# Final Project – Final Report Building Learning Agents to Play the Game of Light Against Zombies

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## Table of Contents

[Final Project – Final Report Building Learning Agents to Play the Game of Light Against Zombies 1](#_Toc75021721)

[1. Table of Contents 1](#_Toc75021722)

[2. Figure List 2](#_Toc75021723)

[3. Table List 4](#_Toc75021724)

[4. Abstract 4](#_Toc75021725)

[5. Introduction 4](#_Toc75021726)

[6. Literature Review 5](#_Toc75021727)

[*6.1* *Insights from the GVG-AI Competition* 5](#_Toc75021728)

[*6.2* *Monte Carlo Tree Search* 6](#_Toc75021729)

[*6.3* *From AlphaGo to AlphaZero* 7](#_Toc75021730)

[7. Our Model - Zombie Invasion Problem 12](#_Toc75021731)

[*7.1* *Assumptions* 12](#_Toc75021732)

[*7.2* *Stochastic game* 13](#_Toc75021733)

[*7.3* *Agents* 13](#_Toc75021734)

[*7.4* *Game rules* 14](#_Toc75021735)

[8. Building the Framework 15](#_Toc75021736)

[8.1 Framework Implementation 16](#_Toc75021737)

[8.2 Implemented Agents 16](#_Toc75021738)

[9. Framework Performance Test 18](#_Toc75021739)

[9.1 The Algorithm – Double Deep Q Network 19](#_Toc75021740)

[9.2 Epsilon Greedy strategy 19](#_Toc75021741)

[9.3 Zombie Player test on a 3x5 board 20](#_Toc75021742)

[9.4 Light Player test on a 3x5 board 22](#_Toc75021743)

[10. Double Deep Q-Network Evaluation 24](#_Toc75021744)

[10.1 Evaluation Configuration 24](#_Toc75021745)

[10.2 Game Scenarios 25](#_Toc75021746)

[10.3 Double Deep Q-Network as Zombie Player 26](#_Toc75021747)

[10.4 Double Deep Q-Network as Light Player 30](#_Toc75021748)

[11. Learning Based Monte Carlo Tree Search 34](#_Toc75021749)

[11.1 MCTS - Implementation and Config 35](#_Toc75021750)

[11.2 Results and Conclusions 36](#_Toc75021751)

[12. Learning Based AlphaZero Algorithm 36](#_Toc75021752)

[12.1 Our Network 36](#_Toc75021753)

[12.2 Training Process of AlphaZero 38](#_Toc75021754)

[12.3 AlphaZero Evaluation 38](#_Toc75021755)

[13. Competing AlphaZero and DDQN 46](#_Toc75021756)

[13.1 Constructing the Composite Agent 47](#_Toc75021757)

[13.2 Final Evaluation 47](#_Toc75021758)

[13.3 The Competition 48](#_Toc75021759)

[13.4 Summary 60](#_Toc75021760)

[14. References 60](#_Toc75021761)

[15. Appendix A – Elaboration of Related Literature 61](#_Toc75021762)

[15.1 Reinforcement Learning 61](#_Toc75021763)

[*15.2* *Stochastic Games* 62](#_Toc75021764)

[*15.3* *Nash Equilibrium in SGs* 63](#_Toc75021765)

[*15.4* *Learning in SGs* 64](#_Toc75021766)

## Figure List

[Figure 1 - Step of Monte Carlo tree search 7](#_Toc75021767)

[Figure 2 - AlphaGo overview 8](#_Toc75021768)

[Figure 3 - Resnet contribution over traditional CNNs 9](#_Toc75021769)

[Figure 4 - AlphaGo Zero training characteristics 9](#_Toc75021770)

[Figure 5 - is the utility function of our game. For some hit-point value we calculate , which determines the chances of a zombie to not pass the right border 15](#_Toc75021771)

[Figure 6- Frameworks' Main configuration example 17](#_Toc75021772)

[Figure 7 - Abstract methods every agent should implement in order to join the Framework 17](#_Toc75021773)

[Figure 8 - Two simple lines of code to run the Framework and get results 17](#_Toc75021774)

[Figure 9 - Summary - three steps for running the Framework 18](#_Toc75021775)

[Figure 10 – Double Deep Q-Network architecture and flow 19](#_Toc75021776)

[Figure 11 - Epsilon greedy values – convex function of 20](#_Toc75021777)

[Figure 12 – environment set-up for zombie Player performance check with optional actions 20](#_Toc75021778)

[Figure 13 – Zombie Player actions distribution along different ranges of episodes 21](#_Toc75021779)

[Figure 14 – Total zombies survived vs. the episodes (blue) with its moving average (orange) 22](#_Toc75021780)

[Figure 15 - environment set-up for light Player performance check with optional actions 22](#_Toc75021781)

[Figure 16 - Light Player actions distribution along different ranges of episodes 23](#_Toc75021782)

[Figure 17 - Total zombies survived vs. the episodes (blue) with its moving average (orange) 24](#_Toc75021783)

[Figure 18 - Example of reward per episode graph, DDQN plays Zombie and Single Action plays Light 26](#_Toc75021784)

[Figure 19 - Scenario Evaluation by Average Test Reward 26](#_Toc75021785)

[Figure 20 - Heat-Map of the Average Test Rewards of all the scenarios that DDQN Agent plays Zombie and Single Action Agent plays Light 27](#_Toc75021786)

[Figure 21 - A summary of all the scenarios that DDQN plays Zombie by Heat-Maps of the Average Test Reward 28](#_Toc75021787)

[Figure 22 - Comparing the results of the DDQN agent as the Zombie Player, with the best parameters over the different four simple competitors 30](#_Toc75021788)

[Figure 23 - Heat-Map of the Average Test Rewards of all the scenarios that DDQN Agent plays Light and Single Action Agent plays Zombie 31](#_Toc75021789)

[Figure 24 - A summary of all the scenarios that DDQN plays Light by Heat-Maps of the Average Test Reward 32](#_Toc75021790)

[Figure 25 - Comparing the results of the DDQN agent as the Light Player, with the best parameters over the different four simple competitors 34](#_Toc75021791)

[Figure 26 - NN Architecture. Red and blue arrows indicate convolutional and fully Connected layers respectevly. 37](#_Toc75021792)

[Figure 27 - A summary of all the scenarios that AlphaZero plays Zombie. Heat-Maps of the Average Test Reward 40](#_Toc75021793)

[Figure 28 - Comparing the results of the AlphaZero agent as the Zombie Player, with the best parameters over the different four simple competitors 42](#_Toc75021794)

[Figure 29 - Comparing the results of the AlphaZero agent as the Light Player, with the best parameters over the different four simple competitors 45](#_Toc75021795)

[Figure 30 - Action and Rewards distribution graph 48](#_Toc75021796)

[Figure 31 - Rewards over Episodes, board 10x10 49](#_Toc75021797)

[Figure 32 - Action and Reward Distributions, board 10x10 50](#_Toc75021798)

[Figure 33 - Rewards over Episodes, board 20x20 51](#_Toc75021799)

[Figure 34 - Action and Reward Distributions, board 20x20 52](#_Toc75021800)

[Figure 35 - Rewards over Episodes, board 30x30 53](#_Toc75021801)

[Figure 36 - Action and Reward Distributions, board 30x30 54](#_Toc75021802)

[Figure 37 - Rewards over Episodes, board 10x10 55](#_Toc75021803)

[Figure 38 - Action and Reward Distributions, board 10x10 56](#_Toc75021804)

[Figure 39 - Rewards over Episodes, board 20x20 57](#_Toc75021805)

[Figure 40 - Action and Reward Distributions, board 20x20 58](#_Toc75021806)

[Figure 41 - Rewards over Episodes, board 30x30 59](#_Toc75021807)

[Figure 42 - Action and Reward Distributions, board 30x30 60](#_Toc75021808)

[Figure 43 – Game tree when there are two actions by player 63](#_Toc75021809)

## Table List

[Table 1 – Learning parameters while evaluating the zombie Player 21](#_Toc75021810)

[Table 2 – learning parameters while evaluating the light Player 23](#_Toc75021811)

[Table 3 – DDQN Evaluation: Game Scenarios 25](#_Toc75021812)

[Table 4 - Best Configurations of all Scenarios in which the DDQN plays Zombie 29](#_Toc75021813)

[Table 5 - Best Configurations of all Scenarios in which the DDQN plays Light 33](#_Toc75021814)

[Table 6 – DDQN Evaluation: Game Scenarios 39](#_Toc75021815)

[Table 7 - Best Configurations of all Scenarios in which AlphaZero plays Zombie 41](#_Toc75021816)

[Table 8 - Best Configurations of all Scenarios in which AlphaZero plays Light 44](#_Toc75021817)

[Table 9 - Summary of competing DDQN Agent vs. Single, Double and Gaussian Agents 46](#_Toc75021818)

[Table 10 – DDQN trained with Composite agents, evaluation vs. all Simple Agents 47](#_Toc75021819)

[Table 11 - AlphaZero trained with Composite Agent, evaluation vs. all Simple Agents 47](#_Toc75021820)

## Abstract

## Introduction

One of the primary goals of the field of artificial intelligence (AI) is to produce fully autonomous agents that interact with their environments to learn optimal behaviors, improving over time through trial and error. Crafting AI systems that are responsive and can effectively learn has been a long-standing challenge, ranging from robots, which can sense and react to the world around them, to purely software-based agents, which can interact with natural language and multimedia.

Toward this project, we were presented with a problem that concerns one of the defense industries, in which we were asked to build a family of stochastic games that characterized with extreme state-space complexity. Each one of the stochastic games consists of two players with non-symmetric action space which are exposed to different types of information.

Our goal is to implement carefully selected algorithms and examine whose performance is superior in each game. We aim to achieve successful results with three types of agents: one that learns using a traditional Reinforcement Learning algorithm, another one from the tree-search area and the that last that uses state-of-the-art algorithm from the field of Reinforcement Learning. The combination and analysis of agents from different domains of research will provide us a broad understanding of the field that will produce the most successful agents for our family of games.

In the following document we present the final project of the Intelligent Systems program at Afeka College of Engineering.

During the past months, we've been busy building the framework of the two-player-game described by the partners in the aerospace industry. A Framework for developing and comparing reinforcement learning algorithms. It is able to evaluate the learning process of the agents, compare between learning agents and more! we will discuss the subject in detail in chapter ‎8 (Building the Framework)‎8.

Furthermore, we implemented two algorithms from the Reinforcement Learning domain called: Double-Deep-Q-Network [17] and AlphaZero [14]. Our framework provides each player the ability to play by the algorithm, DDQN, AlphaZero or any other! Our learning agent based DDQN is used for performance test of the framework in chapter ‎9 (Framework Performance Test). In chapter ‎10 (Double Deep Q-Network Evaluation) and chapter ‎12 (Learning Based AlphaZero Algorithm) we provide in-depth analysis of the learning algorithms that we implemented, in manners of achieving optimal policy in various scenarios.

At the same time of building the algorithms we mentioned, we implemented another algorithm from the field of search algorithms, called Monte-Carlo-Tree-Search (see Learning Based Monte Carlo Tree Search) in favor of comparison with algorithms from the Reinforcement Learning field. The MCTS algorithm did not lead to any success due to RAM limitations and the fact that the state-space in the game is too large to withstand the search tree it builds.

After implementing and testing the performance of all the algorithms individually, we trained them in a number of different board modes (10, 20 and 30 square board) to allow for competition and play between them.

## Literature Review

The area of learning agents that master a particular game or on the other side, agents that seek the highest score over a set of games, grew to huge scales in the past few years. Since we are not facing with a studied problem nor a known game, we will divide our review into three sections:

1. Insights from the GVG-AI Competition
2. Monte Carlo Tree Search
3. From AlphaGo to AlphaZero

All along with elaboration of the potential contribution of each topic to our research due to the successes of similar problems and previous research of the domain.

* Note that we have extra elaboration on basic RL concepts and ideas we won't apply in the scope of the project, all in: Appendix A –

### *Insights from the GVG-AI Competition*

The General Video Game AI competition (GVGAI) was created in order to test these general agents on a multitude of real-time games (both stochastic and deterministic) under the same conditions and constraints. It has received significant international attention in the seven years it has been running and has allowed for many interesting algorithms to be tested on the large number of problems.

The GVG-AI Competition explores the problem of creating controllers for general video game playing, in such platform, researches have an opportunity to test their agents via participating in the competitions.

The past few years have led to some great RL algorithms like 'MaastCTS' and 'OLMCTS' [11], both based the MCTS [12] algorithm. All of these have proven themselves in the competitions, therefor, might be useful and beneficial to our research.

The platform of GVG-AI letting the competitors test their algorithms on some environments built specially for them (by DeepMind) to challenge and push them to their limits. By going over all the proposed environments, I found some games with a lot of resemblance to us, like in our research, there were two players in a zero-sum stochastic game alongside the fact that the action space of the agents is much like ours, for the illustration in this paper I want to elaborate on two games that might be of our interest:

**Ghostbusters**, a version of the known Atari game with improvements to satisfy the competition demands. The game contains two players: one player is the ghost and the other is the hunter.

The ghost can pass through walls and wraps around the level and its aim is to either avoid dying or catch the hunter. While the hunter shoots missiles and moves faster than the ghost and its goal is to avoid the ghost that can hurt him and shoot the ghost.

The algorithms that achieved the best results in the competition are called: MCTS and [MaastCTS2](https://github.com/DennisSoemers/MaastCTS2/tree/master/Two-Player/src/MaastCTS2) which are both a variant of an agent that learns by building Monte Carlo Tree Search.

The game Ghostbusters briefly described above, reminds our game in many manners, the similarity of the actions the agents take, the ghost that is capable of moving around the grid while targeting to catch the hunter, much like our Light Player (described later). And on the other hand, the hunter that shoot missiles in many directions to try and catch the ghost, just like our Zombie Player (described later).

Another game is called **Upgrade-X**, its environment contains two players which both are located in a two-dimensional square which they can't leave. Each player has some laser cannons, which they can move around. Getting hit by the laser cannon of the opponent hurts the player and make him lose some health points. The winner is the player that survives tor the one with the most points at the end of the game.

The algorithms that achieved the best results in the competition are called: OLMCTS, SARSA-UCT and [MaastCTS2](https://github.com/DennisSoemers/MaastCTS2/tree/master/Two-Player/src/MaastCTS2). All of which are also a variant of an agent that learns by building Monte Carlo Tree Search.

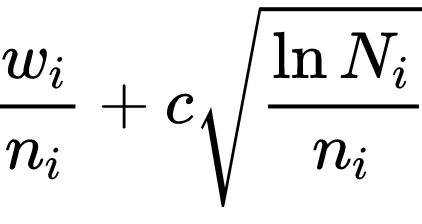
Like in the previous game, the Upgrade-X games have a bunch of similarities with our game like, the two agents moving around with laser cannons are responsible of the canon direction and their goal is to maximize their health/strength points until the end of the game, a process that reminds a lot our Zombie Agent (described later) that is responsible to his zombie's direction and velocities alongside the goal of maximizing their strength throughout the game.

### *Monte Carlo Tree Search*

The GVG-AI competition brought into the RL field brilliant algorithms and ideas. Most of whom have one thing in common – Monte Carlo Tree Search Algorithm.

The focus of MCTS is on the analysis of the most promising moves, expanding the [search tree](https://en.wikipedia.org/wiki/Search_tree) based on [random sampling](https://en.wikipedia.org/wiki/Monte_Carlo_method) of the search space. The application of Monte Carlo tree search in games is based on many playouts, also called roll-outs. In each playout, the game is played out to the very end by selecting moves at random. The final game result of each playout is then used to weight the nodes in the game tree so that better nodes are more likely to be chosen in future playouts.

Thus, each round of Monte Carlo tree search consists of four steps:

* Selection: Start from root R and select successive child nodes until a leaf node L is reached. The root is the current game state and a leaf is any node that has a potential child from which no simulation (playout) has yet been initiated.
  + We traverse according to the upper confidence bound of  Where:
    - is the total reward aggregated in the current node
    - is the number of visits through the current node
    - is the number of visits through parent node
    - is the exploration factor
* *Expansion*: Unless  ends the game decisively (e.g., win/loss/draw) for either player, create one (or more) child nodes and choose node *C* from one of them. Child nodes are any valid moves from the game position defined by *L*.
* *Simulation*: Complete one random playout from node *C*. This step is sometimes also called playout or rollout. A playout may be as simple as choosing uniform random moves until the game is decided (for example in chess, the game is won, lost, or drawn).
* *Backpropagation*: Use the result of the playout to update information in the nodes on the path from *C* to *R*.

All the above can be summed up to Figure 1:

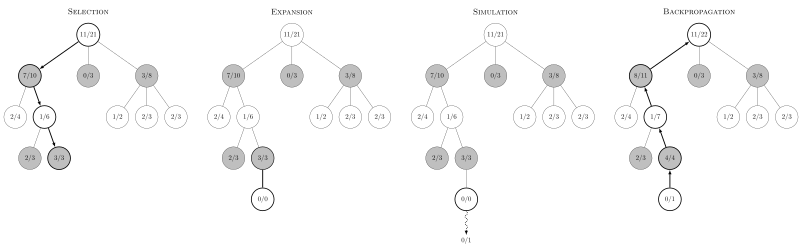


Figure 1 - Step of Monte Carlo tree search

### *From AlphaGo to AlphaZero*

Having briefly explained Monte Carlo Tree Search ideas, we will expand the conversation to a family of outbreaking algorithms that, for today, are considered State of the Art: Known as: Alpha Go, AlphaGo Zero, and Alpha Zero. These are algorithms developed by DeepMind, whom are Google's top artificial intelligence research group.

#### AlphaGo

AlphaGo [16] is the first paper in the series, showing that Deep Neural Networks could play the game of Go by predicting a **policy** (mapping from state to action) and **value estimate** (probability of winning from a given state). These policy and value networks are used to enhance tree-based lookahead search by selecting which actions to take from given states and which states are worth exploring further.

AlphaGo uses 4 Deep Convolutional Neural Networks, 3 policy networks and a value network. 2 of the policy networks are trained with supervised learning on expert moves.

Supervised learning describes loss functions consisting of some kind of . In this case, the is the action the policy network predicted from a given state, and the y is the action the expert human player had taken in that state.

The rollout policy is a smaller neural network that takes in a smaller input state representation as well. As a consequence of this, the rollout policy has a significantly lower modeling accuracy of expert moves than the higher capacity network. However, the rollout policy network’s inference time (time to make a prediction of action given state) is 2 microseconds compared to 3 milliseconds with the larger network, making it useful for Monte Carlo Tree Search simulations.

The SL policy network is used to initialize the 3rd policy network which is trained with self-play and policy gradients. Policy gradients describe the idea of optimizing the policy directly with respect to the resulting rewards, compared to other RL algorithms that learn a value function and then make the policy greedy with respect to the value function. The policy gradient trained policy network plays against previous iterations of its own parameters, optimizing its parameters to select the moves that result in wins. The **self-play dataset** is then used to train a value network to predict the winner of a game from a given state.

