Development of an Economic Scenario Generator for asset liability modelling of UK pension schemes.

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Executive Summary

The purpose of this thesis is to develop a mathematical model that will address a number of issues currently faced by Quantum Advisory LLP, an independent actuarial and employee benefits consultancy.

Pension fund modelling

Quantum Advisory provides a range of investment consultancy services to pension scheme trustees, which includes advising on scheme strategic asset allocation. To provide this service, Quantum Advisory uses Asset and Liability Modelling (ALM) to review the risk and return of different strategies.

ALM software uses stochastic analysis based on a wide range of plausible economic scenarios to quantify the probability of different events occurring over a defined time horizon. For example, the probability of a pension scheme's assets being worth less than its liabilities after five years.

Currently Quantum Advisory purchases economic scenarios from an external source on an annual basis. One drawback of this approach is that the external scenarios do not necessarily reflect the investment team's own views on the economy, causing potential conflicts between ALM outputs and advice. Therefore, the company is seeking to bring this capability in-house.

The aim of the project is to develop a mathematical specification for an Economic Scenario Generator (ESG). An ESG uses Monte-Carlo simulation to produce a large number of plausible economic scenarios, which are to be used by Quantum Advisory for the stochastic analysis of pension scheme assets and liabilities.

The software currently used by Quantum Advisory is provided by a multinational risk management advisory firm. This software uses advanced models to efficiently model pension funds. Within the software, scenarios are generated and results are used in order to simulate the pension fund growth.

Economic Scenario Generator

Quantum Advisory pension fund forecasts are based on the generated scenarios. The ALS provides a large range of scenarios that simulate the long term economy. Since it uses a Vector Auto Regression model (VAR), the same model will be used to generate scenarios. Both models have the ability of including our view on the future. The VAR model is used to find complex relationships among variables; it will measure the correlations, cross-correlations and auto-correlations among variables. However, multicollinearity can become a problem as more variables and lagged values are added to the model as Gordon Schlegel (1985, p.3) observes:

> A high degree of multicollinearity will make difficult to determine which explanatory variables are significant, since the standard errors of the coefficient estimates will tend to be large.

Acknowledgments Page

As a conclusion of my master's programme of Operational Research, Applied Statistics and Risk at the Cardiff University, an internship has been performed at Quantum Advisory, an actuarial and employee benefits consultancy.

The internship lasts three months. It gave me the opportunity to explore the theoretical aspects of finance, actuarial science and it practical aspect with different tasks I have got involved in. Before describing the subject of my project, I would like to thanks Joanna Emery and Paul Harper from Cardiff University, for giving me the opportunity of writing my dissertation into a professional environment, my supervisors Jonathan Thompson and Rhyd Lewis from Cardiff University, Deon Dreyer and Dan Redwood, my supervisors at Quantum that lead me during my internship and my colleagues from the investment team Amanda Burdge, Robert Davies, Scott Edmunds, Jayna Gandhi and Jordan Griffiths.

Last but not least I want to thanks my colleagues at Quantum Advisory, who were always friendly and interested during this internship.

Elie Sadaka, Cardiff, September 8, 2014.

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The aim of the project is to develop an Economic Scenario Generator.

Currently Quantum Advisory LLP are purchasing these scenarios from a multinational risk management advisory firm. Building the model in-house will mean the scenarios generated will reflect investment team's view of the economy and reduce Quantum Advisory's software expenses.

In order to build the model, two software programs will be used; R and Excel. Econometric models will also be analysed and tested in order to get relevant results.

Chapter 1. Introduction

1.1 Problem statement and Economic Scenario Generation

As an actuarial and employee benefits consultancy, Quantum Advisory is required to provide an anticipation value of their clients' funds over time. Anticipation is made by deciding what economic scenarios could happen over time. Currently, Quantum Advisory is renting software that can generate thousands of realistic scenarios based on different economic assumptions. From a multitude of scenarios, a range of them that model their view of the economy will be selected and thus forecast the final fund level after a certain period of time.

The disadvantage of this software is that these scenarios do not always reflect the investment team's own views on the economy, potentially causing conflicts between a scenario's result and the advice provided. Developing such a generator in-house would also reduce expense on the investment department.

To create the ESG, a mathematical plan needs to be developed. The right model has to be used and tested in order to the maximise realism of the model. Different models will be tested like the Vector AutoRegression and the Generalized AutoRegressive Conditional Heteroskedasticity (GARCH). These models are well-used in Finance to characterize and model time series. Their characteristics will be discussed and analysed in this report. Once the right model has been identified, evaluations will be run in order to benchmark its quality.

Chapter 2. Statistical and Financial foundations

To model the variables' volatility, computations will have to be made with the variables' data. Returns will have to be computed from index data.
In this section, definitions and explanations of financial notions will be given.

2.1 Returns

The return, usually quoted as a percentage, is the gain or loss of an asset during a certain period of time. It can be seen as the gain of an investment. The return is calculated as below:

Let the return r_i at time i, where p_j is the price at time j and i $\equiv (i-1)$:

$$r_i = \frac{p_i - p_j}{p_j}$$

Using returns instead of financial variable prices is advantageous as it transforms all variables in comparable metrics and therefore facilitates relationships evaluation.

In financial mathematics, logarithm returns are often used because they are symmetric, on the contrary of ordinary returns. Using ordinary returns means that positive and negative growth of equal amount will not cancel each other out and result in no net change, while logarithmic returns of equal amount will cancel each other out.

This difference can be explained with the example of an investment of £100 that yields an arithmetic return of 50% followed by an arithmetic return of -50% will result in £75, while an investment of £100 that yields a logarithmic return of 50% followed by a logarithmic return of -50% will come back to £100.

When we deal with trades with short durations, returns are usually very small. The approximation below gives values close to initial returns:

$$\log(1 + r) \approx r,\ \ r \ll 1.$$

2.2 Measure of Risk

In finance, volatility gives the variation of price of a financial asset over time. Different tools are used in order to measure the risk or the volatility of assets.

2.2.1 Standard deviation of return

Standard deviation of returns is a measure of volatility. The larger the standard deviation is, the larger the variations you get in returns.

Standard deviation is usually applied to the annual rate of return of an investment in order to measure its volatility. A volatile asset will have a high standard deviation while a stable asset a lower value.

The symbol for Standard Deviation is σ and is calculated using:

$$\sigma = \sqrt{\frac{\sum(x-\mu)^2}{N}}$$

Where x represents the daily return on an asset, μ the mean return throughout a period, and N is the number of days.

The annual standard deviation is defined by:

$$\sigma_{\text{annual}} = \sigma_{\text{daily}} \times \sqrt{260}$$

Where 260 is the number of working days throughout the year.

2.2.2 Value at Risk

Value at risk (or VaR) measures the potential loss in value of an asset or portfolio over a defined period for a given confidence interval. It has three components: a time interval, confidence interval, and a loss amount. The value at risk of the deficit for a pension scheme is one of the key outputs when we perform asset and liability management work.

There are three different methods to predict the value at risk, delta-normal, historical simulation, and the Monte Carlo method. The delta-normal assumes that all asset returns are normally distributed, while the historical simulation method consists of going back in time and applying current weights to a time-series of historical asset returns. The Monte Carlo method assumes that future prices need to be defined according to probability distribution. The ESG will provide the scenarios, enabling the third method to be used in the ALM

The Monte Carlo method

There are two different approaches to forecast the future; deterministic and stochastic. Models that assume a fixed relationship between inputs and that always produce the same output are called deterministic. Models that depend on random inputs which provide a distribution of probable results are called stochastic.

Kritzman (1993, p21) provides an example to explain this difference:

> A model that predicts an eclipse, for example is deterministic, because it relied on known fixed laws governing the motions of the earth, the moon and the sun. You are unlikely to hear an astronomer say that there is a 30% chance of an eclipse next Wednesday. A model that predicts tomorrow's weather, however, is stochastic, because many uncertain elements influence the weather.

The Monte Carlo simulation involves a stochastic process. It will not use past price movement to forecast future prices. This process suggests that forecasting future prices need to be expressed in terms of probability distribution and will not be affected by past prices' value.

However, the process requires acquiring different parameters based on past experiences such as the cross-correlation and auto-correlation between variables and their volatility. These parameters must be obtained in order to create the ESG.

Chapter 3. Volatility Modelling and Forecasting

3.1 Time series analysis concept

Box, Jenkins and Reinsel (2008) define time series as:

…a sequence of observations taken sequentially in time.

And describe its main features:

> An intrinsic feature of a time series is that, typically, adjacent observations are dependent. The nature of this dependence among observations of a time series is of considerable practical interest. Time series analysis is concerned with techniques for the analysis of this dependence. This requires the development of stochastic and dynamic models of time series data and the use of such models in important areas of application.

Financial investors often need predictions of economic variables. If a time series is available for a variable and the data from the past contains information about its future development, a certain function could be used to forecast the future value from collected past data. For example, a high inflation rate in one month is usually followed by high rate the next month. It can be depicted as follows:

Let's denote y_t the value in period t of the variable of interest. At the end of the period T, a forecast period of T+h may have the form of:

$$\hat{y}_{T+h} = f(y_T, y_{T-1}, \dots),$$

Where f(.) expresses a function of past observations y_T, y_{T-1}.

With economic variables, the value of one variable can be related to past values of other variables. Expressing the related variables by $y_{1t}, y_{2t}, \dots, y_{Kt}$ the forecast of $y_{1,T+h}$ at the end of T can be written:

$$\hat{y}_{1,T+h} = f_1\left(y_{1,T}, y_{2,T}, \dots, y_{K,T}, y_{1,T-1}, y_{2,T-1}, \dots, y_{K,T-1}, y_{1,T-2}, \dots\right).$$

Note that the cap sign (^) indicates an estimated value.

In general, a forecast of the k-th variable may be written as:

$$\hat{y}_{k,T+h} = f_k\left(y_{1,T}, \dots, y_{K,T}, y_{1,T-1}, \dots, y_{K,T-1}, \dots\right).$$

A set of time series y_{kt}, k=1,...,K, t =1,...,T, is called a multiple time series and the last formula depicts the forecast $\hat{y}_{k,T+h}$ as a function of a multiple time series.

The number of chosen variables will depend on the aim of the analysis, depending on the interrelationships we want to explore.

As an example, the Swiss National Bank (2006, p5) uses four variables in order to predict the country's inflation, Consumer Price Index, M2, M3 and the real exchange rate of Switzerland.

3.2 Auto Regression

The economic scenario generator is based on a Vector AutoRegression model that is a multivariate variant of the AutoRegression model. We will introduce the basics of the Auto Regression model in order to describe the Vector Auto Regression analysis that will be used by the scenario generator.

The AR model will estimate a variable from its previous value. An AR(p) model is an auto regression model with an order of p, p is the number of its previous values the model will use to estimate a variable. AR(p) is given by this definition:

$$x_t = c + \sum_{i=1}^{p} \varphi_i \, x_{t-i} + \varepsilon_t$$

Where c is a constant, $\varphi_i, \dots, \varphi_p$ are the parameters of the AR(p) model and ε_t is an error term. This error term is a stochastic variable that has no inter-temporal correlation and a mean of 0. A stochastic process with these characteristics is called a white noise process and has the following attributes:

$$\mathbb{E}[\varepsilon_t] = 0$$

$$\mathbb{E}[\varepsilon_t^2] = \sigma_\varepsilon^2$$

$$\mathbb{E}[\varepsilon_t \varepsilon_{t-k}] = 0 \text{ for } k \neq 0$$

The parameters φ_i of the AR(p) model can be calculated using the Yule-Walker equations:

$$\gamma_m = \sum_{k=1}^{p} \phi_k \gamma_{m-k} + \sigma_\epsilon^2 \delta_m \text{ , m = 0,..., p,}$$

with $\gamma_m = \mathbb{E}[x_t x_{t-m}]$ the autocorrelation function of x_t, δm the Kronecker delta function, which equals 1 if m = 0 and 0 otherwise. This result in p + 1 Yule Walker equations that allow to write the set of equations as a matrix equation for m > 0:

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} \gamma_0 & \gamma_{-1} & \gamma_{-2} & \cdots \\ \gamma_1 & \gamma_0 & \gamma_{-1} & \cdots \\ \gamma_2 & \gamma_1 & \gamma_0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \vdots \end{bmatrix}$$

3.3 Vector AutoRegression

A Vector AutoRegression model is a generalization of the AutoRegression model and is used to capture the interdependencies among multiple variables.

Let X be a k x 1 vector where k is a variable of a dataset, X_t is the status of the vector at time t. Then $x_{i,t}$ describes the value of the variable i at time t. VAR(p), a p-th order VAR is defined by:

$$X_t = c + \sum_{i=1}^{p} A_i X_{t-i} + \varepsilon_t$$

where c is a k × 1 vector of constants, A_i are k × k matrices (i = 1, . . . , p) , and ε is a k × 1 vector of white noise error terms. Each element of the error term has no inter-temporal correlation between consecutive error terms, a constant variance and a mean of 0. To illustrate this further, the example below uses a set of equations for a 2 variables VAR(2) model:

$$x_{1,t} = c_1 + a_{11}^{(1)} x_{1,t-1} + a_{12}^{(1)} x_{2,t-1} + a_{11}^{(2)} x_{1,t-2} + a_{12}^{(2)} x_{2,t-2} + \varepsilon_t$$

$$x_{2,t} = c_2 + a_{21}^{(1)} x_{1,t-1} + a_{22}^{(1)} x_{2,t-1} + a_{21}^{(2)} x_{1,t-2} + a_{22}^{(2)} x_{2,t-2} + \varepsilon_t$$

where $a_{ij}^{(k)}$ is the element in row i and column j of parameter matrix A_k . The variable $x_{i,t}$ at time t is explained in terms of two lags of itself ($x_{1,t-1}$ and $x_{1,t-2}$) and by two lags of the other variable ($x_{2,t-1}$ and $x_{2,t-2}$). As with the AR(p) model, we can use the Yule-Walker equations to estimate each parameter of A_i matrices. Let first define $\Gamma_i = \mathbb{E}[X_t X_{t-i}]$ as the cross-correlation function of X_t. The parameter matrices A_i are the solutions to the following system of Yule-Walker equations:

$$\Gamma_i = \sum_{i=1}^{p} A_i \Gamma_{i-1} \text{ for i = 1,…,p;}$$

$$\Gamma_0 = A_1 \Gamma_{-1} + \Sigma$$

where Σ denotes the vector of variances of the white noise error terms.

3.4 ARCH/GARCH

Many financial time series have a tendency to volatility clustering (periods of high and low market uncertainty). On the contrary of the vector autoregression model that assumes a constant variance over time, the AutoRegressive Conditional Heteroskedasticity model (or ARCH) can model this specificity. These models are used to illustrate changing of variance. They are mainly used for financial decisions concerning risk analysis.

There are a large number of acronyms applied to particular structures of models that have a similar basis. According to Reinhard Hansen and Lunde (2005, p27), the most powerful model to forecast future value of a variable is the Generalized AutoRegressive Conditional Heteroskedasticity model developed by Bollerslev (1986) of parameter (1,1). The (1,1) in parentheses is a standard notation where the first number refers to the number of autoregressive lags used in the equation and the second number is the number of lags included in the moving average component of a variable.

According to Richard Minkah (‘Forecasting Volatility’, 2007), GARCH(1,1) provides relevant forecasts only for short forecasting horizons (up to 120 days). As our scenario generator is required to forecast over a very long period of time, the GARCH model is not suitable in this particular scenario and will not be used.

Chapter 4. Data Analysis

4.1 Description of data

The variables defined will depend on the aim of the modelling. Our dataset contains three kinds of variables:

- Equity that contains seven worldwide Morgan Stanley Capital International (MSCI) Indexes,
- Alternatives with three indexes, United Kingdom Property, Hedge Fund of Funds, Commodities,
- Rates with the UK Consumer Price Index, UK Retail Price Index, UK Nominal and index linked bond.

4.1.1 Equity and Alternatives Indexes

By analysing various pension scheme funds, we have found that these indexes are representative of the asset classes that are available to Quantum clients. These Indexes (defined in appendix 2) and bonds return derived from yield curves will tell us how pension scheme assets fluctuate over time.

Table 4.1 outlines equity and alternative index variables used in the ESG and the data source for each.

Variable		Data source
Equity		
	UK equity (GBP)	MSCI UK total return index
	North American equity (USD)	MSCI North America total return index
	Eurozone equity (EUR)	MSCI EMU total return index
	Japanese equity (JPY)	MSCI Japan total return index
	Pacific (ex Japan) equity (USD)	MSCI Pacific (ex Japan) total return index
	Emerging Markets equity (USD)	MSCI Emerging Markets total return index
	Global equity (USD)	MSCI World total return index
Alternatives		
	UK Property (GBP)	UK IPD all property total return index
	Hedge Fund of Funds (USD)	HFRI Fund Weighted Composite index
	Commodities (USD)	DJUBS Commodities total return index

4.1.2 Yields

Our model will include bonds based on short, middle and long term interest rate. A nominal 1, 5, 10, 15, 20 and 25 years Treasury Bonds Yields, and real 5, 10, 20, 25 years Treasury Bond Yields will be added to the dataset.

The data used is based on a yield curve fitted to the yields of UK government bonds (gilts) currently in issue. This is important to the ALM for a number of reasons:

- The discount rates used to value scheme liabilities are based on gilt yields (e.g. a common discount rate is the yield on 20 year gilts).
- These yields are used to model the return on a portfolio of gilts.
- The real yield curve on index-linked gilts can be used to estimate market expectations of future inflation. This can be used to value inflation linked liabilities.
- Finally, the interest rate is used to model the return on derivatives such as swaps used to hedge interest rate.

In our case, real yields are modelled so that we can derive the market's expectations for future inflation (e.g. the market expectation of inflation over the next 20 years equal 20 year gilt yield minus the 20 year real yield).

Table 4.2 outlines yields and spread variables used in the ESG and the data source for each.

Variable		Data source
	Yields, Spreads	
	UK Nominal interest rates Treasury Bond Yields (1, 5, 10, 15, 20, 25 years)	Bank of England
	UK Real interest rates Treasury Bond Yields (5, 10, 20, 25 years)	Bank of England

4.1.3 Inflation indexes

The two main inflation measures used in the UK are the Consumer Prices Index (CPI) and Retail Prices Index (RPI). The estimations of these indexes will vary due to differences in the methodology, services and products used to compute them. The difference between these indexes are that the RPI does not cover the same items as CPI; the CPI excludes items such as charges for financial services. Therefore the CPI does not include housing costs and charges for mortgage interest payments.

Benefits in UK pension schemes are often increased in line with an inflation index – either CPI or RPI, which needs to be allowed for in ALM. These are also key economic variables which may have strong correlations with other variables.

Table 4.3 outlines inflation indices variables used in the ESG and the data source for each

Variable		**Data source**
	Inflation Indices	
	UK Consumer Price Indices	ONS
	UK Retail Price Index	ONS

4.2 <u>Daily vs Monthly</u>

When we perform a VAR analysis, all variables need to have the same frequency and convert higher-frequency data to the frequency of the lowest-frequency data. For example, if we have daily, weekly and monthly data then we will need to convert everything to monthly frequency. In our case, monthly values will be used since alternatives and inflation indices are monthly based. Another reason of using monthly values is that larger numbers of lags also greatly increase the complexity of the model and the computing requirements.

Our dataset will cover more than 14 years (3789 days), from April 1999 to April 2014.

4.3 <u>Fitting VAR(p) Model</u>

In this section, we will describe the different steps for fitting the optimal AutoRegression model and why a different route will be taken. Finally, results of a VAR with a lag of 1 will be explained.

