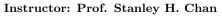
ECE 302: Probabilistic Methods in Electrical and Computer Engineering

Fall 2020





Homework 1

Fall 2020 (Due: 09/04/2020)

Name:	Email:	
	ght) Eastern Daylight Time. Please print this home ou can use a tablet. Submit your homework through	
Exercise 1. Calculate the infinite series	$\sum_{k=0}^{\infty} k \cdot \left(\frac{2}{3}\right)^{k+1}$	

Exercise 2.

Evaluate the integrals

$$\int_{a}^{b} \frac{1}{b-a} \left(x - \frac{a+b}{2} \right)^{2} dx$$

$$\int_0^\infty \lambda x e^{-\lambda x} dx$$

$$\int_{-\log y}^{\log y} \frac{\lambda x}{4} e^{-\lambda |x|} dx,$$

where y > 1.

Exercise 3. Evaluate the infinite series	$\sum_{k=0}^{\infty} (k-\lambda)^2 \frac{\lambda^k e^{-\lambda}}{k!}$	(1

Exercise 4.

Simplify the following sets with the domain of real numbers in mind

- (a) $[2,5] \cap ([1,3] \cup \{0,3,4\})$
- (b) $(1,2)^c \cup [4,6]$
- (c) $\bigcap_{n=1}^{\infty} (2 1/n, 2 + 1/n)$
- (d) $\bigcup_{n=1}^{\infty} [3, 6 \frac{1}{n}]$

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space S and $= \{1, 3, 9, 11\}$	}. Find (a) <i>A</i>	$\cap B \cap C$, (b)	$A^c \cap B$, (c)	$A \setminus C$, and (d	$) (A \setminus B) \cup B.$	
	danast of Dal	J.,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	:l	+1+ (A + + D)($C = AC \cap DC$	
	nd part of Del	Morgan's Law	v, i.e., show	that $(A \cup B)^{\alpha}$	$\dot{c} = A^c \cap B^c.$	
	nd part of Del	Morgan's Lav	v, i.e., show	that $(A \cup B)^{\alpha}$	$C = A^c \cap B^c$.	
	nd part of Del	Morgan's Lav	v, i.e., show	that $(A \cup B)^{\alpha}$	$c = A^c \cap B^c$.	
	nd part of Del	Morgan's Lav	v, i.e., show	that $(A \cup B)^{\alpha}$	$c = A^c \cap B^c$.	
	nd part of Del	Morgan's Lav	v, i.e., show	that $(A \cup B)^{\alpha}$	$S = A^c \cap B^c$.	
	nd part of Del	Morgan's Lav	v, i.e., show	that $(A \cup B)^{\alpha}$	$F = A^c \cap B^c$.	
	nd part of Del	Morgan's Lav	v, i.e., show	that $(A \cup B)^{\alpha}$	$C = A^c \cap B^c$.	
	nd part of Del	Morgan's Lav	v, i.e., show	that $(A \cup B)^{\alpha}$	$A^c = A^c \cap B^c$.	
	nd part of Del	Morgan's Lav	v, i.e., show	that $(A \cup B)^{\alpha}$	$S = A^c \cap B^c$.	
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	nd part of Del	Morgan's Lav	v, i.e., show	that $(A \cup B)^{\alpha}$	$S = A^c \cap B^c$.	
ercise 6. ove the secon	nd part of Del	Morgan's Lav	v, i.e., show	that $(A \cup B)^{\alpha}$	$S = A^c \cap B^c$.	
	nd part of Del	Morgan's Lav	v, i.e., show	that $(A \cup B)^{\alpha}$	$F = A^c \cap B^c$.	
	nd part of Del	Morgan's Lav	v, i.e., show	that $(A \cup B)^{\alpha}$	$F = A^c \cap B^c$.	

Let A	ccise 7. $A = (-\infty, r]$ and $B = A \cup A$	$C = (-\infty, s]$ where C , and $A \cap C = -\infty$	e $r \leq s$. (a) Fin \emptyset .	nd an expression	n for $C = (r, s]$	in terms of A and A
		, and 11110 —	<i>v</i> .			
Exer Show	crise 8. That if $A \cup B = A$	$1 \text{ and } A \cap B = A,$, then $A = B$.			

Exercise 9.

This is a programming exercise. You can use either MATLAB or Python.

(a) Compute the result of the following matrix vector multiplication.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

- (b) Plot a sine function on the interval $[-\pi, \pi]$ with 1000 data points using matplotlib.pyplot.plot in Python or plot in MATLAB.
- (c) Generate 10,000 uniformly distributed random numbers on interval [0, 1). Use hist in MATLAB or matplotlib.pyplot.hist Python to generate a histogram of all the random numbers.

Please insert your code / solution after this page.

Exercise 1	ш	١.

A collection of letters, a-z, is mixed in a jar. Two letters are drawn at random, one after the other.

- (a) What is the probability of drawing a vowel (a,e,i,o,u) and a consonant in either order?
- (b) Write a MATLAB / Python program to verify your answer in part (a). That is, randomly draw two letters without replacement and check whether one is a vowel and the other is a consonant. Compute the probability by repeating the experiment for 10000 times.

Please write your hand-written solution here.

Please insert your code / solution after this page.

Exercise 11.

There are 50 students in a classroom.

- (a) What is the probability that there is at least one pair of students having the same birthday? Show your steps.
- (b) Write a MATLAB / Python program to simulate the event, and verify your answer in (a). Hint: You probably need to repeat the simulation for many times to obtain a probability. Submit your code and result

You may assume that a year only has 365 days. You may also assume that all days have equal likelihood to be taken.

Please write your hand-written solution here.

Please insert your code / solution after this page.