IN4050 Mandatory Assignment 1: Traveling Salesman Problem

Rules

Before you begin the exercise, review the rules at this website: https://www.uio.no/english/studies/examinations/compulsory-activities/mn-ifi-mandatory.html (This is an individual assignment. You are not allowed to deliver together or copy/share source-code/answers with others.)

Especially, notice that you are **not allowed to use code or parts of code written by others** in your submission. We do check your code against online repositories, so please be sure to **write all the code yourself**. Any use of **auto-generated code** must be clearly identified, along with the tool or software used to generate it. Read also the "Routines for handling suspicion of cheating and attempted cheating at the University of Oslo": https://www.uio.no/english/studies/examinations/cheating/index.html By submitting this assignment, you confirm that you are familiar with the rules and the consequences of breaking them.

Delivery

Deadline: Tuesday, October 8 2024, 23:59

Your submission should be delivered in Devilry. You may redeliver in Devilry before the deadline, but include all files in the last delivery, as only the last delivery will be read. You are recommended to upload preliminary versions hours (or days) before the final deadline.

What to deliver?

Deliver one single zipped folder (.zip, .tgz or .tar.gz) which includes:

- PDF report containing:
  - Your name and username (!)
  - Instructions on how to run your program, with example runs.
  - Answers to all questions from assignment.
  - Brief explanation of what you've done.
  - *Your PDF may be generated by exporting your Jupyter Notebook to PDF, if you have answered all questions in your notebook*
- Source code
  - Source code may be delivered as jupyter notebooks or python files (.py)
- The european cities file so the program will run right away.
- Any files needed for the group teacher to easily run your program on IFI linux machines.

Important:

- Include example runs of your code by doing the reports described in the tasks. Simply implementing the code, but never running it will not give many points.
- Include the code that was used to make all reports. Do not include reports of performance and time without also including the code that was used to produce it.
- If you weren't able to finish the assignment, use the PDF report to elaborate on what you've tried

and what problems you encountered. Students who have made an effort and attempted all parts of the assignment will get a second chance even if they fail initially. This exercise will be graded PASS/FAIL.

Introduction

In this exercise, you will attempt to solve an instance of the traveling salesman problem (TSP) using different methods. The goal is to become familiar with evolutionary algorithms and to appreciate their effectiveness on a difficult search problem. You have to use Python to solve the assignment. You must write your program from scratch (but you may use non-EA-related libraries).

	Barcelona	Belgrade	Berlin	Brussels	Bucharest	Budapest
Barcelona	0	1528.13	1497.61	1062.89	1968.42	1498.79
Belgrade	1528.13	0	999.25	1372.59	447.34	316.41
Berlin	1497.61	999.25	0	651.62	1293.40	1293.40
Brussels	1062.89	1372.59	651.62	0	1769.69	1131.52
Bucharest	1968.42	447.34	1293.40	1769.69	0	639.77
Budapest	1498.79	316.41	1293.40	1131.52	639.77	0

Figure 1: First 6 cities from csv file.

Problem

The traveling salesman, wishing to disturb the residents of the major cities in some region of the world in the shortest time possible, is faced with the problem of finding the shortest tour among the cities. A tour is a path that starts in one city, visits all of the other cities, and then returns to the starting point. The relevant pieces of information, then, are the cities and the distances between them. In this instance of the TSP, a number of European cities are to be visited. Their relative distances are given in the data file, *european_cities.csv*, found in the zip file with the mandatory assignment.

(You will use permutations to represent tours in your programs. The **itertools** module in Python provides a permutations function that returns successive permutations, this is useful for exhaustive search)

Helper code for visualizing solutions

Here follows some helper code that you can use to visualize the plans you generate. These visualizations can **help you check if you are making sensible tours or not**. The optimization algoritms below should hopefully find relatively nice looking tours, but perhaps with a few visible inefficiencies.

In [13]:

```
import matplotlib.pyplot as plt
import numpy as np
import time
import pandas as pd
import itertools as it
import random
random.seed(99)

%matplotlib inline
np.random.seed(57)
#Map of Europe
europe_map = plt.imread('map.png')

#Lists of city coordinates
city_coords = {
    "Barcelona": [2.154007, 41.390205], "Belgrade": [20.46, 44.79], "Berlin": [13.40, 52.52],
    "Brussels": [4.35, 50.85], "Bucharest": [26.10, 44.44], "Budapest": [19.04, 47.50],
    "Copenhagen": [12.57, 55.68], "Dublin": [-6.27, 53.35], "Hamburg": [9.99, 53.55],
    "Istanbul": [28.98, 41.02], "Kyiv": [30.52, 50.45], "London": [-0.12, 51.51],
    "Madrid": [-3.70, 40.42], "Milan": [9.19, 45.46], "Moscow": [37.62, 55.75],
    "Munich": [11.58, 48.14], "Paris": [2.35, 48.86], "Prague": [14.42, 50.07],
    "Rome": [12.50, 41.90], "Saint Petersburg": [30.31, 59.94], "Sofia": [23.32, 42.70],
    "Stockholm": [18.06, 60.33], "Vienna": [16.36, 48.21], "Warsaw": [21.02, 52.24]}
```

In [14]:

```
#Helper code for plotting plans
#First, visualizing the cities.
import csv
with open("european_cities.csv", "r") as f:
    data = list(csv.reader(f, delimiter=';'))
    cities = data[0]

fig, ax = plt.subplots(figsize=(10, 10))
ax.imshow(europe_map, extent=[-14.56, 38.43, 37.697 + 0.3, 64.344 + 2.0], aspect="auto")

# Map (long, lat) to (x, y) for plotting
for city, location in city_coords.items():
    x, y = (location[0], location[1])
    plt.plot(x, y, 'ok', markersize=5)
    plt.text(x, y, city, fontsize=12)
```

