Oppgave 1

a) $X \sim b(x; n=20, p_0 = 0.25)$

$P(X \leq 5) = \underline{\underline{0.617}}$

$P(5 \leq X < 10) = P(X \leq 9) - P(X \leq 4) = 0.986 - 0.415 =$

$= \underline{\underline{0.571}}$

b) $L(p; x) = \binom{n}{x} p^x (1-p)^{n-x}$

$l(p; x) = \ln\binom{n}{x} + x \ln p + (n-x)\ln(1-p)$

$\frac{\partial l}{\partial p}(p; x) = \frac{x}{p} + \frac{n-x}{1-p} \cdot (-1) = 0$

$x - \cancel{xp} - pn + \cancel{xp} = 0$

$\Longrightarrow \underline{\underline{\hat{p} = \frac{X}{n}}}$

tabell

$E(\hat{p}) = E\left(\frac{X}{n}\right) = \frac{1}{n}E(X) = \frac{1}{n} np = \underline{\underline{p}}$

$Var(\hat{p}) = Var\left(\frac{X}{n}\right) = \frac{1}{n^2} Var(X) = \frac{1}{n^2} np(1-p) = \underline{\underline{\frac{p(1-p)}{n}}}$

c) $H_0: p = p_0 = 0.25$ mot $H_1: p > p_0 = 0.25$

Gjennomfører hypotesetesten ved bruk av P-verdi.

P-verdi =

P(Observere det vi har observert eller noe mer ekstremt når vi antar H_0 rett)

$$= P\left(\hat{p} \geq \frac{8}{18} \,\middle|\, p = p_0 = 0.25, n = 18\right)$$

$$= P\left(\frac{X}{18} \geq \frac{8}{18} \,\middle|\, p = p_0 = 0.25, n = 18\right)$$

$$= P(X \geq 8 \mid p = p_0 = 0.25, n = 18)$$

$$= 1 - P(X \leq 7) = 1 - 0.943 = \underline{0.057}$$

Forkast H_0 hvis P-verdi $\leq \alpha$

Behold H_0 hvis P-verdi $> \alpha$

Med $\alpha = 0.05$ er P-verdi $> \alpha$ som betyr at vi <u>beholder H_0.</u>

Oppgave 2

X = antall klager

$$p(x;\lambda) = \frac{\lambda^x}{x!}e^{-\lambda} \qquad x = 0,1,2,\ldots$$

a) $P(X>0) = 1-P(X=0) = \underline{1-e^{-\lambda}}$

$$P(X<3 \mid X>0) = \frac{P(X<3 \cap X>0)}{P(X>0)} = \frac{P(X=1 \cup X=2)}{P(X>0)}$$

$$= \frac{\lambda e^{-\lambda} + \frac{\lambda^2}{2}e^{-\lambda}}{1-e^{-\lambda}} = \underline{\underline{\frac{\lambda e^{-\lambda}(1+\frac{\lambda}{2})}{1-e^{-\lambda}}}}$$

$i \in \{A, B, C\}$

$$p(x \mid i) = \frac{\lambda_i^x}{x!}e^{-\lambda_i}$$

$p(i)$

$\lambda_A = 5$
$\lambda_B = 15$
$\lambda_C = 20$
$p(A) = 0.5$
$p(B) = 0.25$
$p(C) = 0.25$

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b)

$$p(x) = \sum_i p(x \mid i)p(i) = \underline{\underline{\frac{\lambda_A^x}{x!}e^{-\lambda_A}\cdot 0.5 + \frac{\lambda_B^x}{x!}e^{-\lambda_B}\cdot 0.25 + \frac{\lambda_C^x}{x!}e^{-\lambda_C}\cdot 0.25}}$$

$$\mu_x = E\{X\} = \sum_x x\,p(x) = \sum_x x \sum_i p(x \mid i)p(i)$$

$$= \sum_i \left[\sum_x x\,p(x \mid i)\right] p(i)$$

$$= \sum_i \lambda_i\, p(i)$$

$$= \underline{\lambda_A \cdot 0.5 + \lambda_B \cdot 0.25 + \lambda_C\, 0.25}$$

$$Var\{X\} = \sum_x (x-\mu_x)^2 p(x)$$

$$\boxed{Var\{X\} = E\{Var\{X|Y\}\} + Var\{E\{X|Y\}\}}$$

$$= \sum_x (x-\mu_x)^2 \sum_i p(x|i)p(i)$$

$$= \sum_i \left[\sum_x (x-\lambda_i)^2 p(x|i) + (\lambda_i-\mu_x)^2\right] p(i)$$