The final workhorse of AlphaGo is the combination of policy and value networks in MCTS, depicted below:

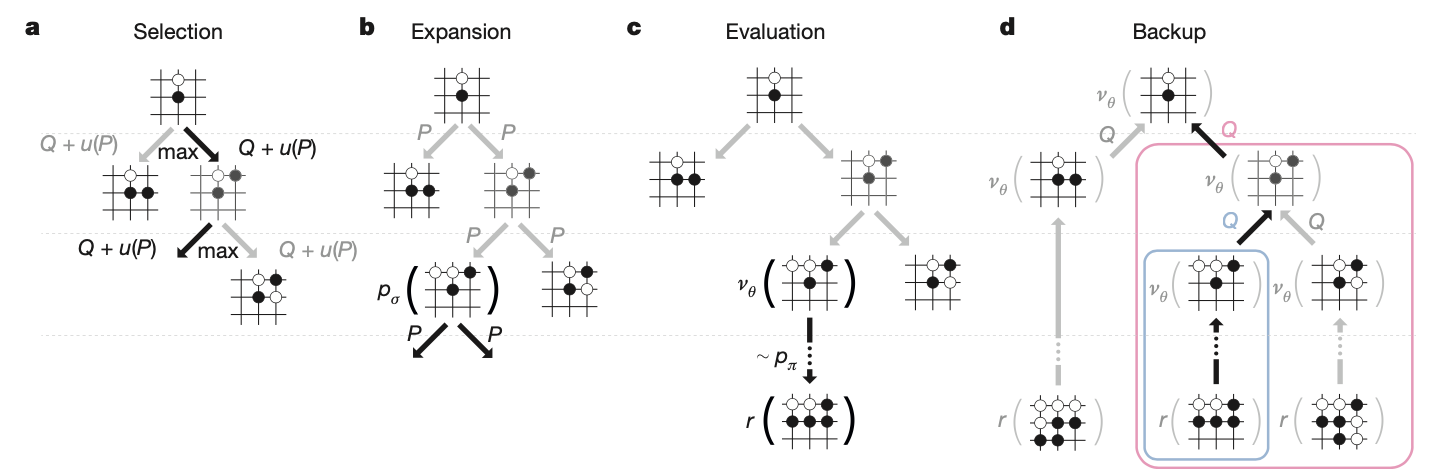


Figure 2 - AlphaGo overview

#### AlphaGo Zero

AlphaGo Zero [15] significantly improves the AlphaGo algorithm by making it more general and starting from **“Zero” human knowledge**. AlphaGo Zero avoids the supervised learning of expert moves initialization and combines the value and policy network into a single neural network. This neural network is scaled up as well to utilize a ResNet compared to a simpler convolutional network in AlphaGo. The contribution of the ResNet performing both value and policy mappings is evident in the diagram below comparing the dual task ResNet to separate task CNNs:



Figure 3 - Resnet contribution over traditional CNNs

One of the most interesting characteristics of AlphaGo Zero is the way it trains its policy network using the action distribution found by MCTS, depicted below:

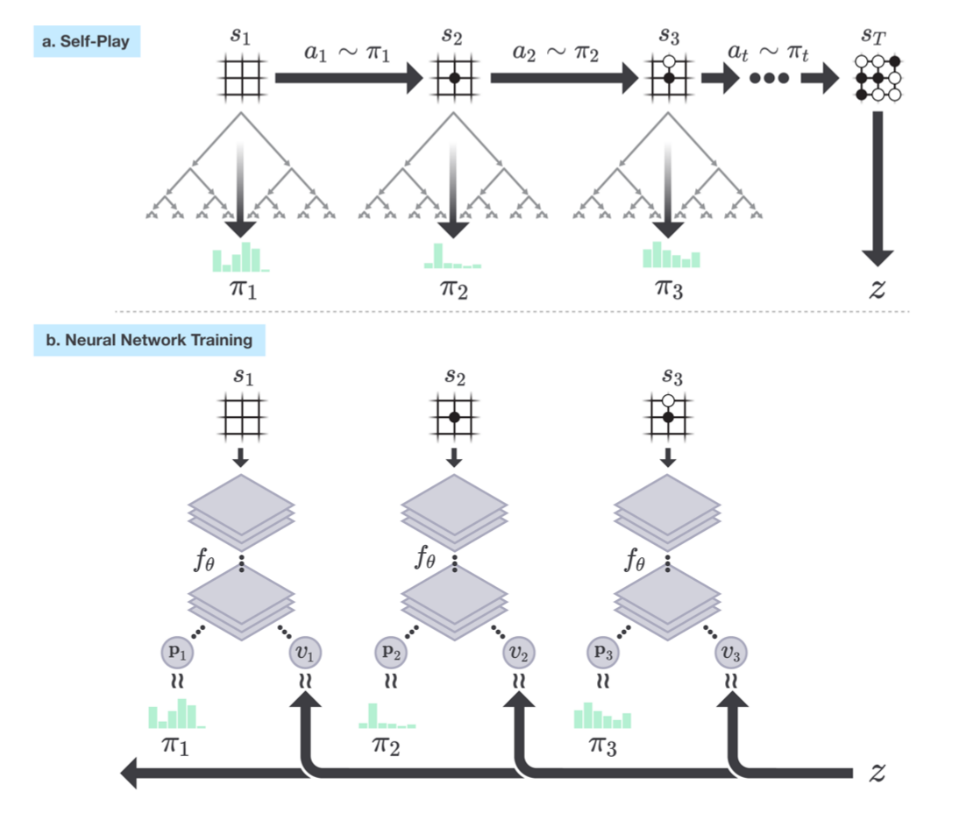


Figure 4 - AlphaGo Zero training characteristics

The MCTS trains the policy network by using it as supervision to update the policy network. This is a clever idea since MCTS produces a better action distribution through lookahead search than the policy network’s instant mapping from state to action.

#### AlphaZero

AlphaZero [14] is the first step towards generalizing the AlphaGo family outside of Go, looking at changes needed to play Chess and Shogi as well. This requires formulating **input state and output action** representations for the residual neural network.

AlphaZero also makes some more subtle changes to the algorithm such as the way the self-play champion is crowned and the eliminations of data augmentation from Go board games such as reflections and rotations.

##### The Neural Network

Unsurprisingly, there's a neural network at the core of things. The neural network is parameterized by θ and takes as input the state of the board. It has two outputs: a continuous value of the board state from the perspective of the current player, and a policy that is a probability vector over all possible actions. When training the network, at the end of each game of self-play, the neural network is provided training examples of the form . is an estimate of the policy from state (we'll get to how πt is arrived at in the next section), is the final outcome of the game from the perspective of the player at . The neural network is then trained to minimize the following loss function (excluding regularization terms):

The underlying idea is that over time, the network will learn what states eventually lead to wins (or losses). In addition, learning the policy would give a good estimate of what the best action is from a given state. The neural network architecture in general would depend on the game. Most board games such as Go can use a multi-layer CNN architecture. In the paper by DeepMind [14], they use 20 residual blocks, each with 2 convolutional layers.

##### Monte Carlo Tree Search for Policy Improvement

Given a state , the neural network provides an estimate of the policy . During the training phase, we wish to improve these estimates. This is accomplished using a Monte Carlo Tree Search (MCTS). In the search tree, each node represents a board configuration. A directed edge exists between two nodes  if a valid action can cause state transition from state  to . Starting with an empty search tree, we expand the search tree one node (state) at a time. When a new node is encountered, instead of performing a rollout, the value of the new node is obtained from the neural network itself. This value is propagated up the search path. Let's sketch this out in more detail.

For the tree search, we maintain the following:

* : the expected reward for taking action aa from state , i.e., the Q values
* : the number of times we took action aa from state  across simulations
* : the initial estimate of taking an action from the state ss according to the policy returned by the current neural network.

From these, we can calculate , the upper confidence bound on the Q-values as:

Here  is a hyperparameter that controls the degree of exploration. To use MCTS to improve the initial policy returned by the current neural network, we initialize our empty search tree with ss as the root. A single simulation proceeds as follows. We compute the action  that maximizes the upper confidence bound . If the next state  (obtained by playing action  on state ) exists in our tree, we recursively call the search on s′. If it does not exist, we add the new state to our tree and initialize  and the value  from the neural network, and initialize  and  to 0 for all . Instead of performing a rollout, we then propagate  up along the path seen in the current simulation and update all  values. On the other hand, if we encounter a terminal state, we propagate the actual reward (+1 if player wins, else -1).

After a few simulations, the  values at the root provide a better approximation for the policy. The improved stochastic policy  is simply the normalized counts . During self-play, we perform MCTS and pick a move by sampling a move from the improved policy .

##### Policy Iteration Through Self-Play

We now have all elements required to train our unsupervised game playing agent! Learning through self-play is essentially a policy iteration algorithm - we play games and compute Q-values using our current policy (the neural network in this case), and then update our policy using the computed statistics.

Here is the complete training algorithm. We initialize our neural network with random weights, thus starting with a random policy and value network. In each iteration of our algorithm, we play a number of games of self-play. In each turn of a game, we perform a fixed number of MCTS simulations starting from the current state . We pick a move by sampling from the improved policy . This gives us a training example . The reward  is filled with value of: +1 if a zombie has passed the board alive, else -1. The search tree is preserved during a game.

At the end of the iteration, the neural network is trained with the obtained training examples. The old and the new networks are pit against each other. If the new network wins more than a set threshold fraction of games (55% in the DeepMind paper), the network is updated to the new network. Otherwise, we conduct another iteration to augment the training examples.

##### Contribution of AlphaZero over AlphaGo-Zero

Our implementation is influenced from the last versions of the algorithm (AlphaZero, AlphaGo Zero). It's important to understand why AlphaZero is more suitable for our problem. Let's dive into the two approaches.

The AlphaZero algorithm described in [14] differs from the original AlphaGo Zero [15] algorithm in several aspects. AlphaGo Zero estimates and optimizes the probability of winning, assuming binary win/loss outcomes. AlphaZero instead estimates and optimizes the expected outcome, **taking account of draws or potentially other outcomes**. The rules of Go are invariant to rotation and reflection. This fact was exploited in AlphaGo and AlphaGo Zero in two ways. First, training data was augmented by generating 8 symmetries for each position. Second, during MCTS, board positions were transformed using a randomly selected rotation or reflection before being evaluated by the neural network, so that the Monte Carlo evaluation is averaged over different biases. **The rules of chess and shogi are asymmetric, and in general symmetries cannot be assumed**. AlphaZero does not augment the training data and does not transform the board position during MCTS. In AlphaGo Zero, self-play games were generated by the best player from all previous iterations. After each iteration of training, the performance of the new player was measured against the best player; if it won by a margin of 55% then it replaced the best player and self-play games were subsequently generated by this new player. **In contrast, AlphaZero simply maintains a single neural network that is updated continually**, rather than waiting for an iteration to complete.

#### MuZero

MuZero presents a very powerful generalization to the algorithm of AlphaZero that allows it to learn without a perfect simulator. Chess, Shogi, and Go are all examples of games that come with a perfect simulator, if you move your pawn forward 2 positions, you know exactly what the resulting state of the board will be.

Since our case consider a perfect simulator, we do not require an algorithm such as MuZero.

## Our Model - Zombie Invasion Problem

Our learning model is based on a two-dimensional game grid on which the zombies live. In this chapter we'll expand beyond the idea and intuition, we will formally review the rules of the agents, and define the principles of stochastic game in our particular case of two-player zero sum game.

Imagine a board of zombies approaching from some locations in the left side of the board towards the right side of the board. Above all that, there is a light that can be positioned anywhere on the board.  
The two agents will be called 'Zombie Player' and 'Light Player' the Zombie Player is responsible of positioning the zombies in the left side and determine their initial angle and speed that will stay constant for each zombie. On the other hand, the Light Player decides where to project his light in every turn. Each zombie that leaves the left side of the board and goes under the light of the Light Player is damaged and his strength meter is lowered by some value.  
In general, the goal of the Zombie Player/Light Player is to maximize/minimize the strength of the zombies that are reaching the right side of the board.

### *Assumptions*

we'll start with some basic assumptions relating the environment and the agent's movement.

Throughout the game, the time and space will be considered discrete while the system operates in discrete time over a horizon T. The system area is represented by N-by-M grid with integer coordinates

Around the board will revolve two types of agents, light and zombies. A zombie and light marking might take coordinates on the integer grid (cells), while at each time moment a zombie can move one cell in right direction.

Furthermore, the light-mark is represented by a square area A-by-A.

Figure 2: grid set-up for example with light-mark of 3x3 and 8 zombies

### *Stochastic game*

As stated above, we deal with two player stochastic game, we will now define the problem we are facing similar to what is stated in the section ‎15.2.

Our stochastic game is defined by the tuple where:

* + 1. is the number of players
    2. is the transition function
    3. is the action set for the player
    4. is the discount factor (for now )
    5. is the reward function for player i

We deal with stochastic two players zero-sum game, i.e.

with limited information on one side (asymmetric information).

### *Agents*

#### Zombie Player

The first player is the Zombie Player, its objective is to maximize the average lifetime of zombies. One way of defining the above is the sum lifetime of all zombies and average over all game rounds.  
In each round, the Zombie Player must decide on a coordinate Y, where the next zombie should start. Action is an integer number from 0 to N (N is the size of the board). Therefore, the actions it can take are the set: .

The Zombie Player bases its decisions on the available information which is the matrix of the zombies' locations N-by-M. Each cell of the matrix is 0 (no zombie) or 1 (occupied by zombie). Denote the collection of variables (i.e. observations, actions) available to player 1 at time t by

, where

Denote a subset of all observations until time t and actions until time t-1 by . Therefore contains a set of t N-by-M matrices and a history of choices from the set .

#### Light Player

Another player in the game is the Light Player. Its objective is to minimize the average lifetime of zombies, that is defined similarly to the definition of the Zombie Player: the sum lifetime of all zombies and average over all game rounds.  
In each round of the game, the Light Player has to choose where to put the center of the light (x, y coordinates):

Thus, the space of action in its possession is:

Again, similarly to the Zombie Player, the Light Player must choose an action based on some available information it has like the N-by-M matrix of zombies and zombie's history as defined above (see Zombie Player).  
In addition, it has information on the strength of the zombies at any given moment and the history of light locations. To sum up, all the information available to him is:

* A tensor (two 2-d matrix) of:
  + A zombie location matrix, such that
  + A zombie strength matrix, with non-empty cells at zombie location, such that

* The mark (light) at time t, , i.e., the player’s action

Therefore, the available information at time t is

Denote a subset of all observations until time t and actions until time t-1 by

### *Game rules*

The game has discrete clock, in each clock tick:

* The Zombie Player decides where a new zombie will appear. The new zombie's hit points is equal to one.  
  All previous existing zombies are moved right 1 cell, i.e. if for   
  In general,
* Zombies that go over the right boundary disappear
* Each zombie that inside marked region (light) got additional hit of the amount c. Meaning the hit points are increased by c (default value c=1)
* Each remaining zombie heal itself by multiplying hit point by (1-epsilon) factor. Thus:
* Once all hit points are calculated and falling zombies are removed from the board, there is a “kill process” that might remove some zombies from the board
* For each zombie a utility function U (see Figure 5) is calculated based on the hit point. U produce values from [0,1] such that zombie with no hit point get 0 and zombies with large value of hit points get utility close to 1

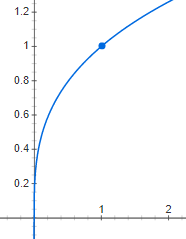


Figure 5 - is the utility function of our game.  
For some hit-point value we calculate , which determines the chances of a zombie to not pass the right border

* If the outcome is positive, zombie is removed from the board
* The reward (for Zombie Player) for the round is computed and its equal to the number of zombies that are still in play
* The round ends by plants selecting a new place for the light
* Note: The healing process and utility function U we mentioned above are responsible for meeting the requirement that each zombie should be hit multiple times.

## Building the Framework

Building a simulation for Reinforcement learning purposes is mostly a manner of creating an environment and allow a variety of agents to access it take action in it and gain some rewards from it.

In this section we will elaborate the characteristics of the environment, the main components inside it and provide details about the type of agents and algorithms we implemented.

As stated in the [Introduction](#_Introduction), the project first came up as a requirement from one of the Israeli Air Industries, to build and evaluate learning agents on top of playing a game with certain rules. The basic requirement from the environment was to examine the success of learning agents versus simpler agents (who play by a fixed, uniform strategy, etc.). In addition, as detailed in [Chapter 7](#_Our_Model_-), the game is between two players, the Zombie Player and the Light Player, with an opposite reward based on the number of zombies survived at the final line of the board. To satisfy all these requirements, in this project we built the game as defined in [Chapter 7](#_Our_Model_-), while we create an independency between the Player type (Zombie or Light) and the algorithm it is playing by.

### Framework Implementation

Building an environment that is able to communicate with an interface of generic type of any agent, turned to be an integrated and significant part of the project:

* More than 40 classes, abstract classes and config files.
* Four simple agents (based on known distribution).
* Three algorithmic agents from two different domains (Reinforcement Learning and Tree Search).

Furthermore, all the results we are going to discuss from now on (chapters ‎9, ‎10 and forth) are automatically generated by the framework after each scenario and batch of scenarios.

### Implemented Agents

As mentioned above, we implemented four simple agents and three algorithmic agents. From a given set of possible actions.

The Simple Agents are:

* Single Action - picks the first action
* Double Action - picks the first and middle actions
* Uniform - picks a uniformly action
* Gaussian - picks an action from the normal distribution with:
  + Mean – middle action
  + Standard deviation –

The algorithmic agents are:

* Double-Deep-Q-Network Agent. More details provided at: "The Algorithm – Double Deep Q Network"
* Monte-Carlo-Tree-Search Agent. More details provided at: "MCTS - Implementation and Config"
* Alpha-Zero Agent. More details provided at: "Learning Based AlphaZero Algorithm"

To be able to run the project, we first must define our desired configurations:

* Interactive mode – Boolean, whether or not we want to visualize the environment
* Display width/height of the visualization
* Number of training and validation episodes
* Number of zombies per episode
* Light size
* Board width/height
* Maximum hit points
* Heal ratio

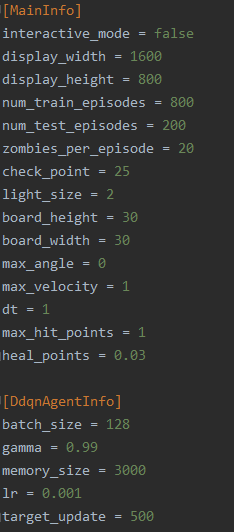


Figure 6- Frameworks' Main configuration example

Next, we have to define the minds behind the competitors:

* For example, we want to let the Double Deep Q-Network agent play as the light player against a zombie player that acts according the uniform distribution
* Each agent must implement some basic methods in order to participate the game:

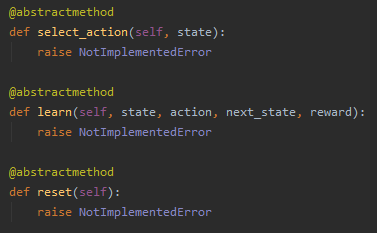


Figure 7 - Abstract methods every agent should implement in order to join the Framework

Finally, we are able to run the framework with the agents we have built:

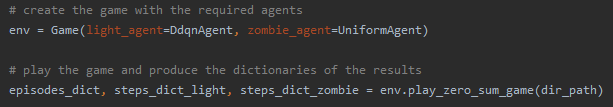


Figure 8 - Two simple lines of code to run the Framework and get results

To sum up:



Figure 9 - Summary - three steps for running the Framework

## Framework Performance Test

Usually in RL projects, we will use some known and tested environment, since that’s not the case, we have to test the performance of the environment with some simple scenarios in order to prove sanity and stability.

Following are the steps of the tests:

* First, we will test the Zombie Player performance with a random-uniform Light Player.
* Therefore, we will test the light Player performance with a random-uniform zombie Player.
* Then, we will test the performance of both the agents trying to learn while playing against each other.

Now, before we start the tests like mentioned, lets introduce the algorithm we are going to use for learning.

### The Algorithm – Double Deep Q Network

For testing the performance we'll use a model known as DDQN, which stands for Double Deep Q-Network.

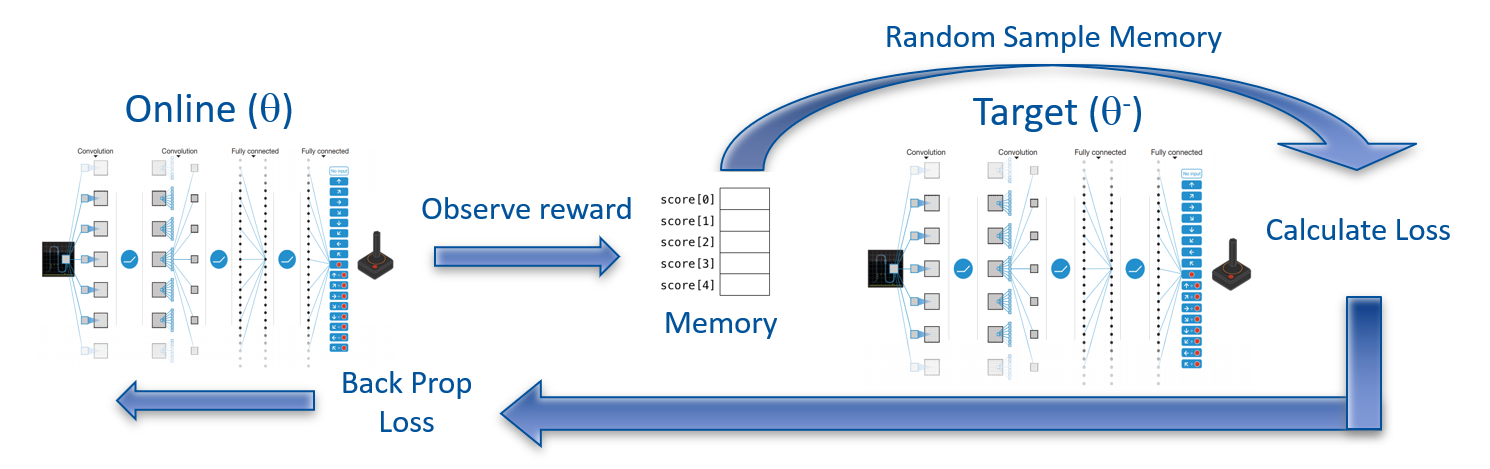


Figure 10 – Double Deep Q-Network architecture and flow

The learning algorithm we used in this project called: "Double Deep Q Network".  
In **Double Deep Q Network**, the agent uses two neural networks to learn and predict what action to take at every step. One network, referred to as the **online network**, is used to predict what to do when the agent encounters a new state. It takes in the state as input and outputs Q values for the possible actions that could be taken.  
The other network, referred to as the **target network**, is used to evaluate what is the best action to take for the next state (the action with the highest Q value).