In [15]:
```python
#A method you can use to plot your plan on the map.
def plot_plan(city_order):
    fig, ax = plt.subplots(figsize=(10, 10))
    ax.imshow(europe_map, extent=[-14.56, 38.43, 37.697 + 0.3, 64.344 + 2.0], aspect="auto")

    # Map (long, lat) to (x, y) for plotting
    for index in range(len(city_order) - 1):
        current_city_coords = city_coords[city_order[index]]
        next_city_coords = city_coords[city_order[index+1]]
        x, y = current_city_coords[0], current_city_coords[1]
        #Plotting a line to the next city
        next_x, next_y = next_city_coords[0], next_city_coords[1]
        plt.plot([x, next_x], [y, next_y])

        plt.plot(x, y, 'ok', markersize=5)
        plt.text(x, y, index, fontsize=12)
    #Finally, plotting from last to first city
    first_city_coords = city_coords[city_order[0]]
    first_x, first_y = first_city_coords[0], first_city_coords[1]
    plt.plot([next_x, first_x], [next_y, first_y])
    #Plotting a marker and index for the final city
    plt.plot(next_x, next_y, 'ok', markersize=5)
    plt.text(next_x, next_y, index+1, fontsize=12)
    plt.show()
```

In [16]:
```python
#Example usage of the plotting-method.
plan = list(city_coords.keys()) # Gives us the cities in alphabetic order
print(plan)
plot_plan(plan)
```

```
['Barcelona', 'Belgrade', 'Berlin', 'Brussels', 'Bucharest', 'Budapest', 'Copenhagen', 'Dublin', 'Hamburg', 'Ist
anbul', 'Kyiv', 'London', 'Madrid', 'Milan', 'Moscow', 'Munich', 'Paris', 'Prague', 'Rome', 'Saint Petersburg',
'Sofia', 'Stockholm', 'Vienna', 'Warsaw']
```

Exhaustive Search

First, try to solve the problem by inspecting every possible tour. Start by writing a program to find the shortest tour among a subset of the cities (say, **6** of them). Measure the amount of time your program takes. Incrementally add more cities and observe how the time increases. Plot the shortest tours you found using the plot_plan method above, for 6 and 10 cities.

Note: To get distances between cities, use the dictionary `data` created by reading the file `european_cities.csv`. *Do not* calculate distances based on the coordinates. The actual distances do not only depend on the differences in the coordinates, but also of the curvature of the earth. The distances available in `data` are corrected for this, and contain the actual true distances.

In [17]:

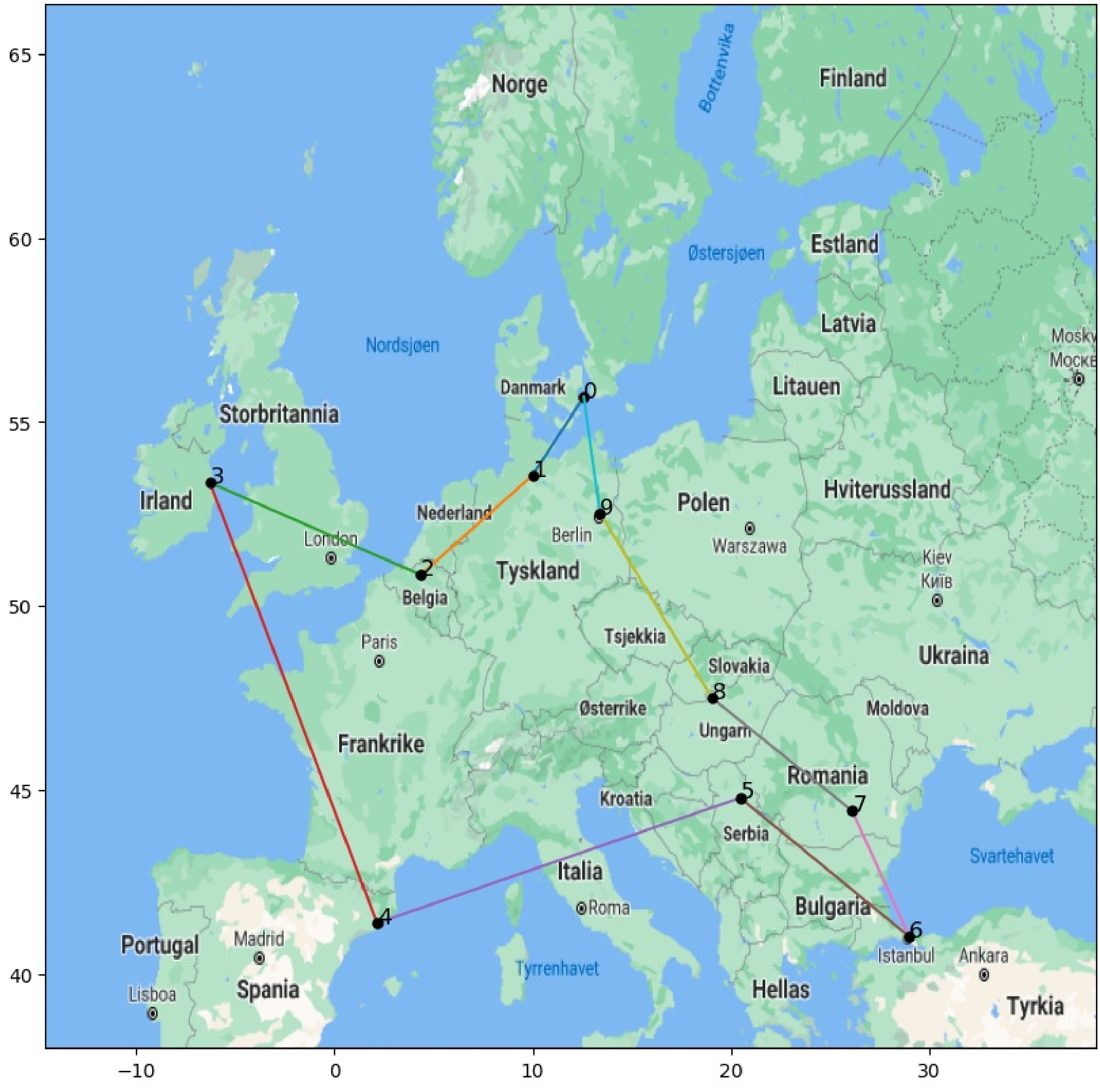
```
# Number of cities
n1 = 6
n2 = 10
n3 = 24

df = pd.read_csv("european_cities.csv", delimiter=';')

# Data in numpy array, used in calc distance
dist_m = df.to_numpy()

# Want to create map of cities to index, to map route later
cities = df.columns.tolist()
cities_dict = {}
for i in range(len(cities)):
    cities_dict[i] = cities[i]
#print(cities_dict)

#List of cities in a path, represented by ints
cities_6 = np.linspace(0, n1-1, n1, dtype='int')
```

```python
cities_10 = np.linspace(0, n2-1, n2, dtype='int')
cities_24 = np.linspace(0, n3-1, n3, dtype='int')

#Generate all permutations for exhaustive search (also used in hill climb, sampled for starting path)
permutations_6 = it.permutations(cities_6)
permutations_10 = it.permutations(cities_10)

perm_6 = []
perm_10 = []

for perm in permutations_6:
    perm_6.append(perm)

for perm in permutations_10:
    perm_10.append(perm)


# For 24 cities, dont need all permutations. Instead generate just n samples.
def gen_perm(cities, n):
    perms = []
    cities_ = list(cities) # swap to list to make use of sample
    m = len(cities)
    for i in range(n):
        perm = random.sample(cities_, m) # E.g for 24 cities: Sample without replacement from cities list 24 ti
        perms.append(perm)
    return perms

perm_24 = gen_perm(cities_24, int(1e3)) #only need 20, but create more for testing
```

