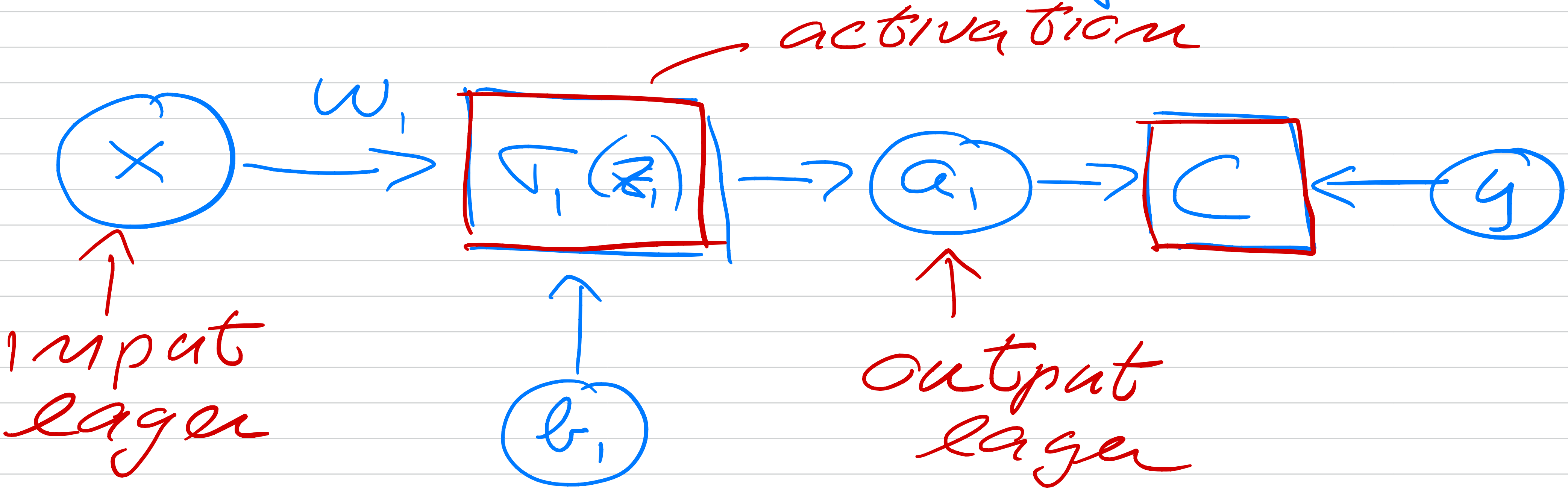
Lecture FYS5429, February 6, 2024

Basic mathematics of NNs



$$z_1 = w_1 x + b_1$$

$$C = C(x, y, w_1, b)$$

$$y(x) = 2x + 1$$

$$C = \frac{1}{2}(a_1 - y)^2$$

$$= \frac{1}{2}\left(\underbrace{\sigma_1(z_1)}_{w_1 x + b_1} - y\right)^2$$

$$\frac{\partial C}{\partial w_1} = \frac{\partial C}{\partial a_1}\,\frac{\partial a_1}{\partial z_1}\,\frac{\partial z_1}{\partial w_1}$$

$$a_1 = \sigma_1(w_1 x + b_1) \quad (= \hat{y})$$

$$= w_1 x + b_1 = z_1$$

$$y(x) = 2x + \underline{1} \sim \delta'$$

$$\frac{\partial C}{\partial w_1} = \underbrace{(a_1 - y)\cdot \underline{1}}\cdot x$$

$$\frac{\partial C}{\partial b_1} = \underbrace{\frac{\partial C}{\partial a_1} \frac{\partial a_1}{\partial z_1}}_{\delta^1} \cdot \underbrace{\frac{\partial z_1}{\partial b_1}}_{=1}$$

$$z_1 = w_1 x + b_1$$

$$\frac{\partial C}{\partial w_1} = \underbrace{\frac{\partial C}{\partial a_1} \frac{\partial a_1}{\partial z_1}}_{\delta^1} \frac{\partial z_1}{\partial w_1}$$