For the evaluation process we use something called **replay memory**, which holds the last history up to sometime in the past. And eventually, for **loss calculation** we sample a **random batch** (with some size smaller than the memory size) from the replay memory and updating by **back propagation** the online network. After some number of rounds called **replace target frequency**, we **update the target** net weights according to the online net. We can look at Figure 10 that sums up the whole idea.

### Epsilon Greedy strategy

**Epsilon greedy policy** is a way of selecting random actions with uniform distribution from a set of available actions. Using this policy either we can select random action with epsilon probability and we can select an action with 1-epsilon probability that gives maximum reward in given state.

During the learning process we will use the epsilon greedy strategy with non-linear decrease in epsilon of:

While the 'start' and 'end' parameters stand for the starting value and ending value of the epsilon function. The 'step' parameter represents the current step of an episode and is multiplied by the 'decay' parameter that is equal to , for achieving the start-end values of the epsilon function.

Which with 100,000 steps looks like:

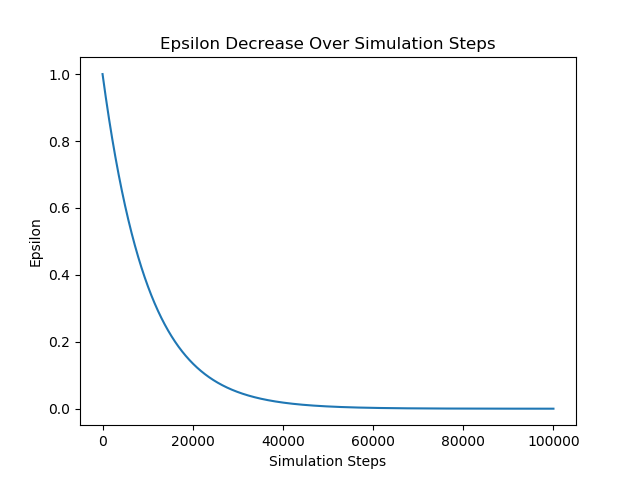
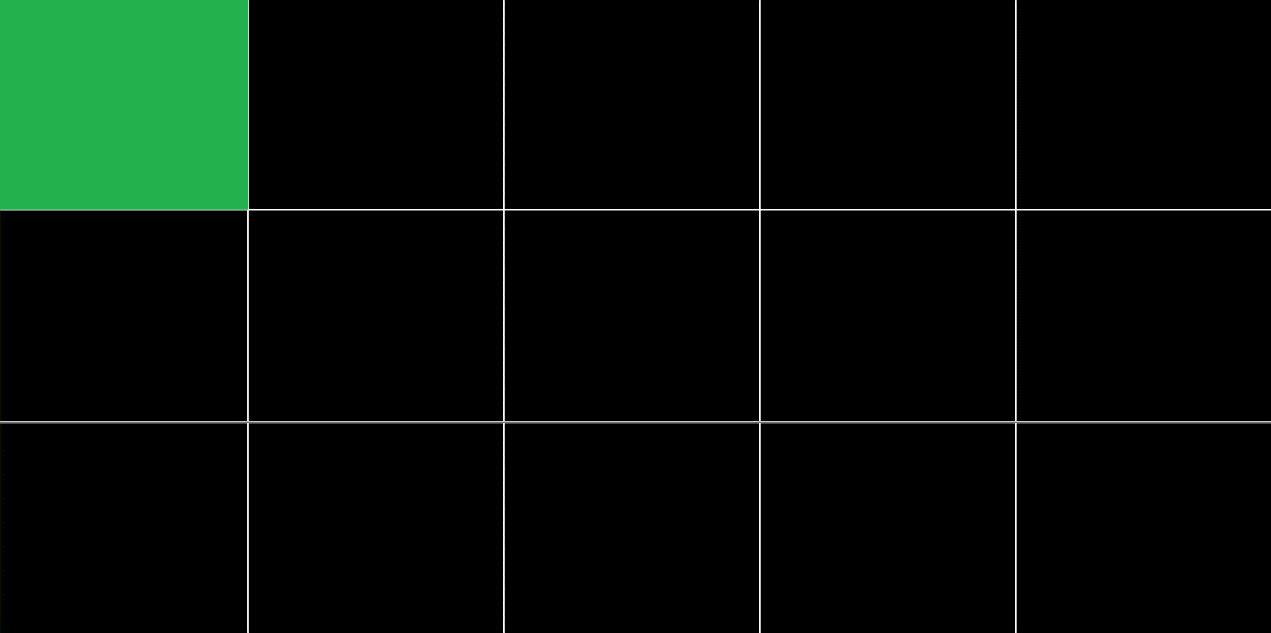


Figure 11 - Epsilon greedy values – convex function of

### Zombie Player test on a 3x5 board

As of the first test of learning, consider the Zombie Player that learns alone while the Light Player is forced to take some predetermined action.

At the beginning, we implemented the DDQN algorithm for the zombie agent with grid of 3x5 that looks like:



0

1

2

Figure 12 – environment set-up for zombie Player performance check with optional actions

As we can see in Figure 12 – environment set-up for zombie Player performance check, the light Player is forced to take the top left cell as action in every step as stated above.

Consider the following parameters for the learning process:

|  |  |
| --- | --- |
| Light Player action | 0 |
| Target update | 10 |
| Num episodes | 100 |
| Steps per episode | 100 |
| Batch size | 264 |
| Gamma (discount factor) | 0.999 |
| Epsilon-greedy start | 1 |
| Epsilon-greedy end | 0.05 |
| Epsilon-greedy decay | 0.000222 |
| Replay memory size | 1000 |
| Learning rate | 0.001 |

Table 1 – Learning parameters while evaluating the zombie Player

With a deep NN of three layers, all fully connected (called 'Linear' in pytorch formulation): Linear (15,128), Linear (128,128), Linear (128,3). As we know, the zombie Player has three possible actions to play – the meaning of the '3' in the last layer.

We achieved a convergence in the number of times the zombie Player chose to send a zombie from the top row (the worst decision it could make):

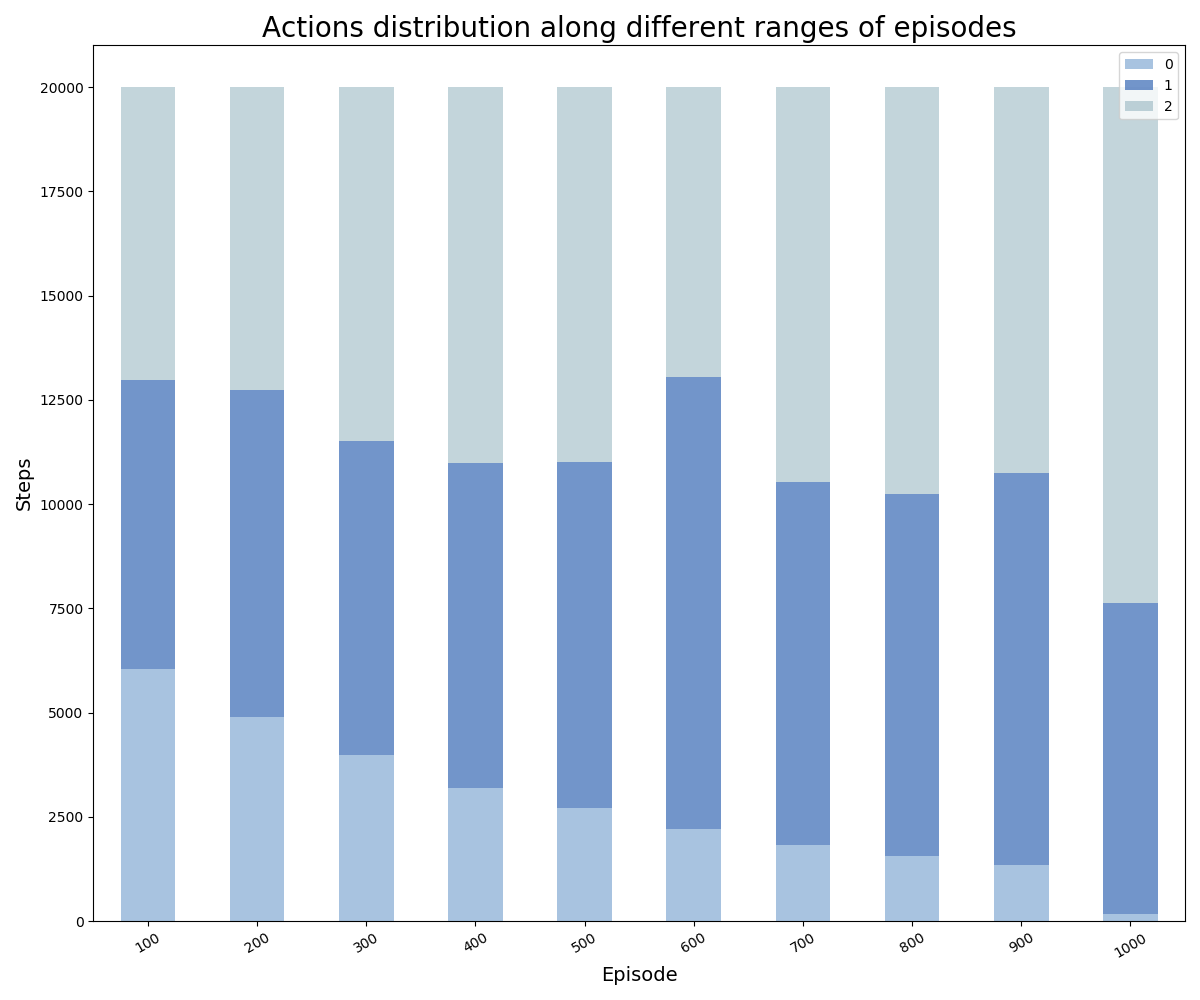


Figure 13 – Zombie Player actions distribution along different ranges of episodes

In Figure 13 – Zombie Player actions distribution along different ranges of episodes we can notice the fading of the lower blue which means that the zombie Player decides to choose action one or two outright as the episodes go on.

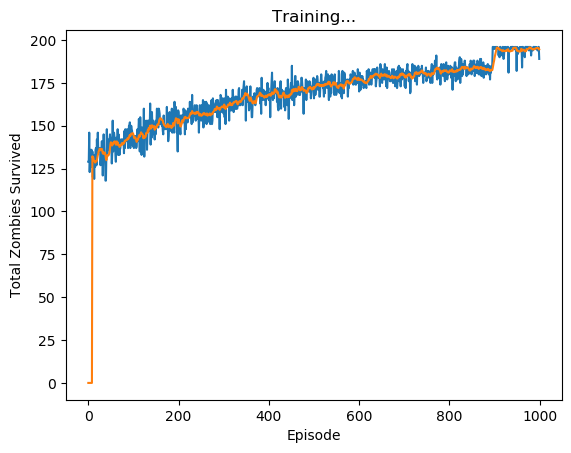
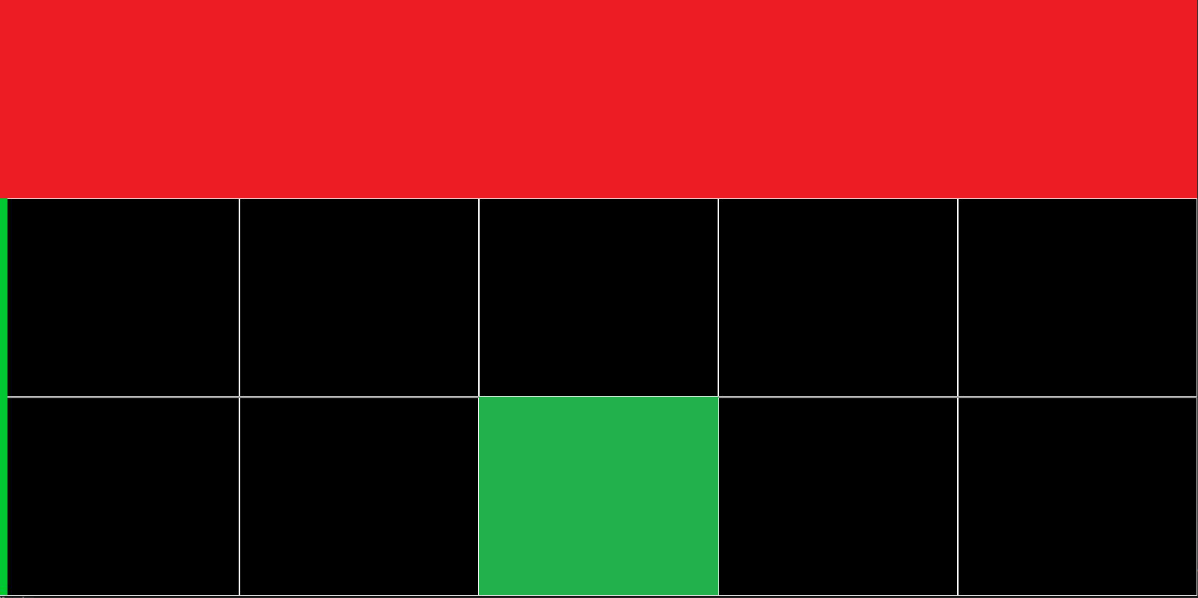


Figure 14 – Total zombies survived vs. the episodes (blue) with its moving average (orange)

Plus, we can tell from Figure 14, the zombie Player reaches the maximum it can get – reaches 196 zombies from possible of 196 (there are 201 steps with grid width of 5), we were able to achieve that thanks to the last 100 episodes with zero epsilon greedy parameter.

### Light Player test on a 3x5 board



7

8

6

5

9

0

1

2

3

4

10

11

13

14

12

Figure 15 - environment set-up for light Player performance check with optional actions

Figure 15 illustrates the simulation while testing the performance of the light Player.   
As we can see, the zombie Player takes only the action 0 (red cells, predetermined for simplicity) what caused the zombies to exit from the upper cell solely. Which after few steps made the entire top row full of zombies. In addition, the green cell represents the light action in the current step.

Furthermore, in general, we can tell in the first episodes there should survive roughly ~130 zombies since the actions are taken random and there is 33% chance for the light Player to light the top row.

Once again, consider the following parameters:

|  |  |
| --- | --- |
| Zombie Player action | 0 |
| Target update | 10 |
| Num episodes | 1000 |
| Steps per episode | 200 |
| Batch size | 256 |
| Gamma (discount factor) | 0.999 |
| Epsilon-greedy start | 1 |
| Epsilon-greedy end | 0.05 |
| Epsilon-greedy decay | 0.00001 |
| Replay memory size | 1000 |
| Learning rate | 0.001 |

Table 2 – learning parameters while evaluating the light Player

With a deep NN of three layers, all fully connected (called 'Linear' in pytorch formulation): Linear (15,128), Linear (128,128), Linear (128,15).  
In this case we have fifteen outputs. Hence the output of the last layer equals to 15.

This time we achieve increase in the amount the light Player chooses to light the first row. The phenomenon indicates the light agent's recognition of the fact that the zombies are coming out of the upper cell.

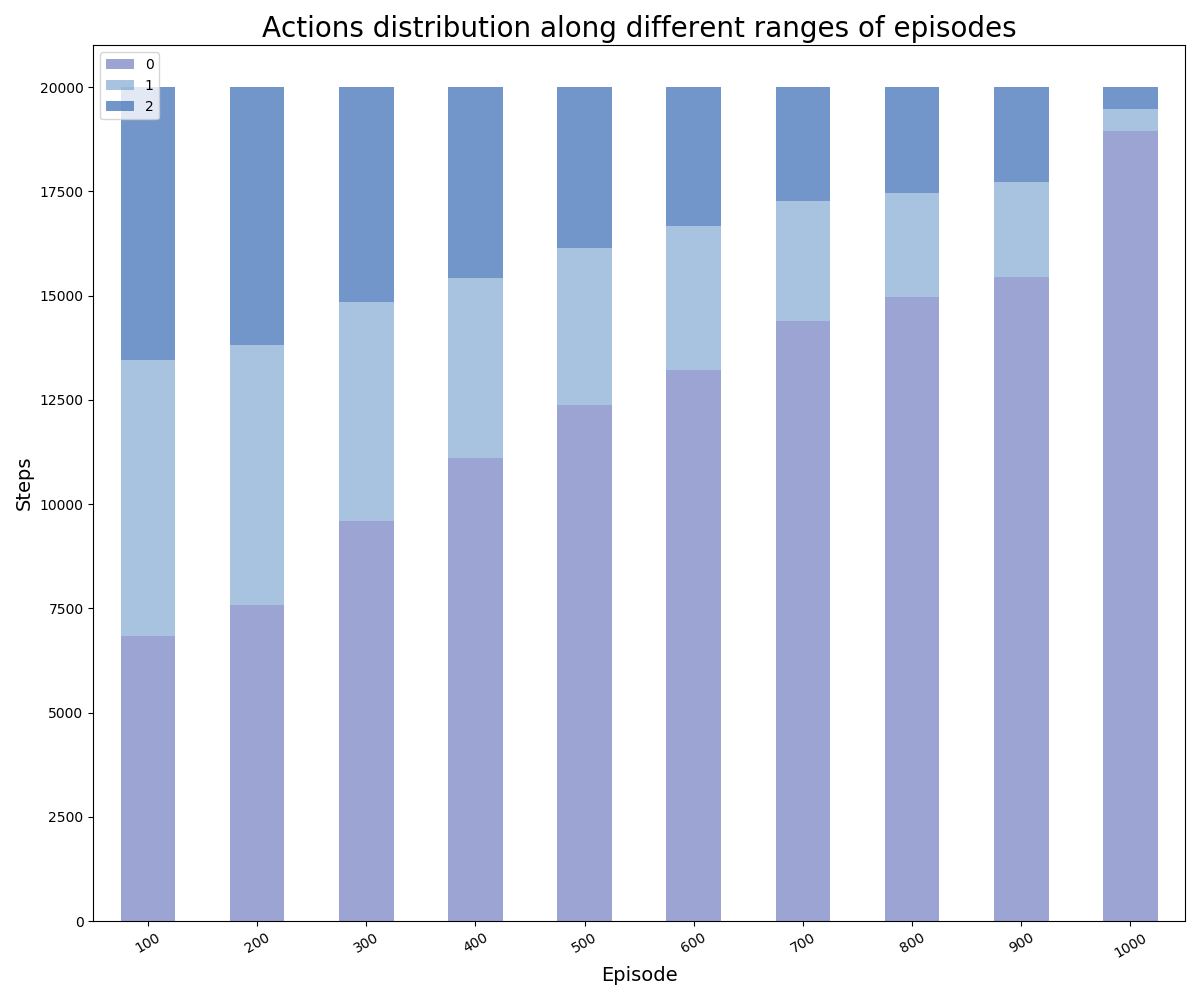


Figure 16 - Light Player actions distribution along different ranges of episodes

In Figure 16 we can see the significant increase in the number of times the light agent selected the top row illumination throughout the simulation progress.  
After 900 episodes, the light Player chooses to light the correct row more than 90% of the time. Note that the value of the greedy epsilon here is decreasing to 0 at the 900th episode.

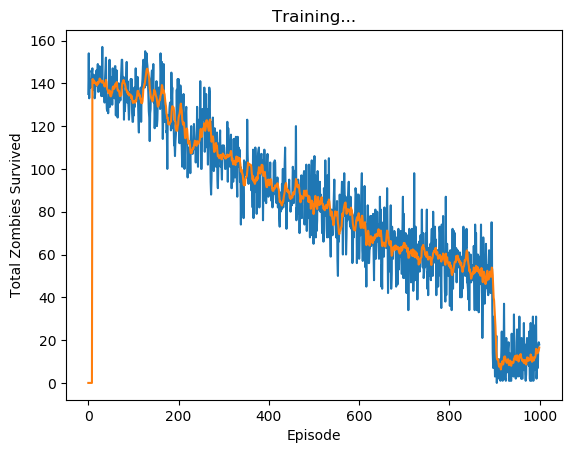


Figure 17 - Total zombies survived vs. the episodes (blue) with its moving average (orange)

In Figure 17 we can clearly see the learning process of the light agent, from the first episodes with 140 zombies survived (out of possible 195 over the episode, it's approximately two thirds), it managed to eliminate over 90% of the zombies by the 1000th episode, note that the noise remains thanks to the stochastic coin we flip every time step which means that zombies can still survive after getting hit.

## Double Deep Q-Network Evaluation

In this chapter we will examine the performance of a DDQN-based agent against the four simple agents and itself. We seek to find the best parameter sets for each scenario and compare their performances side by side.

We will examine the performance of all scenarios for different board sizes and different values of the parameters: **Target network update frequency** and **Replay memory size** (Those two parameters are most influencing when it comes to DDQN).