$$= \sum_i \left[\lambda_i + (\lambda_i-\mu_x)^2\right] p(i)$$

$$= \sum_i \lambda_i p(i) + \sum_i (\lambda_i-\mu_x)^2 p(i)$$

$$= \mu_x + \sum_i (\lambda_i-\mu_x)^2 p(i)$$

$$\boxed{\mu_x = \sum_i \lambda_i p(i)}$$

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c)

$$p(i|x) = \frac{p(x|i)p(i)}{p(x)} = \frac{p(x|i)p(i)}{\sum_i p(x|i)p(i)}$$

$$p(A|x) = \frac{\frac{\lambda_A^x}{x!} e^{-\lambda_A} \cdot 0.5}{\sum_i \frac{\lambda_i^x}{x!} e^{-\lambda_i} \cdot p(i)}$$

$p(B|x)$

$p(C|x)$

Oppgave 3

Modell:

$$Y = \alpha_0 - \beta x + E$$

Y - tilfeldig variabel

x - variabel

α_0 - konstant, kjent

β - konstant, ukjent

E - tilfeldig variabel $n(e; 0, \sigma)$

σ - konstant, ukjent

Herav:

$$[Y|x] \sim n([y|x]; \alpha_0 - \beta x, \sigma)$$

Estimatorer:

$$\beta^* = B = \frac{\alpha_0 \sum_i x_i - \sum_i x_i Y_i}{\sum_i x_i^2}$$

$$\sigma^{2*} = S^2 = \frac{1}{n-1} \sum_i (Y_i - \alpha_0 + B x_i)^2$$

a)

$$E\{B\} = \frac{\alpha_0 \sum_i x_i - \sum_i x_i E\{Y_i\}}{\sum_i x_i^2} = \frac{\alpha_0 \sum x_i - \sum_i x_i (\alpha_0 - \beta x_i)}{\sum_i x_i^2}$$

$$\underline{\underline{= \beta}} \quad \text{forventningsrett}$$

$$Var\{B\} = \frac{\sum_i x_i^2 Var\{Y_i\}}{(\sum_i x_i^2)^2} = \frac{\sigma^2 \sum_i x_i^2}{(\sum_i x_i^2)^2} = \underline{\underline{\frac{\sigma^2}{\sum_i x_i^2}}}$$

B er normalfordelt pga lineær-kombinasjon av normalfordelte variable Y_i.

$$B \sim n\left(b; \beta, \frac{\sigma}{\sqrt{\sum_i x_i^2}}\right)$$

b)

$$T_{n-1} = \frac{\dfrac{B-\beta}{\sigma/\sqrt{\Sigma}} \overset{\sim}{\ } Z}{\underbrace{\sqrt{\dfrac{S^2(n-1)}{\sigma^2(n-1)}}}_{\hat{\sim}\ \chi^2_{n-1}/(n-1)}} = \frac{B-\beta}{\sqrt{S^2/\Sigma x_i^2}}$$

$$\text{Prob}\left\{-t_{n-1,\alpha/2} < T_{n-1} < t_{n-1,\alpha/2}\right\} = 1-\alpha \qquad \alpha = 0.05$$

$$\text{Prob}\left\{-t_{n-1,\alpha/2} < \frac{B-\beta}{\sqrt{\dfrac{S^2}{\Sigma x_i^2}}} < t_{n-1,\alpha/2}\right\} = 1-\alpha$$

$$\text{Prob}\Bigg\{\underbrace{B-\sqrt{\frac{S^2}{\Sigma x_i^2}}\cdot t_{n-1,\alpha/2}}_{B_L} < \beta < \underbrace{B+\sqrt{\frac{S^2}{\Sigma x_i^2}}\cdot t_{n-1,\alpha/2}}_{B_U}\Bigg\} = 1-\alpha$$

95% konfidensinterval for β:

$$[B_L, B_U].$$

For å teste

$$H_0: \beta = \beta_0 \quad \text{mot} \quad H_1: \beta \neq \beta_0$$

med signifikans $\alpha = 0.05$

Hvis $\beta_0 \notin [B_L, B_U]$ forkast H_0!