$$= \delta^1 x$$

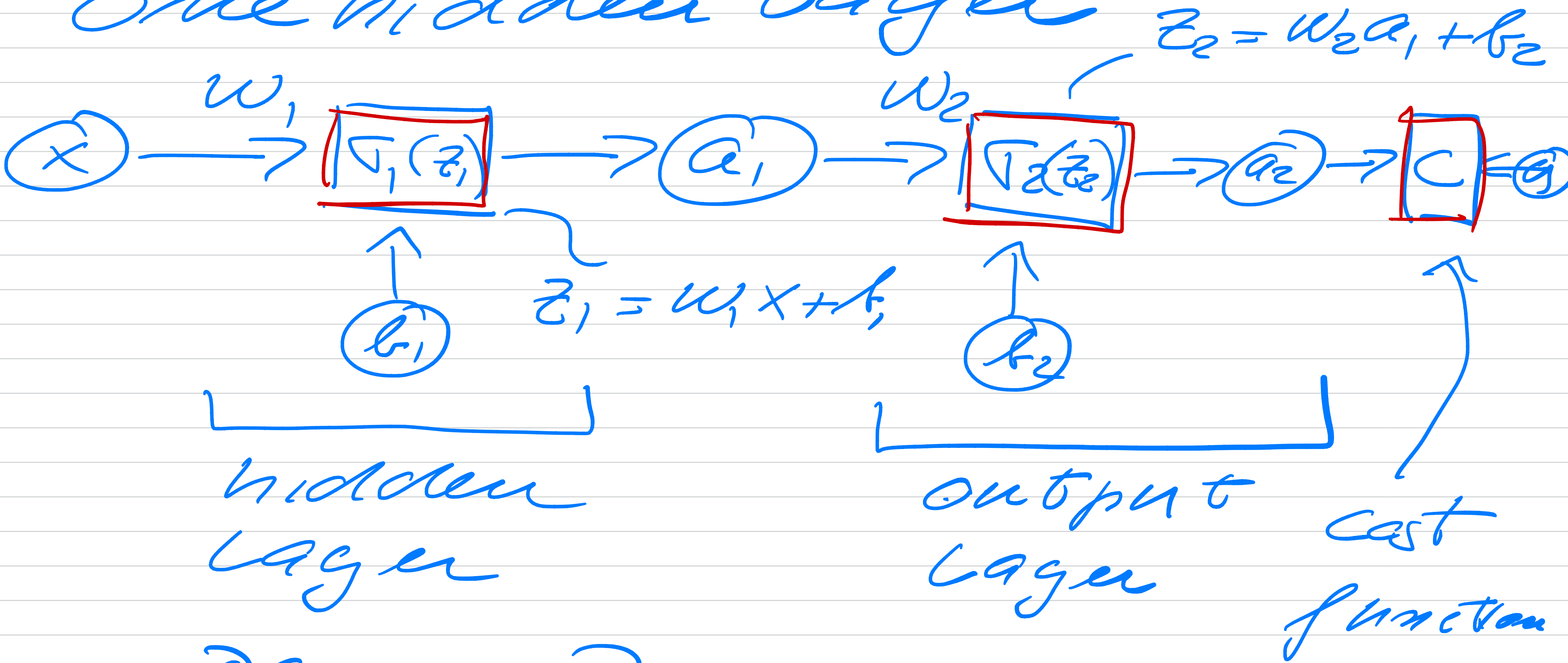
$$\frac{\partial C}{\partial b_1} = \delta^1$$

Training of gradients

$$w_1 \leftarrow w_1 - \eta \underbrace{\delta^1 x}_{\frac{\partial C}{\partial w_1}}$$

$$b_1 \leftarrow b_1 - \eta \underbrace{\delta^1}_{\frac{\partial C}{\partial b_1}}$$

One hidden layer



$$\frac{\partial C}{\partial W_1}, \quad \frac{\partial C}{\partial W_2}, \quad \frac{\partial C}{\partial b_1}, \quad \frac{\partial C}{\partial b_2}$$

$$\frac{\partial C}{\partial W_2} = \delta^2 a_1 = \frac{\partial C}{\partial a_2}\frac{\partial a_2}{\partial z_2}\frac{\partial z_2}{\partial W_2}$$

$$\frac{\partial C}{\partial w_1} = \underbrace{\frac{\partial C}{\partial a_2} \underbrace{\frac{\partial a_2}{\partial z_2}}_{\sigma_2'}}_{\delta^2} \frac{\partial z_2}{\partial z_1} \frac{\partial z_1}{\partial w_1}$$

$$z_2 = w_2 a_1 + b_2$$

$$= w_2 \sigma_1(z_1) + b_2$$

$$= w_2 \sigma_1(\underbrace{w_1 x + b_1}_{z_1}) + b_2$$

$$\frac{\partial z_1}{\partial w_1} = x$$

$$\frac{\partial z_2}{\partial z_1} = \underbrace{\frac{\partial z_2}{\partial a_1}}_{w_2} \underbrace{\frac{\partial a_1}{\partial z_1}}_{\sigma_1'}$$

$$\frac{\partial C}{\partial w_1} = \underbrace{\delta^2 \sigma_1' w_2}_{\delta^1} \cdot x$$

$$a_0 \to a^{(0)}$$

$$\frac{\partial C}{\partial b_1} = \delta^1$$

$$w_2 \Leftarrow w_2 - \eta \delta^2 a_1$$

$$b_2 \Leftarrow b_2 - \eta \cdot \delta^2$$

$$w_1 \Leftarrow w_1 - \eta \delta^1 a_0$$

$$b_1 \Leftarrow b_1 - \eta \delta^1$$