The evaluation process of DDQN is done with three phases:

* Estimating Average Test Reward of each scenario of different sets of parameters on various boards
* Choosing the best sets of parameters
* At last, comparing the results of the best agent over the different competitors

### Evaluation Configuration

For the evaluation process, let's review the hyper-parameters of the model:

* Fixed parameters:
* Number of training episodes: 800
* Number of test episodes: 200
* Zombies per episode: 20
* Light size: 2
  + In case the learning agent plays as Zombie, the light size is a third of the board length - In order to make it less easy
* Minimum hit points of certain death: 1
* Heal ratio: 0.97
* Tuning Parameters:
* Target Policy update frequency – [500, 750, 1000]
* Replay Memory size – [3000, 4000, 5000]

### Game Scenarios

Each scenario consists of two players on top of a board. The scenarios we are going to run are:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Game Board | Light Player | Zombie Player |  | Zombie Player | Light Player |
| 10x10 | DDQN | Single Action |  | DDQN | Single Action |
| Double Action |  | Double Action |
| Uniform |  | Uniform |
| Gaussian |  | Gaussian |
| 20x20 | DDQN | Single Action |  | DDQN | Single Action |
| Double Action |  | Double Action |
| Uniform |  | Uniform |
| Gaussian |  | Gaussian |
| 30x30 | DDQN | Single Action |  | DDQN | Single Action |
| Double Action |  | Double Action |
| Uniform |  | Uniform |
| Gaussian |  | Gaussian |

Table 3 – DDQN Evaluation: Game Scenarios

We repeat each scenario twice to reduce the bias of a single sample. Which means, in total we execute twice four scenarios (four simple agents), for each board size, for each player type. That sums up to .

We do all the above for every combination of the tuning parameters which adds up to scenarios to process.

Let's start with an example of a single evaluation. Consider a scenario of DDQN as Zombie and Single Action Agent as Light.

First, we repeat the scenario for several times while our Framework produces the following reward per episode graphs:

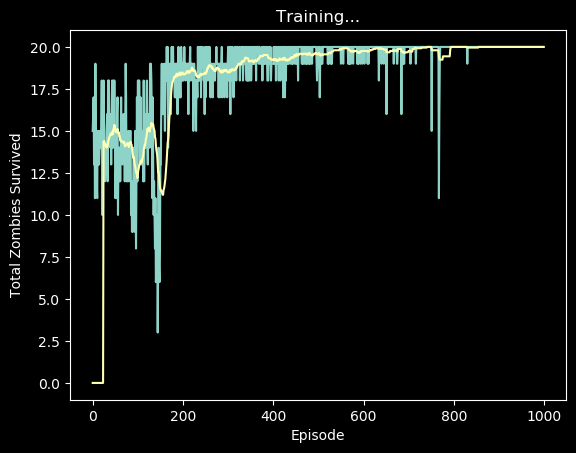


Figure 18 - Example of reward per episode graph, DDQN plays Zombie and Single Action plays Light

Second, we get the average of test rewards (episodes 800 - 1000) from all graphs as in Figure 19:

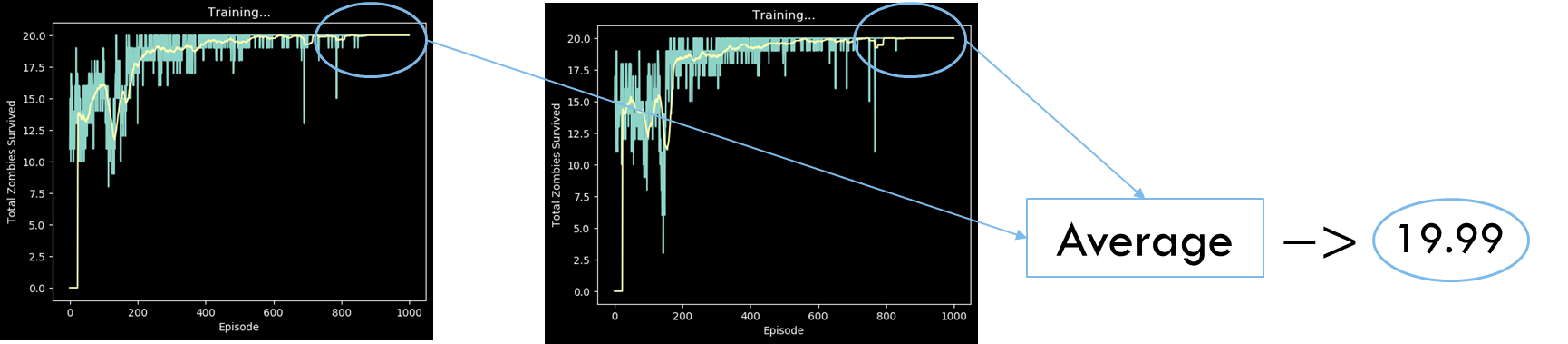


Figure 19 - Scenario Evaluation by Average Test Reward

And finally get an estimate of the performance of the agents in the scenario (the value 19.99 as in Figure 19)

### Double Deep Q-Network as Zombie Player

Before diving in to the results, lets understand a simple scenario of DDQN Agent as Zombie and Single Action Agent as Light:

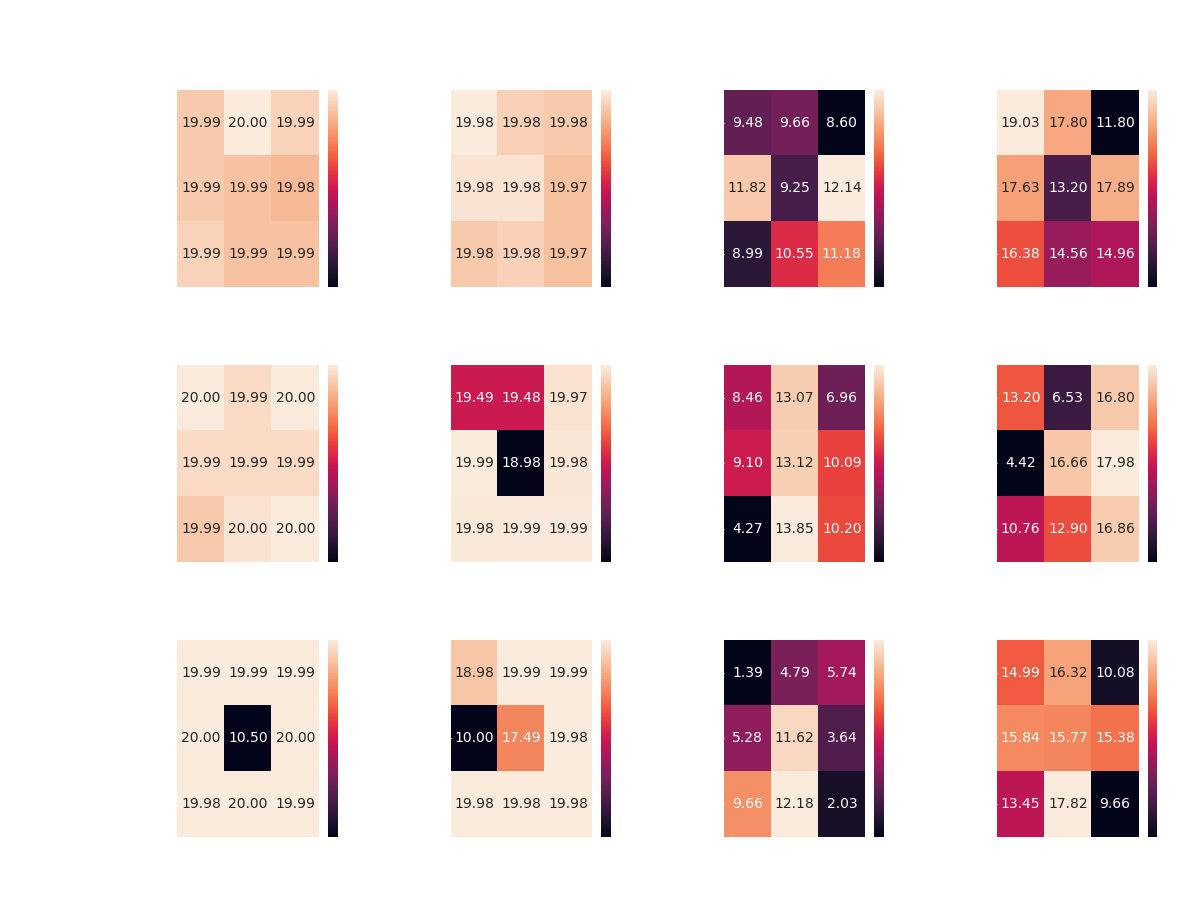


Figure 20 - Heat-Map of the Average Test Rewards of all the scenarios that DDQN Agent plays Zombie and Single Action Agent plays Light

The diagram consists of two axes for each tuning parameter and cells with values of the Average Test Rewards. There is also a matching color axis that describes the absolute value displayed inside each cell.

Since the maximum reward possible in a single episode is 20 (the total number of zombies in episode – [see configuration](#_Evaluation_Configuration)) We can easily conclude that the DDQN agent as Zombie outperformed the Light Player for all parameters.

Next, let's have a look at the rest of the scenarios:

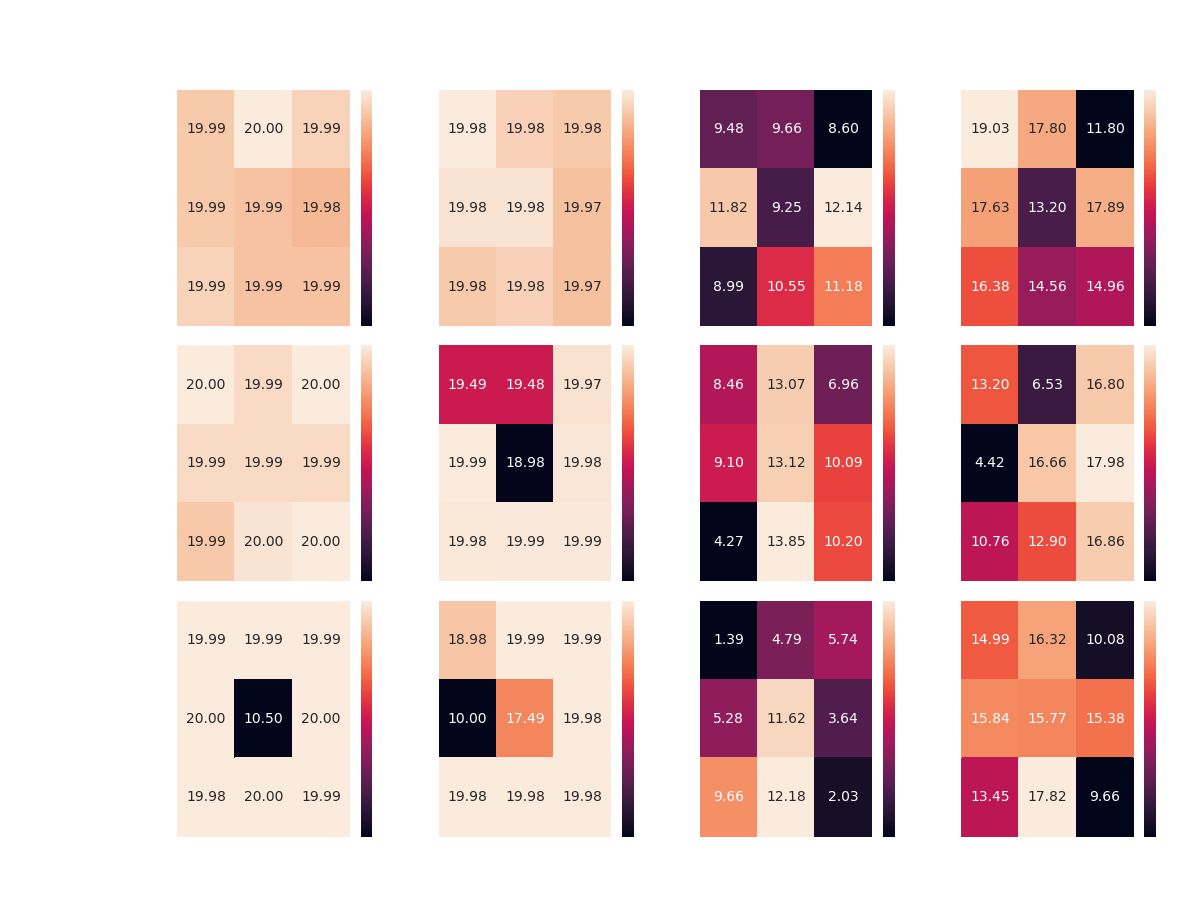


Figure 21 - A summary of all the scenarios that DDQN plays Zombie by Heat-Maps of the Average Test Reward

Overall, it seems that the DDQN agent manages to overcome its competitors in most of the scenarios:

* Achieving optimal reward competing Single and Double Action Agents with most of the configurations, on top all boards
* Achieving 17+ average reward competing the Gaussian Agent on all boards, without achieving optimal reward in any scenario

However, the DDQN agent seems to encounter difficulties when faced with the Uniform Agent – managing to achieve average reward of 12-14.

While Competing against both the Random Agents, the DDQN Agent gets approximately same average reward over all three boards. Perhaps more learning episodes would make a change here?

* Keep in mind that the highest reward against the Uniform Agent as light player is 15 – calculated by an agent which picks the best action exclusively (places zombies at the first row)

To settle the issue, we ran multiple scenarios of the same configuration, DDQN Agent vs. Uniform Agent, this time with 1800 learning episodes and 200 test episodes – all cases yield to the same average test reward of around 12, hence, the DDQN Agent can't achieve better results in those scenarios.

Now we can summarize all results of best parameter configurations:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Board size | Competitor | Memory Size | Target Update | Average Test Reward |
| 10x10 | Single Action | 3000 | 750 | 20 |
| Double Action | 3000 | 500 | 19.98 |
| Uniform | 4000 | 1000 | 12.14 |
| Gaussian | 3000 | 500 | 19.03 |
| 20x20 | Single Action | 3000 | 500 | 20 |
| Double Action | 4000 | 500 | 19.99 |
| Uniform | 5000 | 750 | 13.85 |
| Gaussian | 4000 | 1000 | 17.98 |
| 30x30 | Single Action | 5000 | 750 | 20 |
| Double Action | 3000 | 1000 | 19.99 |
| Uniform | 5000 | 750 | 12.18 |
| Gaussian | 5000 | 750 | 17.82 |

Table 4 - Best Configurations of all Scenarios in which the DDQN plays Zombie

* In cases where the same result was obtained for several scenarios, the best params chosen arbitrarily

From the summary of the results (see Table 4), there does not appear to be any clear preference for a particular configuration in the aspects of the boards and agents.

From here, we can present the rewards graph of each scenario (see Figure 18) side-by-side along different board sizes:



Figure 22 - Comparing the results of the DDQN agent as the Zombie Player, with the best parameters over the different four simple competitors

* note that in the case where the algorithm plays the zombie, the reward is positive

In general, it can be noticed that there is learning against all the agents in all the games.  
There is absolute success (convergence to optimal policy) against the constant agents  
On the other hand, in games against the random agents, there is partial success but a significant upward trend in almost all cases. In all cases except against the random agent in the game in the smallest board.  
However, because at this stage we have not examined the results in relation to any optimal/another method, we cannot say if these are the best results the agent could achieve, but these are good and satisfactory results - there is general learning in all board sizes against all agents.

### Double Deep Q-Network as Light Player

Same of the [last chapter](#_Double_Deep_Q-Network), Before diving in, let's review a simple scenario of DDQN Agent as Light and Single Action Agent as Zombie:

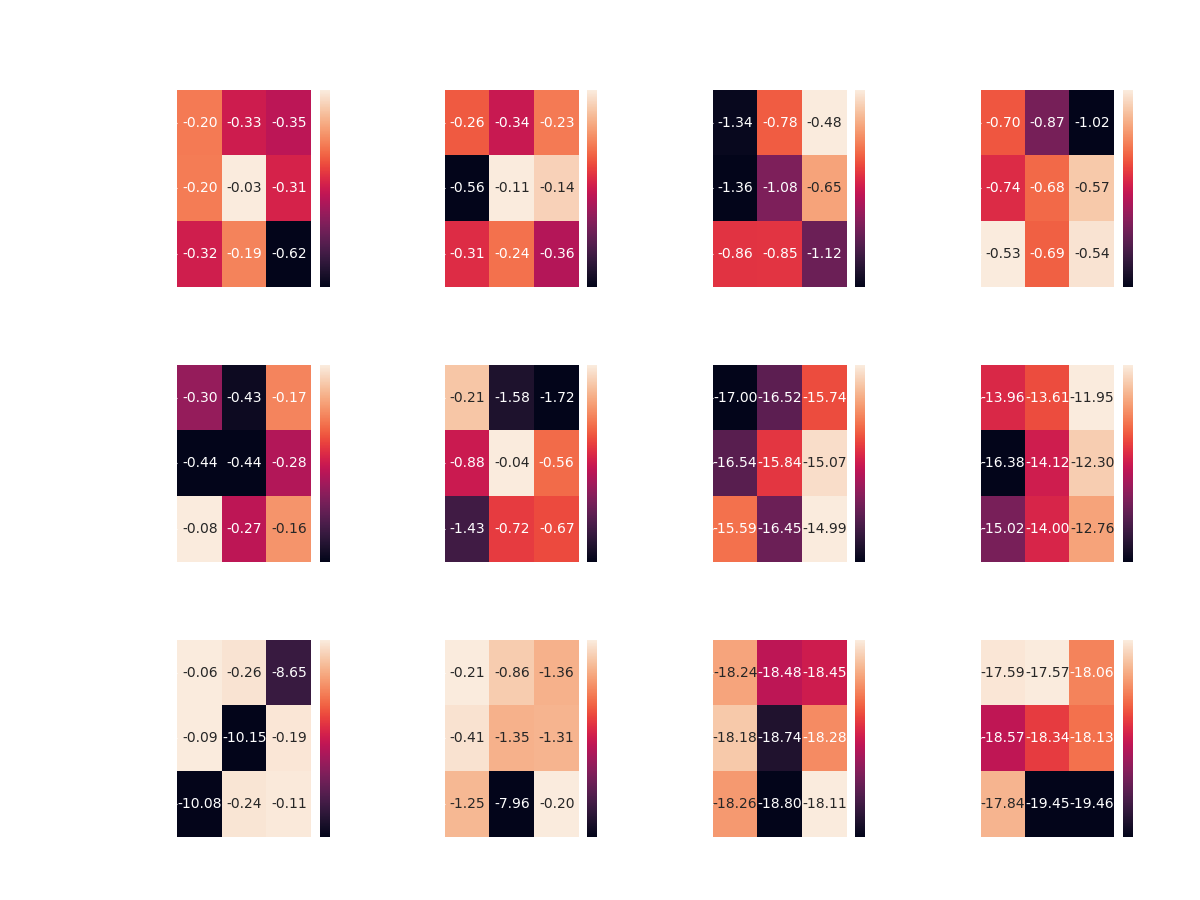


Figure 23 - Heat-Map of the Average Test Rewards of all the scenarios that DDQN Agent plays Light and Single Action Agent plays Zombie

We look at the same diagram with one significant difference – the values inside the cells are negative! Remember that we are playing a zero-sum game in such a way that the light agent will always receive an opposite and necessarily, negative reward. Yet his mission is the same, to maximize it.