In [18]:
```python
#Functions to find distance and fitness

# (row, column), e.g Barcelona distance to Belgrade -> dist_m[1, 0]

def calc_distance(cities):
    '''
    Input:
        - cities (list) : Vector with unique number representing corresponding city

    Output:
        - distance (float) : Total distance, where larger is worse.
    '''
    distance = 0
    for i in range(len(cities)-1):
        distAB = dist_m[cities[i], cities[i+1]]
        distance += distAB
    distance += dist_m[cities[-1], cities[0]] #return to starting point

    return distance


def fitness(distance):
    return 1/distance


def exhaustive_search(perms):
    best_path = []
    best_distance = 1e6
    for perm in perms:
        dist = calc_distance(perm)
        if dist < best_distance:
            best_path = perm
            best_distance = dist

    return best_path, best_distance
```

In [19]:
```python
# Find all differnet combination of 6 and 10 cities and calculate best path


start_time = time.time()
p6, d6 = exhaustive_search(perm_6)
end_time = time.time()
p6_time = end_time - start_time


start_time = time.time()
p10, d10 = exhaustive_search(perm_10)
end_time = time.time()
p10_time = end_time - start_time


def path_toCities(path):
    # Input list/arr of ints and convert to strings
    return [cities_dict[i] for i in path]
```

```
def plot_path(path):
    #Plots path given input as ints
    path = path_toCities(path)
    plot_plan(path)

plot_path(p6)
plot_path(p10)
```



What is the shortest tour (i.e., the actual sequence of cities, and its length) among the first 10 cities (that is, the cities starting with B,C,D,H and I)? How long did your program take to find it? Calculate an approximation of how long it would take to perform exhaustive search on all 24 cities?

In [261…

```
import math
#Answer
```

```python
print(f"Time for 6 cities: {p6_time:.4f} seconds")
print(f"Time for 10 cities: {p10_time:.4f} seconds")
print(f"Relative increase: Cities {(10/6):.2f}, Time increase(p10/p6): {(p10_time/(p6_time+1e-5)):.2f}\n") #avo

print(f"Six cities: path: {path_toCities(p6)}, distance: {d6:.2f}")
print(f"Ten cities: path: {path_toCities(p10)}, distance: {d10:.2f}")

#Estimate 24 cities time
time24 = math.factorial(24)*p10_time/math.factorial(10)
print(f"Number of possible paths for 6 and 10 cities is: 6! = {math.factorial(6)} and 10! = {math.factorial(10)
print(f"I can maybe assume for each iteration we use around {p6_time/math.factorial(6)} and {p10_time/math.fact
print(f"So for 24 cities we'd get {time24/(60 * 60 * 24 * 365):.2e} years ")
```

```
Time for 6 cities: 0.0015 seconds
Time for 10 cities: 5.5774 seconds
Relative increase: Cities 1.67, Time increase(p10/p6): 3681.12

Six cities: path: ['Barcelona', 'Belgrade', 'Bucharest', 'Budapest', 'Berlin', 'Brussels'], distance: 5018.81
Ten cities: path: ['Copenhagen', 'Hamburg', 'Brussels', 'Dublin', 'Barcelona', 'Belgrade', 'Istanbul', 'Buchares
t', 'Budapest', 'Berlin'], distance: 7486.31
Number of possible paths for 6 and 10 cities is: 6! = 720 and 10! = 3628800. For 24 cities we have 24! combinati
osn: 620448401733239439360000
I can maybe assume for each iteration we use around 2.0904673470391l7e-06 and 1.53698066555003e-06 per iteration
So for 24 cities we'd get 3.02e+10 years
```

Hill Climbing

Then, write a simple hill climber to solve the TSP. How well does the hill climber perform, compared to the result from the exhaustive search for the first **10 cities**? Since you are dealing with a stochastic algorithm, you should run the algorithm several times to measure its performance. Report the length of the tour of the best, worst and mean of 20 runs (with random starting tours), as well as the standard deviation of the runs, both with the **10 first cities**, and with all **24 cities**. Plot one of the the plans from the 20 runs for both 10 cities and 24 cities (you can use plot_plan).