<u>4.3.1 The optimal lag</u>

The estimation of the lag length of an auto-regressive process is a decisive exercise. In order to find the optimal lag, many criteria have been employed. In brief, an auto-regressive process of lag p refers to a time series where its current value depend on its first p lagged values. The lag length is always unknown and need to be estimated with the help of various criteria such as the Aikaike's information criterion (AIC) (Akaike 1973), the Schwarz information criterion (SIC) (Schwarz 1978), the Hannan-Quinn criterion (HQC) (Hannan and Quinn 1979), the final prediction error (FPE) (Akaike 1969) and the Bayesian information criterion (BIC) (Akaike 1979). These criteria are defined by:

Akaike information criterion: $\mathrm{AIC_p} = -2\mathrm{T}\left[\ln\left(\widehat{\sigma_p^2}\right)\right] + 2\mathrm{p}$

Schwarz information criterion: $\mathrm{SIC_p} = \ln\left(\widehat{\sigma_p^2}\right) + [\mathrm{p}\ln(\mathrm{T})]/\mathrm{T}$

Hannan-Quinn criterion: $\mathrm{HQC_p} = \ln\left(\widehat{\sigma_p^2}\right) + 2\mathrm{T}^{-1}\mathrm{p}\ln[\ln(\mathrm{T})]$

The final prediction error: $\mathrm{FPE_p} = \widehat{\sigma_p^2}(\mathrm{T}-\mathrm{p})^{-1}(\mathrm{T}-\mathrm{p})$

Bayesian information criterion:

$$\mathrm{BIC_p} = (\mathrm{T}-\mathrm{p})\ln\left[(\mathrm{T}-\mathrm{p})^{-1}\mathrm{T}\sigma_p^2\right] + \mathrm{T}\left[1+\ln\left(\sqrt{2\pi}\right)\right] + \mathrm{p}\ln\left[\mathrm{p}^{-1}\left(\sum_{t=1}^{T} y_t^2 - \mathrm{T}\widehat{\sigma_p^2}\right)\right]$$

Where T is the sample size and $\sigma_p^2 = (\mathrm{T}-\mathrm{p}-1)^{-1}\sum_{t=p}^{T}\widehat{\varepsilon_t^2}$, ε_t is the model's residuals.

As Khim-Sen Liew (2004, p. 1) observes:

> Akaike's information criterion (AIC) and final prediction error (FPE) are superior to the other criteria under study in the case of small sample (60 observations and below), in the manners that they minimize the chance of under estimation while maximizing the chance of recovering the true lag length. One immediate econometric implication of this study is that as most economic sample data can seldom be considered "large" in size, AIC and FPE are recommended for the estimation the autoregressive lag length.

Depending on the dataset we are using (daily or monthly), different coefficient of error will be generated. With our dataset that uses monthly data, the lag 1 gets the lowest HQ and SIC and lag 5 the lowest AIC.

4.3.2 VAR(5) limits

A VAR analysis with a lag of 5 will be computed to show the limit of this method. R software will be used to do the computation; R used the S language of coding.

Figure 4.1 R forecast of the next 260 days of the UK RPI.

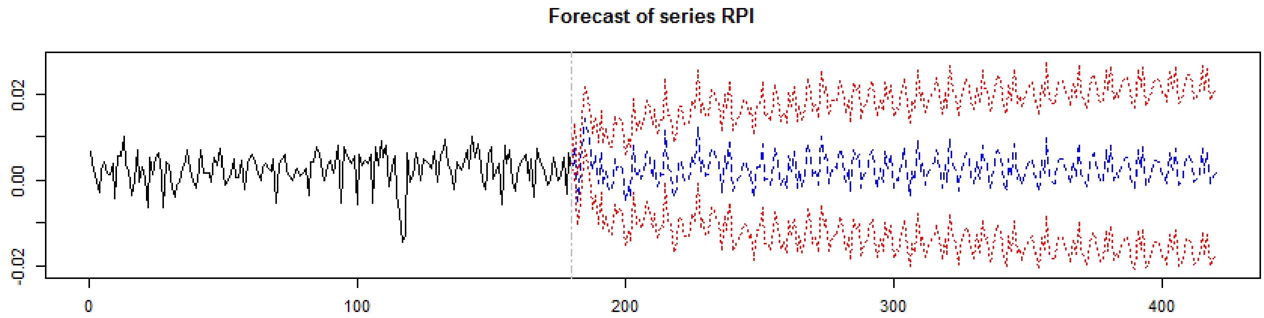
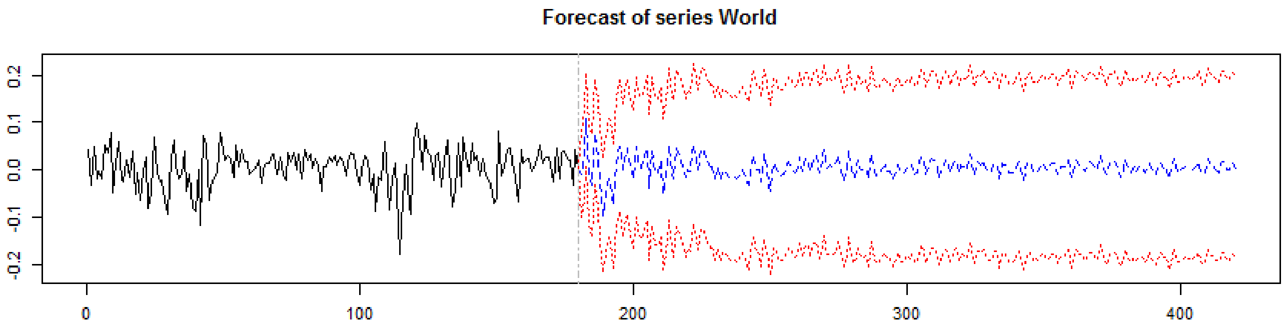


Figure 4.2 R forecast of the next 260 days of the MSCI World Index.



As illustrated in these graphs, the confidence interval of forecasted value is very large, and shows the limitation of using time series modelling for forecasting.

Forecasts in R are not used for the following reasons:

- They don't contain all of the information required in the ALM, since economic scenarios should enable us to calculate in detail the progression of a pension scheme's funding level under each scenario and make estimates of the likelihood of particular events occurring.
- Time series forecasts are purely based on historical data whilst real forecasts will take into consideration other factors.

- The calculation of pension scheme funding level progression under these forecasts is complex. It is much easier to simulate random scenarios and calculate the funding level under each (with the Monte Carlo simulation) than it is to come up with a joint probability distribution of 22 variables and perform calculations based upon that.

Therefore, a different approach will be followed in order to develop the ESG that enable to simulate in detail the progression of pension scheme's funding level.

Chapter 5. Economic Scenario Generation

The Economic Scenario Generation will be divided in two parts. In the first part, a vector auto-regression with a lag of one will be computed and in the second part, outputs from the previous computation will be used in Excel to generate scenarios.

5.1 Vector Auto-Regression

5.1.1 Correlation, auto-correlation and cross-correlation

There are three types of correlations that the VAR model will capture if the model is used with a dataset that contains more than one variable; correlation, auto-correlation, and cross-correlation.

A correlation describes the direction and the strength of the dependency between observations of two random variables. Auto-correlation uses the strength of the dependency between an observation of a random variable and its own previous observations. Instead of analysing previous observations of the variable itself, a cross-correlation analyses previous observations of other variables.

These three correlations gives us crucial information about common variables growth over time. VAR(p) model will evaluate a variable from other variables and the p predecessors of itself. Using this approach enables all of these correlations to be computed.

5.1.2 VAR(1)

A VAR(1) will be computed on R with the dataset of 22 variables. An auto-regression processes with a set of 2 variables is described below; the principle is similar with 22 variables.

Let's consider a VAR(1) model with two variables, Y^1 and Y^2; at time t:

$$Y_t^1 = C^1 + A_{Y^1Y^2}Y_{t-1}^2 + A_{Y^1Y^1}Y_{t-1}^1 + \varepsilon_t^1$$
$$Y_t^2 = C^2 + A_{Y^2Y^1}Y_{t-1}^1 + A_{Y^2Y^2}Y_{t-1}^2 + \varepsilon_t^2$$

Where C^1 and C^2 are constants predicted by the model, $A_{Y^1Y^2}$ is the dependence of variable Y_t^1 on Y_{t-1}^2, $A_{Y^1Y^1}$ is the dependence of variable Y_t^1 on Y_{t-1}^1 and are also predicted values.

Finally, ε_t^i are the simulated residuals, i = 1, 2.

Alternatively, it can be written in matrix form:

$$Y_t = \begin{pmatrix} Y_t^1 \\ Y_t^2 \end{pmatrix} \quad A = \begin{pmatrix} A_{Y^1Y^1} & A_{Y^1Y^2} \\ A_{Y^2Y^1} & A_{Y^2Y^2} \end{pmatrix} \quad C = \begin{pmatrix} C^1 \\ C^2 \end{pmatrix} \quad \varepsilon_t = \begin{pmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \end{pmatrix}.$$

This results in $Y_t = C + AY_{t-1} + \varepsilon_t$ where A is the autoregression matrix.

In addition, residuals $\varepsilon_t \sim N(0, \Sigma)$ where Σ is the covariance matrix of residuals, i.e. ε_t are random variates simulated from a multivariate normal distribution with covariance matrix Σ and a mean of zero.

Therefore, R is fitting a VAR(1) model to the data, finding values for the matrix of constants C, the auto-regression matrix A, and the covariance matrix of residuals.

However, in our model we are discarding C and replacing it with a matrix of constants that will give us long term averages consistent with Quantum's market views, whilst keeping A and using it directly in the ESG.

5.1.3 Auto-regressive matrix

The auto-regression matrix that is used in our model is a VAR(1) estimation of each variable. It is computed with R. R output shows that a large number of parameters are not statistically significant; such a result is not surprising as getting accurate predictions requires a very advanced stochastic model.

In order to simulate ε_t in the ESG, we use the Cholesky decomposition of the covariance matrix Σ.

Using the VAR(1) model, the predicted value of a variable is a constant added to the sum of all dependence on historical variables. We are using the covariance matrix of residuals to simulate the residuals for future months in the ESG since we want to have a range of plausible economic scenarios with which to illustrate the risks of different investment strategies to pension schemes.

In summary, we'll be keeping the auto-regression parameters and the covariance matrix of residuals. The only information we aren't using from the R output are the estimates of the constant parameters for each variable. These are being replaced with constants that give a long term average equal to Quantum's views on the market.

5.1.4 Covariance matrix of residuals

A Cholesky decomposition of the covariance matrix of residuals is used to randomly simulate correlated residuals in the ESG.

Given a symmetric positive definite square matrix A, the Cholesky decomposition is a decomposition of the form

$$A = LL^{T}$$

where L is a lower triangular matrix with positive and real diagonal entries, and L^{T} the conjugate transpose of L.

Covariance is a measure of how changes in one variable are associated with changes in a second variable. Specifically, covariance measures the degree to which two variables are linearly associated.

The residuals are the difference between the model's predictions and the current outcome we are modelling. Very few models produced through auto-regression will have all residuals close to zero unless the auto-regression is being used to analyse a fixed process.

In our model, we 'extract' from the dataset of 22 variables the associations between their residuals, i.e. the relation between each variable's residual.

Finally, we are keeping the covariance matrix of residuals and using it to randomly generate residuals for future time periods in the ESG using the Cholesky decomposition.

5.2 Procedure

A simplification has been made to the model in order to make it functional; the starting values will be equal to the long term average expectation.

There are four steps to generate scenarios:

1. We generate a matrix Z that contains standard normal variates. This matrix is generated by using an Excel function that returns a random probability from a normal distribution of numbers with a mean of 0 and a standard deviation of 1.

2. We simulate correlated normal residual variates by multiplying a vector of the standard normal variates Z by the transpose of the Cholesky decomposition (appendix 4.2) of the covariance matrix of residuals (appendix 4.1). Therefore each scenario will have different variable residuals. The process is repeated for each scenario.

$$\varepsilon_t = Z \text{ x } L^T$$

   $\varepsilon_t \sim N(0, \Sigma)$ where Σ is the covariance matrix of residuals.

3. The auto-regressive component is computed by multiplying the auto-regressive matrix (AR, appendix 4.3) by Y_{t-1}, a vector of the previous month forecast. However, at time 1, due to the model simplification, the first month forecast is the long term average expectations. Starting time 2, this matrix will contain the forecast values at this time period.

4. Finally, the time series vector is computed by the formula:

$$Y_t = D + AR \text{ x } Y_{t-1} + \varepsilon_t.$$

   Where D (appendix 4.4) is the constant element of the time series. This constant should ensure that each variable has the correct long-term average value,
   $\mathbb{E}[Y_t] = \mathbb{E}[AR \text{ x } Y_{t-1}] + D$ and therefore
   $D = \mathbb{E}[Y_t] - \mathbb{E}[AR \text{ x } Y_{t-1}]$
   Where $\mathbb{E}[Y_t]$ is the expectation of the vector Y_t and therefore the long term expectation value of each variable and $\mathbb{E}[AR \text{ x } Y_{t-1}]$ is the expectation of the auto-regression component of the time series.

The ESG will use Excel to generate 100 scenarios will the help of VBA at each time and repeat the four steps in order to generate next time scenarios. The scenarios produced for time 2 will be used to generate time 3 scenarios and so on.

Generating realistic scenarios is crucial in our model. Scenarios generated by the ESG should be distributed in the same way as their historical data. Scatterplots that depict the correlation between two variables have been plotted. As plotted below, when we compare the correlation between historical data and data generated by the ESG, the same pattern is found.

Figure 5.1 Scatter plot of the historical returns between hedge fund of funds and commodities indexes.

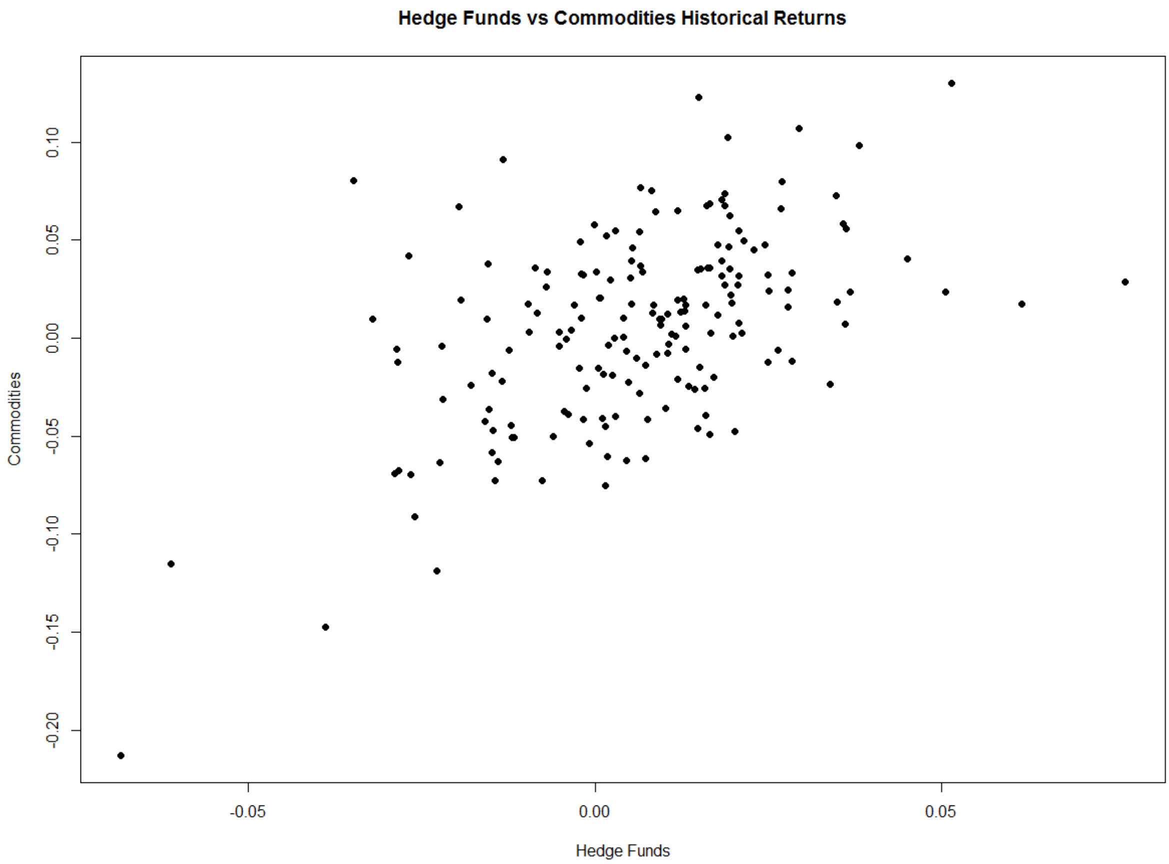


Figure 5.2 Scatter plot of the generated returns between hedge fund of funds and commodities indexes.

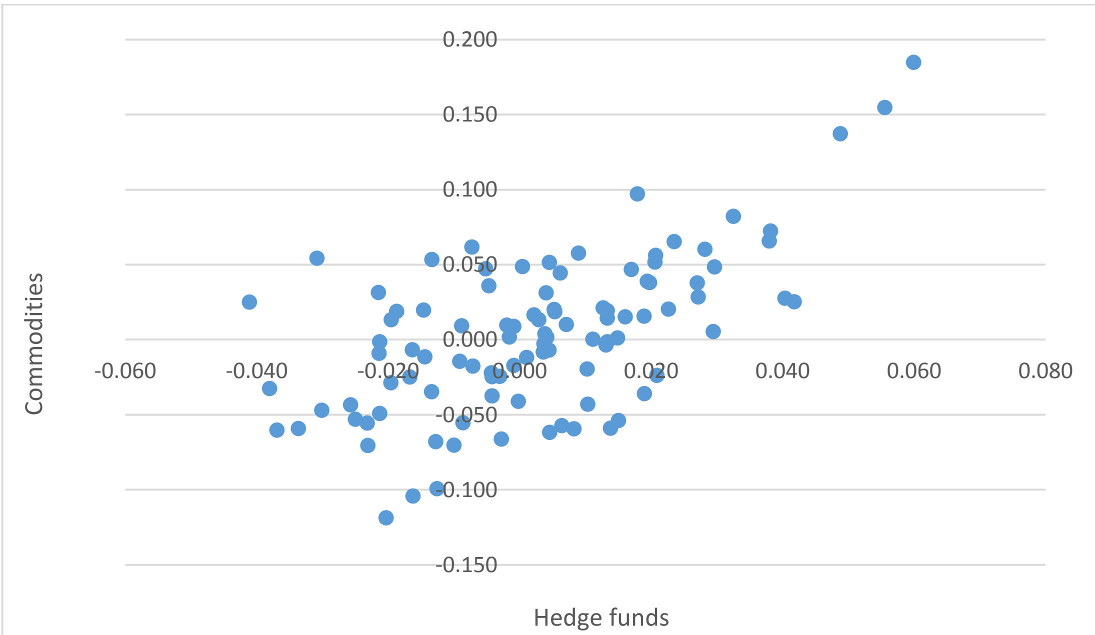


Figure 5.3 Scatter plot of the historical returns between North American equity and UK equity indexes.

