

Mathematical Olympiads Discord Server

2019 May Intermediate Contest

Time: 4 hours

Each problem is worth 7 points

Calculators and protractors are not allowed. Do not write your name on your working. At the end of the contest, please scan your solutions and working and send them to Sharky Kesa#9845 via direct message. Do not discuss the contents of this paper outside the text channel #finished-contestants and the voice channel Post-Contest Banter until notified by staff.

Problem 1. Find all positive integers a and b such that $a^2 + 2b^2$ is a power of 2.

Problem 2. Let \mathbb{R} denote the set of real numbers. Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ such that for all real numbers x and y ,

$$f(xf(x) + f(y)) = xf(x + y).$$

Problem 3. Let n and k be given positive integers. Find the number of k -tuples (S_1, S_2, \dots, S_k) of sets S_i such that $S_i \subseteq \{1, 2, \dots, n\}$ and $S_1 \subseteq S_2 \supseteq S_3 \subseteq S_4 \supseteq S_5 \subseteq \dots \subseteq S_k$.

Problem 4. Let Γ be the circumcircle of $\triangle ABC$. O lies on the internal angle bisector of $\angle BAC$ such that a circle centred at O is tangent to the segment BC at P and the arc BC of Γ without A at Q . Prove that $\angle PAO = \angle QAO$.