

$$x-1 \leq 2 \\ x \leq 3$$

$$B = \{x \mid x \leq 3\}$$

Exercise 4.

A number x is selected at random in the interval $[-1, 3]$. Let the events $A = \{x \mid x < 0\}$, $B = \{x \mid (x-1)^2 < 4\}$, $C = \{x \mid x > 1\}$. Find (a) $P[A|B]$, (b) $P[B|C]$, (c) $P[A|C^c]$, (d) $P[B|C^c]$.

$$A. P[A|B] = P[A \cap B] / P[B] = 1/4$$

$$B. P[B|C] = P[B \cap C] / P[C] = 1/2 / 1/2 = 1$$

$$C. P[A|C^c] = P[A \cap C^c] / P[C^c] = 1/4 / 1/2 = 1/2$$

$$D. P[B|C^c] = P[B \cap C^c] / P[C^c] = 0 / 1/2 = 0$$



$$P[A] = 1/4$$

$$P[B] = 1$$

$$P[C] = 1/2, P[C^c] = 1/2$$

Exercise 5.

Let A, B, C be events with probabilities $P[A] = 0.3$, $P[B] = 0.2$, $P[C] = 0.5$. Find

(a) $P[A \cup B]$ if A and B are independent

(b) $P[A \cup B]$ if A and B are disjoint

(c) $P[A \cup B \cup C]$ if A, B and C are independent

(d) $P[A \cup B \cup C]$ if A, B and C are pairwise disjoint; Can this happen?

$$A. P[A \cup B] = P[A] + P[B] - P[A]P[B] = 0.5 - 0.06 = 0.44$$

$$B. P[A \cup B] = P[A] + P[B] = 0.5$$

$$C. P[A \cup B \cup C] = P[A] + P[B] + P[C] - P[A]P[B] - P[A]P[C] - P[B]P[C] + P[A]P[B]P[C] \\ = 0.3 + 0.2 + 0.5 - 0.06 - 0.15 - 0.1 + 0.03 = 0.72$$

$$D. P[A \cup B \cup C] = P[A] + P[B] + P[C] = 0.3 + 0.2 + 0.5 = 1$$

This is possible since A, B , and C account for all events.
If $P[A \cup B \cup C] > 1$, this would not be possible.