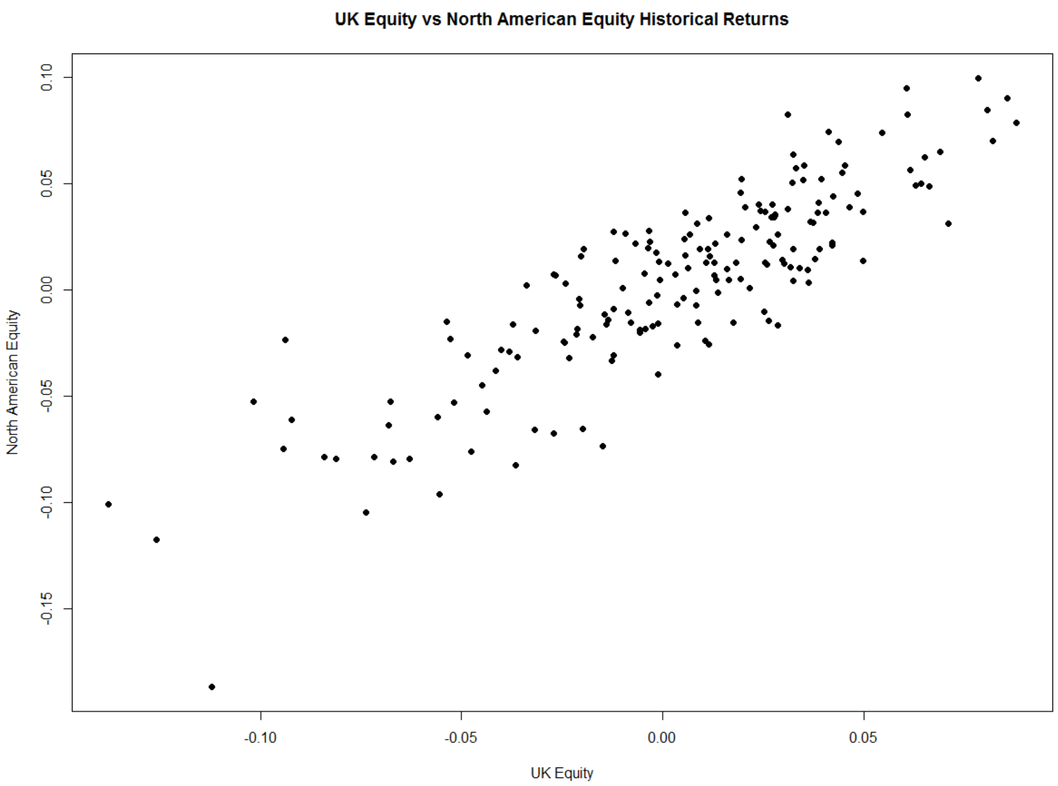


Figure 5.4 Scatter plot of the generated returns between North American equity and UK equity indexes.

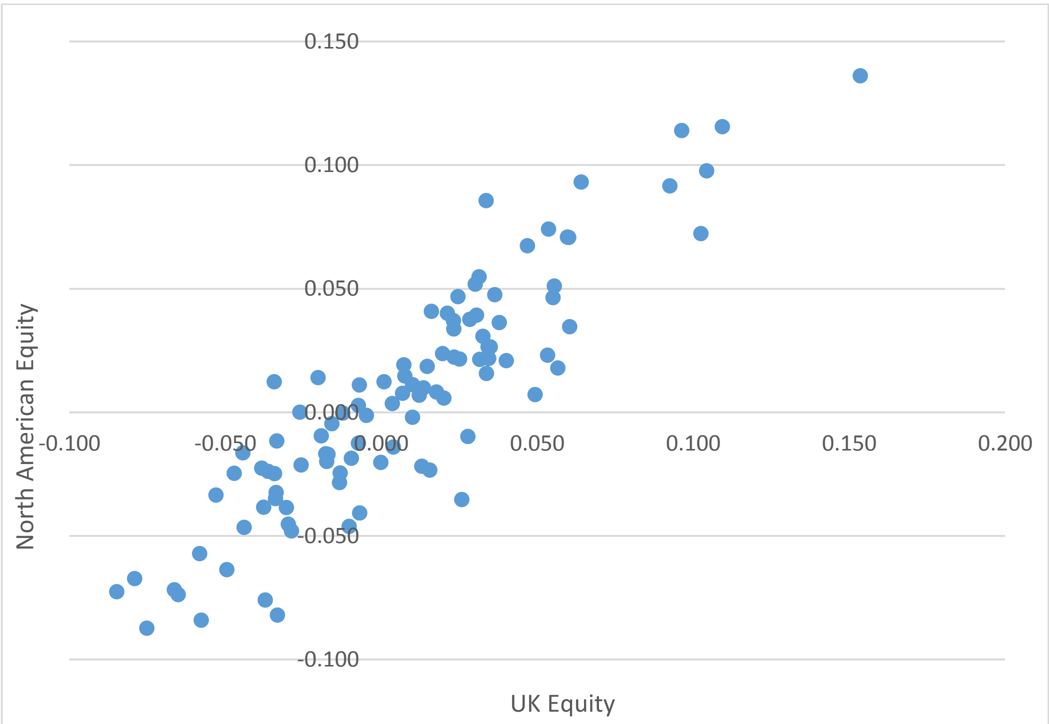


Figure 5.5 Scatter plot of the historical returns between the RPI and UK equity index.

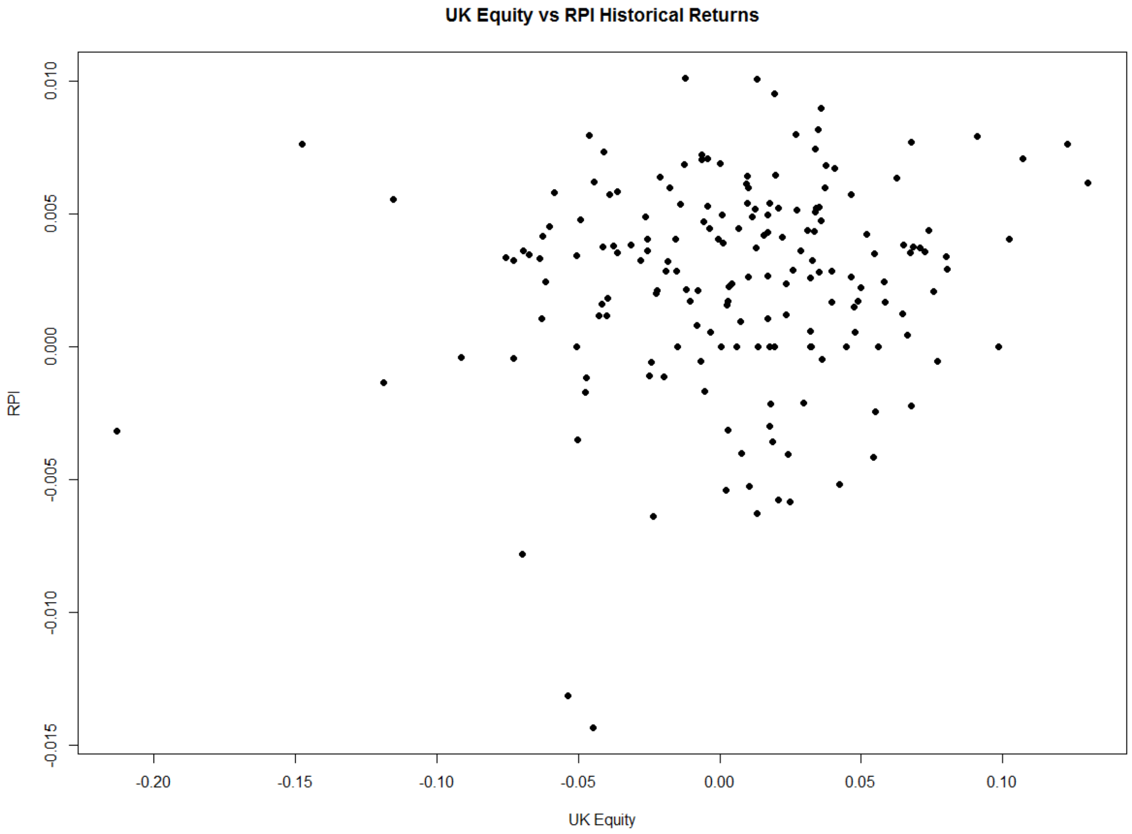
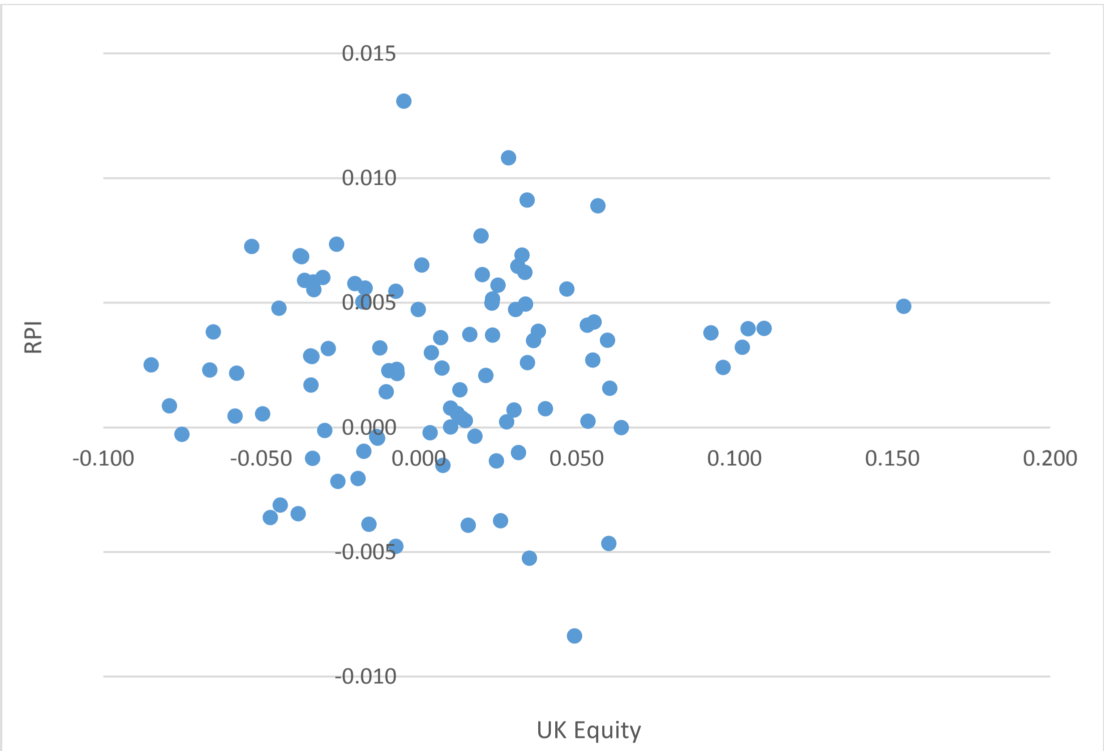


Figure 5.6 Scatter plot of the generated returns between the RPI and UK equity index.



5.3 Scenarios distribution

As shown in the charts below, scenarios generated by the ESG are normally distributed, unlike the historical variable values. The normal distribution of scenarios is due to the generation of normally distributed random numbers.

Note that the skew of the CPI historical returns distribution (Figure 5.9) is negative (-0.736213, Appendix 3, Table A.3.1) and is due to an unusual extended period of low interest rates following 2008, therefore generating normally distributed rates is a solution if we don’t want to replicate that period of low interest rates.

Figure 5.7 Histogram of the historical returns distribution of the World equity index.

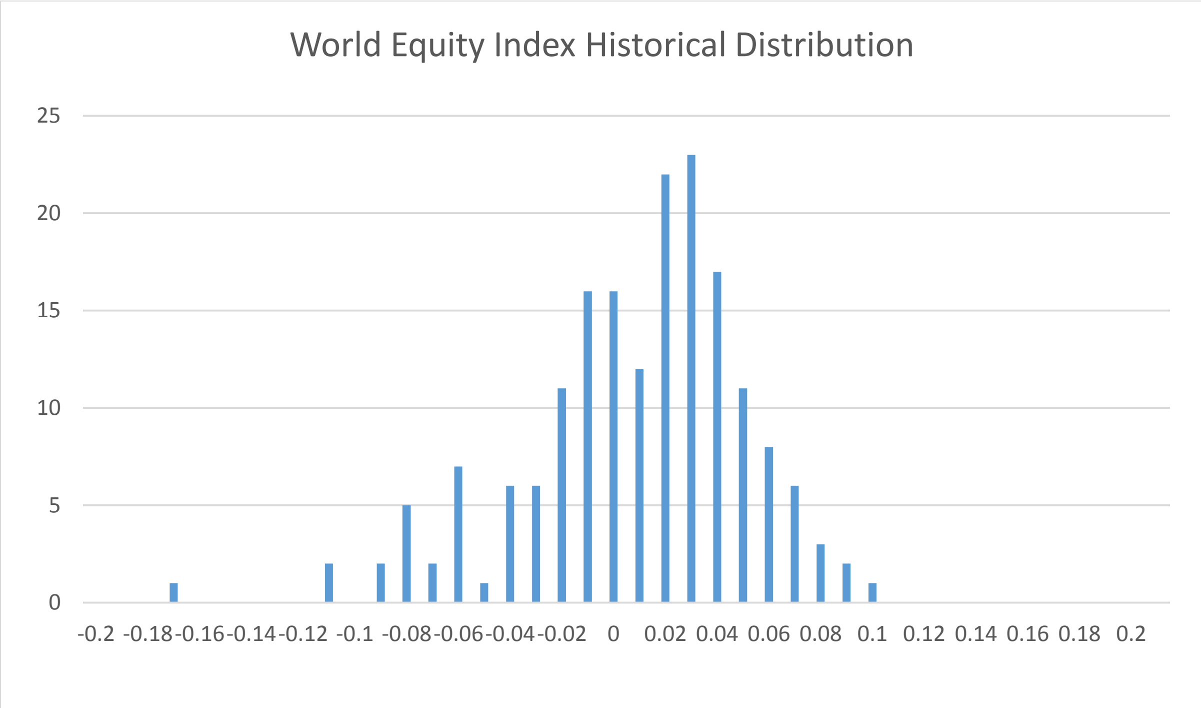


Figure 5.8 Histogram of the generated returns distribution of the World equity index.

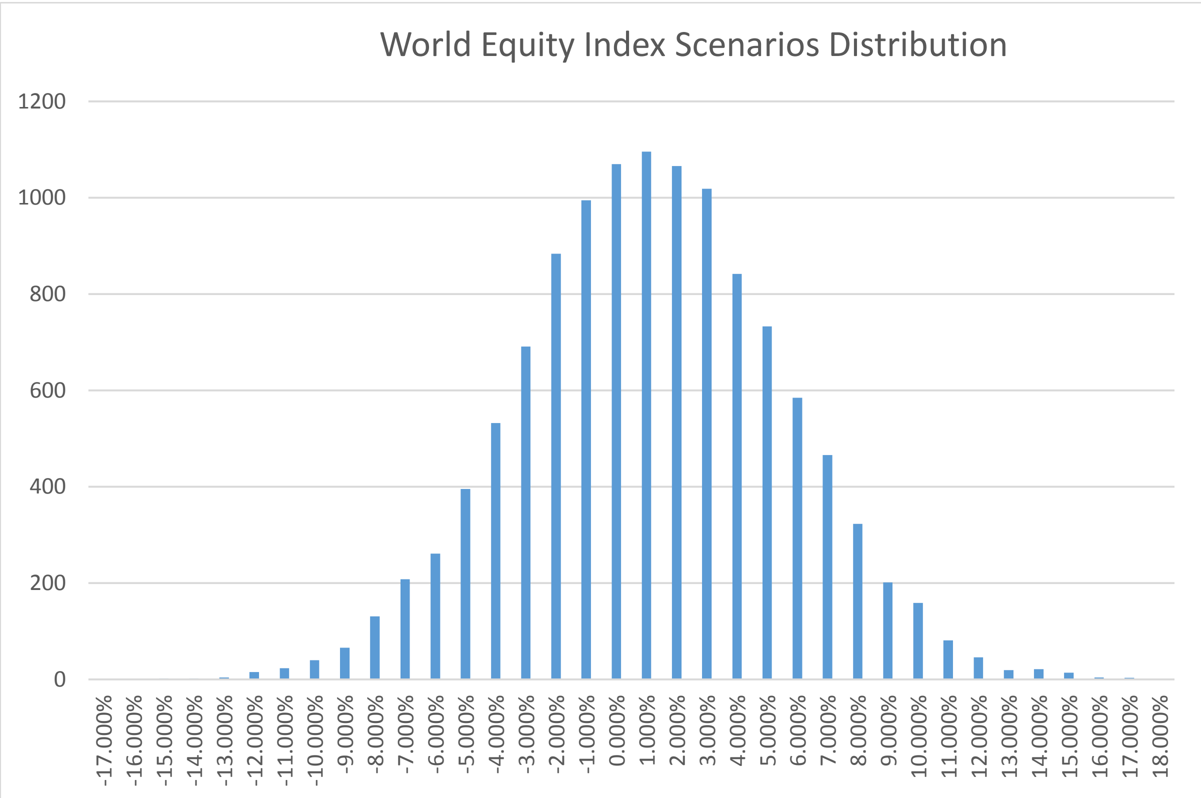


Figure 5.9 Histogram of the historical distribution of the CPI.

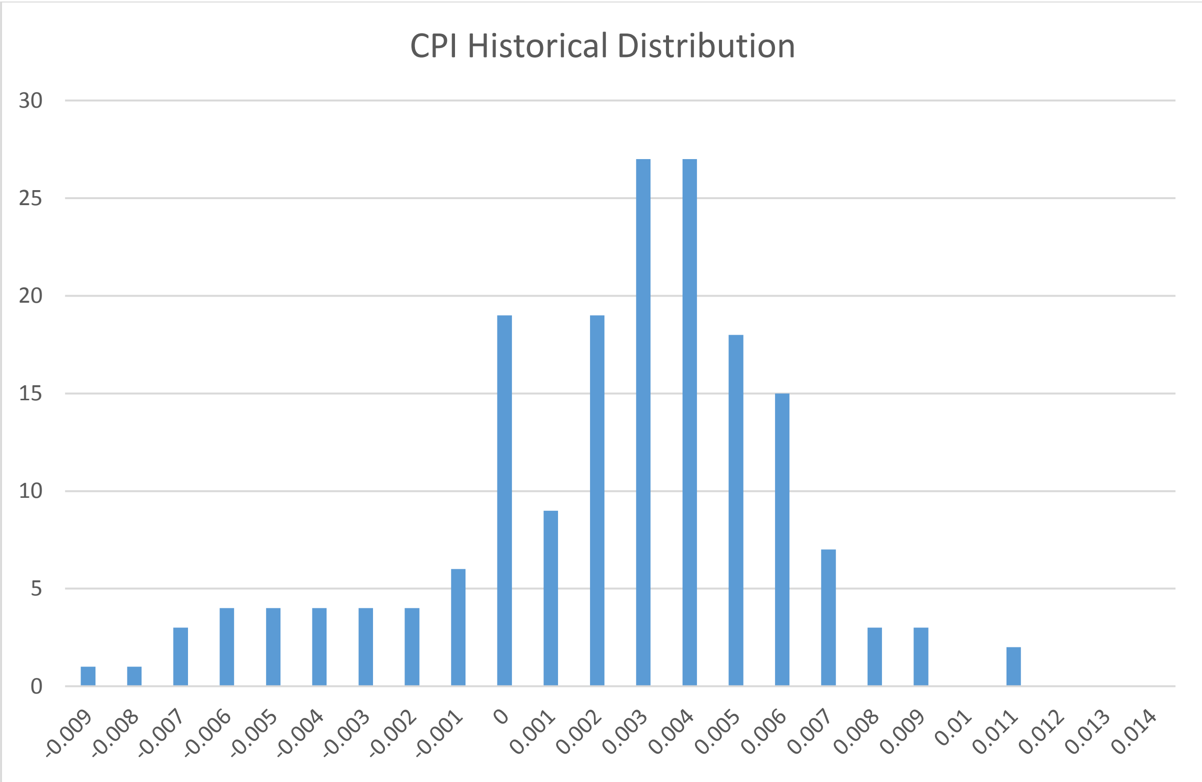


Figure 5.10 Histogram of the generated distribution of the CPI.

