# Lecture FYS5429, April 9, 2024

# FYS5429/9429, APRIL 9, 2024

$$p(x_i, h_j; \Theta) = \frac{f(x_i, h_j; \Theta)}{Z(\Theta)}$$

$$f(x_i, h_j; \Theta) = e^{-E(x_i, h_j; \Theta)}$$

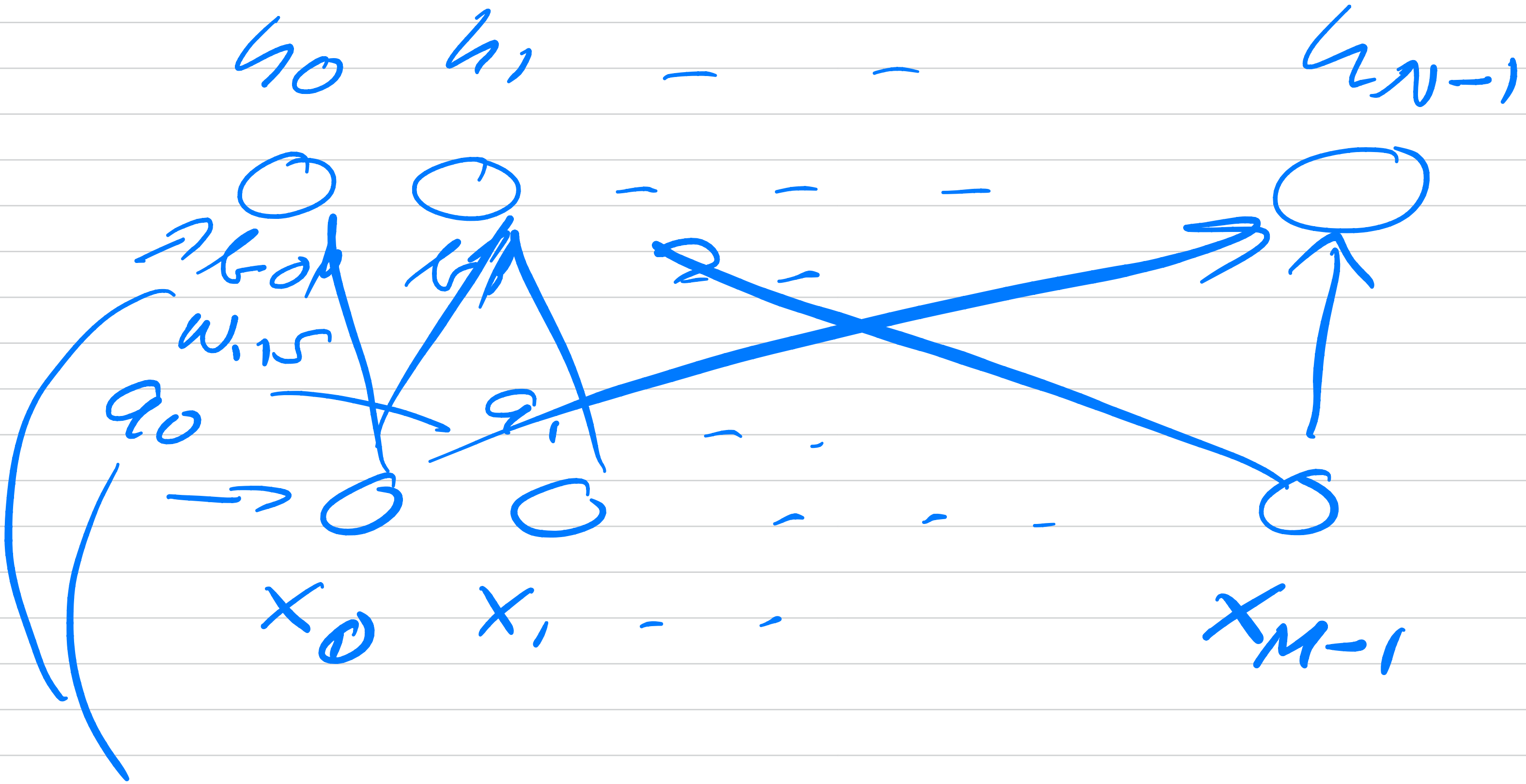
Example of a Boltzmann machine

$$E(x_i; h_j; \Theta) \longrightarrow$$

$$E(x, h; \Theta) = -\left(\sum a_i x_i + \sum b_j h_j + \sum_{ij} x_i w_{ij} h_j\right)$$