Next, let's have a look at the rest of the scenarios:

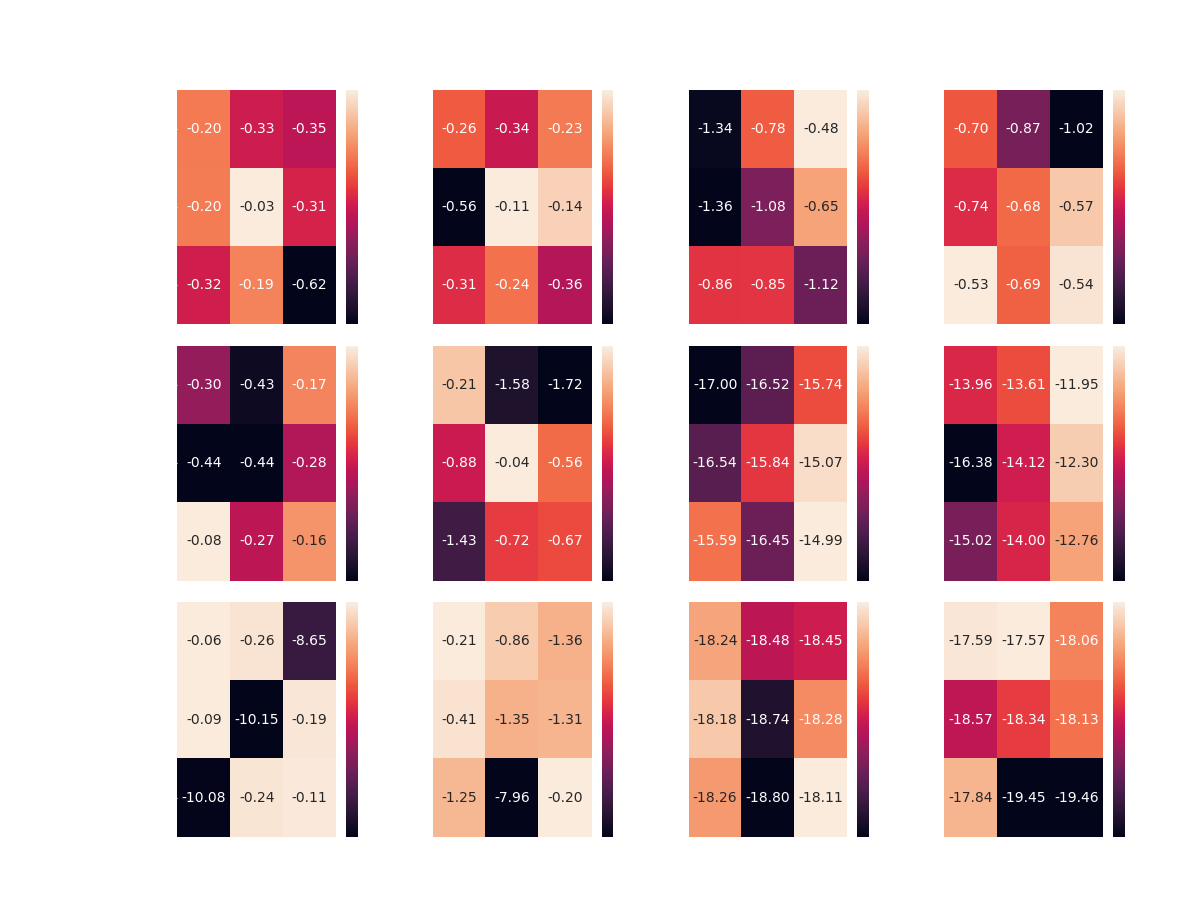


Figure 24 - A summary of all the scenarios that DDQN plays Light by Heat-Maps of the Average Test Reward

This time the DDQN Agent managed to achieve partial success.

Over all boards, the DDQN Agent seems to overcome the two Constant Agents, with some preference of the [750, 4000] configuration in half of the cases.

However, competing the Random Agents didn’t lead to same success. the DDQN Agent was able to reach optimality only in the case of the smallest board, and this time, without any preference of any specific configuration.

In addition, we witness the instability of DDQN Agent while training on large boards, evident in squared board of length 30, the DDQN agent shows that there are combinations of parameters that do not lead to good results against the Constant Agents (Single and Double Agents), yet in most cases it is still a success.

Now we can summarize all the results of best parameter configurations:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Board size | Competitor | Memory Size | Target Update | Average Test Reward |
| 10x10 | Single Action | 4000 | 750 | 0.03- |
| Double Action | 4000 | 750 | 0.11- |
| Uniform | 3000 | 1000 | 0.48- |
| Gaussian | 5000 | 500 | 0.53- |
| 20x20 | Single Action | 5000 | 500 | 0.08- |
| Double Action | 4000 | 750 | 0.04- |
| Uniform | 5000 | 1000 | 14.99- |
| Gaussian | 3000 | 1000 | 11.94- |
| 30x30 | Single Action | 3000 | 500 | 0.06- |
| Double Action | 5000 | 1000 | 0.2- |
| Uniform | 5000 | 1000 | 18.11- |
| Gaussian | 3000 | 750 | 17.57- |

Table 5 - Best Configurations of all Scenarios in which the DDQN plays Light

* In cases where the same result was obtained for several scenarios, the best params chosen arbitrarily

From the summary of above (Table 5), the DDQN Agent seems to prefer the parameters: [750, 4000] while competing against the two Constant Agents (Single and Double Agents).

As for the rest, there is no apparent preference.

From here, we can present the rewards graph of each scenario (see Figure 18) side-by-side along different board sizes:

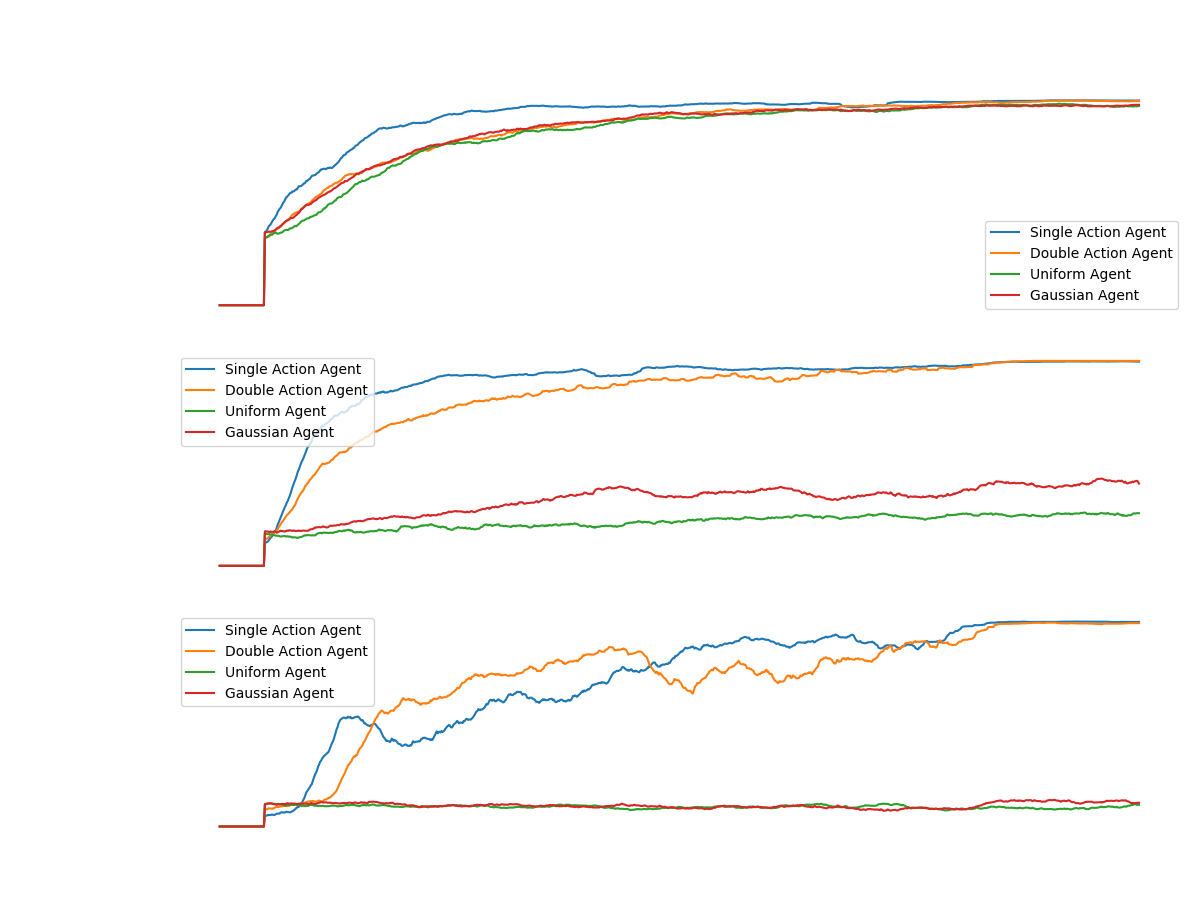


Figure 25 - Comparing the results of the DDQN agent as the Light Player, with the best parameters over the different four simple competitors

* note that in the case where the algorithm plays the zombie, the reward is negative

We can see a beautiful convergence to optimal policy against all agents on a 10x10 board. However, when looking at the success of the light player as the DDQN agent in larger board sizes, one can see controversial success. There is a convergence in all cases in the game against the constant agents. But, if we look at the game against the random agents, we can see a relatively upward trend in the game against the middle Gaussian agent. And in the rest of the games: A complete failure - there is not even a spark of convergence.

## Learning Based Monte Carlo Tree Search

The purpose of this section is to add the Monte Carlo Tree Search algorithm to our project.   
We must take into account that this is the second algorithm we implement during the project, among many more in potential. Therefore, the code we build from now on is fully generic and easily adaptable to absorb any kind of new learning algorithm (see "Building the Framework").

### MCTS - Implementation and Config

As mentioned in "*Monte Carlo Tree Search*" the whole learning process of the algorithm consists of **four stages**. In this chapter we will elaborate on each stage, while presenting conflicts that arose along the way with proposals and the way of realization.

In the first stage the goal is to find a leaf in the tree which we will move on. To do this we will start traversing the nodes of the tree according to UCT strategy until we reach a leaf node.

Now, we would like to expand its children. We can do this in one of two ways, **one is to create all possible nodes** (as the number of possible actions) **and the other is to create only one**.

The consideration here is, On the one hand, to prevent the explosion of the tree, or with other words, the unnecessary expansion of places that will later be found irrelevant, thing that might cause the explosion of memory and killing the process. while the expansion of nodes that are relevant will eventually will cause a positive effect of faster convergence (in terms of number of steps).

On the other hand, when we expand only one leaf, we delay the expansion of the tree in ways that are interesting and relevant.

Imagine that we have reached a particular node, assuming that this node is very relevant to learning, we would like to expand and explore its children. Something we will not be able to do until its fully expanded. The described situation was examined and we finally came to the conclusion that despite the overloading memory in a large tree, **we would prefer to expand all the leaves of a particular node and avoid slowing down the convergence of the algorithm**.

At the next phase, we perform simulations from the selected leaf, in order to estimate the expected reward that an agent will receive from now on. According to that estimation, we update the statistics upwards. Which raises the question: **Should we update the stats up to the tree root or stop at the current game-state-node?**   
According to the literature I have seen and code-libraries concerns the two approaches. Some argue that updating to the root of the tree might bias the game to random directions and therefore it is advisable to update the stats up to the current game-state-node. Others claim that updating to the root of the tree is necessary for the reason that you do not visit this area much (only at beginning of games), therefore, it will take a lot of time to train the agent to choose wise actions at the first moves of each game.

Back to simulation phase. The most significant parameters here are the number of simulations and the depth of a simulation. In each simulation we traverse the tree for some fixed depth, at each step we select random actions for both agents and lastly promoting the environment accordingly, while receiving reward and accumulating it. We continue so until we reach the required depth of simulation. After the process for number-of-simulations time, we average the rewards and backpropagate them.

At this point another assumption was made: **How will we treat a reward with a numerical value greater than 1 in absolute value?** There are few references to such cases in the literature. Accepting such rewards is indeed problematic since a reward received from the environment is 1 or -1 in our game, while reward obtained from simulation gets values from 1 up to the depth-of-simulation (absolute value). **How can we update the statistics of each node with values from different ranges?** Several options were tested and the solution we chose is to normalize the value obtained from the simulation with the depth-of-simulation, which changes its values range back to [-1,1].

After all the above, the backpropagation phase comes, in which, we take the value obtained from the simulation at any given moment and add it to the reward value of each node up to the root node, while increasing the number of visits of the node by 1.

To sum up, during the implementation of Monte Carlo tree search algorithm, some loose ends were discovered:

1. At the selection phase, what upper-confidence-bound constant should we use?
2. At the expansion phase, once we reach a leaf, should we expand all the possible node or just one?
3. After simulation occur, how do we treat a reward greater than 1? Should we update statistics only after simulations to prevent the inconsistency? And if so, how do we use the reward that comes in real time from the environment, since it is a real and accurate reward – we might want to use it.
4. At the backpropagation phase, should we update the stats up to the tree root or the current-game-state?

### Results and Conclusions

For evaluation and test of the algorithm we implemented, we taught it in a game against one of the simple agents. The agent reached an equilibrium in games it played as the zombie agent. Against Simple agents and DDQN agent. While as the Light Player, each learning scenario kept fail duo to the 16 RAM constraint of the machine. It prevented us from playing on any reasonable size of board. The main reason of that is that the action space the light agent has is actually the number of expanded children each node has.

Therefore, we decided to proceed with our research and leaving the MCTS implementation behind, meaning, we won't use it for the following steps and evaluations of the following algorithms.

## Learning Based AlphaZero Algorithm

Following section ‎6.3 (*From AlphaGo to AlphaZero*), the model we use takes the bold ideas that is basically the AlphaZero algorithm and thus actually makes it possible to implement the principles of Multi Agent Reinforcement Learning together with Monte Carlo Tree Search, on top of our game environment: 'Light vs Zombies'.

### Our Network

There are many network architectures regarding the alpha/Go/Zero implementations (see [15],[16],[17]).

In the paper by DeepMind [14], they use 20 residual blocks, each with 2 convolutional layers. We were able to train a 3-layer CNN network followed by a few feedforward layers.

Let's review the architecture of AlphaZero as Light Agent with more details:

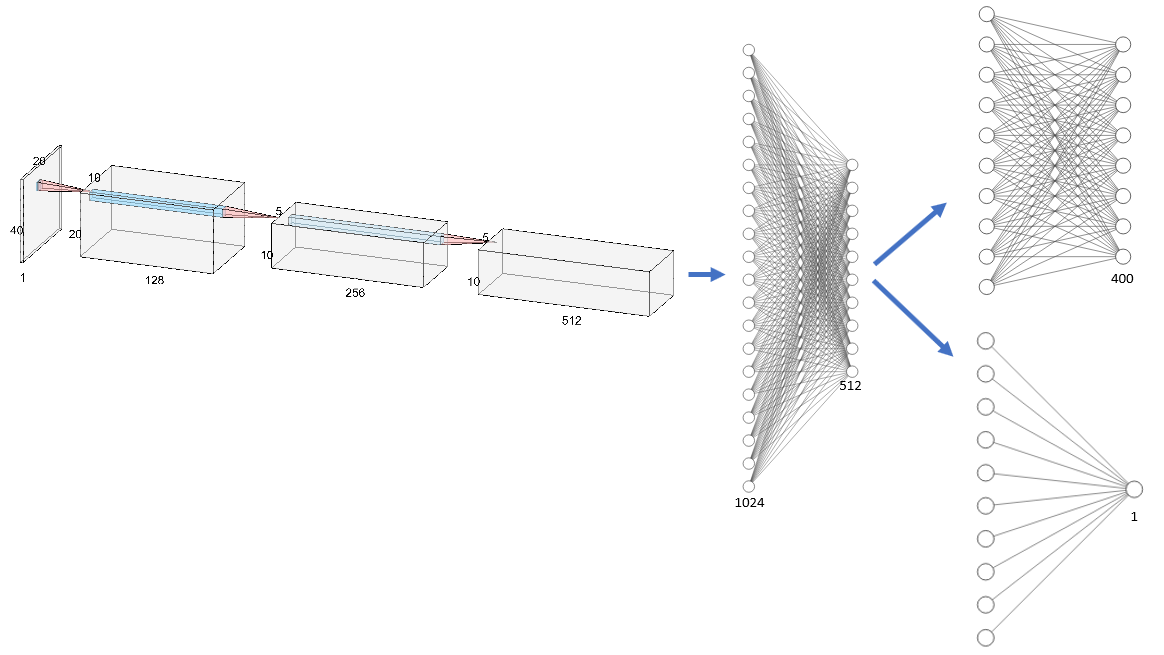


Figure 26 - NN Architecture. Red and blue arrows indicate convolutional and fully Connected layers respectevly.

Our input of 20 by 40 (the light agent has locations and strength as its state) passing forward the following layers:

* 2D-Convolution layer with 128 filters with size of 3 by 3, stride of 1 and padding of 1 to keep the size of the input.
* 2D-MaxPool of stride 2.
* 2D-BatchNorm of the 128 filters.
* 2D-Convolution layer with 256 filters with size of 3 by 3, stride of 1 and padding of 1 to keep the size of the input.
* 2D-MaxPool of stride 2.
* 2D-BatchNorm of the 256 filters.
* 2D-Convolution layer with 512 filters with size of 3 by 3, stride of 1 and padding of 1 to keep the size of the input.
* 2D-BatchNorm of the 512 filters.
* FC layer – 1024 units.
* FC layer – 512 units.
  + Output layer of 400 units – represents the action probability
  + Output layer of 1 unit – represents the value function approximation.

Or with pytorch view:

=================================================================  
Layer (type:depth-idx)                   Param #  
=================================================================  
├─Conv2d: 1-1                            1,280 = 128x(3x3+1) \*plus 1 for bias unit  
├─MaxPool2d: 1-2                         --  
├─BatchNorm2d: 1-3                       256  
├─Conv2d: 1-4                            295,168 = (128x3x3+1)x256 \*biases as number of out-filters  
├─MaxPool2d: 1-5                         --  
├─BatchNorm2d: 1-6                       512  
├─Conv2d: 1-7                            1,180,160 = (256x3x3+1)x512  
├─BatchNorm2d: 1-8                       1,024  
├─Linear: 1-9                            5,243,904 = 1024x(1+512x(3x3+1))   
├─BatchNorm1d: 1-10                      512  
├─Linear: 1-11                           32,896 = 256x(128 +1)  
├─BatchNorm1d: 1-12                      256  
├─Linear: 1-13                           12,900  
├─Linear: 1-14                           129  
=================================================================  
Total params: 6,768,997  
Trainable params: 6,768,997  
Non-trainable params: 0  
=================================================================

### Training Process of AlphaZero

Having gone through the rules of the game, the implementation and special assumptions, we can now give an example that will summary the whole training process with a set of default hyper-parameter:

At first, similarly to MCTS, we build a search tree but this time the tree will be initialized as new in each episode of the game. At each step, we will run 100 simulations of searching at a depth of 2 levels down through the tree, such that after each search, stats will be propagated up the tree. At each episode we will store the state-action-reward triplet. Lastly, At the end of an episode, we initiate a new Monte Carlo Tree Search with the new policy network to get ready for the next episode.

After every 40 episodes we will learn our policy network using all data accumulated from the last 200 episodes.

### AlphaZero Evaluation

Similarly of the evaluation process of DDQN algorithm (described in section ‎10), we let an agent based AlphaZero algorithm play as Light and Zombie players, vs the four simple agents (Constant, Double Constant, Gaussian and Uniform) as its competitor.

#### Evaluation Configuration

For the evaluation and tuning process, we used the following set of hyper parameters of the AlphaZero algorithm:

Fixed parameters:

* Episode's history – 100. The number of recent episodes of which we can sample a batch of training data.
* Number of episodes per training – 20
* Depth of search – 2
* Learning rate – 0.001
* Epochs per training – 10
* Batch size – 128

Tuning Parameters:

* Number of searches – 5, 10, 15
* Exploration rate – 0.5, 1, 1.5

#### Game Scenarios

Similar to ‎10.2, The scenarios we are going to run are:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Game Board | Light Player | Zombie Player |  | Zombie Player | Light Player |
| 10x10 | AlphaZero | Single Action |  | AlphaZero | Single Action |
| Double Action |  | Double Action |
| Uniform |  | Uniform |
| Gaussian |  | Gaussian |
| 20x20 | AlphaZero | Single Action |  | AlphaZero | Single Action |
| Double Action |  | Double Action |
| Uniform |  | Uniform |
| Gaussian |  | Gaussian |
| 30x30 | AlphaZero | Single Action |  | AlphaZero | Single Action |
| Double Action |  | Double Action |
| Uniform |  | Uniform |
| Gaussian |  | Gaussian |

Table 6 – DDQN Evaluation: Game Scenarios

For each scenario we calculate the Average Test Reward, again, as we already explained in section ‎10.2.

#### Alpha Zero as Zombie Player

Similar to the evaluation process of DDQN algorithm (see: *Double Deep Q-Network as Zombie Player*), we start with the summary of the average test reward of each scenario with heatmaps.