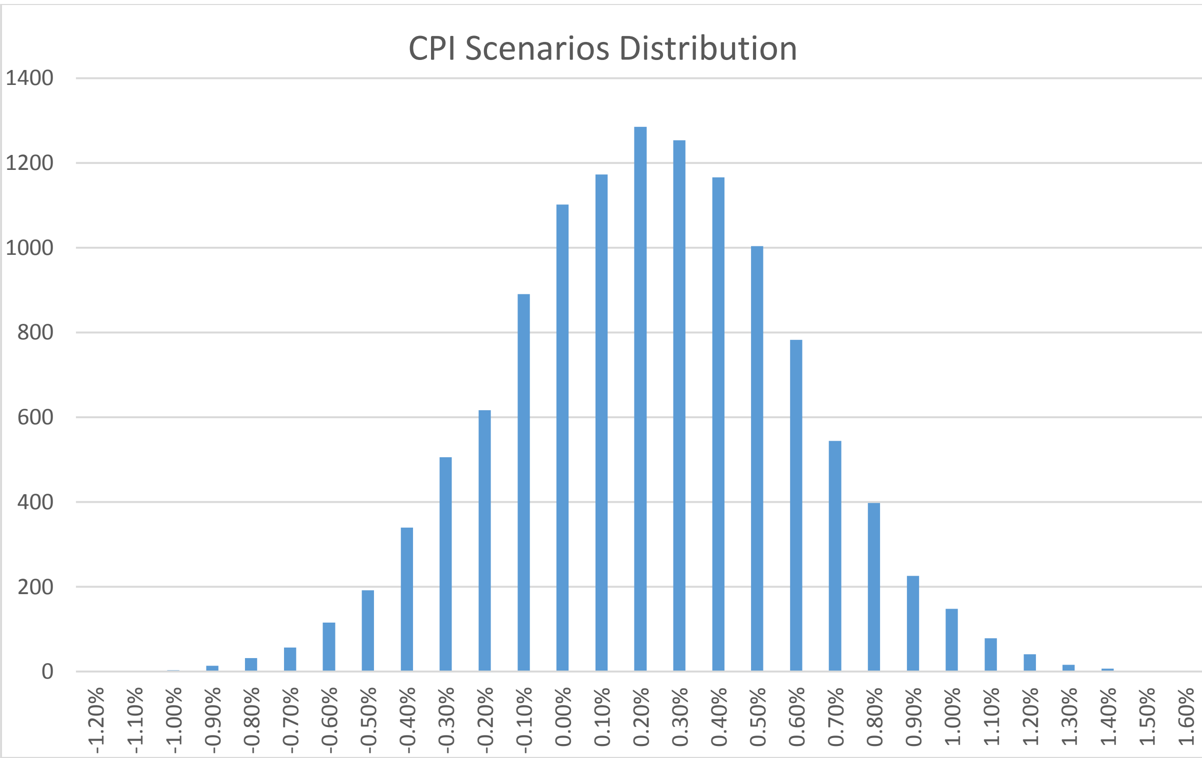


Figure 5.11 Histogram of the historical returns distribution of 10 Years Nominal Interest Rate.

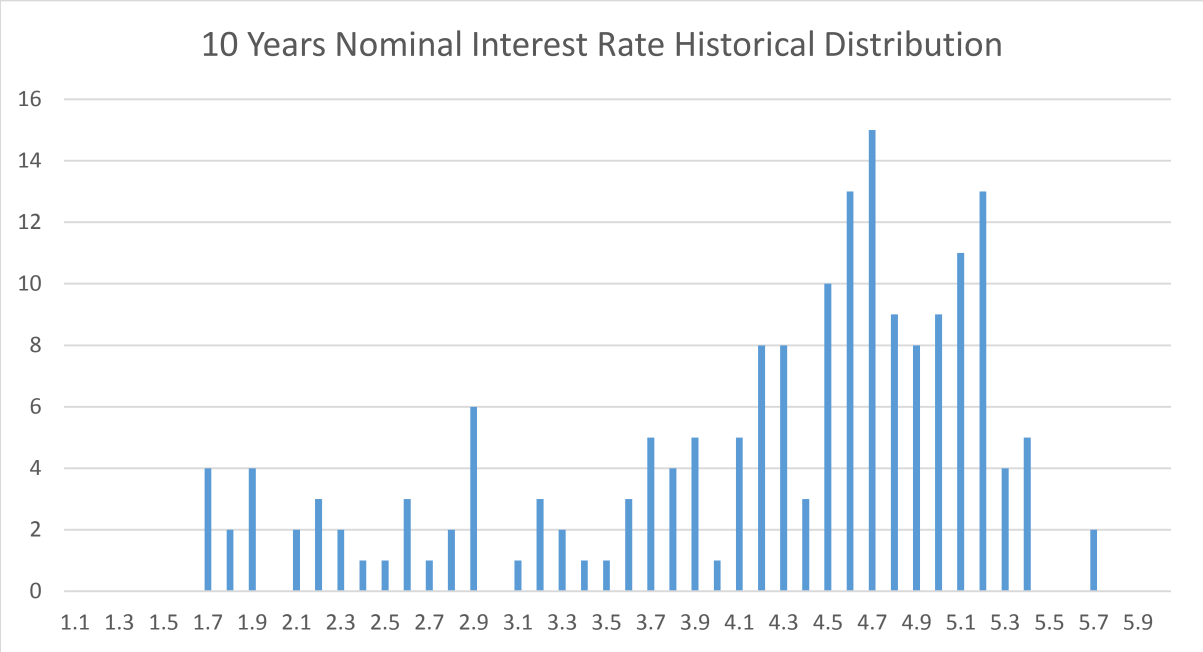
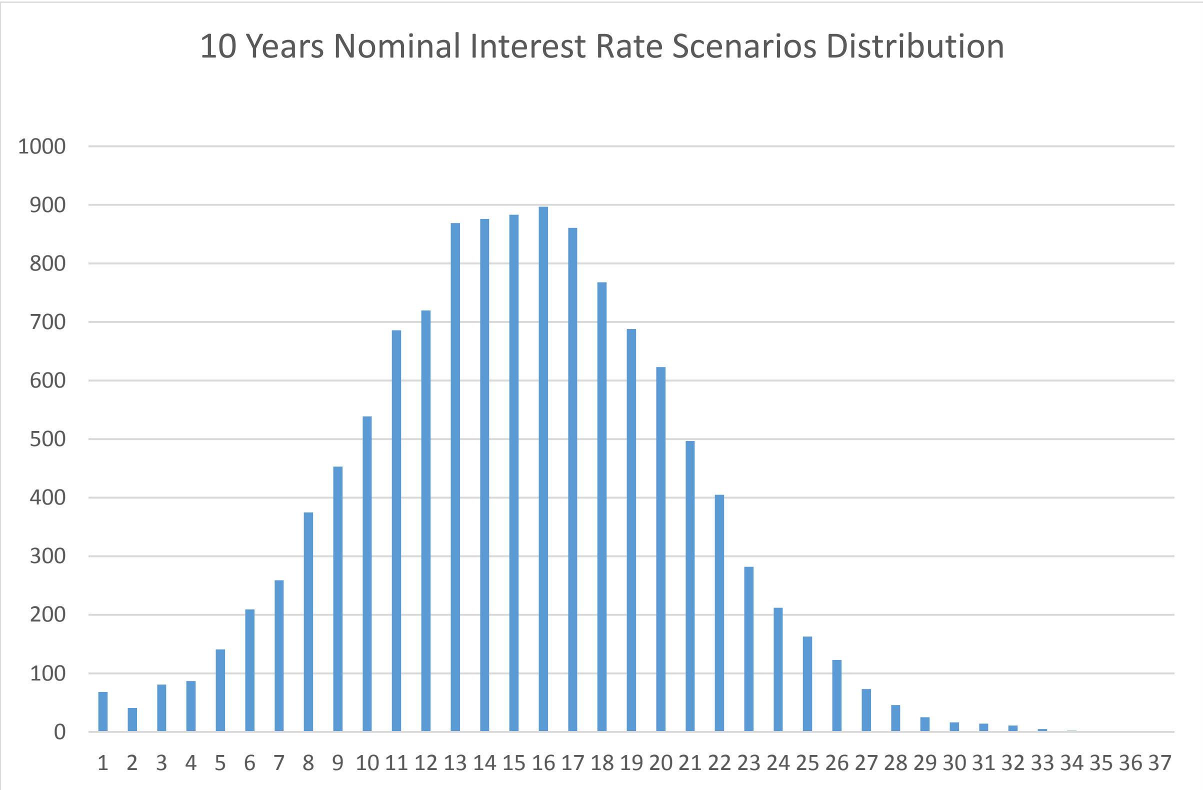


Figure 5.12 Histogram of the generated returns distribution of 10 Years Nominal Interest Rate.



5.4 <u>Results</u>

Below are plotted scenarios that have been generated by the ESG over 120 months (further figures in appendix 5). The red curve depicts a scenario that has been randomly selected over scenarios generated. The black curve represent the average progression of all scenarios. Note that the mean of scenarios starts and end at the same point due to the limitation of the model.

Figure 5.13 Generation of hedge fund of funds index scenarios.

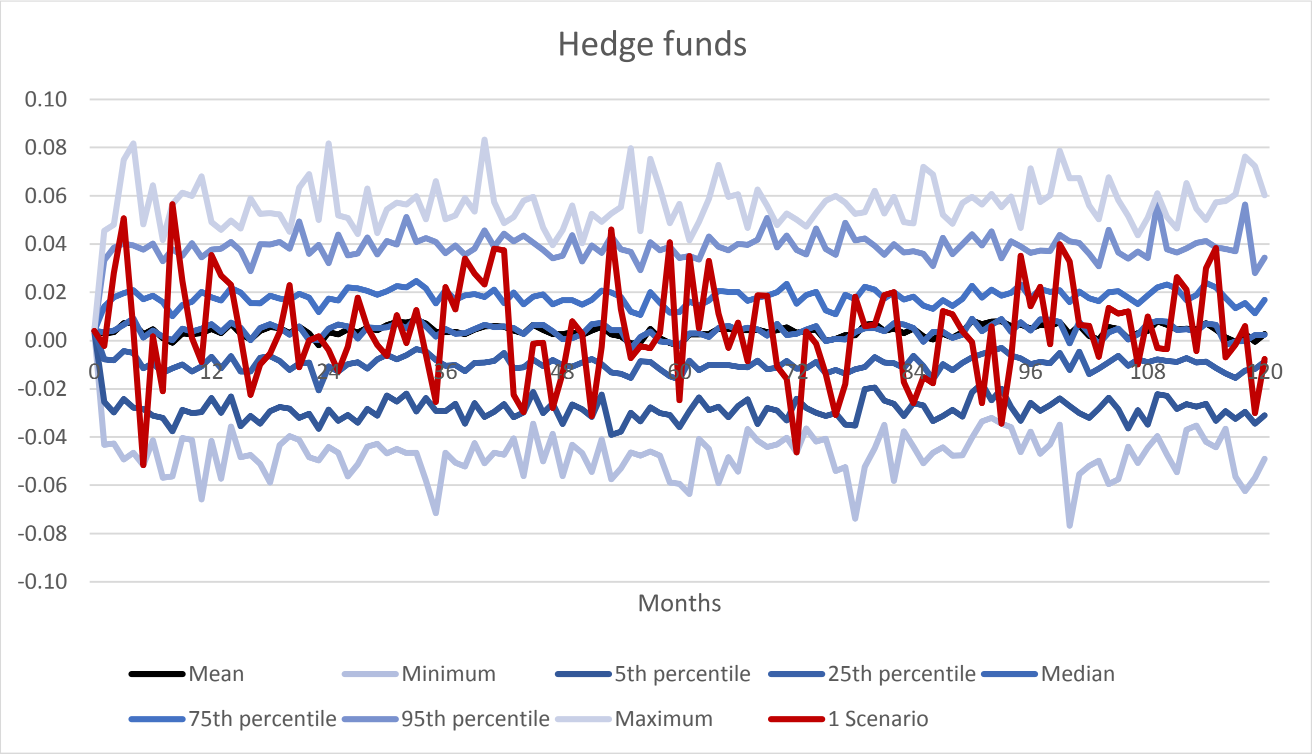


Figure 5.14 Generation of UK property index scenarios.

Figure 5.15 Generation of commodities index scenarios.

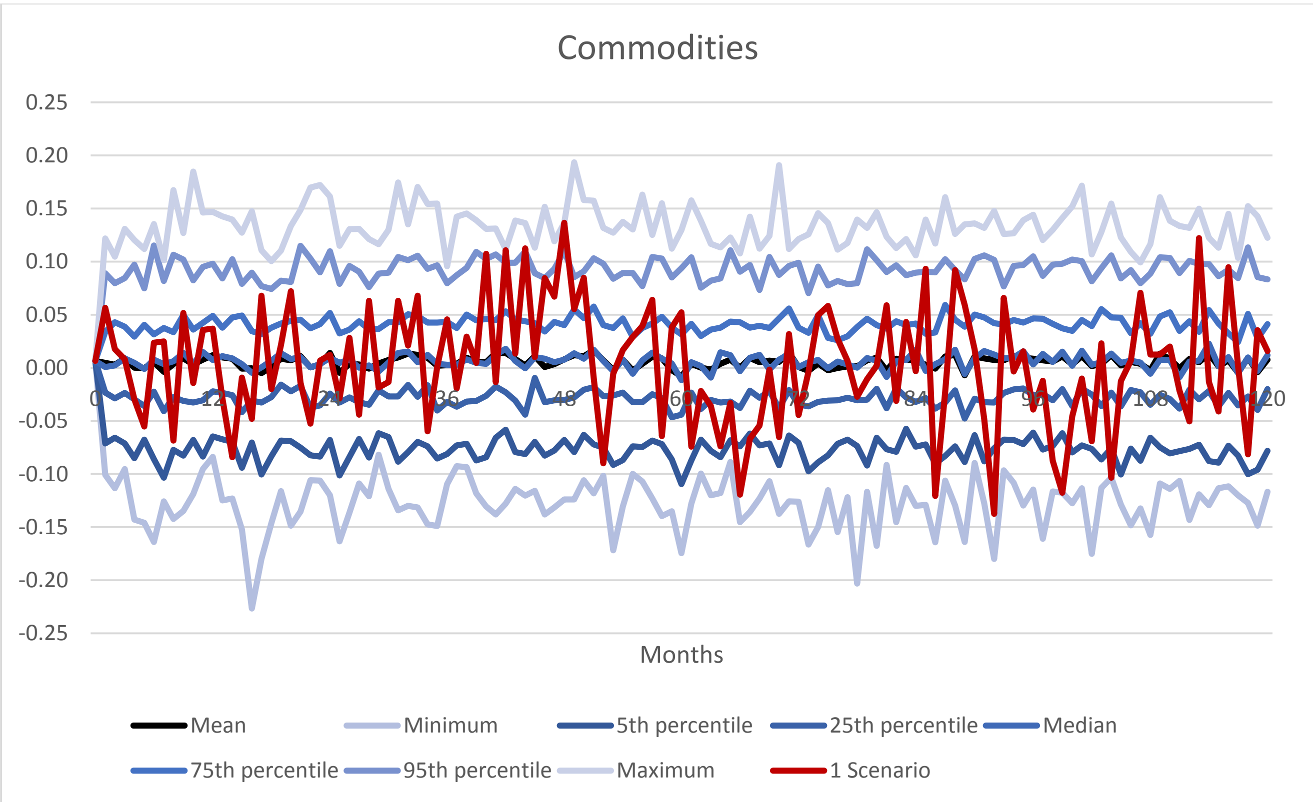
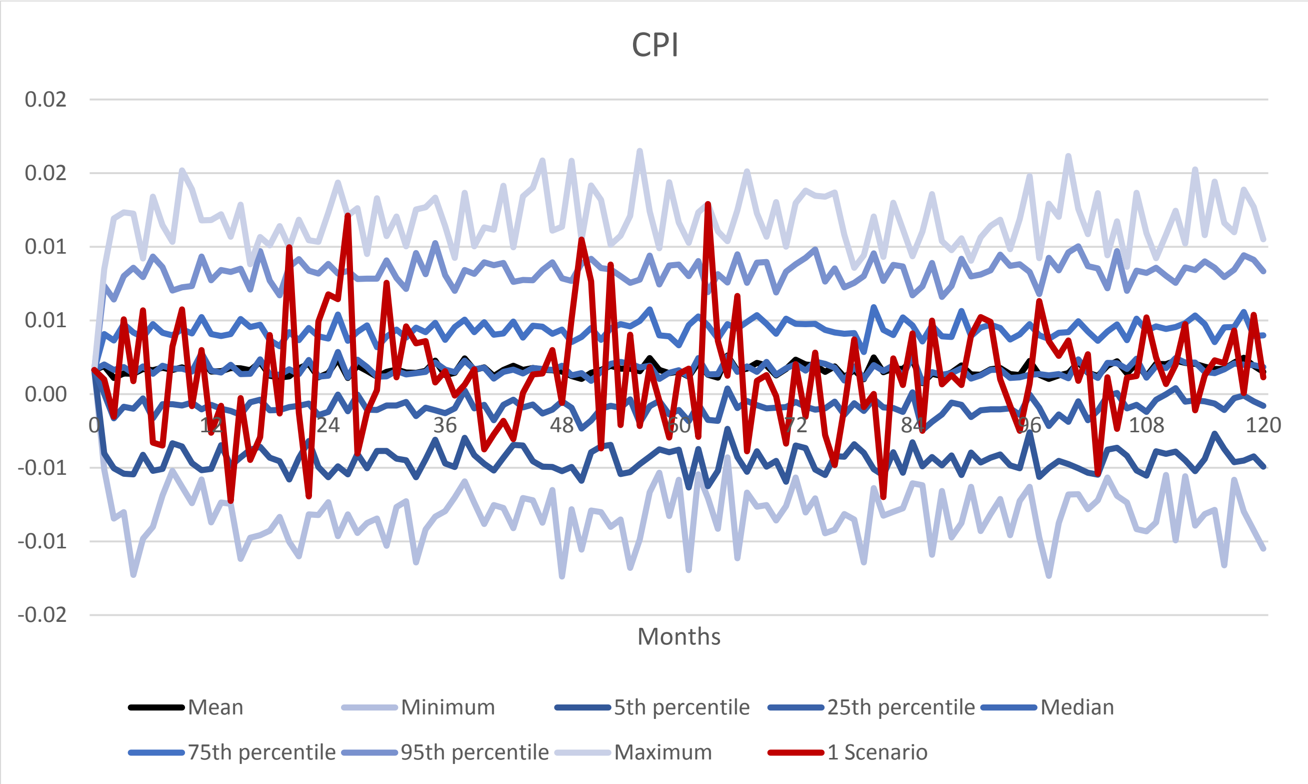


Figure 5.16 Generation of UK CPI index scenarios.



An interesting use of the ESG is the generation of thousands of scenarios in order to quantify the probability of different events occurring over a defined period of time, and to simulate the evolution of a pension scheme's assets based on these different investment strategies.

However, the ESG used in this project requires further development in order to be able to generate larger numbers of scenarios, which should improve the stability of the results.

5.5 <u>Limits of the model</u>

5.5.1 Model simplifications

In order to get realistic scenarios, compromises have to be made due to the complexity of the model. The extended period of low interested rate started 2008 was generated scenarios with very high returns and making economic scenarios unrealistic. Another problem was the number of variables, in fact the bigger the dataset is, the more complex the model will be. We have decided to keep the model as it is and to make a decisive assumption: the starting values will be equal to the long term average expectation. As stated before, one of the problems is the complexity of the dataset and it may be due to parsimony

5.5.2 Parsimony

In research, a parsimony theory is a theory that is simpler than others because it introduces the fewest assumptions. It is often associated with the rule of Ockham's razor that says the simplest explanation is usually the correct one.

