FYS5429/9429,
January 30, 2025

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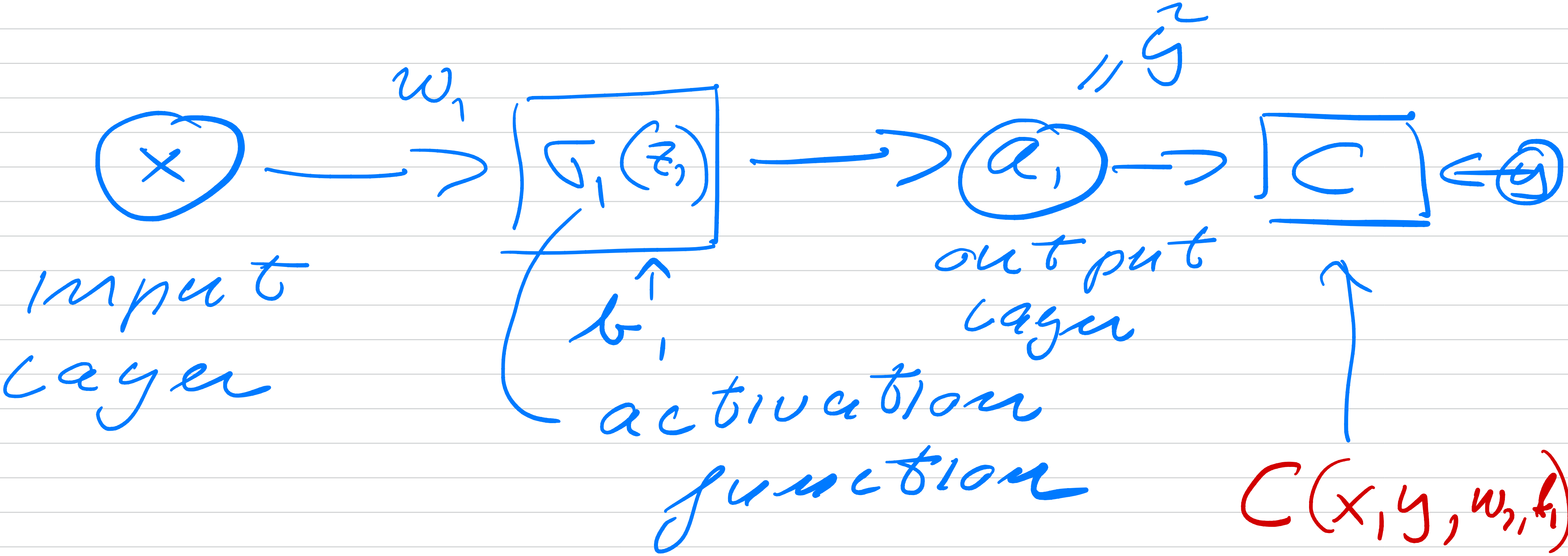


$$z_1 = w_1 x + b_1$$

$$a_1 = \sigma_1(z_1)$$

$$C = \frac{1}{2}(a_1 - y)^2$$

$$= \frac{1}{2}(\sigma_1(z_1) - y)^2 = \frac{1}{2}(\hat{y} - y)^2$$

$$z_1 = w_1 x + b_1$$

$$\theta = \{w_1, b_1\}$$

$$\frac{\partial C}{\partial w_1} = \frac{\partial C}{\partial a_1} \frac{\partial a_1}{\partial z_1} \frac{\partial z_1}{\partial w_1} =$$

$$\sigma_1'$$

$$(y(x) = 2x + 1)$$

$$(a_1 - y)\sigma_1' \cdot x$$

$$\underbrace{(a_1 - y)\,\sigma_1'}_{\delta_1}\, x$$

$$\frac{\partial C}{\partial b_1} = \underbrace{\frac{\partial C}{\partial a_1} \frac{\partial a_1}{\partial z_1}}_{\delta_1} \underbrace{\frac{\partial z_1}{\partial b_1}}_{=1}$$

$$= \delta_1$$

Training of gradients

$$w_1 \leftarrow w_1 - \eta \underbrace{\frac{\partial c}{\partial w_1}}_{\delta_1 x}$$

$$b_1 \leftarrow b_1 - \eta \underbrace{\frac{\partial c}{\partial b_1}}_{\delta_1}$$

