Mathematical Olympiads Discord Server

2019 July Beginner Contest

Time: 4 hours *Each problem is worth 7 points*

Calculators and protractors are not allowed. Do not write your name on your working. At the end of the contest, please scan your solutions and working and send them to `Tony Wang#6285` *via direct message. Do not discuss the contents of this paper outside the text channel* `#finished-contestants` *and the voice channel* `Post-Contest Banter` *until notified by staff.*

Problem 1. Amy writes a 1 or a -1 in each cell of an $n \times 2$ grid, where n is not a multiple of 4. Below each column she writes the product of the 2 entries in that column in blue, and to the right of each row she writes the product of the n entries in that row in green. Amy notices that the sum of the blue numbers is 0. Prove that the sum of the green numbers is 0.

Problem 2. Let $ABCD$ be a parallelogram. The internal angle bisectors of ABC and BCD meet at P, and the internal angle bisectors of CDA and DAB meet at Q. Prove that $PQ \parallel AB$.

Problem 3. Determine all positive integers that can be written as the sum of 2 or more consecutive positive integers.

Problem 4. Let $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ be positive real numbers with the property that $a_1 + a_2 + \cdots + a_n = b_1 + b_2 + \cdots + b_n$. Prove that

$$\sum_{i=1}^{n} \frac{a_i^2}{a_i + b_i} \geq \frac{1}{2} \sum_{i=1}^{n} a_i.$$