Statistically insignificant auto regression parameters calculated when fitting the VAR(1) were retained in the economic scenario generator to ensure that simulated variable values remained stable i.e. that they were mean reverting. However, a drawback of this is that it can potentially lead to unrealistic results. In particular, it was found that the expectation that nominal and real yields would rise significantly from their current low level, caused strong negative effects on short term equity returns in all scenarios.

Further research is required into how these insignificant parameters can be removed from the model i.e. producing a more parsimonious model, without compromising the stability of simulated variable values.

CHAPTER 6. CONCLUSION

Developing strategic software in-house is crucial for companies because it gives them the possibility of personalising and updating them when they need to. Quantum Advisory needed to develop an Economic Scenario Generator because their current generator was not reflecting the investment team's perspective on the economy and led to conflict between the ALM and advice given to pension scheme manager. A dataset of variables has been created, these variables representative of the asset classes that are available to Quantum clients and reflect the global economy. An autoregression analysis has been performed in order to extract three kind of correlations from the dataset and a procedure has been followed in order to generate scenarios. The procedure includes a generation of normally distributed random numbers, a simulation of correlated normal residuals variates, an auto-regressive component being computed, and finally scenarios generated by summing up vectors created previously. By generating thousands of scenarios, investment team members will be able to choose a range of scenarios that most reflect their expected economy future development and to simulate the evolution of funds institution. However, the current Economic Scenario Generator has limitations. In order to address such limitations, further research on the ESG is required.

CHAPTER 7. DISCUSSION

Different problems arose during the development of the scenario generator. Unusually low rates during the previous decade conducted to scenarios with very high indexes' returns due to the low level of newly generated RPI and CPI scenarios. In order to fix such a problem, adjustments had to be made to the model, e.g. the starting values of each variable must be equal to the long term average expectation of each variable. Furthermore, the initial dataset was too large and added more complexity to the model. Further research will have to be conducted in order to find a solution to these problems. Also, benchmarks will have to be run in order to perform a comparison between the model developed and the software provider model by putting the model in the Asset Liability Modelling.

Despite the limitations, as a first version, the model developed is a good start but will need to be adjusted and updated in order to address existing limitations and have a fully operational economic scenario generator.

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APPENDICES

Appendix 1 - Definitions

Skewness is a measure of the asymmetry of the distribution of a random variable about its mean. A curve is skewed to the right (or positively skewed) if it tails off toward the high end of the scale (right tail longer than the left). A curve is skewed to the left (or negatively skewed) if it tails off toward the low end of the scale.

Multicollinearity occurs when two or more predictors in the model are correlated and provide redundant information about the response.

Heteroskedasticity occurs when the variance of the error terms differ across observations.

M2 is a measure of money supply that includes checking deposits (M1) and cash.

M3 is a measure of money supply that includes M2, institutional money market funds, large time deposits, short-term repurchase agreements and other larger liquid assets.

A bond is a debt investment issued for a period of time in which an investor loans money to a governmental or corporate entity.

The nominal interest rate is the actual interest rate without adjusting for inflation. In this project the nominal interest rate at different maturity has been taken to the yield on UK government fixed interest debt at that maturity.

Appendix 2 – Indexes Definition

The MSCI United Kingdom Index is designed to measure the equity market performance of the United Kingdom. The index represents companies within this country that are available to investors worldwide.

The MSCI North America Index measures the performance of the large and mid-cap segments of the US and Canada markets; it has 712 constituents.

The MSCI EMU Index (European Economic and Monetary Union) captures large and mid-cap representation across the ten Developed Markets countries in the EMU; it has 239 constituents.

The MSCI Japan Index is designed to track the equity market performance of Japanese securities listed on the Tokyo Stock Exchange, JASDAQ, Osaka Stock Exchange and Nagoya Stock Exchange.

The MSCI Pacific Index captures large and mid-cap representation across five developed market countries in the Pacific region; it has 454 constituents.

The MSCI Emerging Markets Index captures large and mid-cap representation across 23 Emerging Markets countries and has 834 constituents.

The MSCI World Index captures large and mid-cap representation across 23 Developed Markets countries with 1,612 constituents.

The IPD UK Monthly Property Index measures ungeared total returns to directly held standing property investments from one open market valuation to the next.

The Dow Jones-UBS Commodity Index represents 20 commodities, which are weighted to account for economic significance and market liquidity.

Appendix 3 – Database Statistics

SE Means is the Standard error of the mean

The first quartile (1. Quartile) is the middle number between the smallest number and the median of the dataset

The third quartile (3. Quartile) is the middle value between the median and the highest value of the dataset.

Stdev is the standard deviation.

Table 3.1 – Data statistics n.1.

	Hedge Fund of Funds	UK Property	Commodities	UK CPI	UK RPI	World equity
nobs	180	180	180	180	180	180
Minimum	-0.068421	-0.052678	-0.212832	-0.009677	-0.014352	-0.178437
Maximum	0.076501	0.036377	0.130045	0.010381	0.010095	0.099233
1. Quartile	-0.004196	0.005176	-0.022694	0	0	-0.019229
3. Quartile	0.018248	0.011572	0.035177	0.004132	0.005069	0.031163
Mean	0.006079	0.006037	0.005794	0.00183	0.002455	0.003215
Median	0.007252	0.00769	0.010008	0.002159	0.003225	0.011834
SE Mean	0.001473	0.000916	0.003629	0.000273	0.000292	0.003212
Variance	0.000391	0.000151	0.002371	1.30E-05	1.50E-05	1.86E-03
Stdev	0.019768	0.01229	0.048693	0.003661	0.003918	0.043098
Skewness	-0.248653	-2.119105	-0.63181	-0.736213	-1.086138	-0.857793
Kurtosis	1.74958	6.656192	1.9018	0.679312	2.250466	1.302393

Table 3.2 – Data statistics n.2.

	Japanese equity	UK equity	Eurozone equity	North America Equity	Pacific (ex Japan) equity
nobs	180	180	180	180	180
Minimum	-0.236417	-0.137712	-0.19271	-0.186663	-0.213944
Maximum	0.119515	0.087884	0.149933	0.099438	0.098904
1. Quartile	-0.03441	-0.020031	-0.027038	-0.019051	-0.027374
3. Quartile	0.035945	0.031193	0.038835	0.033682	0.033081
Mean	0.000682	0.003117	0.002712	0.003601	0.002084
Median	0.002136	0.008543	0.01229	0.0102	0.005866
SE Mean	0.003959	0.003114	0.004085	0.003343	0.003442
Variance	2.82E-03	1.75E-03	0.003003	0.002012	0.002133
Stdev	0.053113	0.041784	0.054802	0.044852	0.04618
Skewness	-0.522234	-0.720281	-0.689866	-0.726556	-0.792515
Kurtosis	1.421583	0.666675	1.301809	1.239408	1.965018

Appendix 3 – Database Statistics

Table 3.3 – Data statistics n.3.

	Emerging Markets equity	UK NOM 1 years	UK NOM 5 years	UK NOM 10 years	UK NOM 15 years
nobs	180	180	180	180	180
Minimum	-0.256322	0.17	0.57	1.61	2.310844
Maximum	0.172548	6.32	6.29	5.63	5.2
1. Quartile	-0.022152	0.6675	2.56	3.6925	4.139644
3. Quartile	0.045845	4.895	4.945	4.8725	4.745929
Mean	0.009854	3.233278	3.765111	4.139778	4.280396
Median	0.011718	4.21	4.365	4.49	4.499885
SE Mean	0.004218	0.156695	0.116931	0.075233	0.052577
Variance	0.003203	4.419589	2.461129	1.018806	0.497581
Stdev	0.056592	2.102282	1.568799	1.009359	0.705394
Skewness	-0.81538	-0.35663	-0.596806	-0.999775	-1.312793
Kurtosis	2.505959	-1.546706	-0.950658	-0.026913	0.762764

Table 3.4 – Data statistics n.4.

	UK NOM 20 years	UK NOM 25 years	UK REAL 5 years	UK REAL 10 years	UK REAL 15 years	UK REAL 20 years
nobs	180	180	180	180	180	180
Minimum	2.793238	3.114167	-2.37	-1.39	-0.78	-0.47
Maximum	5.14	5.03	4.26	2.8	2.58	2.47
1. Quartile	4.161413	4.088164	-0.195	0.6475	0.81	0.8175
3. Quartile	4.65	4.56	2.255	2.09	2.01	1.9525
Mean	4.297233	4.257673	1.143389	1.2925	1.311778	1.256333
Median	4.455	4.37	1.69	1.645	1.47	1.305
SE Mean	0.039933	0.032249	0.110373	0.077505	0.063664	0.057622
Variance	0.287033	0.187198	2.19278	1.081256	0.729551	0.597659
Stdev	0.535755	0.432664	1.480804	1.039834	0.854138	0.773084
Skewness	-1.218584	-0.910793	-0.72653	-0.814036	-0.639516	-0.396799
Kurtosis	0.670991	0.101919	-0.748727	-0.468287	-0.649588	-0.930003

Appendix 4 - Economic Scenario Generator Input

Table 4.1 - Covariance matrix of residuals

	HF	PPT	CM	CPI	RPI	World	JPN	UK	EMU	NAm	PACFIC	EM	NOM.1	NOM.5	NOM.10	NOM.15	NOM.20	NOM.25	REAL.5	REAL.10	REAL.15	REAL.20
HF	3.485e-04	2.159e-05	4.383e-04	4.644e-06	3.075e-06	6.332e-04	5.833e-04	5.297e-04	7.466e-04	6.353e-04	5.958e-04	8.605e-04	3.795e-04	5.237e-04	4.949e-04	3.223e-04	1.730e-04	1.806e-04	-9.113e-04	-5.574e-04	-4.023e-04	-3.602e-04
PPT	2.159e-05	2.593e-05	3.996e-05	2.160e-06	2.411e-06	5.338e-05	4.608e-05	5.305e-05	4.903e-05	5.859e-05	4.666e-05	4.360e-05	-7.251e-05	1.622e-05	1.081e-04	1.109e-04	9.494e-05	1.029e-04	-3.061e-04	-1.133e-04	-4.844e-05	-2.440e-05
CM	4.383e-04	3.996e-05	2.266e-03	9.089e-06	1.856e-05	7.407e-04	7.047e-04	6.207e-04	6.776e-04	8.064e-04	7.534e-04	1.248e-03	1.554e-03	1.800e-03	1.392e-03	7.539e-04	2.880e-04	3.140e-04	-2.673e-03	-1.557e-03	-1.066e-03	-8.285e-04
CPI	4.644e-06	2.160e-06	9.089e-06	1.203e-05	1.081e-05	1.890e-05	2.958e-05	2.469e-05	1.976e-05	1.589e-05	2.441e-05	1.896e-05	1.619e-04	1.255e-04	7.949e-05	7.025e-05	6.655e-05	6.668e-05	-8.651e-06	-3.904e-05	-1.833e-05	5.567e-06
RPI	3.075e-06	2.411e-06	1.856e-05	1.081e-05	1.191e-05	1.327e-05	2.513e-05	1.781e-05	7.384e-06	1.168e-05	2.093e-05	1.239e-05	1.027e-04	1.027e-04	9.047e-05	8.884e-05	8.630e-05	9.196e-05	-1.934e-05	-2.027e-05	7.616e-06	3.524e-05
World	6.332e-04	5.338e-05	7.407e-04	1.890e-05	1.327e-05	1.727e-03	1.426e-03	1.541e-03	2.038e-03	1.768e-03	1.438e-03	1.797e-03	2.611e-03	1.995e-03	1.507e-03	9.623e-04	5.268e-04	4.823e-04	-1.676e-03	-1.099e-03	-8.354e-04	-7.620e-04
JPN	5.833e-04	4.608e-05	7.047e-04	2.958e-05	2.513e-05	1.426e-03	2.592e-03	1.156e-03	1.566e-03	1.262e-03	2.174e-03	1.624e-03	4.577e-03	3.135e-03	1.903e-03	1.155e-03	6.451e-04	5.105e-04	-1.239e-03	-1.526e-03	-1.486e-03	-1.426e-03
UK	5.297e-04	5.305e-05	6.207e-04	2.469e-05	1.781e-05	1.541e-03	1.156e-03	1.596e-03	1.845e-03	1.553e-03	1.198e-03	1.549e-03	1.932e-03	1.360e-03	1.169e-03	7.781e-04	4.259e-04	3.654e-04	-1.787e-03	-1.150e-03	-8.404e-04	-7.515e-04
EMU	7.466e-04	4.903e-05	6.776e-04	1.976e-05	7.384e-06	2.038e-03	1.566e-03	1.845e-03	2.772e-03	1.987e-03	1.607e-03	2.170e-03	2.832e-03	2.313e-03	1.882e-03	1.251e-03	7.499e-04	6.933e-04	-1.311e-03	-6.435e-04	-4.129e-04	-3.932e-04
NAm	6.353e-04	5.859e-05	8.064e-04	1.589e-05	1.168e-05	1.768e-03	1.262e-03	1.553e-03	1.987e-03	1.889e-03	1.325e-03	1.786e-03	2.452e-03	1.916e-03	1.509e-03	9.694e-04	5.241e-04	4.887e-04	-1.945e-03	-1.197e-03	-8.734e-04	-7.783e-04
PACFIC	5.958e-04	4.666e-05	7.534e-04	2.441e-05	2.093e-05	1.438e-03	2.174e-03	1.198e-03	1.607e-03	1.325e-03	1.931e-03	1.662e-03	3.351e-03	2.403e-03	1.457e-03	8.444e-04	4.166e-04	3.547e-04	-1.378e-03	-1.369e-03	-1.248e-03	-1.192e-03
EM	8.605e-04	4.360e-05	1.248e-03	1.896e-05	1.239e-05	1.797e-03	1.624e-03	1.549e-03	2.170e-03	1.786e-03	1.662e-03	3.009e-03	2.585e-03	1.897e-03	1.076e-03	3.709e-04	-1.859e-04	-2.281e-04	-2.207e-03	-1.531e-03	-1.289e-03	-1.319e-03
NOM.1	3.795e-04	-7.251e-05	1.554e-03	1.619e-04	1.027e-04	2.611e-03	4.577e-03	1.932e-03	2.832e-03	2.452e-03	3.351e-03	2.585e-03	2.010e-01	8.807e-02	4.415e-02	2.481e-02	1.335e-02	4.692e-03	2.924e-02	8.320e-04	-9.592e-03	-1.154e-02
NOM.5	5.237e-04	1.622e-05	1.800e-03	1.255e-04	1.027e-04	1.995e-03	3.135e-03	1.360e-03	2.313e-03	1.916e-03	2.403e-03	1.897e-03	8.807e-02	6.074e-02	4.400e-02	3.238e-02	2.395e-02	1.766e-02	3.163e-02	1.847e-02	1.081e-02	7.303e-03
NOM.10	4.949e-04	1.081e-04	1.392e-03	7.949e-05	9.047e-05	1.507e-03	1.903e-03	1.169e-03	1.882e-03	1.509e-03	1.457e-03	1.076e-03	4.415e-02	4.400e-02	4.465e-02	3.856e-02	3.191e-02	2.671e-02	2.249e-02	2.181e-02	1.805e-02	1.527e-02
NOM.15	3.223e-04	1.109e-04	7.539e-04	7.025e-05	8.884e-05	9.623e-04	1.155e-03	7.781e-04	1.251e-03	9.694e-04	8.444e-04	3.709e-04	2.481e-02	3.238e-02	3.856e-02	3.659e-02	3.239e-02	2.845e-02	1.818e-02	2.100e-02	1.910e-02	1.724e-02
NOM.20	1.730e-04	9.494e-05	2.880e-04	6.655e-05	8.630e-05	5.268e-04	6.451e-04	4.259e-04	7.499e-04	5.241e-04	4.166e-04	-1.859e-04	1.335e-02	2.395e-02	3.191e-02	3.239e-02	3.031e-02	2.773e-02	1.469e-02	1.894e-02	1.828e-02	1.723e-02
NOM.25	1.806e-04	1.029e-04	3.140e-04	6.668e-05	9.196e-05	4.823e-04	5.105e-04	3.654e-04	6.933e-04	4.887e-04	3.547e-04	-2.281e-04	4.692e-03	1.766e-02	2.671e-02	2.845e-02	2.773e-02	2.654e-02	1.061e-02	1.635e-02	1.678e-02	1.638e-02
RETAIL.5	-9.113e-04	-3.061e-04	-2.673e-03	-8.651e-06	-1.934e-05	-1.676e-03	-1.239e-03	-1.787e-03	-1.311e-03	-1.945e-03	-1.378e-03	-2.207e-03	2.924e-02	3.163e-02	2.249e-02	1.818e-02	1.469e-02	1.061e-02	7.229e-02	4.350e-02	2.704e-02	1.867e-02
RETAIL.10	-5.574e-04	-1.133e-04	-1.557e-03	-3.904e-05	-2.027e-05	-1.099e-03	-1.526e-03	-1.150e-03	-6.435e-04	-1.197e-03	-1.369e-03	-1.531e-03	8.320e-04	1.847e-02	2.181e-02	2.100e-02	1.894e-02	1.635e-02	4.350e-02	3.516e-02	2.717e-02	2.197e-02
RETAIL.15	-4.023e-04	-4.844e-05	-1.066e-03	-1.833e-05	7.616e-06	-8.354e-04	-1.486e-03	-8.404e-04	-4.129e-04	-8.734e-04	-1.248e-03	-1.289e-03	-9.592e-03	1.081e-02	1.805e-02	1.910e-02	1.828e-02	1.678e-02	2.704e-02	2.717e-02	2.453e-02	2.198e-02
RETAIL.20	-3.602e-04	-2.440e-05	-8.285e-04	5.567e-06	3.524e-05	-7.620e-04	-1.426e-03	-7.515e-04	-3.932e-04	-7.783e-04	-1.192e-03	-1.319e-03	-1.154e-02	7.303e-03	1.527e-02	1.724e-02	1.723e-02	1.638e-02	1.867e-02	2.197e-02	2.198e-02	2.116e-02

Appendix 4 - Economic Scenario Generator Input

Table 4.2 – Cholesky matrix

(V1-V22 legend is on appendix 4.4)

