Homework 1

Fall 2020
(Due: 09/04/2020)

Name: _____ Email: _____

Homework is due at 11:59pm (midnight) Eastern Daylight Time. Please print this homework, write your solution, and scan the solution. Or you can use a tablet. Submit your homework through Gradescope. No late homework will be accepted.

Exercise 1.
Calculate the infinite series

$$\sum_{k=0}^{\infty} k \cdot \left(\frac{2}{3}\right)^{k+1}$$

Exercise 2.
Evaluate the integrals

(a)

$$\int_a^b \frac{1}{b-a}\left(x-\frac{a+b}{2}\right)^2 dx$$

(b)

$$\int_0^\infty \lambda x e^{-\lambda x} dx$$

(c)

$$\int_{-\log y}^{\log y} \frac{\lambda x}{4} e^{-\lambda|x|} dx,$$

where $y > 1$.

Exercise 3.
Evaluate the infinite series

$$\sum_{k=0}^{\infty}(k-\lambda)^2\frac{\lambda^k e^{-\lambda}}{k!} \tag{1}$$

Exercise 4.
Simplify the following sets with the domain of real numbers in mind

(a) $[2,5] \cap ([1,3] \cup \{0,3,4\})$

(b) $(1,2)^c \cup [4,6]$

(c) $\bigcap_{n=1}^{\infty}(2-1/n, 2+1/n)$

(d) $\bigcup_{n=1}^{\infty}[3, 6-\frac{1}{n}]$

Exercise 5.
A space S and three of its subsets are given by $S = \{1, 3, 5, 7, 9, 11\}$, $A = \{1, 3, 5\}$, $B = \{7, 9, 11\}$, and $C = \{1, 3, 9, 11\}$. Find (a) $A \cap B \cap C$, (b) $A^c \cap B$, (c) $A \backslash C$, and (d) $(A \backslash B) \cup B$.

Exercise 6.
Prove the second part of DeMorgan's Law, i.e., show that $(A \cup B)^c = A^c \cap B^c$.

Exercise 7.
Let $A = (-\infty, r]$ and $B = (-\infty, s]$ where $r \leq s$. (a) Find an expression for $C = (r, s]$ in terms of A and B. (b) Show that $B = A \cup C$, and $A \cap C = \emptyset$.

Exercise 8.
Show that if $A \cup B = A$ and $A \cap B = A$, then $A = B$.

Exercise 9.
This is a programming exercise. You can use either MATLAB or Python.

(a) Compute the result of the following matrix vector multiplication.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

(b) Plot a sine function on the interval $[-\pi, \pi]$ with 1000 data points using `matplotlib.pyplot.plot` in Python or `plot` in MATLAB.

(c) Generate 10,000 uniformly distributed random numbers on interval [0, 1). Use `hist` in MATLAB or `matplotlib.pyplot.hist` Python to generate a histogram of all the random numbers.

Please insert your code / solution after this page.

Exercise 10.
A collection of letters, a-z, is mixed in a jar. Two letters are drawn at random, one after the other.

(a) What is the probability of drawing a vowel (a,e,i,o,u) and a consonant in either order?

(b) Write a MATLAB / Python program to verify your answer in part (a). That is, randomly draw two letters without replacement and check whether one is a vowel and the other is a consonant. Compute the probability by repeating the experiment for 10000 times.

Please write your hand-written solution here.

Please insert your code / solution after this page.

Exercise 11.
There are 50 students in a classroom.

(a) What is the probability that there is at least one pair of students having the same birthday? Show your steps.

(b) Write a MATLAB / Python program to simulate the event, and verify your answer in (a). Hint: You probably need to repeat the simulation for many times to obtain a probability. Submit your code and result.

You may assume that a year only has 365 days. You may also assume that all days have equal likelihood to be taken.

Please write your hand-written solution here.

Please insert your code / solution after this page.