# Binary-Binary

$x_i = \{0, 1\}$ $\qquad$ $h_j = \{0, 1\}$




N hidden nodes

M visible nodes

Biases $a_i, b_j$ and weights $w_{ij}$ $\Longrightarrow$

unknown parameter

$G = \{a, b, W\}$

$M = 2$ : $\{x_0, x_1\}$ $N = 2$

$\{h_0, h_1\}$

configuration

$X = \{0, 0\}, \{0, 1\}, \{1, 0\}$

$\{1, 1\} \Rightarrow 2^M$ configr for $X$

$$E(x, h; \theta) = -a^T x - b^T h - x^T W h$$

$$= -\sum_{i=0}^{M-1} a_i x_i - \sum_{j=0}^{N-1} b_j h_j$$

$$- \sum_{i,j}^{M-1, N-1} x_i w_{ij} h_j$$

$$Z(\theta) = \sum_{x=0}^{2^M - 1} \sum_{h=0}^{2^N - 1} f(x, h; \theta)$$

$$= \sum_{x=0}^{2^M - 1} \sum_{h=0}^{2^N - 1} e^{a^T x + b^T h + x^T W h}$$

$$P_{BB}(x,h;\theta) = \frac{e^{a^T x + b^T h + x^T W h}}{Z(\theta)}$$

$$P(x,h;\theta) = \frac{f(x,h;\theta)}{Z(\theta)}$$

Marginal probability

$$P(x;\theta) = \sum_h P(x,h;\theta)$$

$$P(h;\theta) = \sum_x P(x,h;\theta)$$

conditional probability

$$P(h|x) = \frac{P(x,h)}{P(x)}$$

(skipping $\theta$)