In [262…

```python
# Implement the algorithm here

# Swapping just neighbhor cities, save if it produces a better fit (shorter path).

# Start with 1 random route
#  swap 2 cities positions, (either neighbor or not) can try both
# (swap n times, which we can choose e.g 1e2 -> 1e4)
# claculate fitness/distance -> update if improved

# Do 20 differnet initalizations and run it.

# perm_1

def hill_climb(starting_route, max_iter):
    n = len(starting_route) # nr. of cities
    curr_path = np.array(starting_route) # current best path
    best_dist = calc_distance(curr_path)

    for i in range(max_iter):
        city1, city2 = random.sample(range(n), 2) #pick 2 random indices to swap
        curr_path[city1], curr_path[city2] = curr_path[city2], curr_path[city1]
        curr_dist = calc_distance(curr_path)

        if curr_dist < best_dist:
            best_dist = curr_dist
        else:
            curr_path[city1], curr_path[city2] = curr_path[city2], curr_path[city1] #swap back if not improved

    return curr_path, best_dist


def hill_iterate(permutations, n, max_iter=1000, printInfo = True):
    # Do hill climb n times
    # Find best, worst and mean path distance
    starting_paths = random.sample(permutations, n)
    m_cities = len(starting_paths[0])
    best_paths =  []
    best_dists = []

    for path in starting_paths:
        path, dist = hill_climb(path, max_iter)
        best_paths.append(path)
        best_dists.append(dist)

    best_paths = np.array(best_paths)
    best_dists = np.array(best_dists)

    min_i = np.argmin(best_dists)
```

```python
    max_i = np.argmax(best_dists)

    mean = np.mean(best_dists)
    std_dev = np.std(best_dists)

    if printInfo:
        print(f"For {m_cities} cities:")
        print(f"Best distance: {best_dists[min_i]:.0f}, worst distance: {best_dists[max_i]:.0f}")
        print(f"Best path: {best_paths[min_i]}")
        print(f"Mean: {mean:.2f} , STD: {std_dev:.2f}\n")

    return best_paths[min_i], best_dists[min_i], best_paths[max_i], best_dists[max_i]


b6, _, _, _ = hill_iterate(perm_6, 20)
b10, _, _, _ = hill_iterate(perm_10, 20)
b24, _, _, _ = hill_iterate(perm_24, 20, int(1e4))

#plot_path(b6)
plot_path(b10)
plot_path(b24)

# Best run for 24 for 1e4 iterations with 20 samples: (12417, 12972, 12785, 13492, 12835, 13209, 12417, 13102)
# Best run for 24 for 1e5: (12718, 12691, 13011, 13137, )  # seemingly dont get much better than 1e4, and gets 

#Could make a plot showing distance in n x n matrix with iterations and number of samples in row and cols
def grid_search():
    n = 5
    m = 4
    iters = np.logspace(0, n-1, n, dtype=int)  # 1, 10, ..., 1e5
    print(iters)
    samples = np.linspace(1, 300, m, dtype=int)
    print(samples)

    dist_m = np.zeros((n, m))
    path_m = np.zeros((n, m), dtype=object)
    time_m = np.zeros((n, m))

    for i in range(len(iters)):
        for j in range(len(samples)):
            start_time = time.time()
            path, dist, _, _ = hill_iterate(perm_24, samples[j], iters[i], printInfo=False)
            end_time = time.time()
            time_m[i, j] = end_time - start_time
            dist_m[i, j] = dist
            path_m[i, j] = path


    print(f"Distance:\n{dist_m}")
    print(f"Time:\n{time_m}")
    print(f"Paths:\n{path_m}")

    return 0



# I wanted to see effect of increasing samples of runs vs number of iterations
#grid_search()

#Shortest path found from simple gridsearch (200 samples, 1e5 iterations)
path_best = np.array([21,  6,  8,  3, 16, 11,  7, 12,  0, 18, 13, 15, 17,  2, 23, 22,  5, 1, 20,  9,  4, 10, 14
dist = calc_distance(path_best)
print(f"Best distance for 24 cities: {dist:.0f}, fitness: {fitness(dist)}")
plot_path(path_best)


print(f"For 24 cities")
```

```
For 6 cities:
Best distance: 5019, worst distance: 5019
Best path: [5 4 1 0 3 2]
Mean: 5018.81 , STD: 0.00

For 10 cities:
Best distance: 7486, worst distance: 8419
Best path: [3 8 6 2 5 4 9 1 0 7]
Mean: 7645.29 , STD: 273.97

For 24 cities:
Best distance: 12874, worst distance: 16357
Best path: [10 23 17 22  5  4  9 20  1 18 13  0 12 16  7 11  3 15  2  8  6 21 19 14]
Mean: 14459.72 , STD: 906.89
```

65
60
55
50
45
40
−10
0
10
20
30
Norge
Finland
Bottenvika
Estland
Østersjøen
Latvia
Nordsjøen
Danmark
Litauen
Moskva
Storbritannia
Irland
Polen
Hviterussland
Nederland
Berlin
Warszawa
London
Tyskland
Belgia
Kiev
Київ
Paris
Tsjekkia
Ukraina
Slovakia
Østerrike
Ungarn
Moldova
Frankrike
Romania
Kroatia
Serbia
Svartehavet
Italia
Bulgaria
Roma
Portugal
Madrid
Istanbul
Ankara
Tyrrenhavet
Lisboa
Spania
Hellas
Tyrkia
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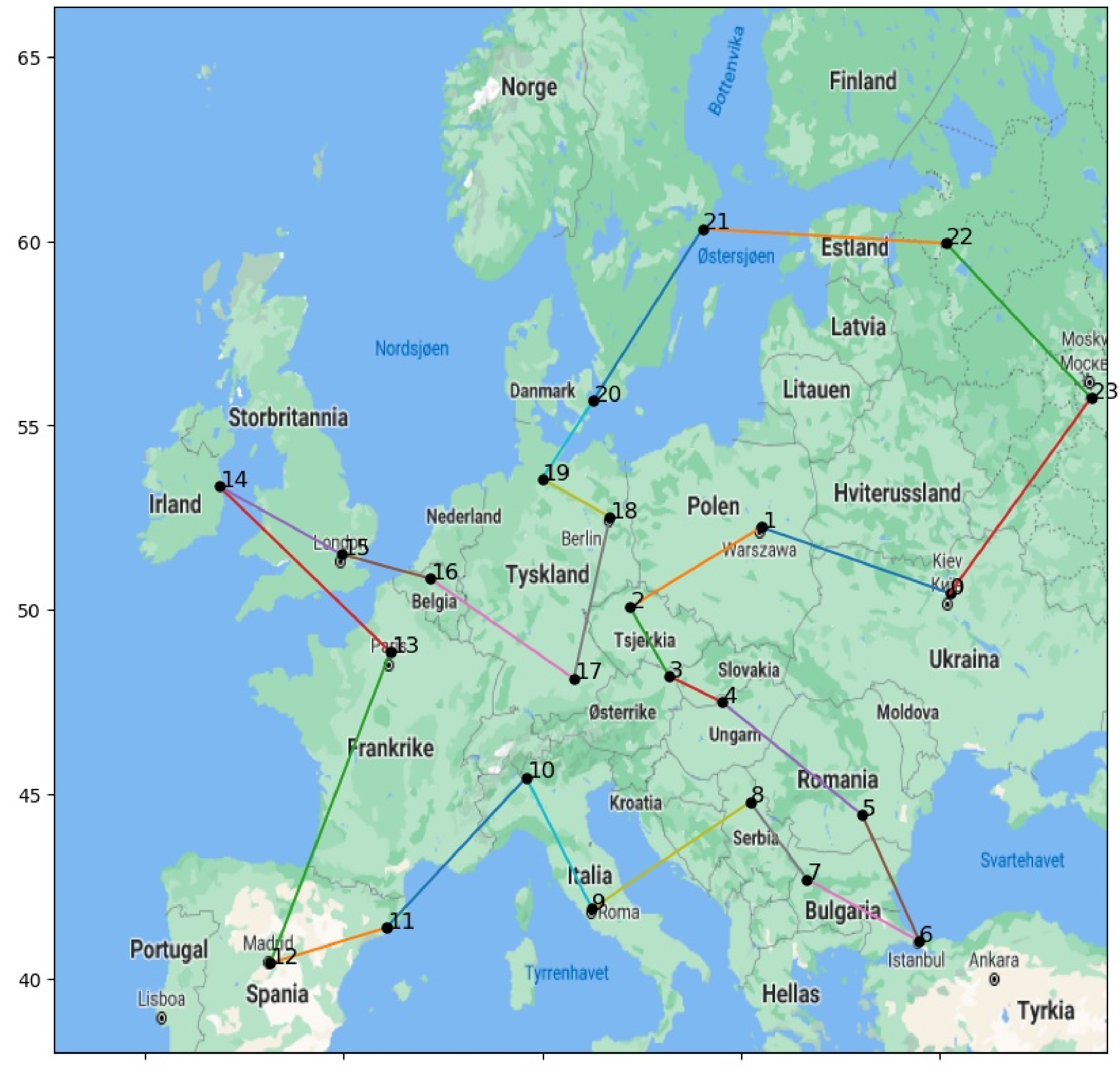


