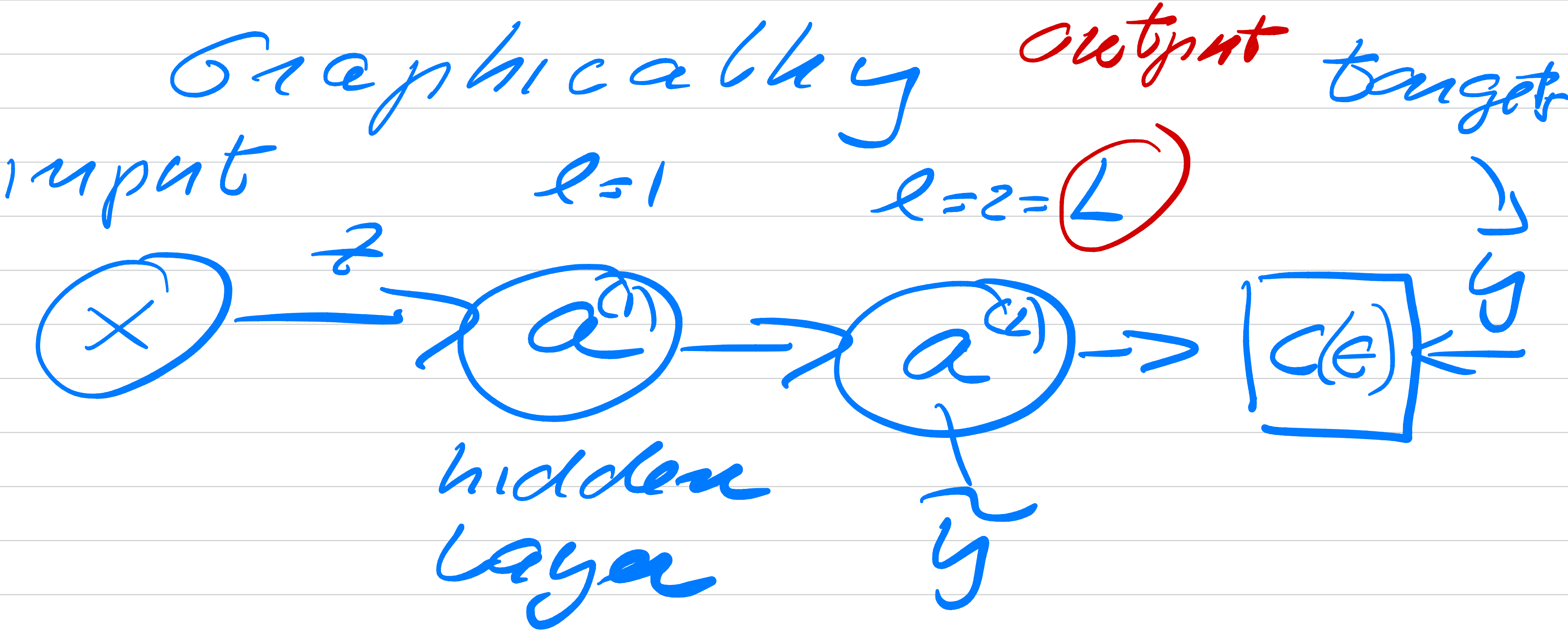
$$x \in \mathbb{R}^{m}$$

$$x \mapsto \sigma(w^T x + b)$$

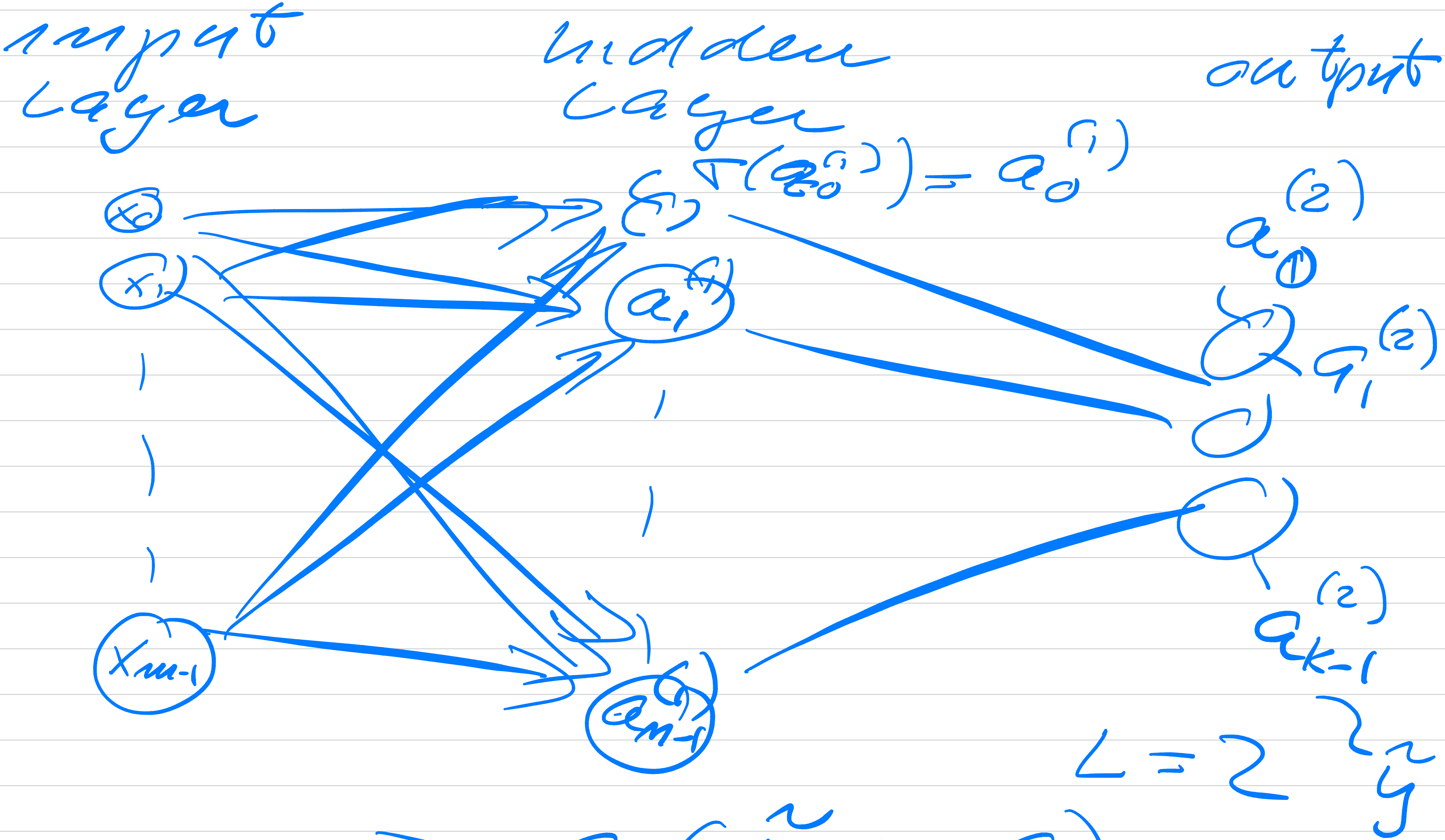
$$= \sigma(A(x))$$

$$= [a_0(x), a_1(x), \ldots, a_{n-1}(x)]$$

Graphically



open up



$\longrightarrow C(\tilde{y}, y; \theta)$

with many layers

$$A_\ell \qquad 1 \leq \ell \leq L$$

for $2 \leq \ell \leq L$

$$\hat{y} = f(x;\Theta) =$$

$$\sigma_L(A_L(\sigma_{L-1}(\dots \sigma_1(A_1(x)))$$

- same activations for all hidden layers
- different activation functions for output

Technicalities