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14	V15	V16	V17	V18	V19	V20	V21	V22
V1	0.019768	0.003543	0.0259	4.32E-05	0.000127	0.034712	0.034242	0.028567	0.041123	0.034286	0.034267	0.048039	0.110728	0.107916	0.04529	0.003736	-0.01528	-0.01304	-0.00097	0.01128	0.036155	0.04989
V2	0	0.011769	0.00331	0.00016	0.001103	0.00258	0.004908	0.001136	0.002537	0.002294	0.003928	0.001488	0.154246	0.139072	0.083534	0.023937	-0.0091	-0.01096	-0.10408	0.011226	0.062544	0.093459
V3	0	0	0.0411	0.000214	0.000453	-0.00257	-0.00095	-0.00141	-0.00774	-0.00147	-0.00027	0.003023	0.139455	0.112735	0.081592	0.056517	0.039559	0.032488	0.055347	0.058914	0.055602	0.055605
V4	0	0	0	0.003651	0.00333	0.003151	0.005383	0.005836	0.003065	0.002332	0.004184	0.002272	-0.20137	-0.17408	-0.08368	-0.02719	-0.00014	0.012016	-0.22776	-0.16208	-0.13275	-0.11576
V5	0	0	0	0	0.001681	-0.00244	-0.00146	-0.00524	-0.00422	-0.00196	-0.00142	-0.00324	0.3266	0.186822	0.111808	0.056806	0.023599	0.013644	0.031036	0.011706	0.009021	0.011109
V6	0	0	0	0	0	0.024966	0.015645	0.024474	0.028744	0.027098	0.015089	0.009726	-0.43416	-0.39324	-0.26638	-0.17756	-0.12286	-0.08786	-0.38078	-0.28122	-0.23912	-0.21439
V7	0	0	0	0	0	0	0.036709	-0.00372	-0.00393	-0.00474	0.025575	0.000923	-0.13211	-0.13415	-0.11033	-0.0791	-0.05304	-0.03459	-0.13802	-0.11158	-0.08587	-0.06598
V8	0	0	0	0	0	0	0	0.015882	0.00317	-0.00412	0.000781	0.001331	0.177819	0.118431	0.07242	0.046473	0.031407	0.015372	0.187268	0.11774	0.077855	0.055787
V9	0	0	0	0	0	0	0	0	0.019153	-0.00616	-0.00044	0.003677	0.33756	0.227957	0.114311	0.041647	0.003123	-0.01209	0.167369	0.095314	0.071067	0.057822
V10	0	0	0	0	0	0	0	0	0	0.002781	-0.00376	-0.00331	0.103469	0.079226	0.071658	0.05477	0.04029	0.031145	0.036954	0.030857	0.025257	0.021771
V11	0	0	0	0	0	0	0	0	0	0	0.005163	0.00177	0.017415	0.014355	0.009769	0.017089	0.021808	0.021006	0.081151	0.053824	0.031672	0.018832
V12	0	0	0	0	0	0	0	0	0	0	0	0.027261	-0.00345	0.000869	0.002242	0.004994	0.005728	0.005459	0.013744	0.031739	0.031598	0.031143
V13	0	0	0	0	0	0	0	0	0	0	0	0	1.963247	1.384	0.807397	0.485685	0.30031	0.189949	1.225829	0.801218	0.617203	0.528226
V14	0	0	0	0	0	0	0	0	0	0	0	0	0	0.437791	0.453218	0.413494	0.358216	0.305232	0.511546	0.45305	0.392846	0.342058
V15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.140154	0.183039	0.176975	0.156688	-0.09952	-0.00349	-0.00968	-0.02226
V16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.066833	0.113601	0.128032	0.141568	0.124299	0.115084	0.125627
V17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.028819	0.064983	-0.00948	0.077547	0.145576	0.192959
V18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.019192	-0.0658	-0.04486	-0.01041	0.006638
V19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.303448	0.222644	0.17153	0.146831
V20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.107358	0.151676	0.180974
V21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.053839	0.083879
V22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.017972

Appendix 4 - Economic Scenario Generator Input

Table 4.3 – Auto-Regressive matrix

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14	V15	V16	V17	V18	V19	V20	V21	V22
V1	3.49E-04	2.16E-05	4.38E-04	4.64E-06	3.08E-06	6.33E-04	5.83E-04	5.30E-04	7.47E-04	6.35E-04	5.96E-04	8.61E-04	3.80E-04	5.24E-04	4.95E-04	3.22E-04	1.73E-04	1.81E-04	-9.11E-04	-5.57E-04	-4.02E-04	-3.60E-04
V2	2.16E-05	2.59E-05	4.00E-05	2.16E-06	2.41E-06	5.34E-05	4.61E-05	5.31E-05	4.90E-05	5.86E-05	4.67E-05	4.36E-05	-7.25E-05	1.62E-05	1.08E-04	1.11E-04	9.49E-05	1.03E-04	-3.06E-04	-1.13E-04	-4.84E-05	-2.44E-05
V3	4.38E-04	4.00E-05	2.27E-03	9.09E-06	1.86E-05	7.41E-04	7.05E-04	6.21E-04	6.78E-04	8.06E-04	7.53E-04	1.25E-03	1.55E-03	1.80E-03	1.39E-03	7.54E-04	2.88E-04	3.14E-04	-2.67E-03	-1.56E-03	-1.07E-03	-8.29E-04
V4	4.64E-06	2.16E-06	9.09E-06	1.20E-05	1.08E-05	1.89E-05	2.96E-05	2.47E-05	1.98E-05	1.59E-05	2.44E-05	1.90E-05	1.62E-04	1.26E-04	7.95E-05	7.03E-05	6.66E-05	6.67E-05	-8.65E-06	-3.90E-05	-1.83E-05	5.57E-06
V5	3.08E-06	2.41E-06	1.86E-05	1.08E-05	1.19E-05	1.33E-05	2.51E-05	1.78E-05	7.38E-06	1.17E-05	2.09E-05	1.24E-05	1.03E-04	1.03E-04	9.05E-05	8.88E-05	8.63E-05	9.20E-05	-1.93E-05	-2.03E-05	7.62E-06	3.52E-05
V6	6.33E-04	5.34E-05	7.41E-04	1.89E-05	1.33E-05	1.73E-03	1.43E-03	1.54E-03	2.04E-03	1.77E-03	1.44E-03	1.80E-03	2.61E-03	2.00E-03	1.51E-03	9.62E-04	5.27E-04	4.82E-04	-1.68E-03	-1.10E-03	-8.35E-04	-7.62E-04
V7	5.83E-04	4.61E-05	7.05E-04	2.96E-05	2.51E-05	1.43E-03	2.59E-03	1.16E-03	1.57E-03	1.26E-03	2.17E-03	1.62E-03	4.58E-03	3.14E-03	1.90E-03	1.16E-03	6.45E-04	5.11E-04	-1.24E-03	-1.53E-03	-1.49E-03	-1.43E-03
V8	5.30E-04	5.31E-05	6.21E-04	2.47E-05	1.78E-05	1.54E-03	1.16E-03	1.60E-03	1.85E-03	1.55E-03	1.20E-03	1.55E-03	1.93E-03	1.36E-03	1.17E-03	7.78E-04	4.26E-04	3.65E-04	-1.79E-03	-1.15E-03	-8.40E-04	-7.52E-04
V9	7.47E-04	4.90E-05	6.78E-04	1.98E-05	7.38E-06	2.04E-03	1.57E-03	1.85E-03	2.77E-03	1.99E-03	1.61E-03	2.17E-03	2.83E-03	2.31E-03	1.88E-03	1.25E-03	7.50E-04	6.93E-04	-1.31E-03	-6.44E-04	-4.13E-04	-3.93E-04
V10	6.35E-04	5.86E-05	8.06E-04	1.59E-05	1.17E-05	1.77E-03	1.26E-03	1.55E-03	1.99E-03	1.89E-03	1.33E-03	1.79E-03	2.45E-03	1.92E-03	1.51E-03	9.69E-04	5.24E-04	4.89E-04	-1.95E-03	-1.20E-03	-8.73E-04	-7.78E-04
V11	5.96E-04	4.67E-05	7.53E-04	2.44E-05	2.09E-05	1.44E-03	2.17E-03	1.20E-03	1.61E-03	1.33E-03	1.93E-03	1.66E-03	3.35E-03	2.40E-03	1.46E-03	8.44E-04	4.17E-04	3.55E-04	-1.38E-03	-1.37E-03	-1.25E-03	-1.19E-03
V12	8.61E-04	4.36E-05	1.25E-03	1.90E-05	1.24E-05	1.80E-03	1.62E-03	1.55E-03	2.17E-03	1.79E-03	1.66E-03	3.01E-03	2.59E-03	1.90E-03	1.08E-03	3.71E-04	-1.86E-04	-2.28E-04	-2.21E-03	-1.53E-03	-1.29E-03	-1.32E-03
V13	3.80E-04	-7.25E-05	1.55E-03	1.62E-04	1.03E-04	2.61E-03	4.58E-03	1.93E-03	2.83E-03	2.45E-03	3.35E-03	2.59E-03	2.01E-01	8.81E-02	4.42E-02	2.48E-02	1.34E-02	4.69E-03	2.92E-02	8.32E-04	-9.59E-03	-1.15E-02
V14	5.24E-04	1.62E-05	1.80E-03	1.26E-04	1.03E-04	2.00E-03	3.14E-03	1.36E-03	2.31E-03	1.92E-03	2.40E-03	1.90E-03	8.81E-02	6.07E-02	4.40E-02	3.24E-02	2.40E-02	1.77E-02	3.16E-02	1.85E-02	1.08E-02	7.30E-03
V15	4.95E-04	1.08E-04	1.39E-03	7.95E-05	9.05E-05	1.51E-03	1.90E-03	1.17E-03	1.88E-03	1.51E-03	1.46E-03	1.08E-03	4.42E-02	4.40E-02	4.47E-02	3.86E-02	3.19E-02	2.67E-02	2.25E-02	2.18E-02	1.81E-02	1.53E-02
V16	3.22E-04	1.11E-04	7.54E-04	7.03E-05	8.88E-05	9.62E-04	1.16E-03	7.78E-04	1.25E-03	9.69E-04	8.44E-04	3.71E-04	2.48E-02	3.24E-02	3.86E-02	3.66E-02	3.24E-02	2.85E-02	1.82E-02	2.10E-02	1.91E-02	1.72E-02
V17	1.73E-04	9.49E-05	2.88E-04	6.66E-05	8.63E-05	5.27E-04	6.45E-04	4.26E-04	7.50E-04	5.24E-04	4.17E-04	-1.86E-04	1.34E-02	2.40E-02	3.19E-02	3.24E-02	3.03E-02	2.77E-02	1.47E-02	1.89E-02	1.83E-02	1.72E-02
V18	1.81E-04	1.03E-04	3.14E-04	6.67E-05	9.20E-05	4.82E-04	5.11E-04	3.65E-04	6.93E-04	4.89E-04	3.55E-04	-2.28E-04	4.69E-03	1.77E-02	2.67E-02	2.85E-02	2.77E-02	2.65E-02	1.06E-02	1.64E-02	1.68E-02	1.64E-02
V19	-9.11E-04	-3.06E-04	-2.67E-03	-8.65E-06	-1.93E-05	-1.68E-03	-1.24E-03	-1.79E-03	-1.31E-03	-1.95E-03	-1.38E-03	-2.21E-03	2.92E-02	3.16E-02	2.25E-02	1.82E-02	1.47E-02	1.06E-02	7.23E-02	4.35E-02	2.70E-02	1.87E-02
V20	-5.57E-04	-1.13E-04	-1.56E-03	-3.90E-05	-2.03E-05	-1.10E-03	-1.53E-03	-1.15E-03	-6.44E-04	-1.20E-03	-1.37E-03	-1.53E-03	8.32E-04	1.85E-02	2.18E-02	2.10E-02	1.89E-02	1.64E-02	4.35E-02	3.52E-02	2.72E-02	2.20E-02
V21	-4.02E-04	-4.84E-05	-1.07E-03	-1.83E-05	7.62E-06	-8.35E-04	-1.49E-03	-8.40E-04	-4.13E-04	-8.73E-04	-1.25E-03	-1.29E-03	-9.59E-03	1.08E-02	1.81E-02	1.91E-02	1.83E-02	1.68E-02	2.70E-02	2.72E-02	2.45E-02	2.20E-02
V22	-3.60E-04	-2.44E-05	-8.29E-04	5.57E-06	3.52E-05	-7.62E-04	-1.43E-03	-7.52E-04	-3.93E-04	-7.78E-04	-1.19E-03	-1.32E-03	-1.15E-02	7.30E-03	1.53E-02	1.72E-02	1.72E-02	1.64E-02	1.87E-02	2.20E-02	2.20E-02	2.12E-02

Appendix 4 - Economic Scenario Generator Input

Table 4.4 – D matrix

Var	Code	Description	D
V1	HF	Hedge funds	-0.00221
V2	PPT	Property	0.00259
V3	CM	Commodities	-0.01441
V4	CPI	CPI	-0.00009
V5	RPI	RPI	-0.00456
V6	World	World Equity	-0.01507
V7	JPN	Japanese Equity	-0.02506
V8	UK	UK Equity	-0.01073
V9	EMU	European Monetary Union Equity	-0.02351
V10	NAm	North American Equity	-0.01582
V11	PACFIC	Pacific (Ex Japan) Equity	-0.01316
V12	EM	Emerging Markets Equity	-0.00322
V13	NOM.1	Nominal interest rate (1 year)	-0.36190
V14	NOM.5	Nominal interest rate (5 year)	2.16223
V15	NOM.10	Nominal interest rate (10 year)	3.85289
V16	NOM.15	Nominal interest rate (15 year)	3.92319
V17	NOM.20	Nominal interest rate (20 year)	4.01950
V18	NOM.25	Nominal interest rate (25 year)	4.10601
V19	REAL.5	Real interest rate (5 year)	-0.05865
V20	REAL.10	Real interest rate (10 year)	-0.05133
V21	REAL.15	Real interest rate (15 year)	0.01459
V22	REAL.20	Real interest rate (20 year)	0.06155

Appendix 5 – Economic Scenarios

Figure 5.1 Generation of Japanese equity index scenarios.

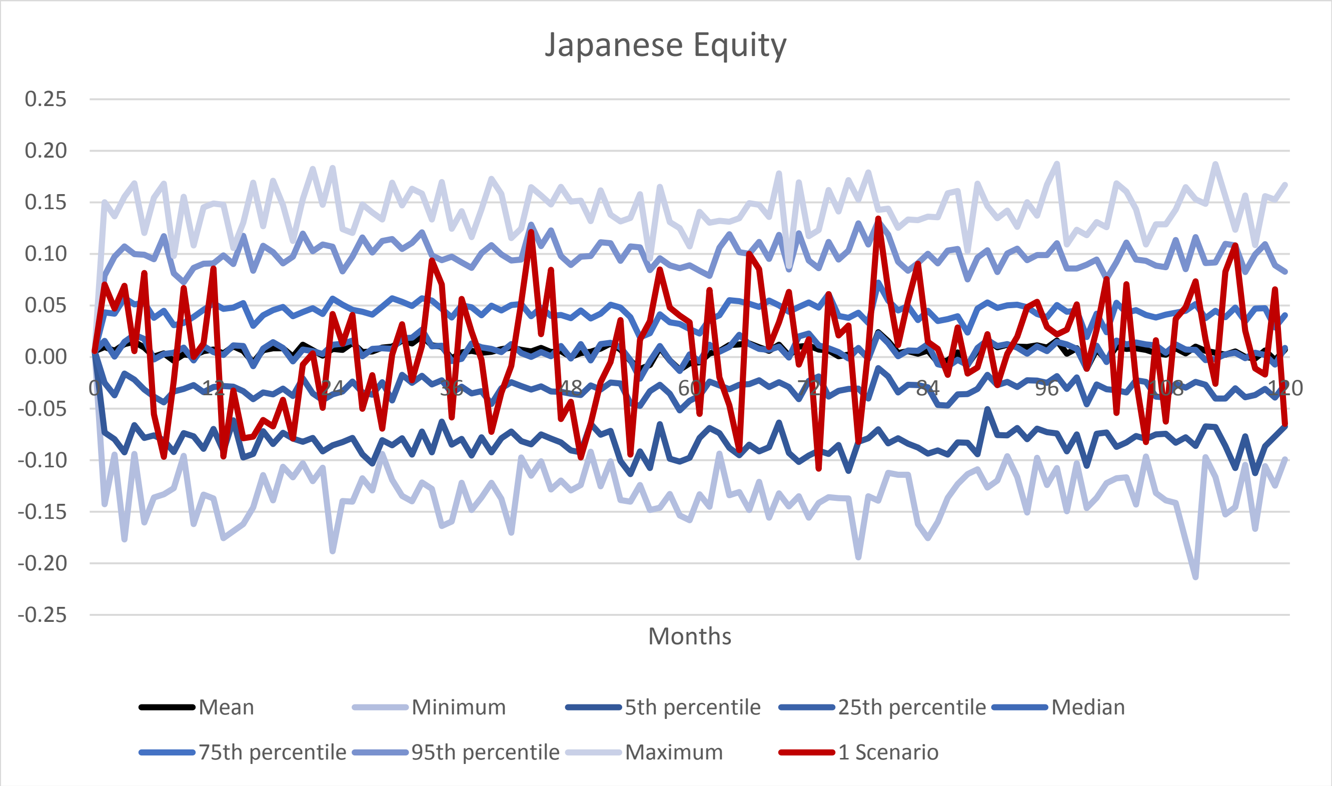
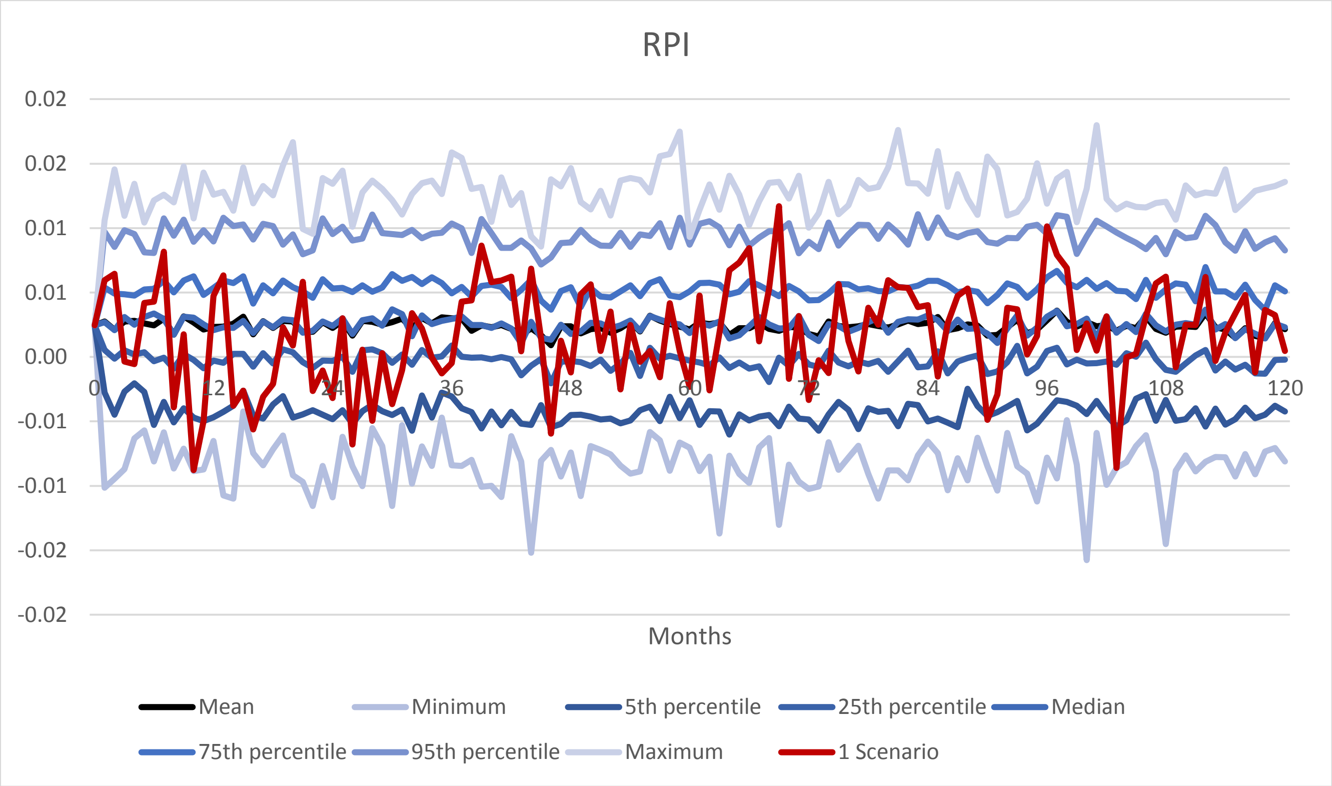


Figure 5.2 Generation of RPI index scenarios.



Appendix 5 – Economic Scenarios

Figure 5.3 Generation of UK equity index scenarios.