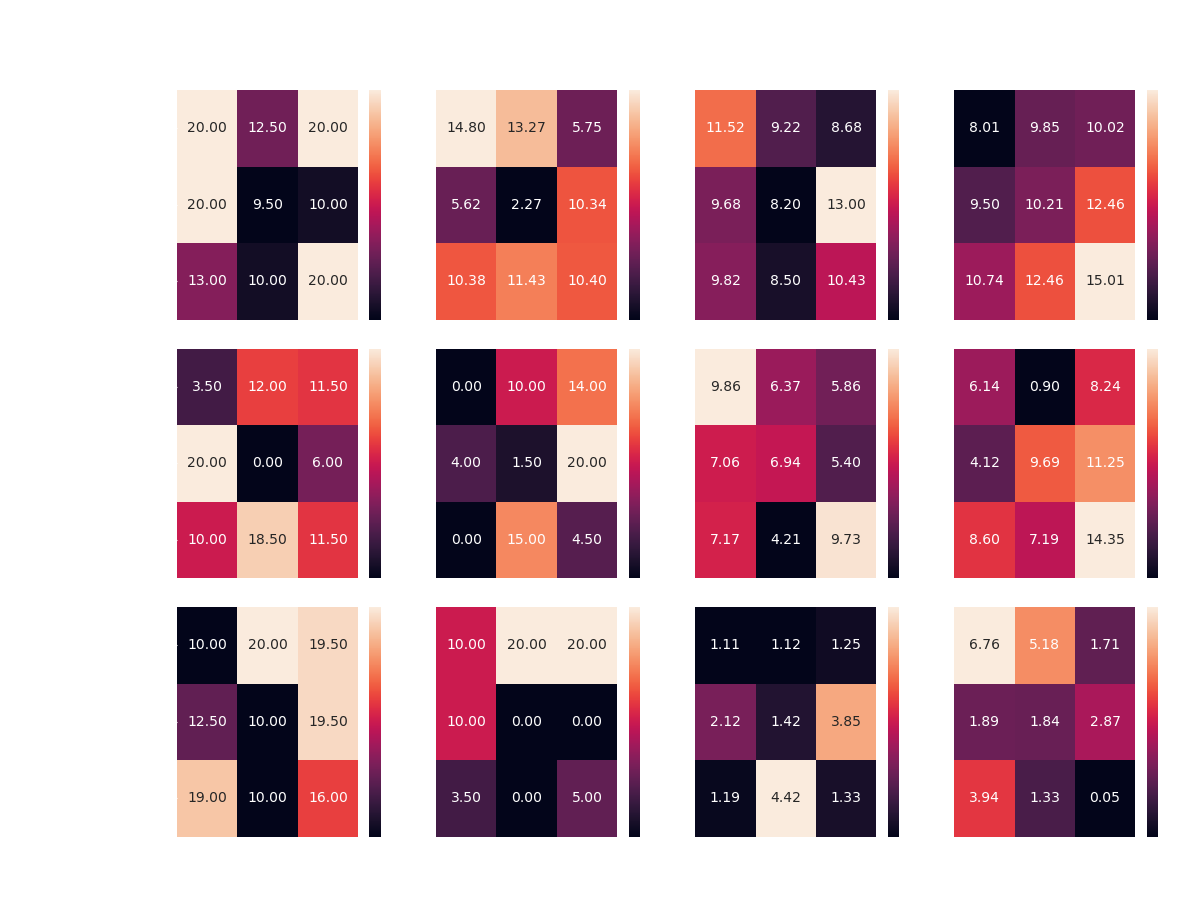


Figure 27 - A summary of all the scenarios that AlphaZero plays Zombie. Heat-Maps of the Average Test Reward

As you can see, unsurprisingly the AlphaZero agent manages to find a way to beat the simple agents in any board configuration.

In contrast to the simple cases, the agent has a hard time coming up with a good strategy (compared to DDQN) against the random agents.

In addition, it is apparent that there is a high variability in the agent's performance across the various configurations, unlike DDQN.

Now, we can summary the results like we did with DDQN (Table 4):

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Board size | Competitor | Monte Carlo searches | Exploration rate | Average Test Reward |
| 10x10 | Single Action | 5 | 0.5 | 20 |
| Double Action | 5 | 0.5 | 14.8 |
| Uniform | 10 | 1.5 | 13 |
| Gaussian | 15 | 1.5 | 15 |
| 20x20 | Single Action | 10 | 0.5 | 20 |
| Double Action | 10 | 1.5 | 20 |
| Uniform | 5 | 0.5 | 9.86 |
| Gaussian | 15 | 1.5 | 14.34 |
| 30x30 | Single Action | 5 | 1 | 20 |
| Double Action | 5 | 1 | 20 |
| Uniform | 15 | 1 | 4.42 |
| Gaussian | 5 | 0.5 | 6.76 |

Table 7 - Best Configurations of all Scenarios in which AlphaZero plays Zombie

We can nicely see the tendency of the amount of searches to stick with the lowest possible for most simple agents (Single and Double agents), while the number of searches increases as the complexity of the agent increase (Gaussian and Uniform agents).

* Note that the claim above is relevant for the case of the Uniform agent in the 20x20 board. The optimal number of searches could be 15 as the difference is neglegible (9.73 and 9.86).

For complete closure, the mean reward graph of the best agents is presented:

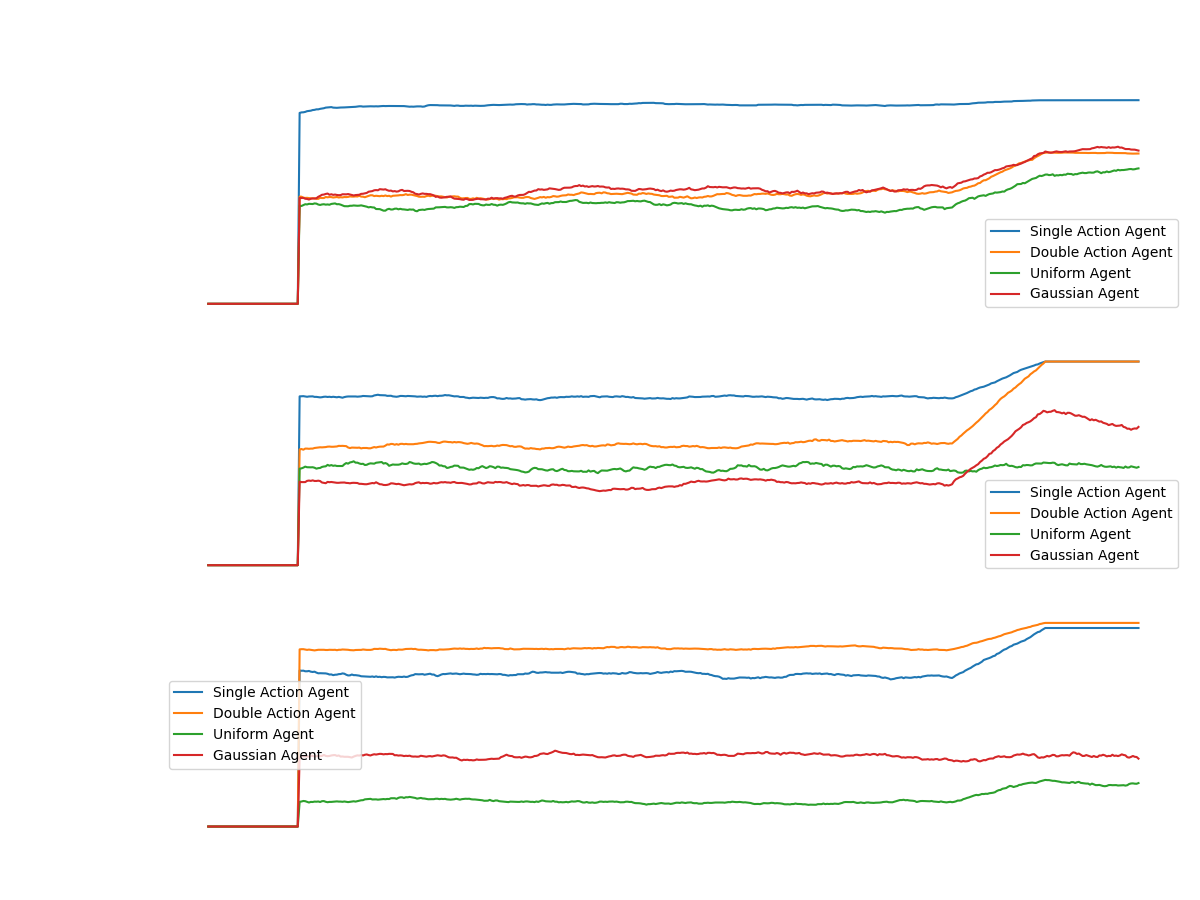


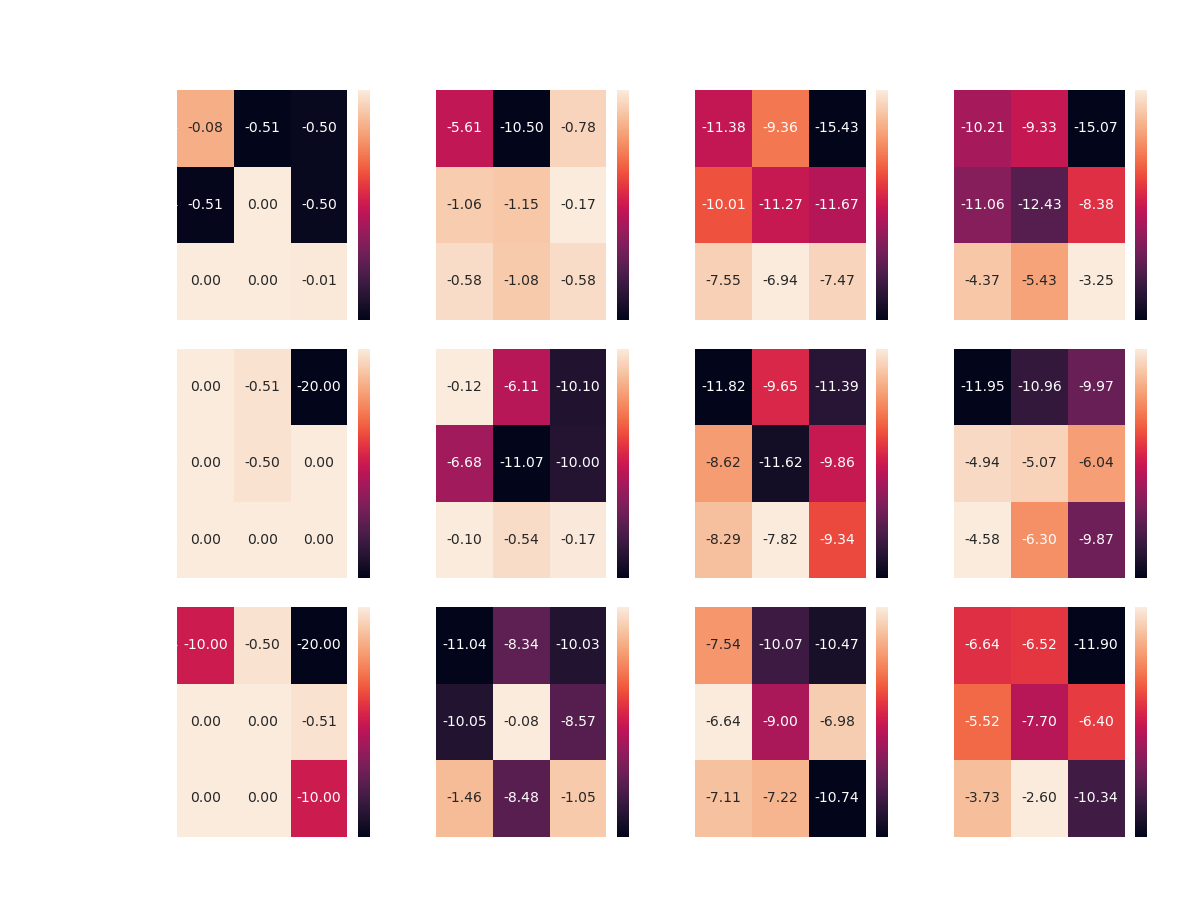
Figure 28 - Comparing the results of the AlphaZero agent as the Zombie Player, with the best parameters over the different four simple competitors

From the graph above, it seems that in cases where the agent fails to converge to successful strategy (compared to DDQN), there is a learning barrier at the beginning of the process.

This barrier could be due to a poor configuration of the AlphaZero algorithm. As well as the partial observability compared to the success the agent reaches as the Light Player (next section).

#### Alpha Zero as Light Player

Now, we begin testing the results of the AlphaZero algorithm, remember that as a Light Player, the agent gets information about the strength of the zombies at each step of the environment - unlike the zombie player.



As expected, the agent comes up with an optimal strategy against the simple agents.

In addition, in an unprecedented way so far in the project, the agent manages to come up with a successful strategy against the random agents!

In both games with the 20 and 30 square boards, the agent surpassed the DDQN agent so that in the game against the Gaussian agent he gained almost the maximum possible reward.

Again, let's review take a look over the summary of best configurations:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Board size | Competitor | Monte Carlo searches | Exploration rate | Average Test Reward |
| 10x10 | Single Action | 15 | 0.5 | 0 |
| Double Action | 10 | 1.5 | -0.17 |
| Uniform | 15 | 1 | -6.94 |
| Gaussian | 15 | 1.5 | -3.25 |
| 20x20 | Single Action | 5 | 0.5 | 0 |
| Double Action | 15 | 0.5 | -0.1 |
| Uniform | 15 | 1 | -7.82 |
| Gaussian | 15 | 0.5 | -4.58 |
| 30x30 | Single Action | 10 | 0.5 | 0 |
| Double Action | 10 | 1 | -0.08 |
| Uniform | 10 | 0.5 | -6.64 |
| Gaussian | 15 | 1 | -2.6 |

Table 8 - Best Configurations of all Scenarios in which AlphaZero plays Light

As the light player, we notice that the algorithm performs well with the greater amounts of tree searches, which it is reasonable since the light player action space is larger (exponential greater compared to zombie player).

At last, the mean of the rewards for the best agents for all boards:

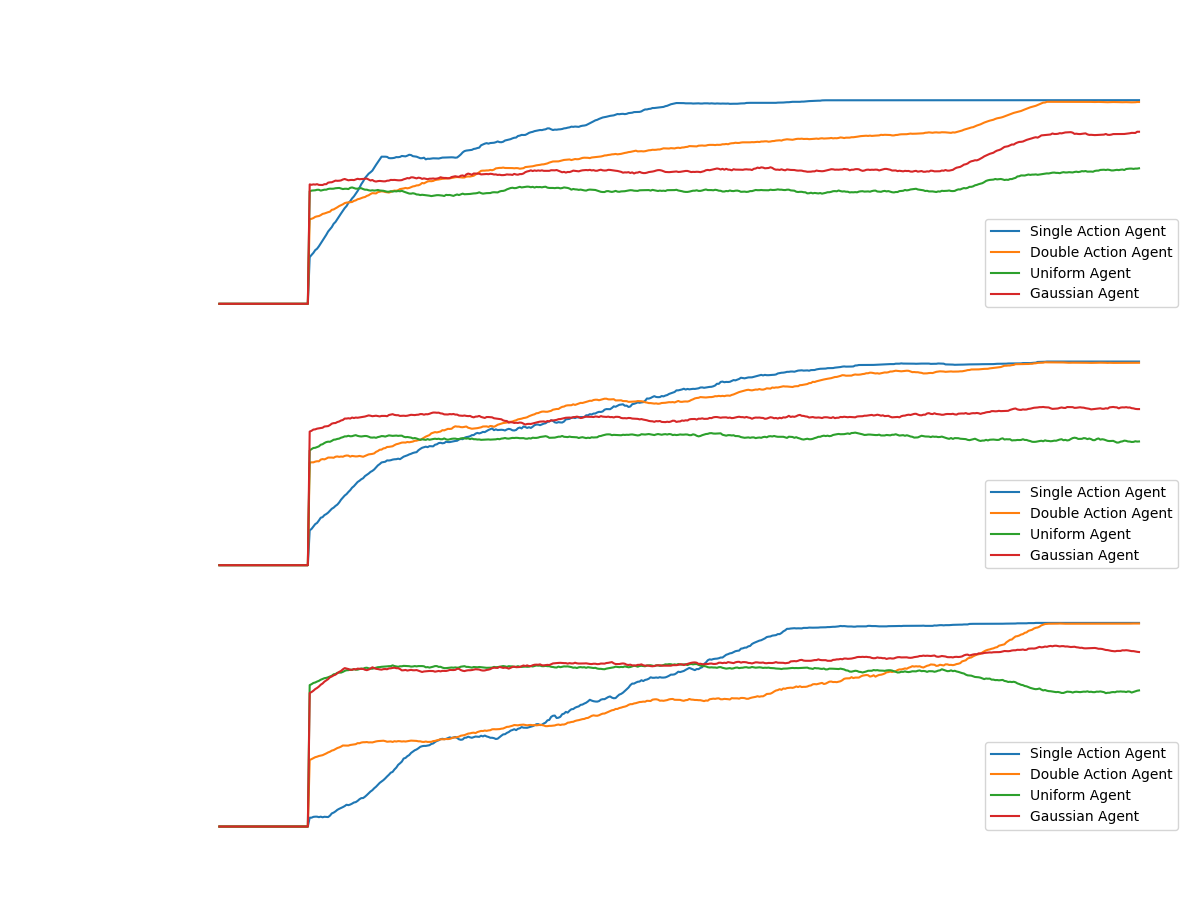


Figure 29 - Comparing the results of the AlphaZero agent as the Light Player, with the best parameters over the different four simple competitors

According to Figure 29, we can see the learning curve of the most successful agents in each game.

The most striking thing that catches our eyes in games against the random agents, is the fact that AlphaZero agent quickly converged into a strategy unlike those against the simple agents.

The main reason for such cases is the fact that the algorithm is based on the Monte Carlo Tree. Which updates the State-Action values ​​according to the rewards from the environment.

In games against the simple agents, AlphaZero agent spends most of the time looking for actions which will reward him positively and finds this only in a very limited number of situations, since the zombies travel in unique places over the board.

In contrast to the games against random agents. Which the rewards are more likely to appear much more frequently and the craft of searching is simpler.

* Note, Once the agent finds a variety of states that are rewarding in different ways, the network can learn. Which happens very quickly (as seen in Figure 29) in games against the random agents.

## Competing AlphaZero and DDQN

We have now reached the final stage of the project, a competition between learning algorithms!

Continuing from the previous stages where we taught the agents and pushed them to the edge of the ability limit and we looked for the best set of parameters for them in different situations of the game (different boards).

Now, it's time to take the best agents in each game board and let them compete with each other, once as the light player and once as the zombie player.

To do so, we will perform the following steps:

1. Save the models (network modes) with the best configurations of the two agents in all boards (from tables: Table 4, Table 5, Table 7 and Table 8), once as the light player and once as the zombie player (12 models in total)
   * Each time we gather the best configuration of a scenario, we give great preference to the configurations of the games against the Uniform Agent (since, this is the most complex) while still considering the others.
2. Prove the convergence of the chosen agent relative to the rest of the simple agents.
3. Build Metrics and graphs to describe the learning process (considering networks parameters and agents' actions).
4. Run and analyze the scenarios of the competition:
   1. AlphaZero as light vs DDQN as Zombie – 3 scenarios
   2. DDQN as light vs AlphaZero as Zombie – 3 scenarios

After gathering the best models of the DDQN agent against the Uniform Agent, we will now compare his performance against the three agents: Single, Double and Gaussian. Below is a summary of the results in binary view, significant success as 1 or not as 0:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Board Size | Model | Single | Double | Gaussian |
| As light | 10x10 | DDQN | 1 | 1 | 1 |
| 20x20 | DDQN | 1 | 1 | 1 |
| 30x30 | DDQN | 0 | 0 | 0 |
| As zombie | 10x10 | DDQN | 0 | 0 | 1 |
| 20x20 | DDQN | 0 | 0 | 1 |
| 30x30 | DDQN | 0 | 0 | 1 |

Table 9 - Summary of competing DDQN Agent vs. Single, Double and Gaussian Agents

As can be seen in Table 9, there is significant success against the Gaussian Agent! This is in contrast to a very limited success against the simplest agents. This is because the best model was trained against the Uniform agent which is very similar to the Gaussian agent.

Hence, we would like to expand the experience of our final model with the goal of defeating all simple agents. Which, as we have seen before (see Figure 22 and Figure 25), is achievable.

### Constructing the Composite Agent

The original idea that stands behind preparing the learning agents for battle, was to train them against the most complex not-learning' agent we have and test their performance against the others, hasn't led to any success.  
Therefore, we had to take another approach in order to generify the agent strategy and its total potential against other unknown agents (ex. The battle they are facing).

For that effort, we built a new agent called 'Composite Agent' that is the combination of all simple agents.

Its' strategy set to imitate one of the four simple agents at a time, for full episode period (from the first zombie enters the board until the last exit). After that, the agent will replace the strategy to the next in line, keep it for a full episode and continue so.

The order of strategies loop is: SingleAction -> DoubleAction -> Gaussian -> Uniform -> SingleAction etc...

By that, we get to learn from data samples generated from constant AND random agents, all at the same train process. With the expectation for the Composite-Agent's strategy to out-perform each of the simple agents, separately.

### Final Evaluation

After training the DDQN Agent against the Composite Agent, we will examine his success against the other simple agents:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Board Size | Model | Single | Double | Gaussian | Uniform |
| As light | 10x10 | DDQN | 1 | 1 | 1 | 1 |
| 20x20 | DDQN | 1 | 1 | 1 | 1 |
| 30x30 | DDQN | 0 | 0 | 1 | 1 |
| As zombie | 10x10 | DDQN | 1 | 1 | 1 | 1 |
| 20x20 | DDQN | 1 | 1 | 1 | 0 |
| 30x30 | DDQN | 0 | 0 | 1 | 0 |

Table 10 – DDQN trained with Composite agents, evaluation vs. all Simple Agents

In general, there has been significant success in training against the uniformed agent (see Table 9).

