Lecture FYS5429, February 27, 2024

FYS 5429, RNNs

Newton's motion law

$$m\frac{d^2x}{dt^2}+\eta\frac{dx}{dt}+x(t)k=F(t)$$

rewrite as two 1st-order eqs

$$v(t)=\frac{dx}{dt}$$

$$m\frac{dv}{dt}+\eta v+x(t)=F(t)$$

$t \rightarrow$ dimless time t

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + x(t) = F(t)$$

$$v = \frac{dx}{dt}$$

$$\frac{dv}{dt} = F - \gamma v - x$$

Discretize

$$x \rightarrow x_i = x(t_i)$$
$$t \rightarrow t_i = t_0 + i\Delta t$$
$$v \rightarrow v_i = v(t_i)$$

Euler's **method** (only v)

$$v_{i+1} \overset{\sim}{=} v_i + \Delta t \left.\frac{dv}{dt}\right|_{v=v_i}$$

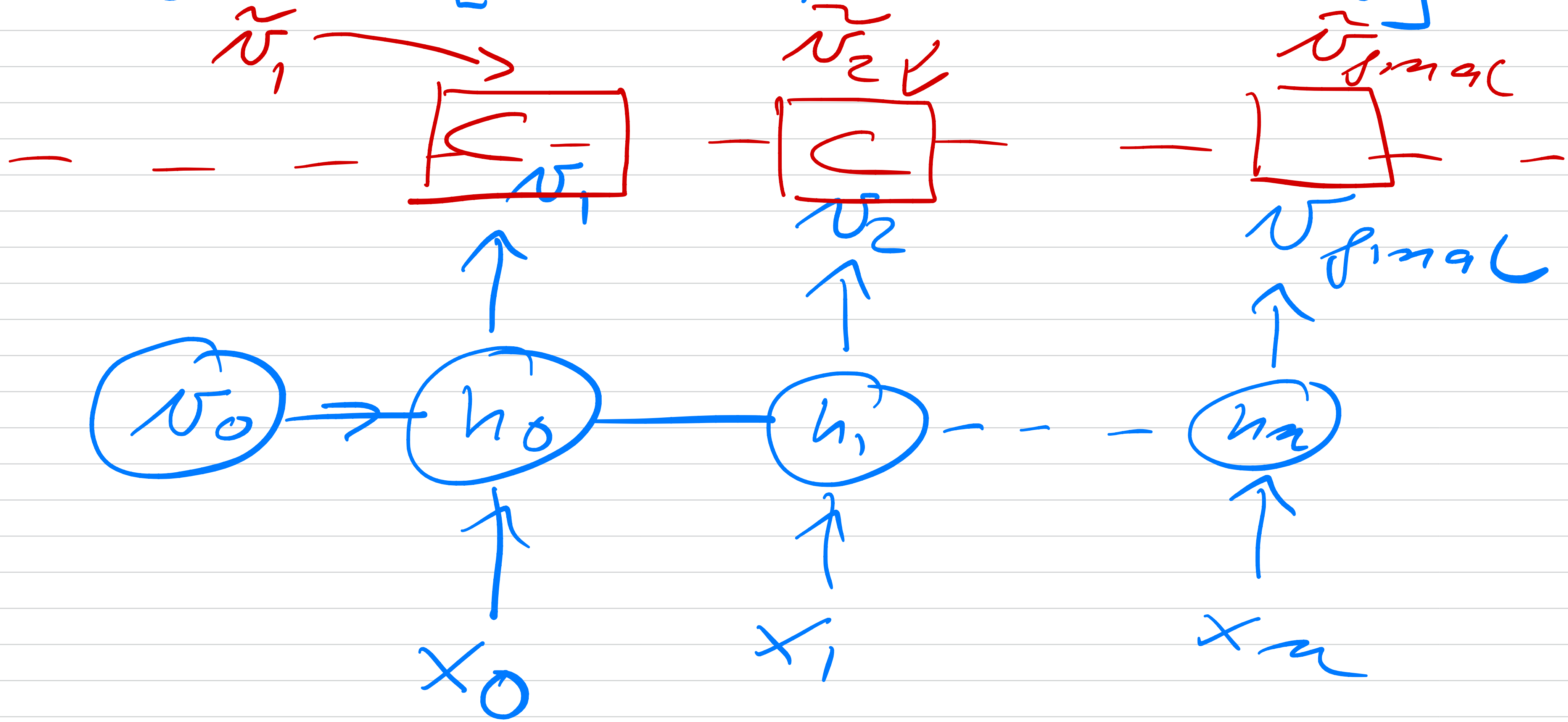
$$v_{i+1} = \underbrace{v_i + \Delta t(F_i - v_i\gamma - x_i)}_{h(x_i,\, v_i,\, F_i)}$$