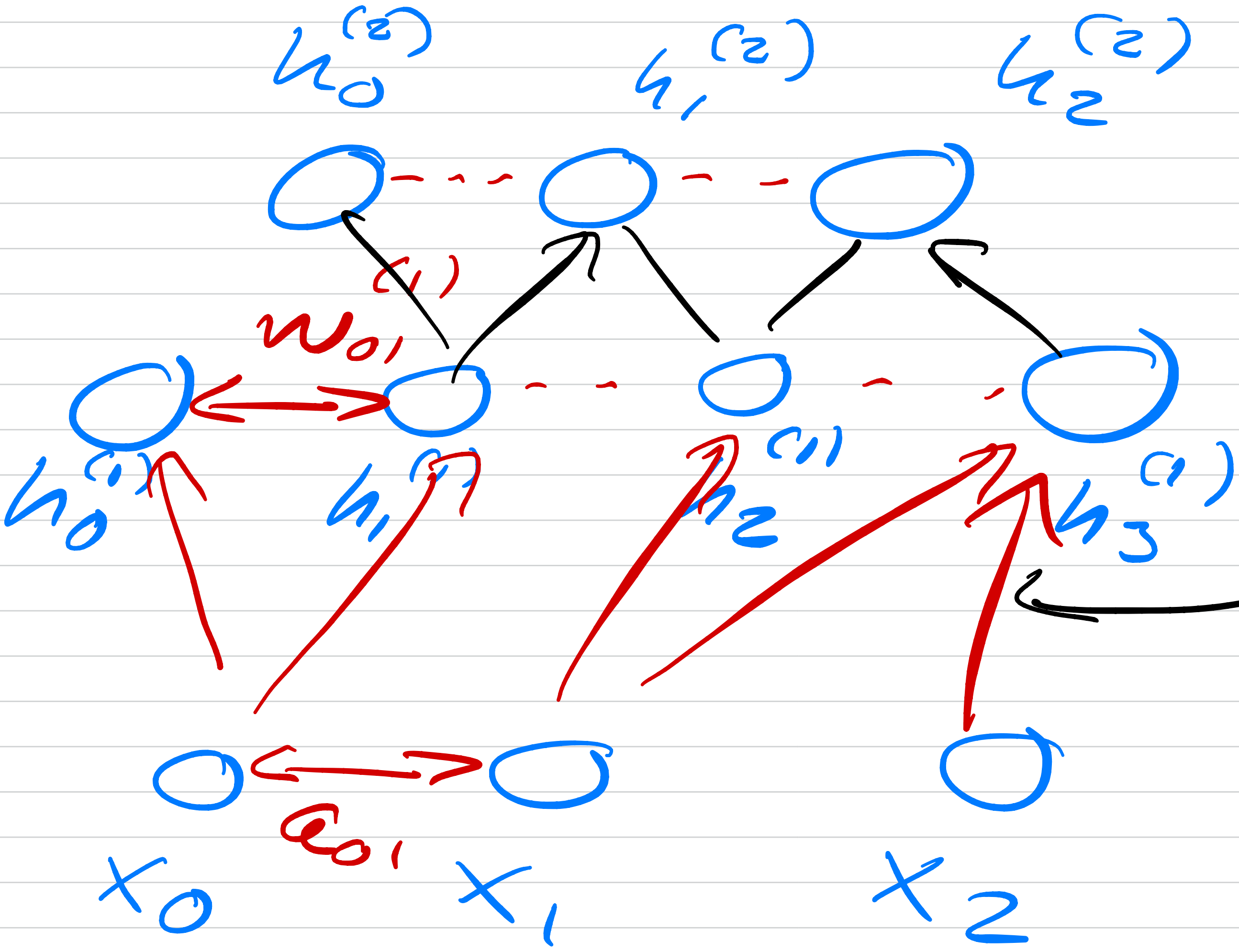
$$P(x|h) = \frac{P(x,h)}{P(h)}$$

$$= \frac{P(h|x)\,P(x)}{P(h)}$$

(Bayes' theorem)

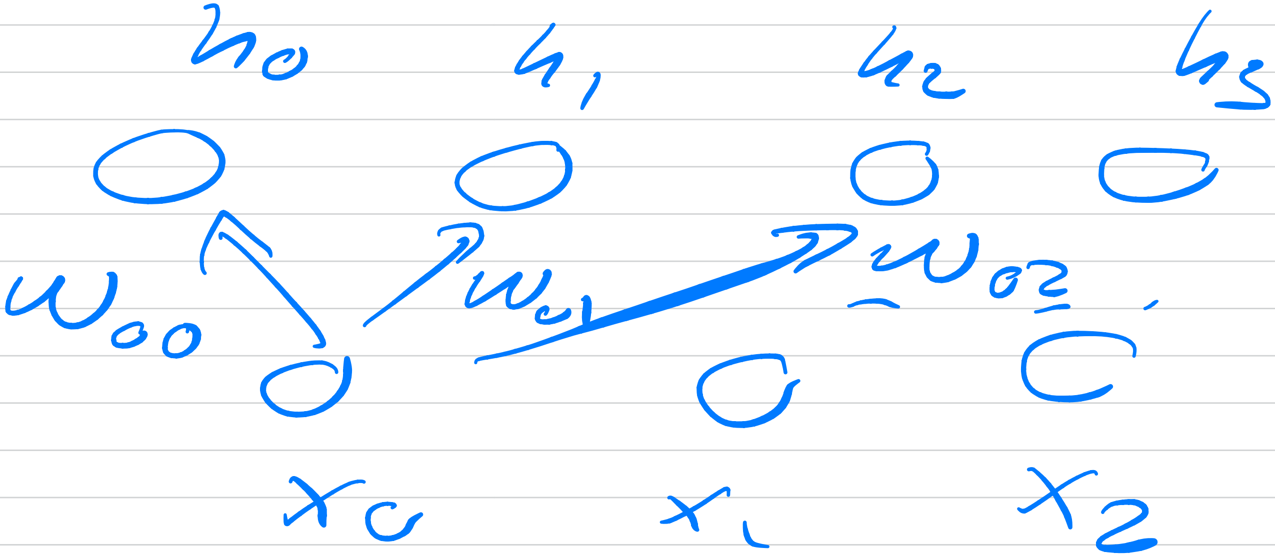
$$E_{BB}(x,h) = a^T x + b^T h + x^T W h$$