In the same way, we will take the AlphaZero agent and we will train it in games against the Composite agent.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Board Size | Model | SingleAction | DoubleAction | Gaussian | Uniform |
| As light | 10x10 | AlphaZero | 1 | 0 | 0 | 0 |
| 20x20 | AlphaZero | 0 | 0 | 1 | 1 |
| 30x30 | AlphaZero | 0 | 1 | 1 | 1 |
| As zombie | 10x10 | AlphaZero | 0 | 0 | 1 | 1 |
| 20x20 | AlphaZero | 1 | 0 | 1 | 1 |
| 30x30 | AlphaZero | 1 | 0 | 0 | 0 |

Table 11 - AlphaZero trained with Composite Agent, evaluation vs. all Simple Agents

Again, we achieved successful and satisfying results.

### The Competition

Now that we have trained our two learning agents: DDQN and AlphaZero to the best we could, in every game board and on each side of it. We will examine the success of the agents against each other.

#### Action and Reward Distribution

Before we dive into the competition, we need to understand the visualization we are going to use throughout the games. In addition to the reward graphs that we already familiar with (see "Game Scenarios"), we will show the distribution of agents' actions in the following graph (for example):

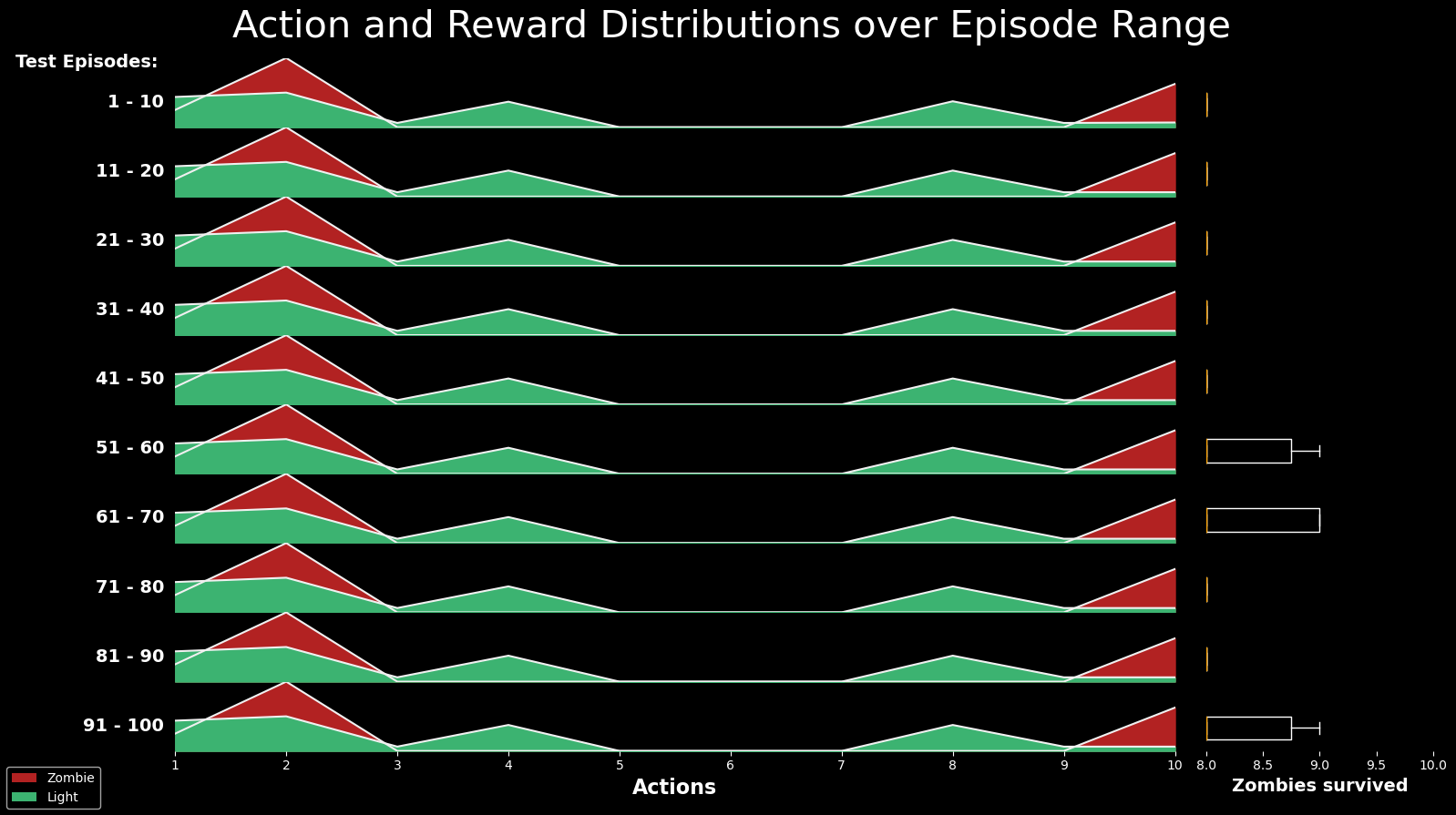


Figure - Action and Rewards distribution graph

For understanding Figure 30, we will elaborate on each of the three graphs separately:

1. In the left graph, we present the range of episodes on which the data is based.
2. The middle graph shows the distribution of each player actions. The action space of the light player is two-dimensional while the zombie player's action space is one-dimensional - as the number of rows of the board. Therefore, to bridge between these two, we decided to present the row of each selected action, for the two players.  
   In this way, the two-dimensional action of the light player is summed up to the row in which it was taken, and for the zombie player, the action is also the row.
   * Note for the graph in Figure 30, the actions the Zombie Players play are - rows 2 and 10. While the Light Player the actions 1,2,4 and 8 (mainly). The light mark is with size of 3x3, which means that expect from the Light Agent to place its mark on the rows of the zombies and the two above them (row 1 for action 2 and rows 8,9 for action 10 of the Zombie Player), from the reason that it is likely to hit the zombies and eventually profit from it. Conversely, in the absence of light, the zombie agent's profit could be observed with the same logic.
3. The right graph shows the box-plot of rewards for the episodes in current range.

Finally, we have to say that this is an insightful and yet incomplete graph, with regards to the light agent. Since, it is not possible to understand in which column he chose to place the cursor.

Now, to compare the performance of the agents, we will take the converged models and pass them on to the agents, each with its own set of parameters (see Table 4 and Table 5).   
We will start by presenting the reward throughout the game of each game board. Following, we will analysis the actions the agents have chosen.

#### DDQN as Light Player vs AlphaZero as Zombie Player

##### Analysis of 10x10 board

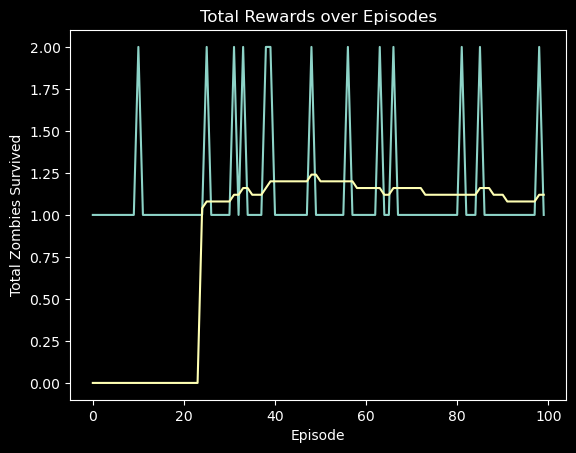


Figure 31 - Rewards over Episodes, board 10x10

There is a significant defeat of the Zombie Player - AlphaZero. With victory of the Light Player - the DDQN, since, the maximum reward that can be received in the game is 20 while on average only one zombie has survived each game.

We will now look at the distribution of agents' actions:

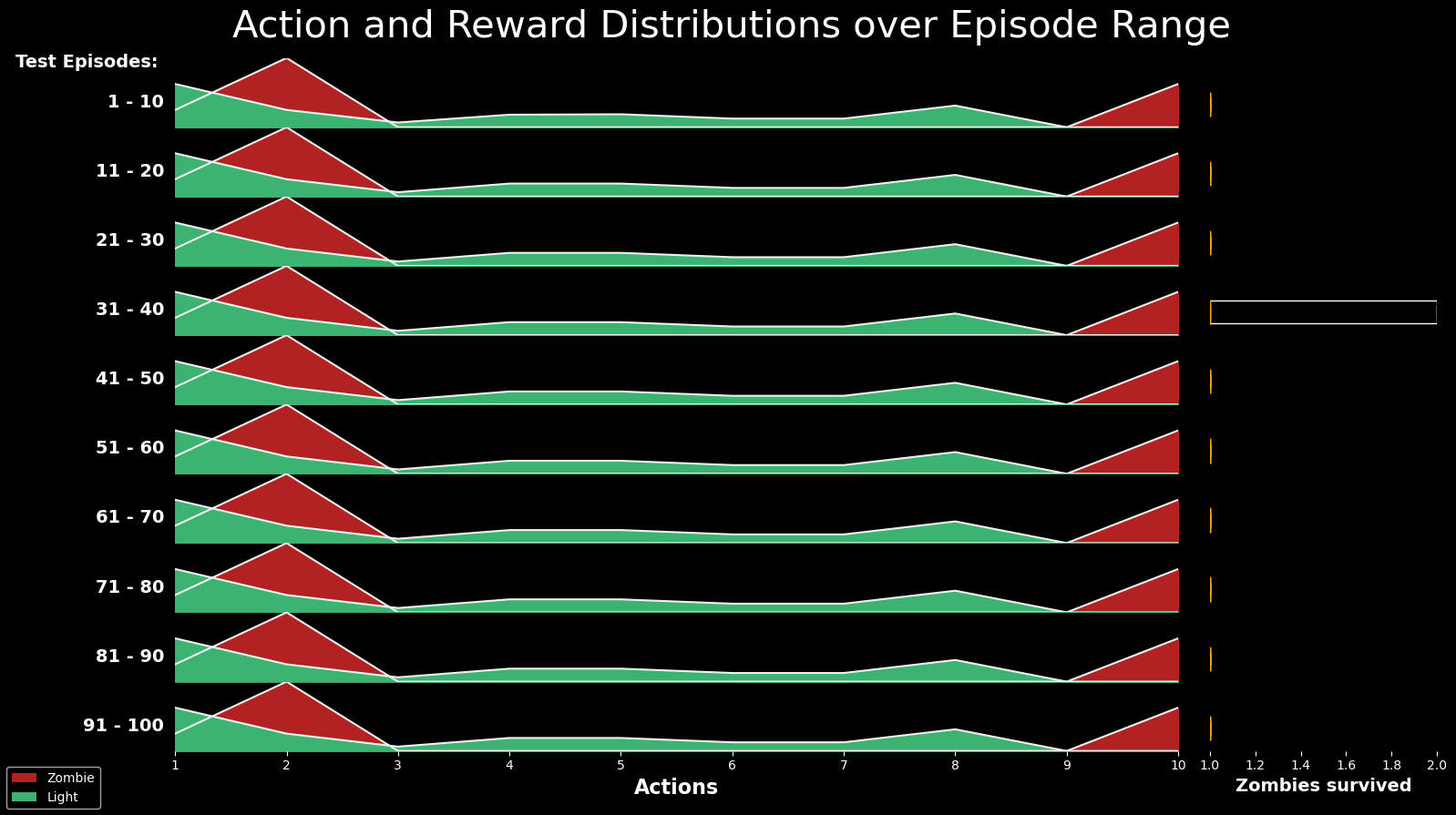


Figure 32 - Action and Reward Distributions, board 10x10

The Zombie Player seems to have chosen to take out zombies from the rows: 2,10 pretty exclusively. In contrast, the light player chose to mark a variety of lines in the range of 1-8, these are smart choices. It is not surprising that the light player has achieved more significant results throughout all games.

In general, it seems that the light player knew how to hit all the lines that would lead to significant damage to the zombies, since the light size is 3X3.

On the right side of the graph, you can see the number of zombies (this value is based on the reward in all games) that survived throughout the episode range, in most runs, the value is stable on one zombie surviving in each episode.

##### Analysis of 20x20 board

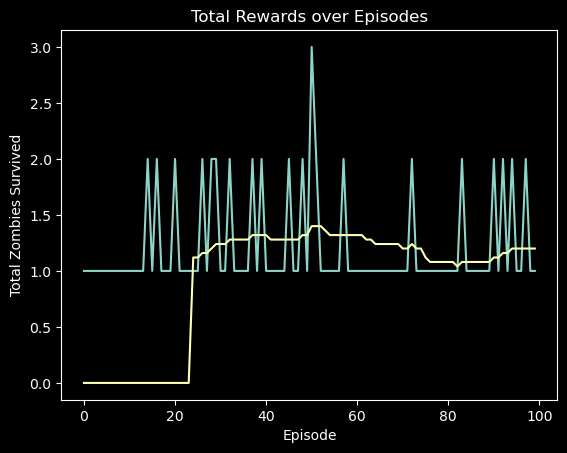


Figure 33 - Rewards over Episodes, board 20x20

This time, the picture looks almost the same, there is a significant victory in favor of the light player – DDQN agent. The players' rewards are stable throughout the scenario on the mean of 1.

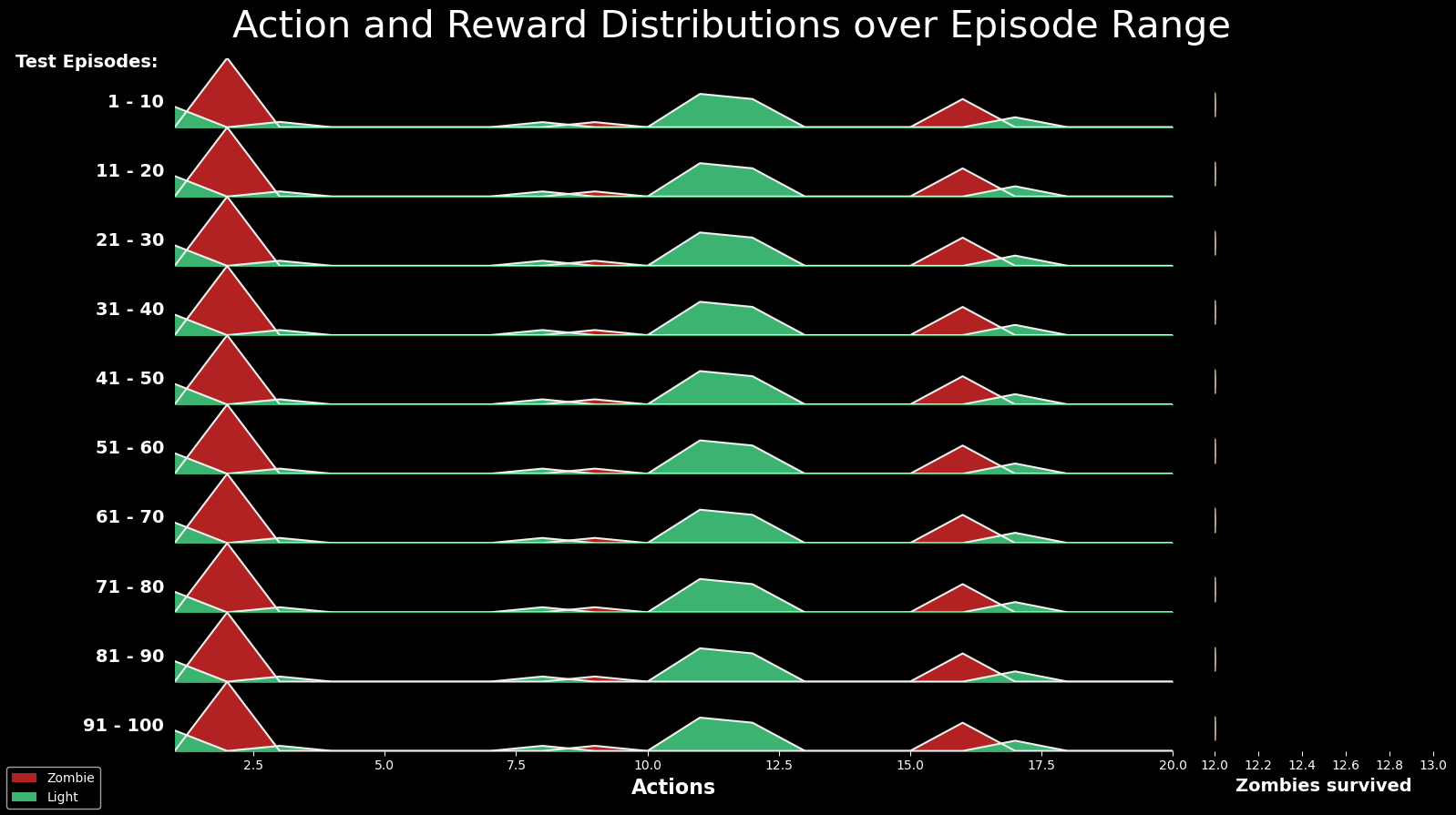
We will now look at the distribution of agents' actions: 

Figure 34 - Action and Reward Distributions, board 20x20

The Zombie Agent (AlphaZero) has chosen two main strategic locations: the second and 16th rows. While the light player has chosen to place the light mainly in rows 1, 11 and 12. these rows have the potential to critically hit zombies, since the light size in this board is 6X6.

##### Analysis of 30x30 board

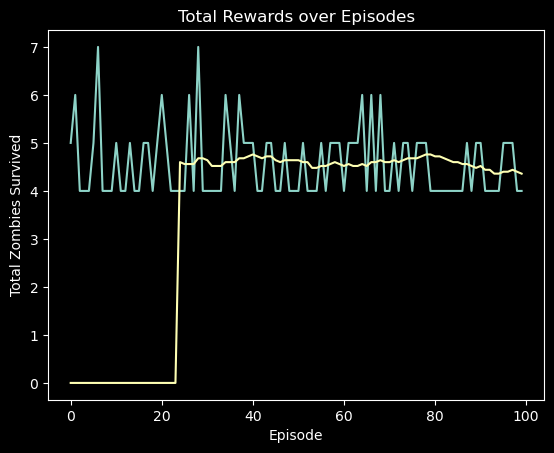


Figure 35 - Rewards over Episodes, board 30x30

The reward graph shows that once again, the scenario tends in favor of the light player – DDQN. The average of the zombies that survive in the episode, throughout the games is stable at 4.5 out of 20.

We will now look at the distribution of agents' actions: 

Figure 36 - Action and Reward Distributions, board 30x30

The Light Player, DDQN, chose to place the light in several rows on the board and significantly in: 4, 8, 16, 18, 22, 30. While the Zombie Player chose to send zombies from row 22 exclusively.

The Light Player's state processing was not good enough in this case. Zombies came out from line 22 and yet the Light Player chose to mark mainly the lines won't reward him at all (line 8 was chosen the most, with no ability to hit zombies).

Despite this, the light marking hit the zombies when placed in lines 12-18, and that was enough to get a significant advantage and win this scenario.

#### AlphaZero as Light Player vs DDQN as Zombie Player

##### Analysis of 10x10 board

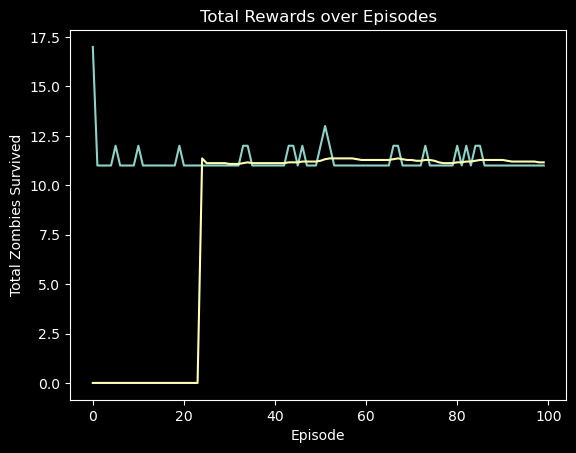


Figure 37 - Rewards over Episodes, board 10x10

Unlike the analyzes in the [previous section](#_DDQN_as_Light), this time one can see the advantage of the zombie player whom is none other than - DDQN. The reward value in each episode stabilizes at 11.5 which means that on average there are 11.5 zombies who successfully pass the game board out of 20.