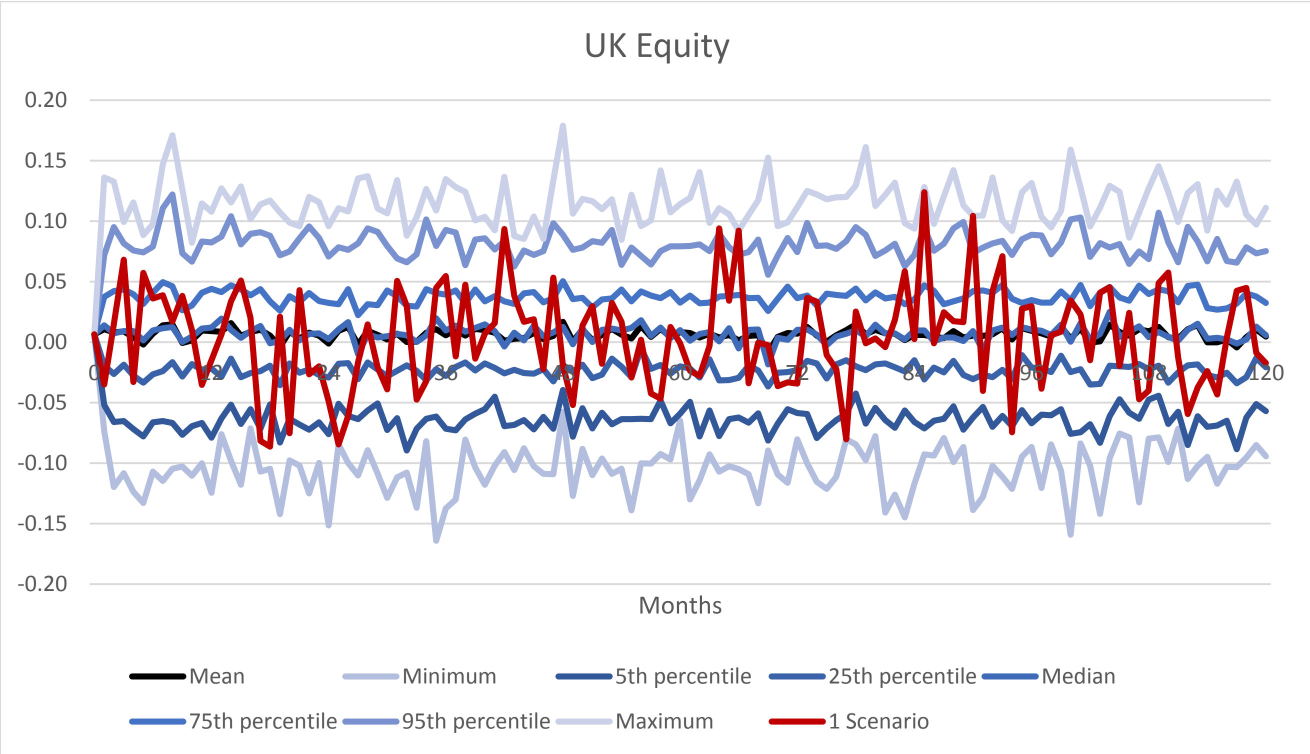


Figure 5.4 Generation of UK equity index scenarios.

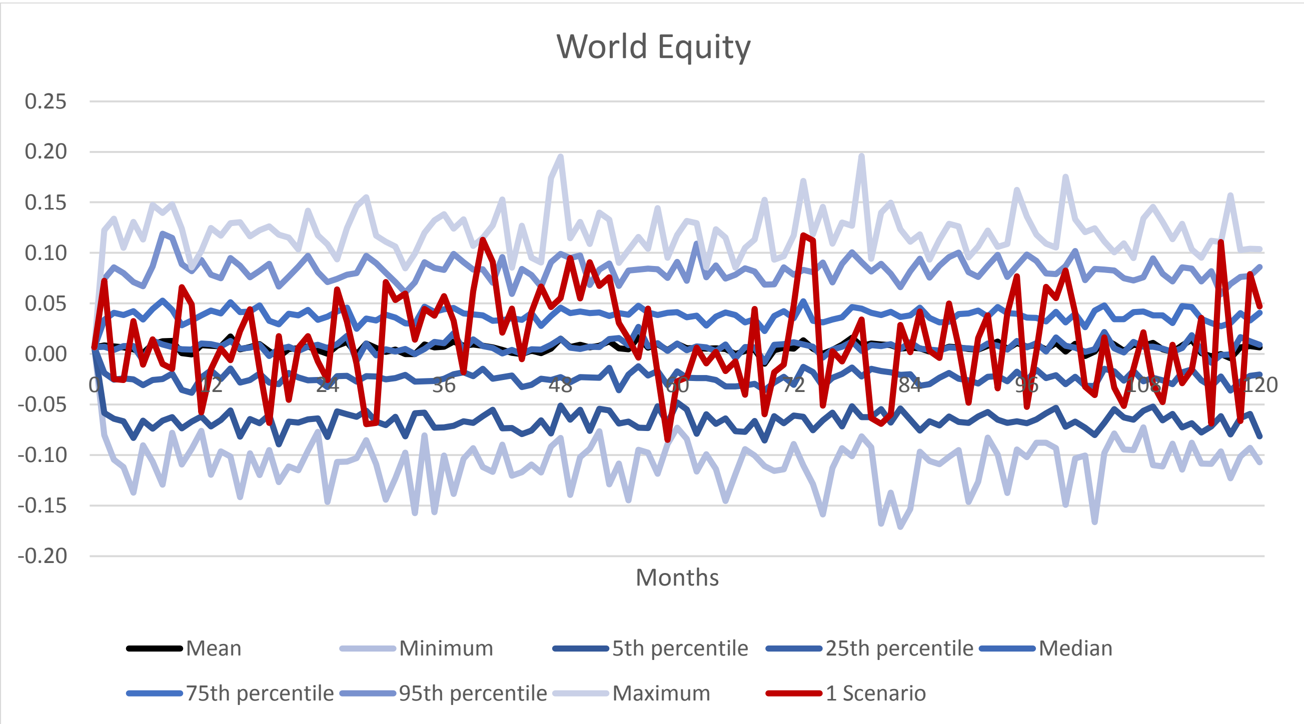


Figure 5.5 Generation of Pacific (Ex Pacific) equity index scenarios.

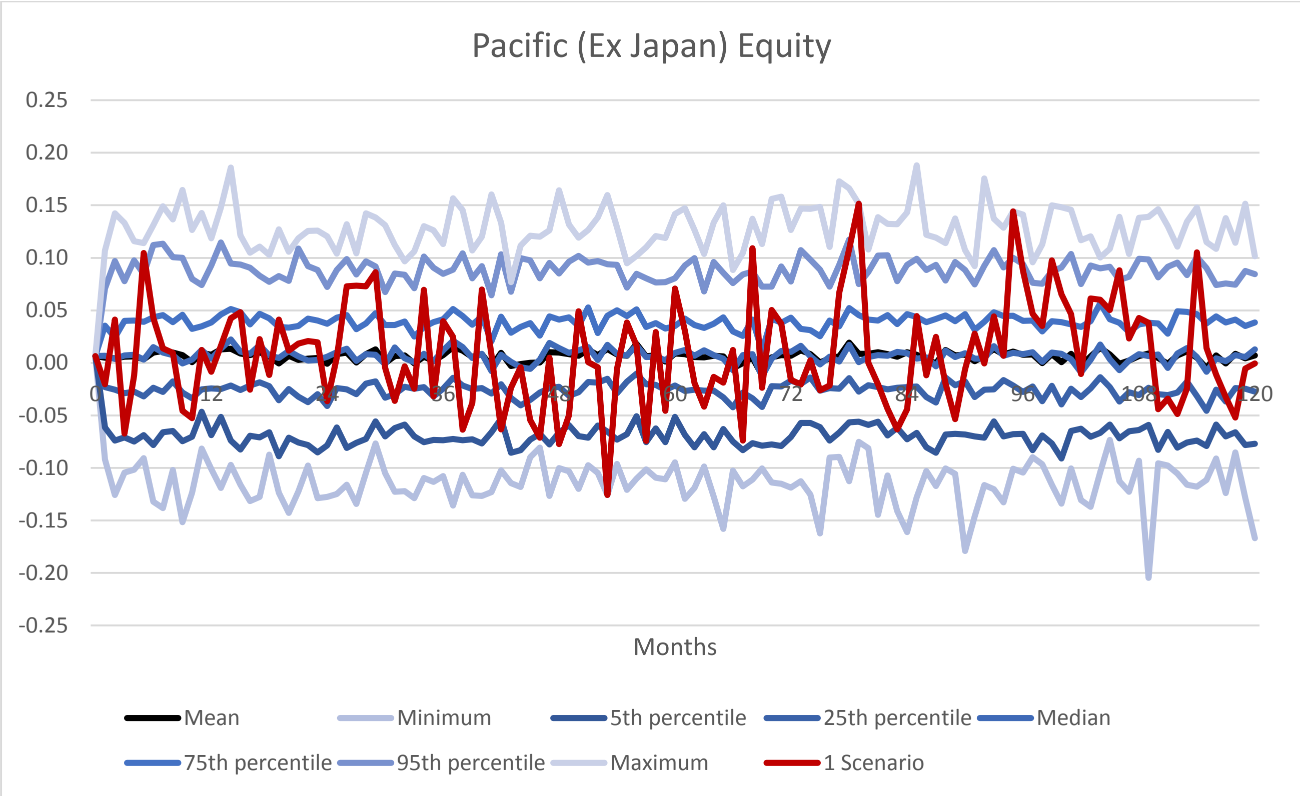


Figure 5.6 Generation of the European Monetary Union equity index scenarios.

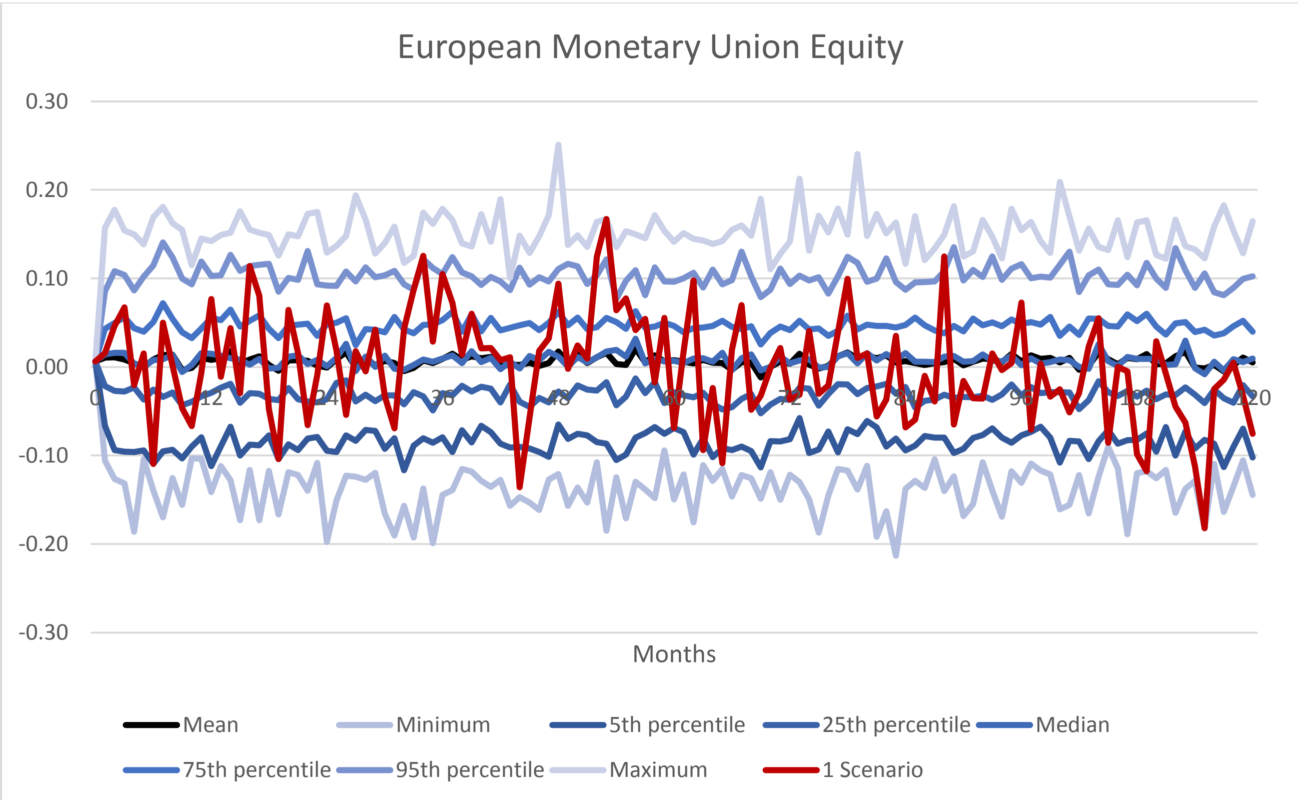


Figure 5.7 Generation of the North American equity index scenarios.

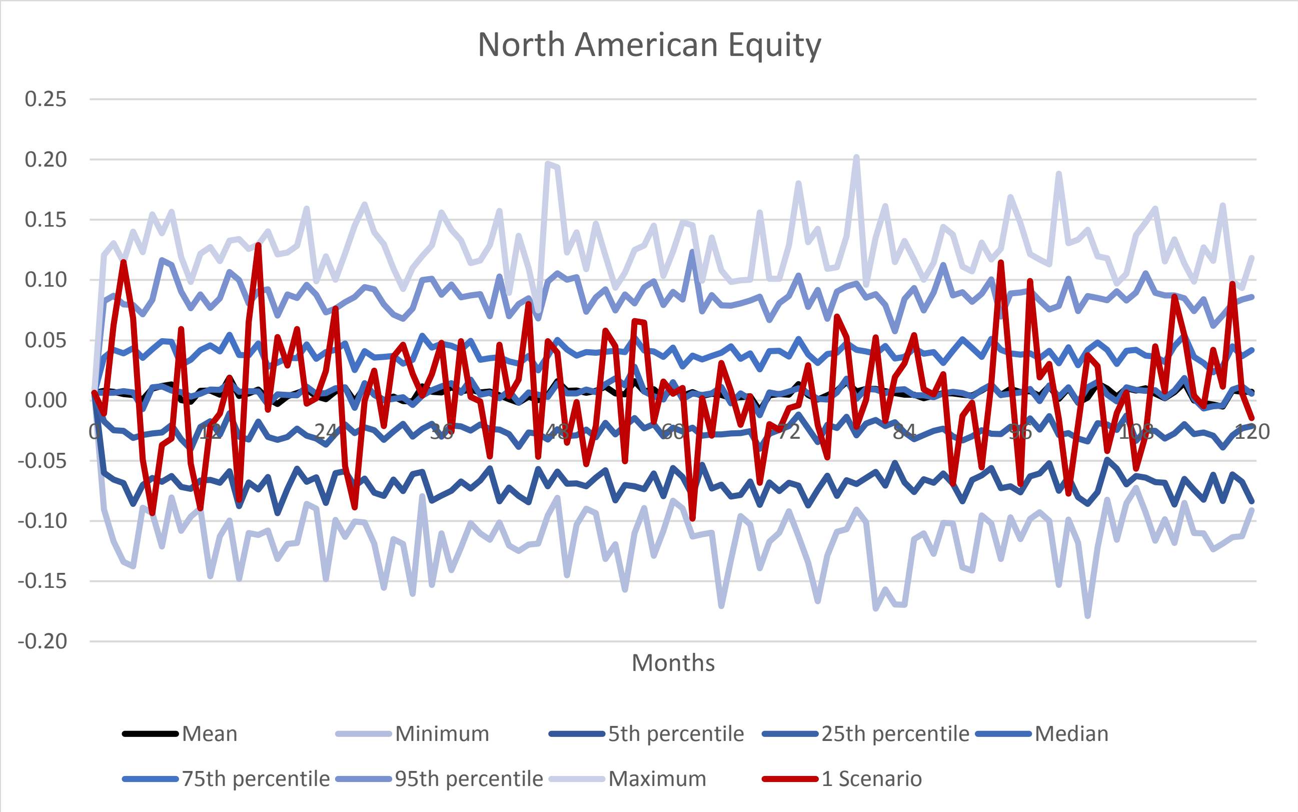
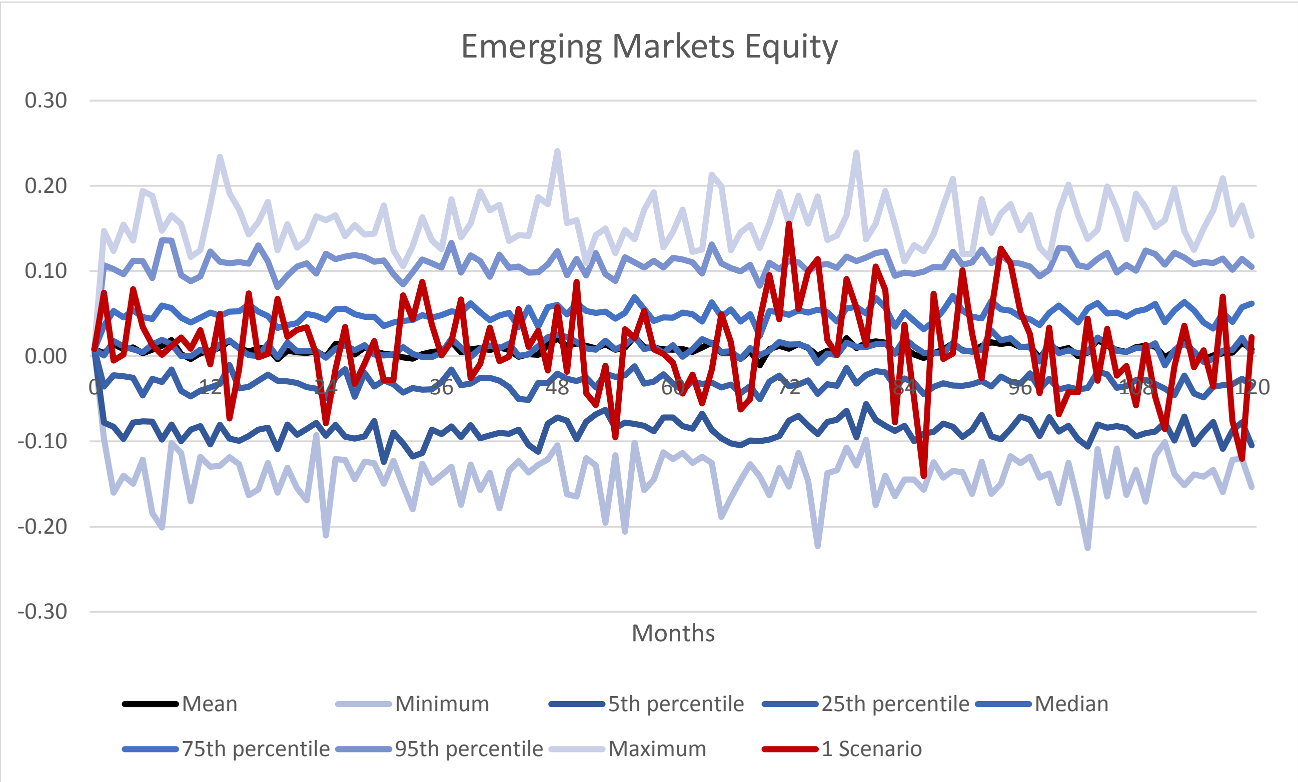


Figure 5.8 Generation of the Emerging Market equity index scenarios.



Appendix 5 – Economic Scenarios

Figure 5.9 Generation of the 1 year Normal Interest rate scenarios.