Best distance for 24 cities: 12287, fitness: 8.138636794614175e-05


```
For 24 cities
```

Did basic search to see effect of paramters, distance matrix with time table

Commment in grid_search() to run. But takes some time to run, and isnt asked for so i left it. Ideally id wanna run same values for x,y but took too long to run 1000 x 1000.

The best run i found one time was 12287 (200 samples and 1e5 iterations). With path: (21, 6, 8, 3, 16, 11, 7, 12, 0, 18, 13, 15, 17, 2, 23, 22, 5, 1, 20, 9, 4, 10, 14, 19)

(used gpt to create nicely formatted table) **Distance Table (Iterations: 1, 10, 100, 1000, 10000; Samples: 1, 100, 200, 300)**

	1	100	200	300
1	34959	25667	25309	25423
10	27489	22059	23575	22265
100	20816	16511	17436	16083
1000	13706	13224	12615	12951
10000	14329	12620	12287	12363

Time Table (Iterations: 1, 10, 100, 1000, 10000; Samples: 1, 100, 200, 300)

	1	100	200	300
1	0.00	0.00	0.00	0.00
10	0.00	0.01	0.03	0.04
100	0.00	0.12	0.24	0.35
1000	0.01	1.20	2.37	3.63

10000 0.12 11.86 23.89 35.10

Genetic Algorithm

Next, write a genetic algorithm (GA) to solve the problem. Choose mutation and crossover operators that are appropriate for the problem (see chapter 4.5 of the Eiben and Smith textbook). Choose three different values for the population size. Define and tune other parameters yourself and make assumptions as necessary (and report them, of course).

For all three variants: As with the hill climber, report best, worst, mean and standard deviation of tour length out of 20 runs of the algorithm (of the best individual of last generation). Also, find and plot the average fitness of the best fit individual in each generation (average across runs), and include a figure with all three curves in the same plot in the report. This means that the x-axis should be the generations over time and the y-axis should be the average (over the 20-runs) fitness of the best gene in that generation. Conclude which is best in terms of tour length and number of generations of evolution time.

Finally, plot an example optimized tour (the best of the final generation) for the three different population sizes, using the plot_plan method.