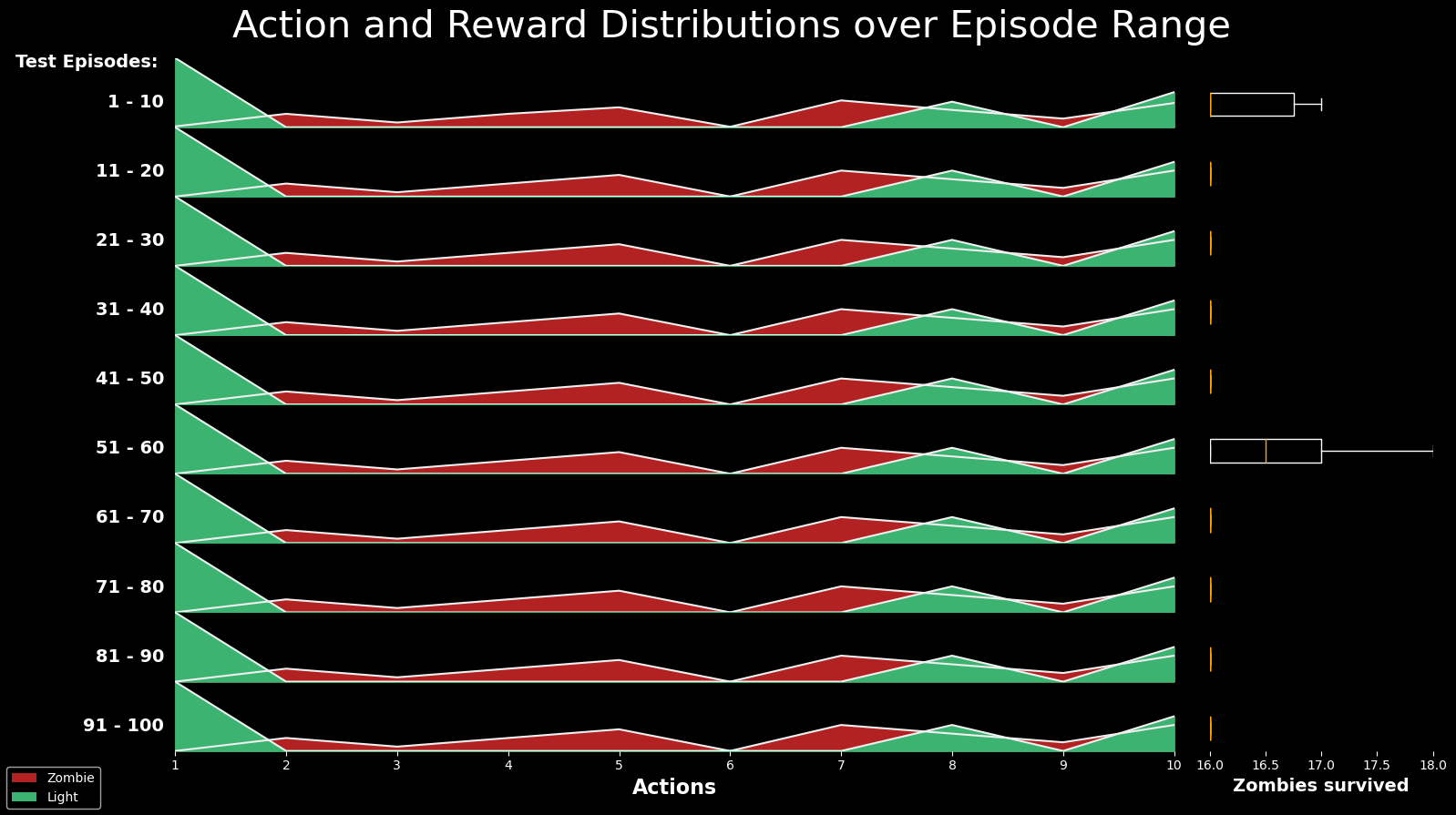
We will now look at the distribution of agents' actions: 

Figure 38 - Action and Reward Distributions, board 10x10

According to Figure 39, the Zombie Player chose to take zombies out of rows 2,4,5,7,9,10 mostly. While the light player chose to mark the lines: 1, 8 and 10 exclusively.

Most of the zombies in the game advanced on lines 5 and 7, the light player did not choose to mark these lines at all for the adjacent ones (remember that the light size in this case is 3X3).

##### Analysis of 20x20 board

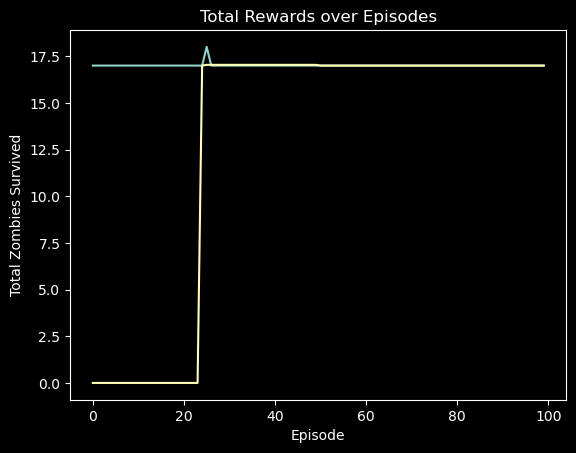


Figure 39 - Rewards over Episodes, board 20x20

A significant victory of the Zombie Player can be seen in the case of a 20X20 board, there is a stabilization of the number of zombies survived in each game, 17 out of a maximum of 20.

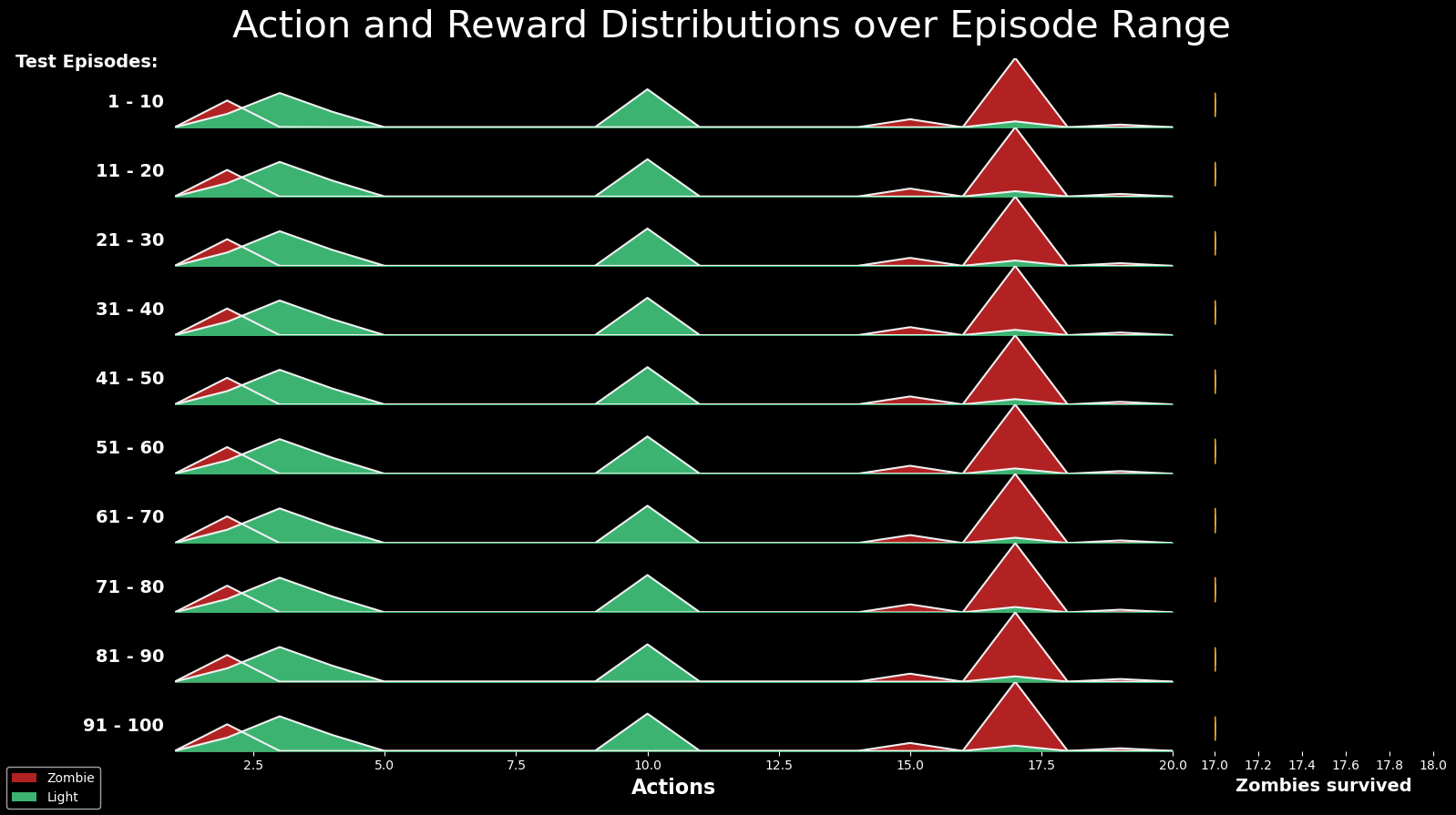
We will now look at the distribution of agents' actions: 

Figure 40 - Action and Reward Distributions, board 20x20

According to Figure 41, The light player chooses to place his cursor in positions 2,3,4 and 10 while the Zombie Player takes an absolute majority of the zombies out of lines 2,15 and especially 17.

Since the Light marker size is 6X6 in the current scenario - Indeed, the Light Player (AlphaZero Agent) fails to find and hasn't harm zombies almost at all.

##### Analysis of 30x30 board



Figure 41 - Rewards over Episodes, board 30x30

Once again, there is almost a balance between the successes of the agents with a tendency in favor of the Zombie Player (the DDQN agent).

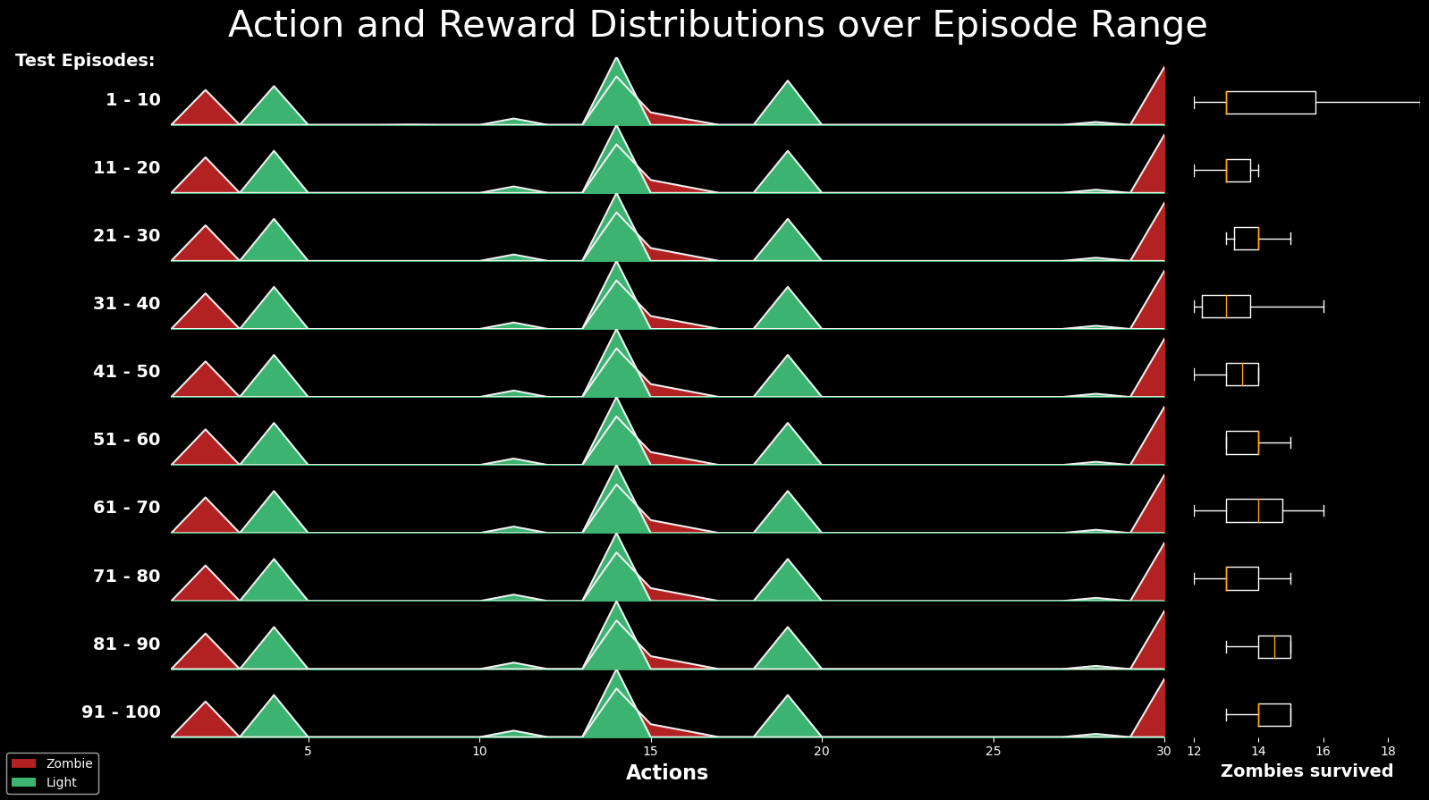
We will now look at the distribution of agents' actions: 

Figure 42 - Action and Reward Distributions, board 30x30

According to Figure 42, the light agent chose to place the cursor in rows 4,14,19,27 while the zombie agent chose to take out the zombies in rows 2,14,15,30.

If we look at the light agent's row selection zones, there seems to be a high overlap with the zombies coming out, in rows 14-15. This overlap led to the same relatively balanced result but the victory of the zombies is still significant, since, the light player did not treat / hit at all two major concentrations of the zombies in rows 2 and 30.

### Summary

By taking into consideration all the results we have seen, the DDQN agent seems to overcome the AlphaZero agent, where they play as Zombie and Light players accordingly.

The DDQN's victory in these scenarios is suspiciously unequivocal, since its [performance against the simple agents](#_Double_Deep_Q-Network) was not as good. This is explained by the fact that the AlphaZero agent was trained, but apparently did not reach his potential peak because his training course is more complex and takes much longer.

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## Appendix A – Elaboration of Related Literature

This section provides further elaboration on the basic assumptions of modeling in the field of reinforcement learning. All the ideas presented here have been researched but will not be reflected in our project.

### Reinforcement Learning

Reinforcement learning is an area of Machine Learning. It is about taking suitable action to maximize reward in a particular situation. Essentially an agent (or several) is built such that it can perceive and interpret the environment in which is placed, furthermore, it can take actions and interact with it.  
Basic reinforcement learning problems are modeled as a Markov Decision process (MDP) which is a 4-tuple , where:

* is a finite set of states.
* is a finite set of actions.
* is the probability that action  in state at time will lead to state , due to action .
* is the immediate reward (or expected immediate reward) received after transitioning from state to state , due to action .

The goal is to learn a policy that maximizes the cumulative sum of discounted rewards

where are the rewards and is a discount factor, tuning parameter through which we can influence the amount of weight we give to future awards in relation to the immediate reward.

We can split the subject of RL into two main partitions: ***Model-Free*** and ***Model-Based***. In Model-Free RL,  
the agent does not have access to a model of the environment (The agent couldn’t estimate the consequences of his actions). In Model-Based RL, the agent has access to a model of the environment.  
Our focus is on the Model-Free type of learning mainly due to the advantage that it doesn’t require a model of the environment.

The Model-Free learning can be considered as two parts of ***off-policy***learning and ***on-policy***learning. an agent might be acting using one or two control policies. In *on-policy* control the agent is evaluating and simultaneously improving the exact policy that it follows. Conversely, in *off-policy* control, the agent is following one policy, but may be evaluating another – it is following a behavior policy while evaluating a target policy. In our work we will implement some off-policy algorithms alongside an algorithm from the tree search area called MCTS for comparison and evaluation.

### *Stochastic Games*

In this paper, two-player zero-sum Stochastic Games (SGs) are considered. These games proceed like MDPs, with the exception that in each state, both players select their own actions simultaneously, which jointly determine the transition probabilities and their rewards. The zero-sum property restricts that the two players’ payoffs sum to zero.

A *Stochastic Game* (SG) is a tuple , Where:

* N is the number of the players/agents
* T: is the transition function
* is the action set for the player
* is the discount factor
* : is the reward function for player

The objective of the n agents is to find a deterministic joint policy (aka. joint strategy aka. strategy profile) (where ) so as to maximize the expected sum of their discounted payoffs. The Q-function, , is the expected sum of discounted payoffs given that the agents play joint action in state and follow policy thereafter. The optimal -function ­ is the -function for (each) optimal policy . So, captures the game structure. The agents generally do not know in advance. Sometimes, they know neither the payoff structure nor the transition probabilities.

For example, consider a zero-sum game with two players, one player (Player 1) wants to maximize his/her total reward, the other (Player 2) would like to minimize that amount. Similar to the case of MDPs, the reward can be discounted or undiscounted, and the game can be episodic or non-episodic.

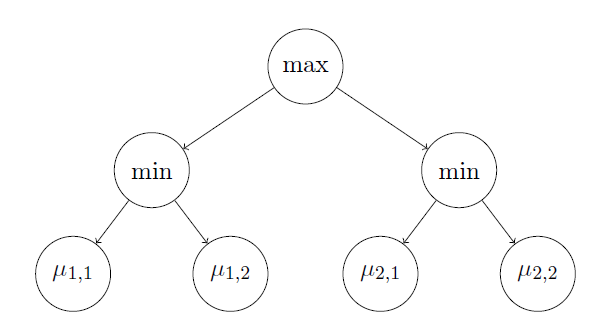


Figure 43 – Game tree when there are two actions by player

We consider a two-player two-round zero-sum game, in which player A has available actions. For each of these actions, indexed by , player B can then choose among possible actions, indexed by . , when player A chooses action and then player B chooses action j, the probability that player A wins is . We investigate the situation (see Figure 43 for an example) from the perspective of Player A, who wants to identify a maximin action

Assuming that Player B is strategic and picks, whatever A’s action , the action minimizing , this is the best choice for A.

### *Nash Equilibrium in SGs*

"In Game Theory, A Nash Equilibrium is a stable state of a system that involves several interacting participants in which no participant can gain by a change of strategy as long as all the other participants remain unchanged"   
Princeton University

A Nash equilibrium is a joint strategy where each agent’s is a best response to the others. For a stochastic game, each agent’s strategy is defined over the entire time horizon of the game.

Given a with players, a Nash Equilibrium is a tuple of strategies such that for all and ,

Where, is the set of strategies available to agent , And,

.

Is the discounted sum of rewards, with discount factor .

A Nash equilibrium is strict if the inequality above is strict. An optimal Nash equilibrium is a Nash equilibrium that gives the agents the maximal expected sum of discounted payoffs.

In the literature, SGs are typically learned under two different settings, and we will call them online and offline settings, respectively. In the offline setting, the learner controls both players in a centralized manner, and the goal is to find the equilibrium of the game [9]. This is also known as finding the worst-case optimality for each player (a.k.a. maximin or minimax policy). In this case, we care about the sample complexity, i.e., how many samples are required to estimate the worst-case optimality such that the error is below some threshold. In the online setting, the learner controls only one of the players, and plays against an arbitrary opponent [10]. In this case, we care about the learner’s regret, i.e., the difference between some benchmark measure and the learner’s total reward earned in the learning process. This benchmark can be defined as the total reward when both players play optimal policies [3], or when Player 1 plays the best stationary response to Player 2. Some of the above online-setting algorithms can find the equilibrium simply through self-playing.

### *Learning in SGs*

Learning in stochastic games can be formalized as a multi-agent reinforcement learning (MARL) problem. we can say that the goal of RL is to learn equilibrium strategies through interaction with the environment.

Our work focuses on competitive settings with partially-observable MARL that has received limited attention throughout the years [2]. There were works include model-free gradient-ascent based method [3][4], simulator-supported methods to improve policies using a series of linear programs [5], Recent scalable methods use Expectation Maximization to learn finite state controller (FSC) policies [6].

The most interesting approach I've found related to our problem of competitive relation between the agents and partial observability framework is described in DEC-HDRQNS [2], that means a Decentralized Hysteretic Deep Recurrent Q-Networks model. Their approach is model-free and decentralized, learning Q-values for each agent. In contrast to policy tables or FSCs, Q-values are amenable to the multi-task distillation process as they inherently measure quality of all actions, rather than just the optimal action.

The proposed approach takes into consideration the concept of Hysteresis (lag) [8].  
Overly-optimistic MARL approaches completely ignore low returns, which are assumed to be caused by teammates’ exploratory actions. This causes severe overestimation of Q-values in stochastic domains.  
Hysteretic Q-learning, instead, uses the insight that low returns may also be caused by domain stochasticity, which should not be ignored. This approach uses two learning rates: nominal learning rate, α, is used when the TD-error is non-negative; a smaller learning rate, β, is used otherwise (where 0 < β < α < 1).