$$v_i = v_{i-1} + \Delta t(F_{i-1} - v_{i-1}\gamma - x_{i-1})$$

$$= h(x_{i-1},\, v_{i-1},\, F_{i-1}) = h_{i-1}$$

$$v_{i+1} = h(x_i,\, h_{i-1},\, F_i) = h_i$$

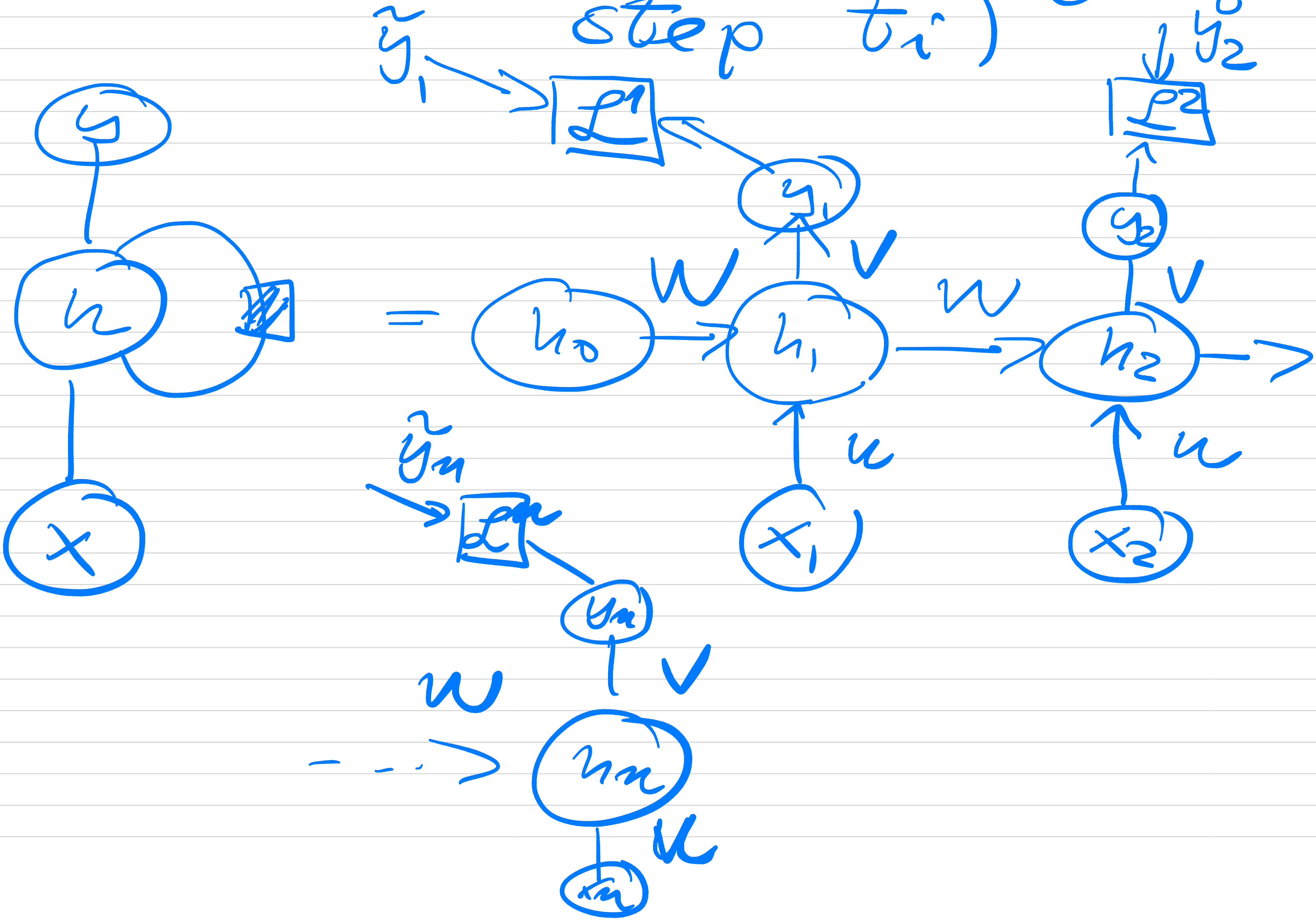
$$v = [v_0 \; v_1 \; \ldots \; v_n]$$



Further simplification

$F = 0$

$v_i \Rightarrow y_i$ (output at given step t_i)



Training :

Feed Forward

bias

$$z_{t_i} = U x_{t_i} + W h_{t_{i-1}} + b$$

$$h_{t_i} = \sigma_h (z_{t_i})$$ (often tanh)

$$r_{t_i} = V h_{t_i} + c$$ ~ bias

final output

$$y_{t_i} = \sigma_y (r_{t_i})$$

Back propagation in time