In [326…

```
# Implement the algorithm here

# Plot three curves, fitness function. Three populations: e.g: 15, 25, 35.
# Generations, e.g: 60
# Break main operations into functions: init_pop, fitness, select_parents, recombine, mutate

#Already have fitness-  Use by calc_distance(path) -> fitness(distance)

def fitness_pop(pop):
    # Calculate fitness function function and return list of whole population
    fit = [fitness(calc_distance(path)) for path in pop] # find fitness of each path in population
    return np.array(fit)

def init_pop(size, cities):
    '''
    Use already existing function to generate a new population of size with cities as path.
    '''
    pop = gen_perm(cities, size)
    return pop

def select_parents(pop, eval_pop):
    # Select which parents to pair up (exploration/exploitation), (rank based/roulette wheel)

    #Use roulette wheel selection: Individuals are selected with a probability proportional to their fitness.
    #                              Fitness values are normalised to create a probability distribution
    parent_pairs = []

    # Followed this structure for roulette: https://cratecode.com/info/roulette-wheel-selection

    tot_fit = np.sum(eval_pop)
    prob_scaled = eval_pop/tot_fit
    cumulative = np.cumsum(prob_scaled) # wheel slice sizes
    n = int(len(pop) / 2) # could vary but right now set as half of population, since i have replacement could 

    # Pick from roulette n pairs, could vary but right now set as half of population
    pairs = np.zeros((n, 2), dtype=object)

    for i in range(n):
        par1 =  select_one_roulette(pop, cumulative)
        par2 = select_one_roulette(pop, cumulative)
        pairs[i] = [par1, par2]

    return pairs

def select_one_roulette(pop_, cumulative_): # from code linked
        # Used to select one individual from the population
        rand = np.random.rand()  # generate a random number between 0 and 1
        for i, cp in enumerate(cumulative_):
            if rand <= cp:
                return pop_[i].copy() # ensure no issues with changing old individuals

def recombine(parent_pairs):
    # Recombine parent pairs with a method: Multi-point crossover/PMX/OX/Edge recombination

    '''
    Function to crossover parent pairs, only with ordered crossover implemented. But easy to add more.
    Returns array of childs
    '''
    def n_point_cross(p1, p2):
        return p1, p2

    # Not fully implemented, began but moved on to OX
```

```python
    def pmx(p1, p2):
        index1,index2 = random.choice(p1) , random.choice(p1)
        p1[p1.index(index1):p1.index(index2)] , p2[p1.index(index1):p1.index(index2)] = p2[p1.index(index1):p1.

        for i in p1[p1.index(index1):p1.index(index2)] not in p2[p1.index(index1):p1.index(index2)]:
            print(i)
        return p1, p2


    def ordered_cross(p1, p2):
        # Odrdered crossover which returns 1 child.
        # Take one substring out or parent one, then fill out rest as in order from p2. No duplicates
        n = len(p1)  # number of genes/cities in path
        child = np.full(n, -1)  # fill with -1
        i1, i2 = sorted(random.sample(range(n), 2))  # crossover points
        child[i1:i2+1] = p1[i1:i2+1].copy()  # not sure if i need copy, but to be safe i did here. As dont want

        # Used help from gpt to get a smarter logic to check for duplicates, my first idea of manually checking
        p2_i = 0
        for i in range(n):  # Ensure you loop over all positions in the child
            if child[i] == -1:  #meaning not filled in yet
                while p2[p2_i] in child:  # check for duplicates
                    p2_i += 1
                child[i] = p2[p2_i] # fill the spot with gene from parent 2

        return child

    #Implement the method for ccrossover on all parent pairs
    size = parent_pairs.shape[0] # same number of childs as parent pairs
    size_path = parent_pairs[0][0].shape[0] # length of path
    childs = np.zeros((size, size_path), dtype=int)

    for i, (p1_, p2_) in enumerate(parent_pairs):
        childs[i] = ordered_cross(p1_, p2_)
        #childs[i] = pmx(p1_, p2_)
        #print(f"Parent 1 {p1_}, Parent 2 {p2_}. Child: {childs[i]}") # seems to working correctly

    return childs

def mutate(childs, mut_rate):
    # Mutate child with mutation rate (mut_rate) with one method of: Bit flip/swap/inversion/gaussian
    n = len(childs[0]) # number of genes/cities in a path
    # Mutate by flipping two genes.
    for i in range(len(childs)):
        rand_ = random.uniform(0, 1)
        if mut_rate > rand_: #e.g if 0.2, want to flip 20% of the times
            index1, index2  = random.sample(range(n), 2) #pick 2 random indices to swap
            #Then swap the genes
            temp = childs[i][index1]
            childs[i][index1] = childs[i][index2]
            childs[i][index2] = temp
            #print(childs[i]) # seems to working correctly


    return childs

def select_new_(parents, childs , evaled_parents , evaled_childs):
    # Combine whole old population (where parens are from) and combine with childs then use roulette again to se
    # population. Make sure we keep same number of individuals each generation
    comb_pop = np.concatenate((parents, childs))
    comb_eval = np.concatenate((evaled_parents, evaled_childs))

    tot_fit = np.sum(comb_eval)
    prob_scaled = comb_eval/tot_fit
    cumulative = np.cumsum(prob_scaled) # wheel slice sizes

    # Want to keep pop size constant, like parents size
    new_pop = np.zeros_like(parents)

    for i in range(parents.shape[0]):
        new_pop[i] = select_one_roulette(comb_pop, cumulative)

    new_eval = fitness_pop(new_pop) # prob a smarter way to make use of alrady existing evals, but awkward when

    return new_pop, new_eval


def GA(cities, pop_size, mutation_rate, generations, printInfo=False):
    '''
    Genetic algorith follow this scheme, where each main process is seperated by a function.
    Runs through generations and updates the best path and fit if found. Also saves best fit for each generatio

    # initialize population randomly -> pick 100-500 paths
```

```
    # EVALUATE each candidate -> use fitness function

    # REPEAT (while or for) until condition is satisfied:
        # SELECT parents -> choosing which parents to pair up (explore -> rank based worst+best), (exploit -> re
                         -> (remove parents from dataset to select)
                         -> new population (new list), with elitism keep some best parents into new list
        # RECOMBINE pairs of parents -> (PMX / n-point crossover)
        # MUTATE resulting offspring -> e.g 30% -> 0.3 *(swap/insert/scramble)
        # EVALUATE new candidates ->
        # SELECT new individuals for next generation

    # Keep parents and childs, but maintain same population size each generation.
    '''
    best_path = []
    best_fit = -1 # higher is better (1/distance)
    fit_hist = []

    print_interval = generations // 20  # used to track fitness when running

    pop = init_pop(pop_size, cities) # init population
    pop = np.array(pop) # change to array for operations
    eval_pop = fitness_pop(pop) # eval each candidate

    for i in range(generations):
        # SELECT PARENTS
        parent_pairs = select_parents(pop, eval_pop)

        # RECOMBINE
        children = recombine(parent_pairs)

        # MUTATE
        children = mutate(children, mutation_rate)

        # EVALUATE
        eval_children = fitness_pop(children)

        # SELECT NEW (old + new), and return evaluation of all
        new_population, new_eval = select_new_(pop, children, eval_pop, eval_children)

        #Find best path
        index = np.argmax(new_eval)
        fit = new_eval[index]

        #print(f"Fit: {fit}, pop_size = {new_population.shape[0]}")

        if fit > best_fit:
            best_fit = fit
            best_path = new_population[index]

        fit_hist.append(best_fit)

        if printInfo:
            if(i % 100 == 0):
                print(f"Generation: {i}, Fitness: {best_fit}")

    return best_path, best_fit, fit_hist



# Params
pop_size = 50
generations = 500
mutation_rate = 0.2

path, fit, fitness_history = GA(cities_24, pop_size, mutation_rate, generations, printInfo=True)

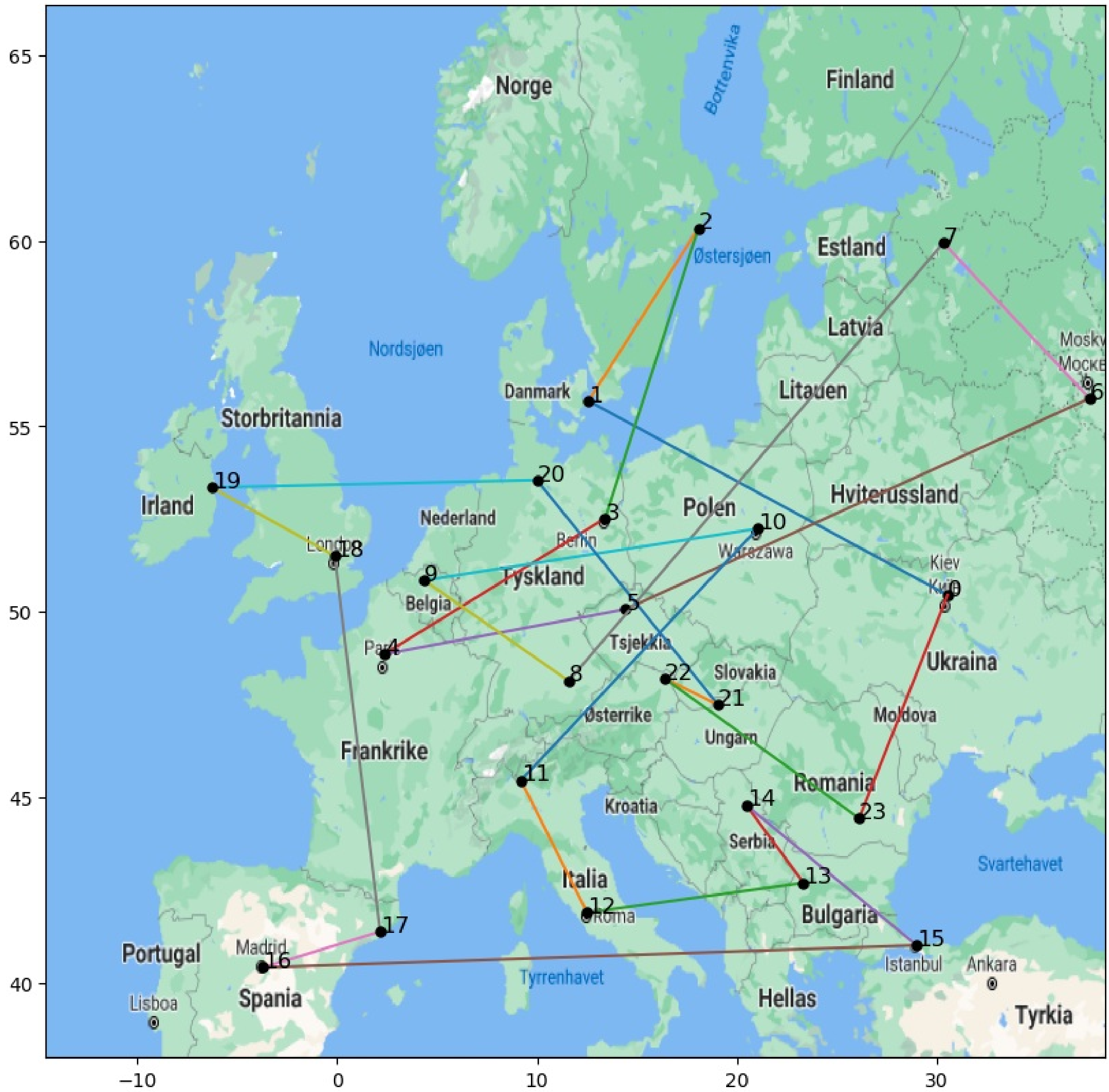
print(f"Best path found: {path}")
print(f"Distance for path: {calc_distance(path):.2f}")

plot_path(path) #basically best plot i get, pointless plotting more paths with 24 cities as it dont work. Howeve

# Goal for fitness from hill climb: 8.138636794614175e-05

# GA is seemingly working for 6 and 10 cities. However it cannot find a good solution for 24.
```

```
Generation: 0, Fitness: 3.751666677921667e-05
Generation: 100, Fitness: 4.429298213132515e-05
Generation: 200, Fitness: 4.429298213132515e-05
Generation: 300, Fitness: 4.432142785911993e-05
Generation: 400, Fitness: 4.432142785911993e-05
Best path found: [10  6 21  2 16 17 14 19 15  3 23 13 18 20  1  9 12  0 11  7  8  5 22  4]
Distance for path: 22562.45
```



