Homework 8

Fall 2020
(Due: Dec 4, 2020, Friday)

Name: ____ Email: __________________________________

Homework is due at 11:59pm (midnight) Eastern Time. Please print this homework, write your solution, and scan the solution. Submit your homework through Gradescope. No late homework will be accepted.

Exercise 1.
Let $Y(t) = X(t) - X(t-d)$.

(a) Find $R_{X,Y}(\tau)$ and $S_{X,Y}(\omega)$.

(b) Find $R_Y(\tau)$.

(c) Find $S_Y(\omega)$.

Exercise 2.
Let $X(t)$ be a zero-mean WSS process with autocorrelation function $R_X(\tau)$. Let $Y(t) = X(t)\cos(\omega t + \Theta)$, where $\Theta \sim \text{uniform}(-\pi, \pi)$ and Θ is independent of the process $X(t)$.

(a) Find the autocorrelation function $R_Y(\tau)$.

(b) Find the cross-correlation function of $X(t)$ and $Y(t)$.

(c) Is $Y(t)$ WSS? Why?

Exercise 3.
Consider the system

$$Y(t) = e^{-t} \int_{-\infty}^{t} e^{\tau} X(\tau) d\tau.$$

Assume that $X(t)$ is zero mean white noise with power spectral density $S_X(\omega) = N_0/2$. Find

(a) $S_{XY}(\omega)$

(b) $R_{XY}(\tau)$

(c) $S_Y(\omega)$

(d) $R_Y(\tau)$

Exercise 4.
Consider the random process

$$X(t) = 2A\cos(t) + (B-1)\sin(t),$$

where A and B are two independent random variables with $\mathbb{E}[A] = \mathbb{E}[B] = 0$, and $\mathbb{E}[A^2] = \mathbb{E}[B^2] = 1$.

(a) Find $\mu_X(t)$

(b) Find $R_X(t_1, t_2)$

(c) Find $C_X(t_1, t_2)$

Exercise 5.
Find the autocorrelation function $R_X(\tau)$ corresponding to each of the following power spectral densities:

(a) $\delta(\omega - \omega_0) + \delta(\omega + \omega_0)$

(b) $e^{-\omega^2/2}$

(c) $e^{-|\omega|}$

Exercise 6.
A WSS process $X(t)$ with autocorrelation function $R_X(\tau) = e^{-\tau^2/(2\sigma_T^2)}$ is passed through an LTI system with transfer function $H(\omega) = e^{-\omega^2/(2\sigma_H^2)}$. Denote the system output by $Y(t)$. Find

(a) $S_{XY}(\omega)$

(b) $R_{XY}(\tau)$

(c) $S_Y(\omega)$

(d) $R_Y(\tau)$

Exercise 7.
A WSS process $X(t)$ with autocorrelation function

$$R_X(\tau) = 1/(1+\tau^2)$$

is passed through an LTI system with impulse response

$$h(t) = 3\sin(\pi t)/(\pi t).$$

Let $Y(t)$ be the system output. Find $S_Y(\omega)$. Sketch $S_Y(\omega)$

Exercise 8.
Consider a WSS process $X(t)$ with autocorrelation function

$$R_X(\tau) = \text{sinc}(\pi\tau).$$

The process is sent to an LTI system, with input-output relationship

$$2\frac{d^2}{dt^2}Y(t) + 2\frac{d}{dt}Y(t) + 4Y(t) = 3\frac{d^2}{dt^2}X(t) - 3\frac{d}{dt}X(t) + 6X(t).$$

Find the autocorrelation function $R_Y(\tau)$.

Exercise 9.
Let $X(t)$ be a WSS process with correlation function

$$R_X(\tau) = \begin{cases} 1 - |\tau|, & \text{if } -1 \le \tau \le 1 \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

It is known that when $X(t)$ is input to a system with transfer function $H(\omega)$, the system output $Y(t)$ has a correlation function

$$R_Y(\tau) = \frac{\sin \pi\tau}{\pi\tau}. \tag{2}$$

Find the transfer function $H(\omega)$.