PURDUE UNIVERSITY

Mid Term 1

Fall 2020

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Session: MWF 3:30-4:20

Please copy and write the following statement:

I certify that I have neither given nor received unauthorized aid on this exam.

I certify that I have neither given nor received unauthorized aid on this exam.

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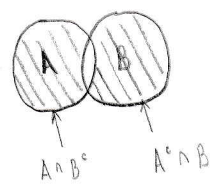
Eli Talcott

(Signature)

Exercise 1. (40 POINTS)

(a)

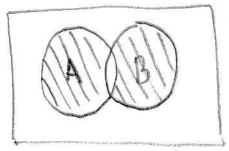
$A \Delta B = (A - B) \cup (B - A)$



$A \Delta B = (A \cap B^c) \cup (A^c \cap B)$

(b)

Find $\mathbb{P}[A \Delta B]$ in terms of $\mathbb{P}[A]$, $\mathbb{P}[B]$ when independent



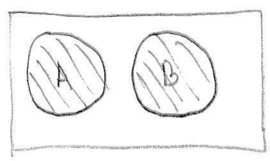
$$\mathbb{P}[A \Delta B] = \mathbb{P}[A \cap B^c] + \mathbb{P}[A^c \cap B]$$

$$= \mathbb{P}[A]\mathbb{P}[B^c] + \mathbb{P}[A^c]\mathbb{P}[B]$$

$$\mathbb{P}[A \Delta B] = \mathbb{P}[A](1-\mathbb{P}[B]) + (1-\mathbb{P}[A])\mathbb{P}[B]$$

(c)

Find $\mathbb{P}[A \Delta B]$ when disjoint



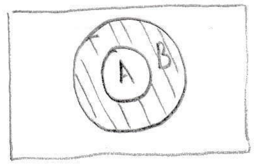
$$\mathbb{P}[A \Delta B] = \mathbb{P}[A \cap B^c] + \mathbb{P}[A^c \cap B]$$

no overlap $\therefore$

$$\mathbb{P}[A \Delta B] = \mathbb{P}[A] + \mathbb{P}[B]$$

(d)

Find $\mathbb{P}[A \Delta B]$ when A contained in B



$$\mathbb{P}[A \Delta B] = \underbrace{\mathbb{P}[A \cap B^c]}_{= 0} + \mathbb{P}[A^c \cap B]$$

$$\mathbb{P}[A \Delta B] = \mathbb{P}[A^c \cap B]$$

$$\mathbb{P}[A \Delta B] = \mathbb{P}[B] - \mathbb{P}[A]$$

Exercise 2. (60 POINTS)

(a)

$\Omega = \{X=0, X=1\}$ sample space is all results

Event space has 6 elements
0 sent, 1 sent, 0 received, 1 received,
error occurs, no error occurs

(b)

$\mathbb{P}(\text{send } 0) = 1-p \quad \mathbb{P}(r=1 \mid s=0) = \varepsilon_0$

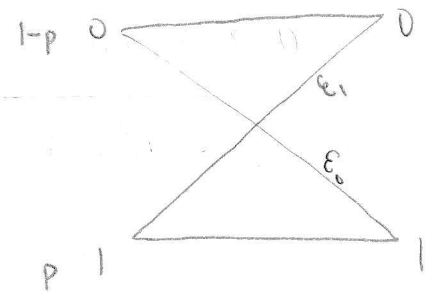
$\mathbb{P}(\text{send } 1) = p \quad \mathbb{P}(r=0 \mid s=1) = \varepsilon_1$

$$P_E = \mathbb{P}[X=1] = \mathbb{P}(r=1 \mid s=0)\mathbb{P}(s=0) + \mathbb{P}(r=0 \mid s=1)\mathbb{P}(s=1)$$

$$P_E = \varepsilon_0(1-p) + \varepsilon_1 p$$

(c)

$\mathbb{P}[X=1 \mid r=1] =$



$\mathbb{P}[r=1] = (1-p)\varepsilon_0 + p(1-\varepsilon_1)$

$$\mathbb{P}[X=1 \mid r=1] = \frac{\mathbb{P}[r=1 \mid X=1]\,\mathbb{P}[r=1]}{\mathbb{P}[r=0 \mid X=1]\,\mathbb{P}[r=0] + \mathbb{P}[r=1 \mid X=1]\,\mathbb{P}[r=1]}$$

$\mathbb{P}[X=1 \mid r=1] =$

(d)

Show that {error occurs} and {1 received} are independent

↳ implies $\mathbb{P}[X=1 \cap r=1] = \mathbb{P}[X=1]\mathbb{P}[r=1]$

$\mathbb{P}[X=1] = P_E = \varepsilon_0(1-p) + \varepsilon_1 p = \varepsilon_0 - \varepsilon_0 p + \varepsilon_1 p$

$\mathbb{P}[r=1] = \mathbb{P}[r=1 | s=0]\mathbb{P}[s=0] + \mathbb{P}[r=1 | s=1]\mathbb{P}[s=1]$

$= \varepsilon_0(1-p) + (1-\varepsilon_1)(p) = \varepsilon_0 - \varepsilon_0 p + p - \varepsilon_1 p$

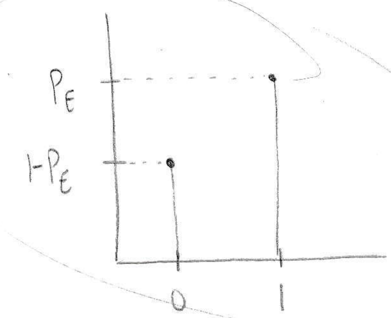
$\mathbb{P}[X=1 \cap r=1] = \mathbb{P}[$

(e)

$p_X(0) = 1 - p_E$

$p_X(1) = p_E$

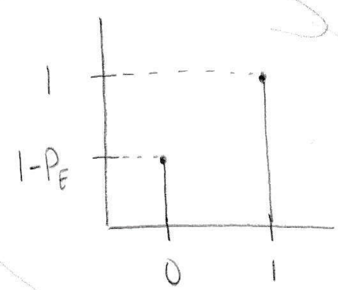
PMF of X



Note: p_E not necessarily higher than $1-p_E$

(f)

CDF of X



(g)

$$\mathbb{E}[X] = 0(1-P_E) + 1(P_E)$$

$$\mathbb{E}[X] = P_E$$

$$\text{Var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 \qquad \mathbb{E}[X^2] = 0^2(1-P_E) + 1^2(P_E) = P_E$$

$$\text{Var}(X) = P_E - P_E^2$$

The END