Mathematical Olympiads Discord Server

2020 January Beginner Contest

Problem 1. What is the minimum number of folds needed to fold a paper circle into a polygon?

Problem 2. The Euclidean plane is initially white. On each move, Luciano chooses a system of Cartesian coordinates and paints the region where $y \geq x^2$ in red. Is it possible for the Euclidean plane to be painted completely red after a finite number of moves?

(A system of Cartesian coordinates may be chosen at any angle or scale, and with any point as the origin, but the two axes must be perpendicular.)

Problem 3. Prove that for any positive real number c, there exists a positive integer n such that the sum of the factors of n is greater than cn.

Problem 4. Let $\mathbb{R}^+$ denote the set of positive real numbers. Find all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ such that $\lfloor f(x)f(y) \rfloor = \lfloor xy \rfloor$ for all positive real numbers x and y.

Language: English

Time: 4 hours
Each problem is worth 7 points