In [327…

```
print(f"Fitness history per generation: {fitness_history}")
plt.plot(fitness_history)
plt.show()
```

```
Fitness history per generation: [3.751666677921667e-05, 3.751666677921667e-05, 3.9747839704912026e-05, 3.9747839
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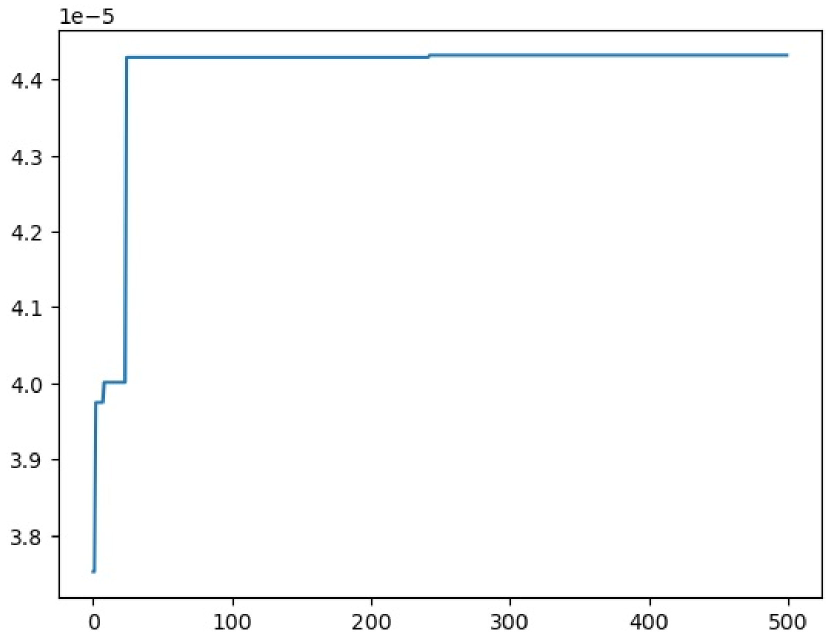


In [328…