(i) Feed Forward

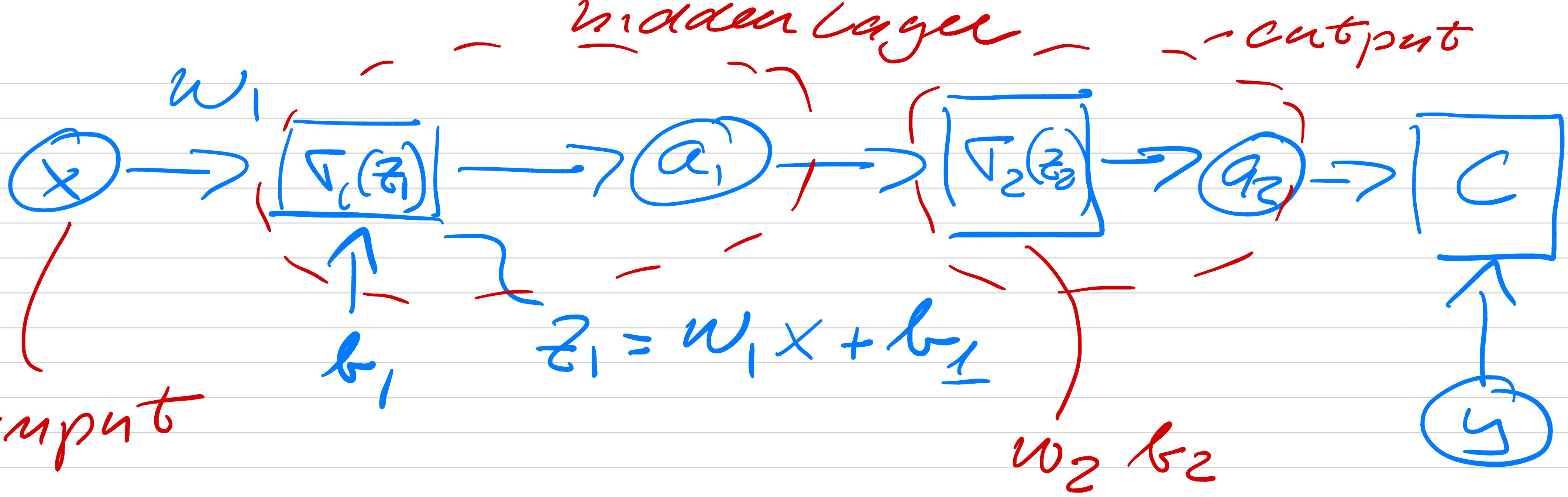
(ii) Backpropagation

$$C(y, \theta, x) = \frac{1}{2}(a_1 - y)^2 + \lambda \sum_{i=0}^{m-1} w_i^2$$

ℓ_2 norm

$$\lambda \sum_{i=0}^{m-1} |w_i|$$

ℓ_1 norm



layer

$$z_2 = w_2 a_1 + b_2$$

$$\theta = \{ w_1, b_1, w_2, b_2 \}$$

$$\frac{\partial C}{\partial w_2} = \underbrace{\delta_2}_{\frac{\partial C}{\partial a_2}\frac{\partial a_2}{\partial z_2}} \underbrace{a_1}_{\frac{\partial z_2}{\partial w_2}}$$

$$\frac{\partial C}{\partial b_2} = \delta_2$$

$$z_2 = w_2 a_1 + b_2$$

$$= w_2 \sigma_1(z_1) + b_2$$

$$= w_2 \sigma_1(w_1 x + b_1) + b_2$$

$$\frac{\partial C}{\partial w_1} = \underbrace{\frac{\partial C}{\partial a_2} \frac{\partial a_2}{\partial z_2}}_{\delta_2} \frac{\partial z_2}{\partial z_1} \frac{\partial z_1}{\partial w_1}$$

$$\frac{\partial z_2}{\partial z_1} = \overbrace{\frac{\partial z_2}{\partial a_1}}^{w_2} \overbrace{\frac{\partial a_1}{\partial z_1}}^{\sigma_1'}$$

$$\frac{\partial C}{\partial w_1} = \underbrace{\delta_2 \sigma_1' w_2}_{\delta_1} \cdot x$$

a_0

$$\frac{\partial C}{\partial b_1} = \delta_1$$

$$w_2 \leftarrow w_2 - \eta \delta_2 a_1$$

$$b_2 \leftarrow b_2 - \eta \delta_2$$

$$w_1 \leftarrow w_1 - \eta \delta_1 a_0 \quad a_0 = x$$

$$b_1 \leftarrow b_1 - \eta \cdot \delta_1$$