Exercise 6.

(a) By using the fact that $\mathbb{P}[A \cup B] \leq \mathbb{P}[A] + \mathbb{P}[B]$, show that $\mathbb{P}[A \cup B \cup C] \leq \mathbb{P}[A] + \mathbb{P}[B] + \mathbb{P}[C]$.

(b) By using the fact that $\mathbb{P}\left[\bigcup_{k=1}^{n} A_k\right] \leq \sum_{k=1}^{n} \mathbb{P}[A_k]$, show that $\mathbb{P}\left[\bigcap_{k=1}^{n} A_k\right] \geq 1 - \sum_{k=1}^{n} \mathbb{P}[A_k^c]$.

A. Set $D = A \cup B$

$P[D \cup C] \leq P[D] + P[C]$ treat union of A and B as a single set D

$\therefore P[A \cup B \cup C] \leq P[A] + P[B] + P[C]$

B. $P[\bigcup_{k=1}^{n} A_k] \leq \sum_{k=1}^{n} P[A_k]$