```
def GA_runs(pop_size, runs=20, cities=cities_24, mutation_rate=0.2, generations=500):
    best_fits = []
    worst_fits = []
    tour_len = []
    all_fit_hists = []

    for i in range(runs):
        path, fit, fit_hist = GA(cities, pop_size, mutation_rate, generations)
        best_fits.append(fit)
        worst_fits.append(np.min(fit_hist))
        tour_len.append(calc_distance(path))
        all_fit_hists.append(fit_hist)

    return best_fits, worst_fits, tour_len, all_fit_hists

avg_fits = []
pop_sizes = [10, 50, 100]

for pop_ in pop_sizes:
    best_fits, worst_fits, tour_len, all_fit_hists = GA_runs(pop_size=pop_, cities=cities_24, mutation_rate=0.2
    print(f"For population: {pop_}")
    best_tour_len = np.min(tour_len)
    worst_tour_len = np.max(tour_len)
    mean_tour_len = np.mean(tour_len)
    std_tour_len = np.std(tour_len)

    print(f"Best Tour len: {best_tour_len:.2f}")
    print(f"Worst Tour len: {worst_tour_len:.2f}")
    print(f"Mean Tour len: {mean_tour_len:.2f}")
    print(f"Standard Deviation of Tour Length: {std_tour_len:.2f}\n")

    avg_fits.append(np.mean(all_fit_hists, axis=0))

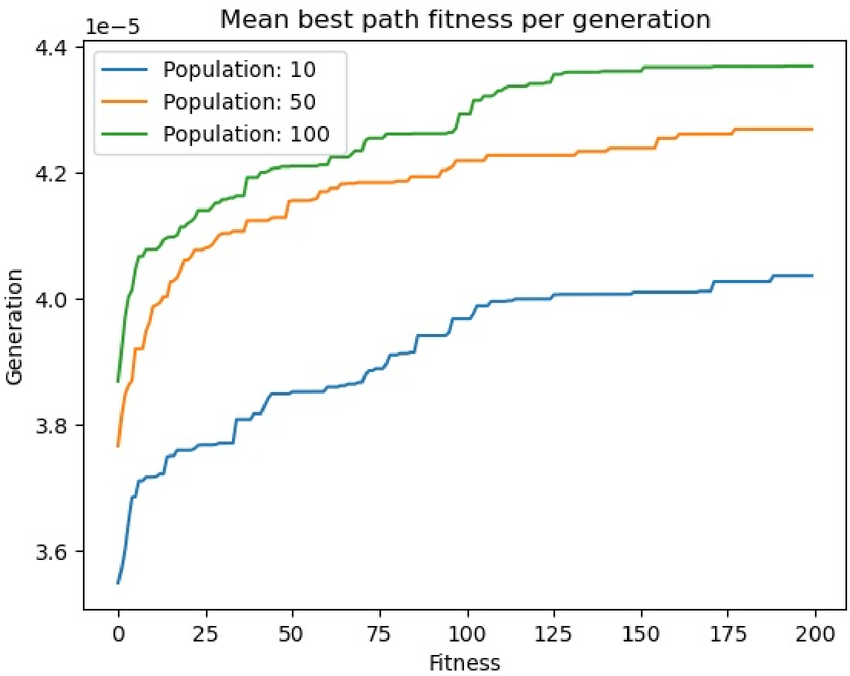

for i in range(len(pop_sizes)):
    plt.plot(avg_fits[i], label=f"Population: {pop_sizes[i]} ")
plt.title(f"Mean best path fitness per generation")
plt.xlabel("Fitness")
plt.ylabel("Generation")
plt.legend()
plt.show()

#see it does imrpove a bit when increase population size, however time increases a lot.
```

```
For population: 10
Best Tour len: 21773.33
Worst Tour len: 27093.38
Mean Tour len: 24840.40
Standard Deviation of Tour Length: 1225.86

For population: 50
Best Tour len: 21654.04
Worst Tour len: 24903.45
Mean Tour len: 23458.84
Standard Deviation of Tour Length: 817.21

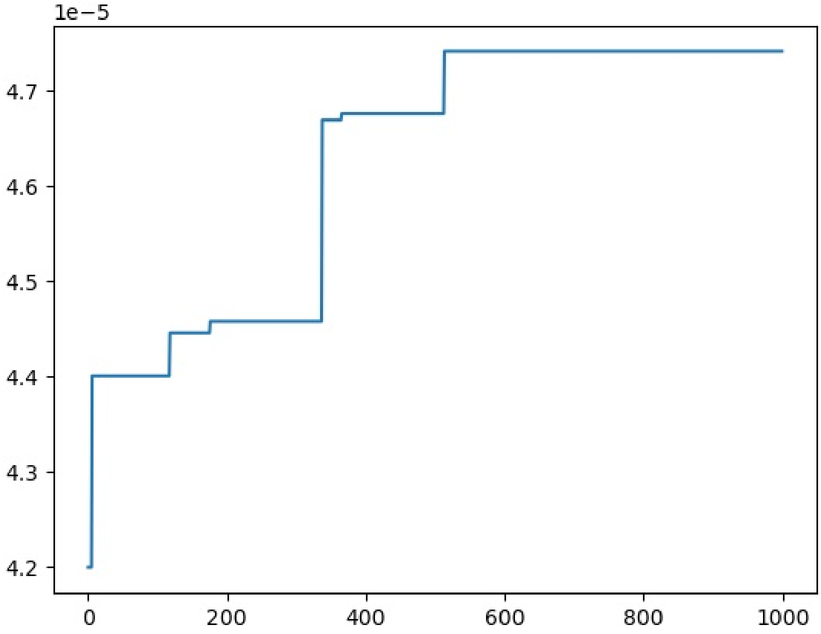
For population: 100
Best Tour len: 21720.24
Worst Tour len: 23894.25
Mean Tour len: 22910.88
Standard Deviation of Tour Length: 629.37
```



In [ ]:

In [329…

```
# Example of fitness history with params, can see it dont improve much.
'''
# Params
pop_size = 200
generations = 1000
mutation_rate = 0.2
'''
hist = [4.198972511426454e-05, 4.198972511426454e-05, 4.198972511426454e-05, 4.198972511426454e-05, 4.19897251l4
plt.plot(hist)
plt.show()
```



Among the first 10 cities, did your GA find the shortest tour (as found by the exhaustive search)? Did it come close?

For both 10 and 24 cities: How did the running time of your GA compare to that of the exhaustive search?

How many tours were inspected by your GA as compared to by the exhaustive search?

In [330…

```
# Answer
```