$$z_j^{(\ell)} = \sum_{i=0}^{m-1} w_{ij}^{(\ell)} \, a_i^{(\ell-1)} + b_j^{(\ell)}$$

$$a_i^{(\ell-1)} = \sigma_{\ell-1}\left(z_i^{\ell-1} \right)$$

$$= \sigma\left(z_i^{\ell-1} \right)$$

We want $\nabla_{\Theta} C(y, \underbrace{\tilde{y}}_{a^{(L)}}; \Theta) = 0$

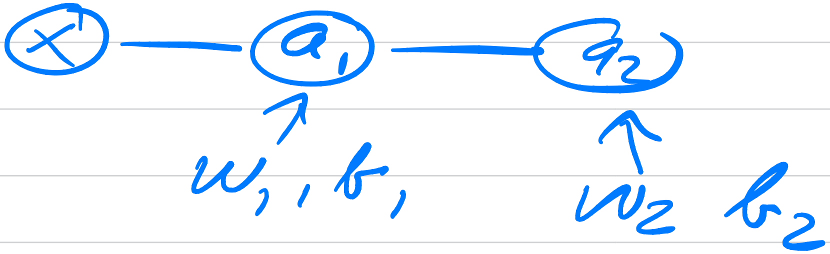
Consider simple example;

x, w, b are all scalars

$L = 2$

$$f(x; \theta) = \sigma_2(w_2 \cdot a_1 + b_2)$$

$$= \sigma_2(w_2 \sigma_1(w_1 x + b_1) + b)$$



$$= \sigma_2(w_2 a_1 + b_2)$$

we want $\frac{\partial C}{\partial w_2}$, $\frac{\partial C}{\partial w_1}$, $\frac{\partial C}{\partial b_2}$

