ECE 302: Probabilistic Methods in Electrical and Computer Engineering

Fall 2020

Instructor: Prof. Stanley H. Chan



Homework 4

 $\begin{array}{c} \text{Fall 2020} \\ \text{(Due: October 12, 2020, Monday)} \end{array}$

Name:			Email:	
				nework, write your solution, mework will be accepted.
Exercise 1. Two dice are tossed. Let	t X be the absolu	te difference in	the number of dots fac	cing up. Let
	$g(X) = \begin{cases} X^2, \\ 0, \end{cases}$	if $X > 2$ otherwise.	nd h(X) = - X - 2	
(a) Find $\mathbb{E}[g(X)]$.				
(b) Find $\mathbb{E}[h(X)]$.				

A modem transmits a +2 voltage signal into a channel. The channel adds to this signal a noise term that is drawn from the set $\{0, -1, -2, -3\}$ with respective probabilities $\{1/10, 2/10, 3/10, 4/10\}$.
(a) Find and sketch the PMF and the CDF of the output Y of the channel.
(b) What is the probability that the output of the channel is equal to the input of the channel?
(c) What is the probability that the output of the channel is positive?
(d) Find the expected value and variance of Y .

Exercise 2.

Exercise 3.
A voltage X is uniformly distributed in the set $\{0, 1, 2, 3\}$.
(a) Find the mean and variance of X .
(b) Find the mean and variance of $Y = X^2 - 2$.
(c) Find the mean of $W = \sin(\pi X/4)$.
(d) Find the mean of $Z = \sin^2(\pi X/4)$.

(a) If X is Poisson (λ) , compute $\mathbb{E}[3/(X+1)]$ (b) If X is Bernoulli (p) and Y is Bernoulli (p)	$[H(q)]$, compute $\mathbb{E}[(2X+Y)^2]$ if X and Y are independent.

Exercise 4.

Let X be the number of photons counted by a receiver in an optical communication system. It is known

Exercise 5.

Exercise 6.

A random variable X has PDF:

$$f_X(x) = \begin{cases} cx(4-x^2), & 0 \le x \le 2\\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find c
- (b) Find $F_X(x)$
- (c) Find $\mathbb{E}[X]$

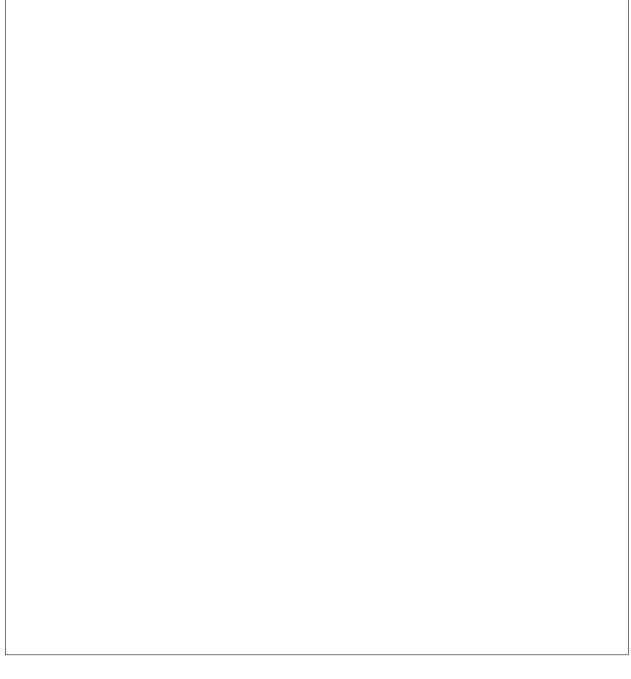
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Exercise 7.

Consider a CDF

$$F_X(x) = \begin{cases} 0, & \text{if } x < -2\\ 0.25, & \text{if } -2 \le x < 0\\ (x+1)/2, & \text{if } 0 \le x < 1\\ 1, & \text{otherwise.} \end{cases}$$

- (a) Find $\mathbb{P}[X < -2]$, $\mathbb{P}[X = 0]$ and $\mathbb{P}[X > 0.5]$.
- (b) Find $f_X(x)$.



Exercise	8.
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Consider a discrete PMF $p_X(k) = \begin{bmatrix} 0.3 & 0.1 & 0.15 & 0.25 & 0.1 & 0.08 & 0.02 \end{bmatrix}$. Write a MATLAB / Python
function that takes this PMF and generates $N = 100,000$ realizations of X . Your function can only us
the uniform random number generator rand in MATLAB (or numpy.random.rand in Python) and no other
random number generators. Submit your code and the empirical histogram of X .

Please attach your code and plot after this page.	