$$E(x,h) = x^T A x + h^T B h + x^T W^{(1)} h^{(1)} + h^{(1)T} W^{(2)} h^{(2)}$$

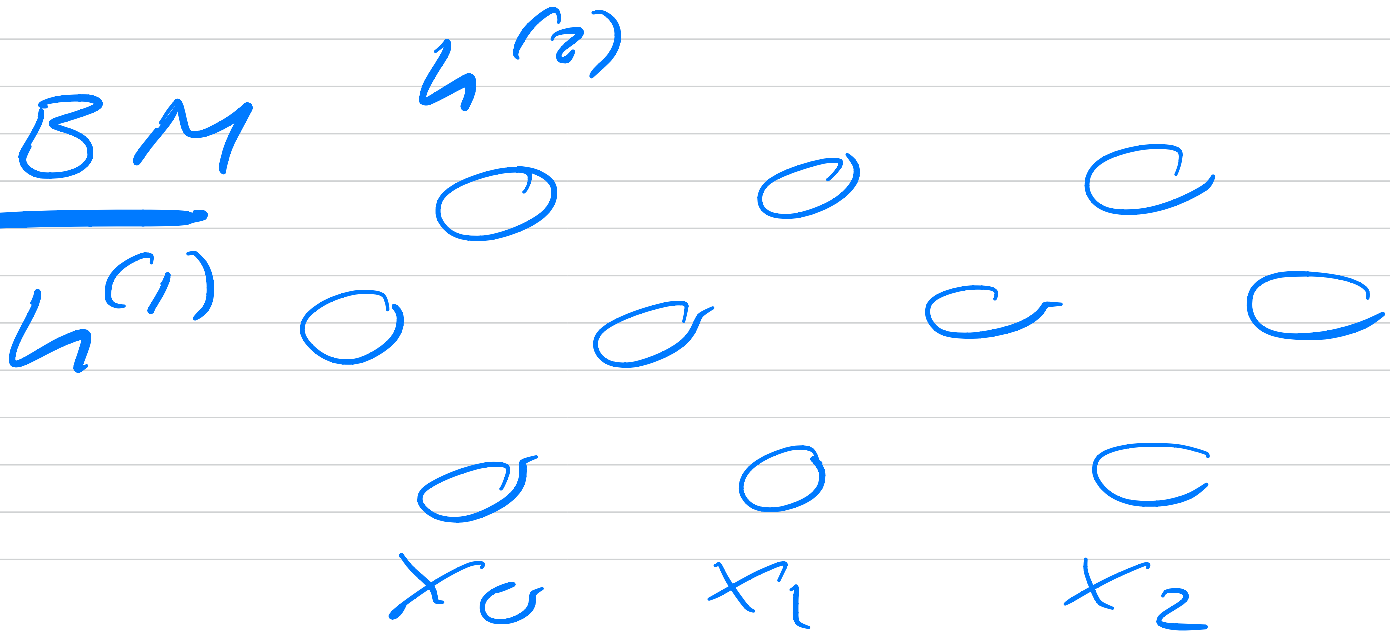



DBN

# RBM




# DBM



$$
\begin{aligned}
E = -\Big( & \sum_{i} a_i x_i + \sum_{j} b_j^{(1)} h_j^{(1)} \\
& + \sum_{i,j} x_i W_{ij}^{(1)} h_j^{(1)} \\
& + \sum_{k} c_k^{(2)} h_j^{(2)} \\
& + \sum_{jk} h_j^{(1)} W_{ij}^{(2)} h_k^{(2)}
\end{aligned}
$$

$$\sum_{i,j} x_i w_{ij} h_j$$

$$= \sum_{j} x \, w_{*j} h_j$$

$$\begin{bmatrix} x_0 & x_1 & \cdots & x_{M-1} \end{bmatrix} \begin{bmatrix} w_{00} & w_{01} & \cdots \\ w_{10} & & \\ \vdots & & \\ & & \end{bmatrix}$$