Exercise 2.

A space S is defined as $S = \{1, 3, 5, 7, 9, 11\}$, and three subsets as $A = \{1, 3, 5\}$, $B = \{7, 9, 11\}$, $C = \{1, 3, 9, 11\}$. Assume that each element has probability $1/6$. Find the following probabilities: (a) $P[A]$, (b) $P[B]$, (c) $P[C]$, (d) $P[A \cup B]$, (e) $P[A \cup C]$, (f) $P[(A \backslash C) \cup B]$.

A. $P[A] = 3\left(\frac{1}{6}\right) = \frac{1}{2}$

B. $P[B] = 3\left(\frac{1}{6}\right) = \frac{1}{2}$

C. $P[C] = 4\left(\frac{1}{6}\right) = \frac{2}{3}$

D. $P[A \cup B] = 6\left(\frac{1}{6}\right) = 1$

E. $P[A \cup C] = 5\left(\frac{1}{6}\right) = \frac{5}{6}$ 　　　　　 $A \cup C = \{1, 3, 5, 9, 11\}$

F. $P[(A \backslash C) \cup B] = 0\left(\frac{1}{6}\right) = 0$ 　　 $(A \backslash C) \cup B = \{5\} \cup \{7, 9, 11\} = \emptyset$

Exercise 3.

Let the events A and B have $P[A] = x$, $P[B] = xy$ and $P[A \cup B] = x^2 z$. Find the following probabilities.

(a) $P[A \cap B]$

(b) $P[A^c \cap B^c]$

(c) $P[A^c \cup B^c]$

(d) $P[A \cap B^c]$

(e) $P[A^c \cup B]$

A. $P[A \cap B] = P[A] + P[B] - P[A \cup B] = x + xy - x^2 z$

B. $P[A^c \cap B^c] = 1 - P[A \cup B] = 1 - x^2 z$

C. $P[A^c \cup B^c] = 1 - P[A \cap B] = 1 - x - xy + x^2 z$

D. $P[A \cap B^c] = P[A \backslash B] = P[A] - P[A \cap B] = x - x - xy + x^2 z = x^2 z - xy$

E. $P[A^c \cup B] = 1 - P[A \backslash B] = 1 - (x^2 z - xy) = 1 + xy - x^2 z$