and $\frac{\partial C}{\partial b_1}$

$$C(y, \tilde{y}; \theta) = \frac{1}{2} \sum_{i=0}^{n-1} (y_i - \tilde{y}_i)^2$$

not an explicit dependence
on $\theta = \{ w_1, b_1, w_2, b_2 \}$

Example of derivative

$$\frac{\partial C}{\partial \tilde{y}} \frac{\partial \tilde{y}}{\partial w_2} \quad \dots \quad \frac{\partial \tilde{y}}{\partial w_1}$$

$$\frac{\partial \tilde{y}}{\partial w_1} = \sigma_2'(w_2 \sigma_1(w_1 x + b_1) + b) \times w_2 \sigma_1'(w_1 x + b_1) x$$

with L - layers

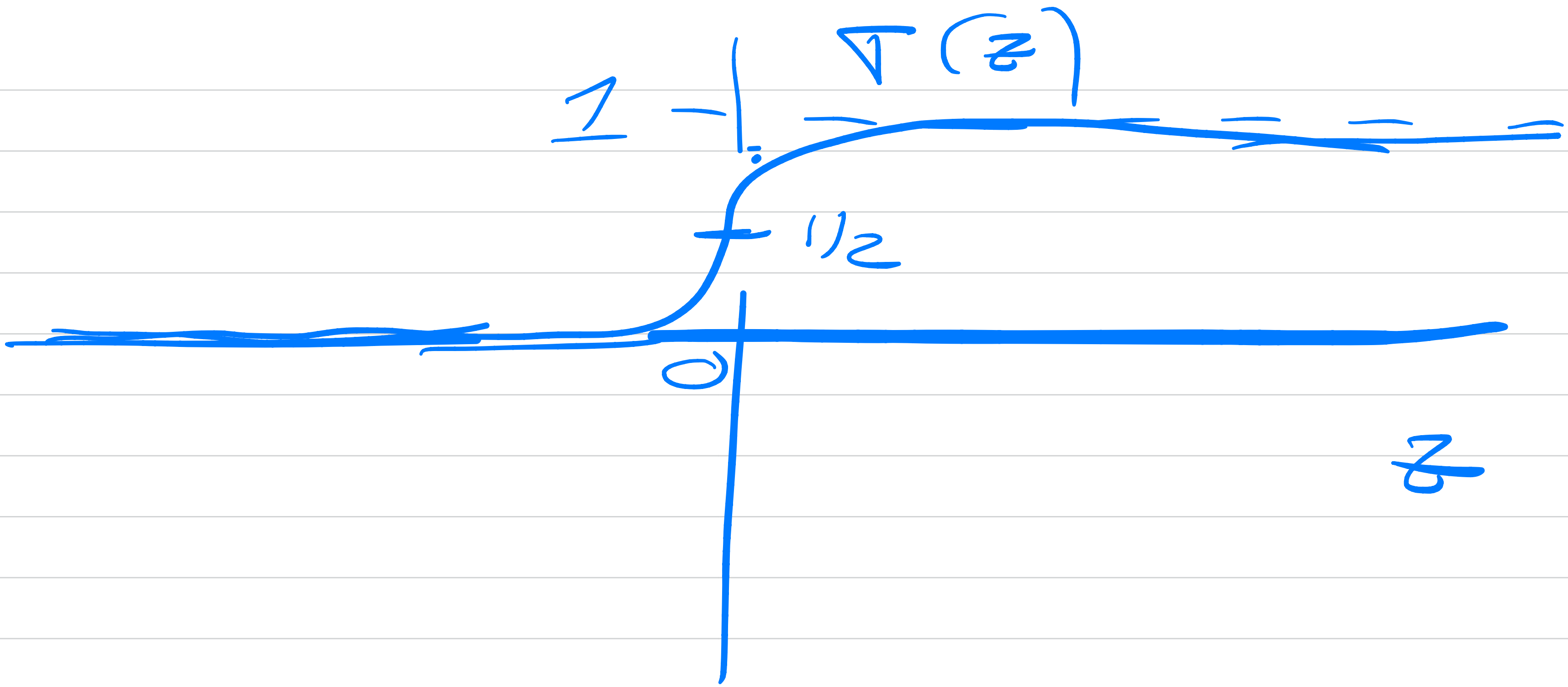
$$\frac{\partial \hat{y}}{\partial w_1} = \left[\prod_{l=2}^{L} w_l \right] \otimes \left[\prod_{l=1}^{L} \sigma_l'(z_l) \right]$$

$\otimes$ X

multiply.

if σ is the sigmoid function $\sigma(z) = \frac{1}{1+e^{-z}}$

$$\sigma(z) = \begin{cases} 1 & z \to \infty \\ 0 & z \to -\infty \end{cases}$$



σ(z)
1
1/2
0
z