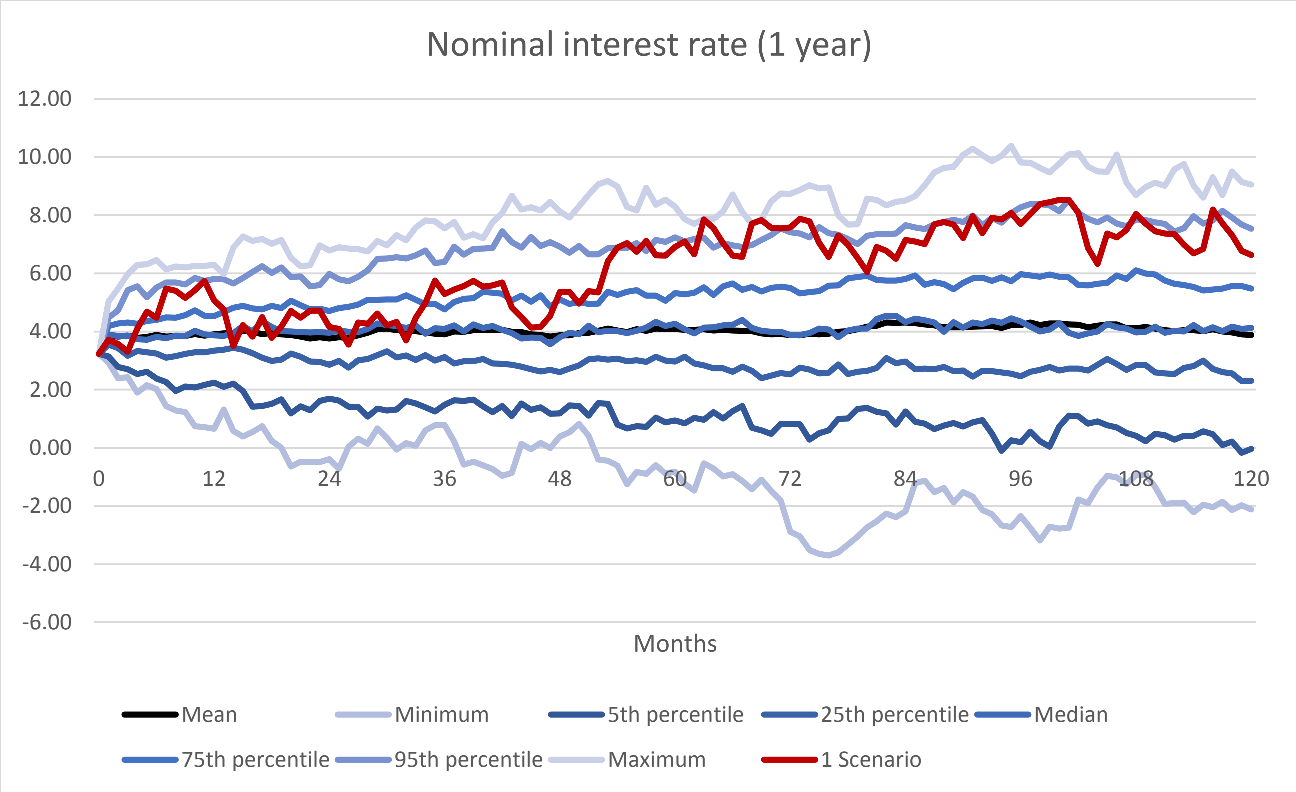
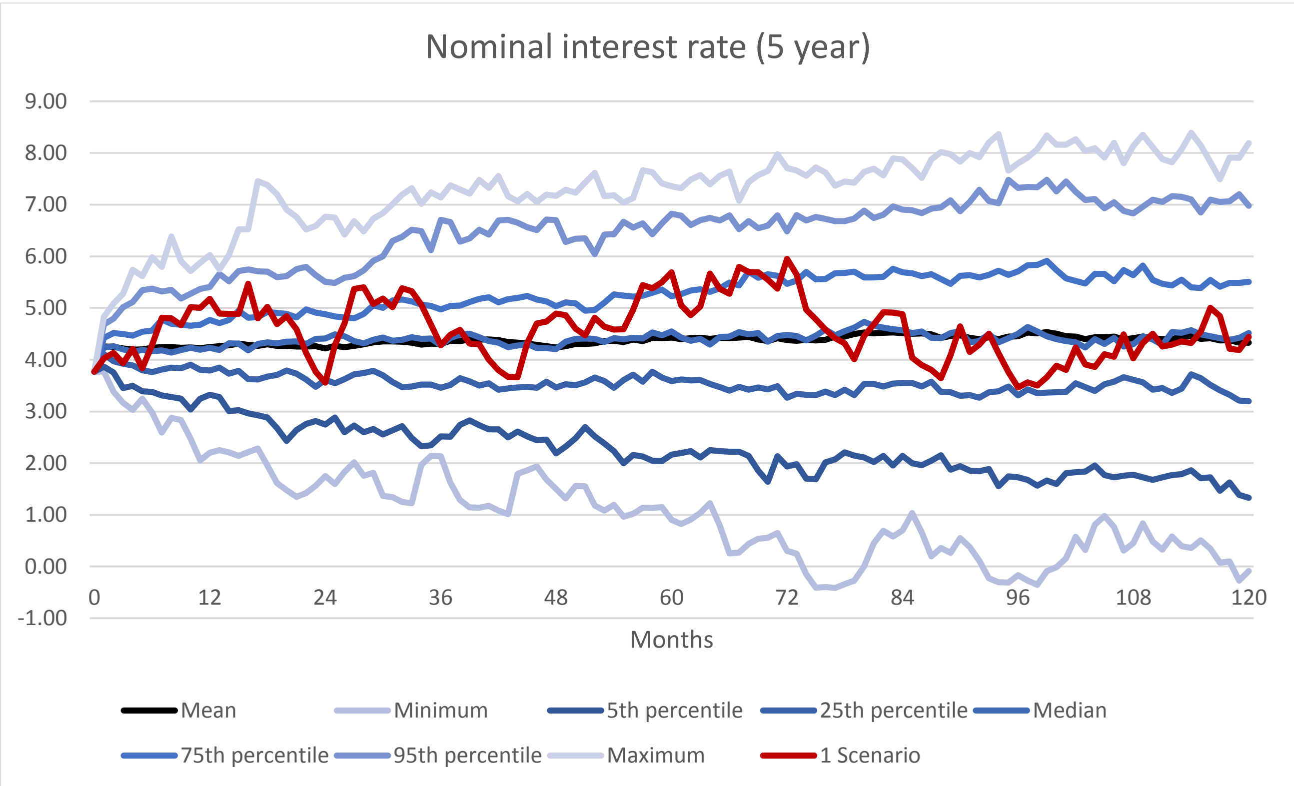


Figure 5.10 Generation of the 5 years Normal Interest rate scenarios.



Appendix 5 – Economic Scenarios

Figure 5.11 Generation of the 10 years Normal Interest rate scenarios.

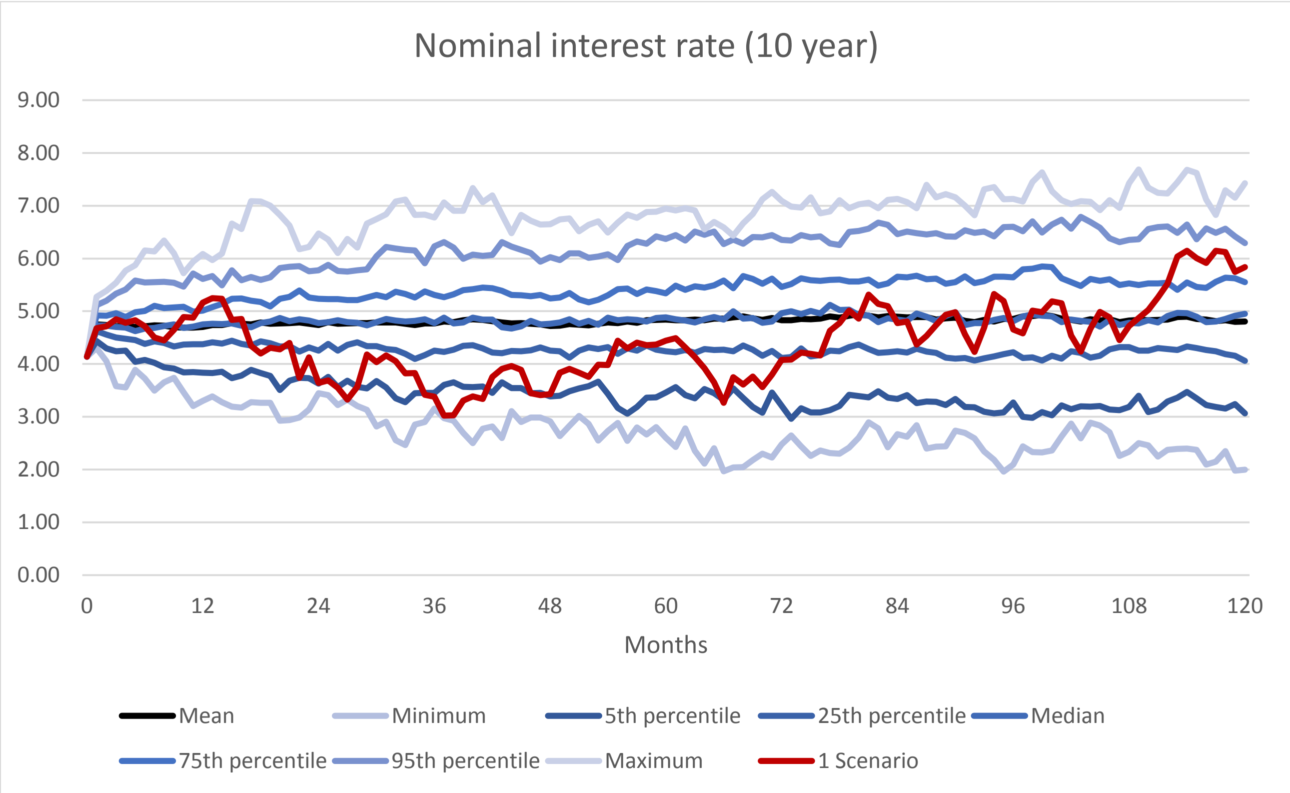


Figure 5.12 Generation of the 15 years Normal Interest rate scenarios.

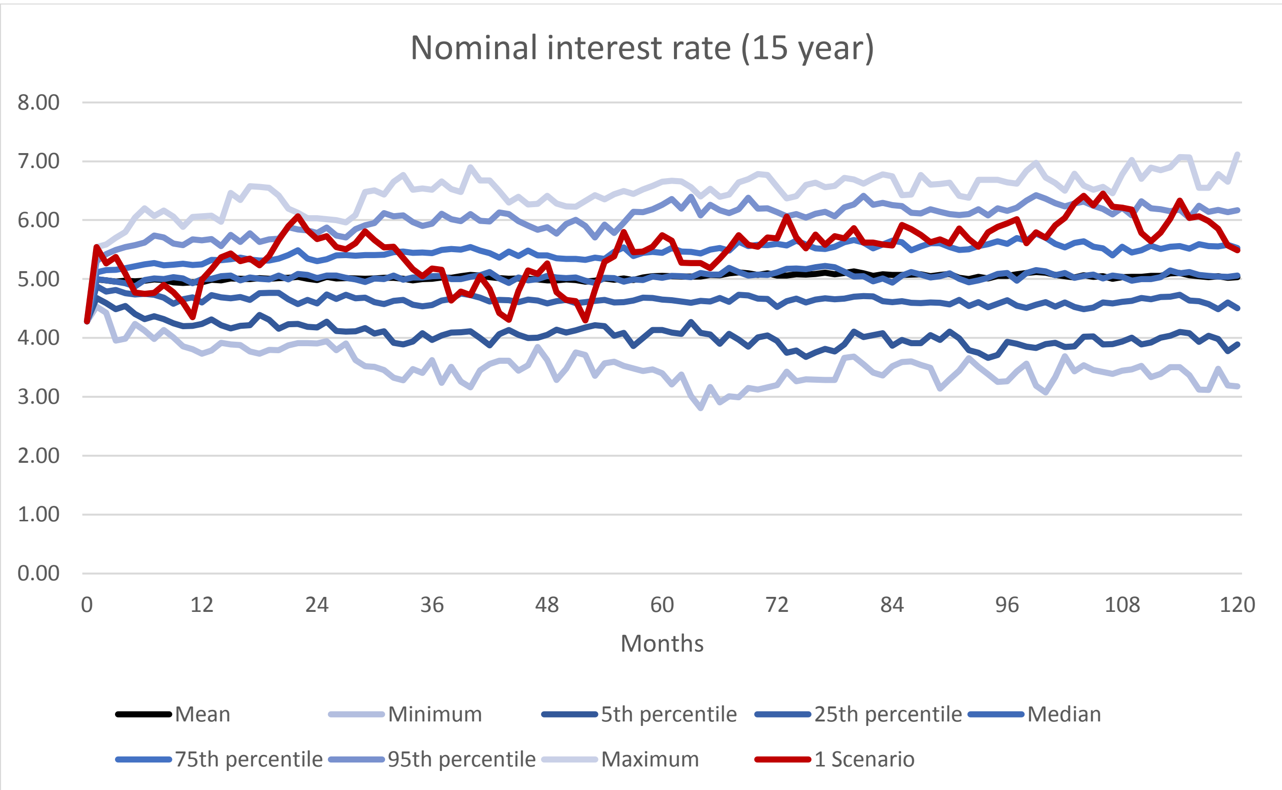


Figure 5.13 Generation of the 20 years Normal Interest rate scenarios.

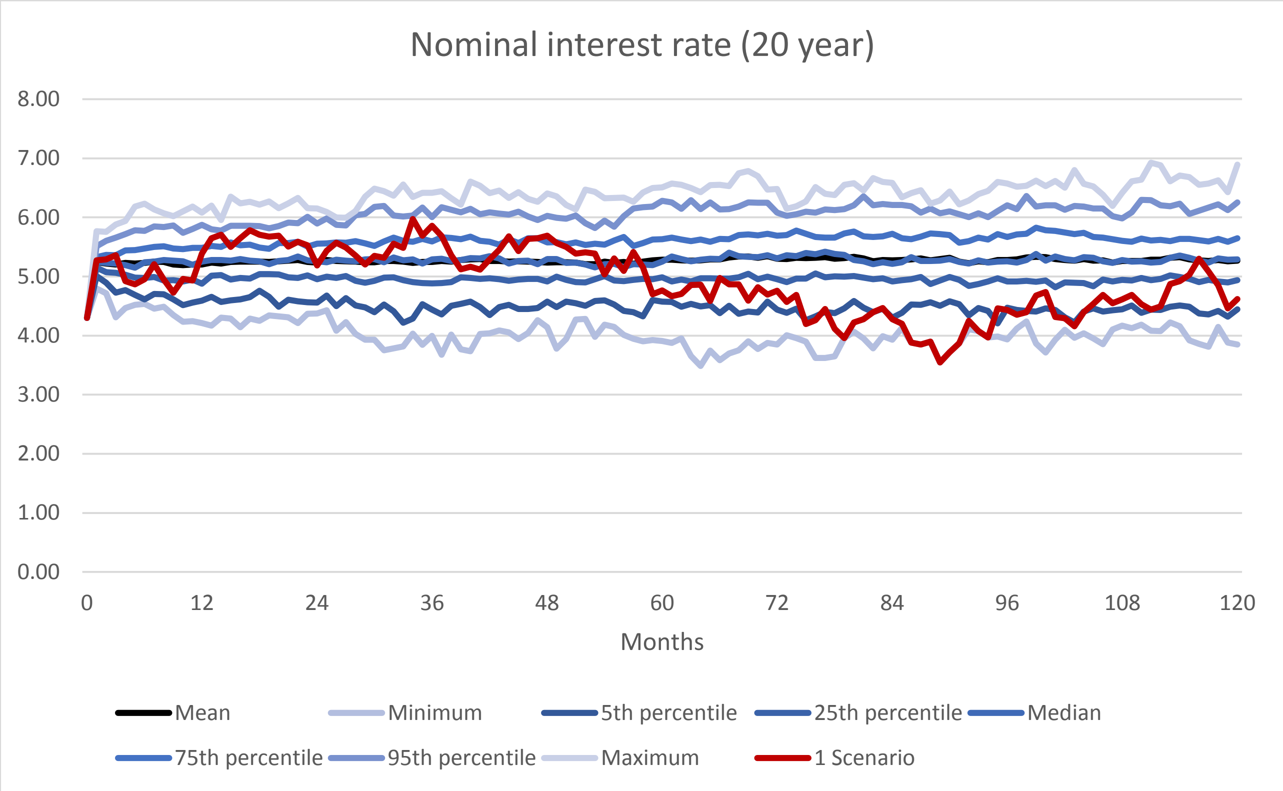
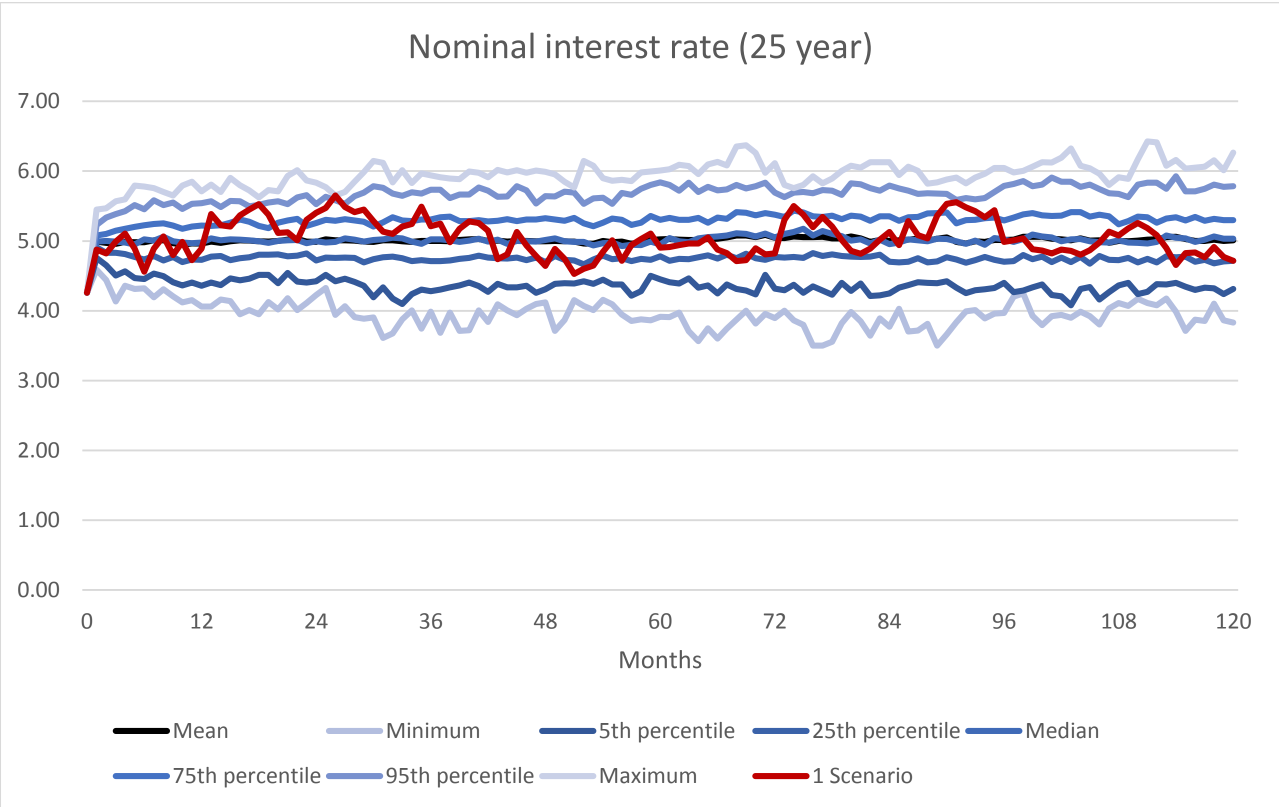


Figure 5.14 Generation of the 25 years Normal Interest rate scenarios.



Appendix 5 – Economic Scenarios

Figure 5.15 Generation of the 5 years Real Interest rate scenarios.

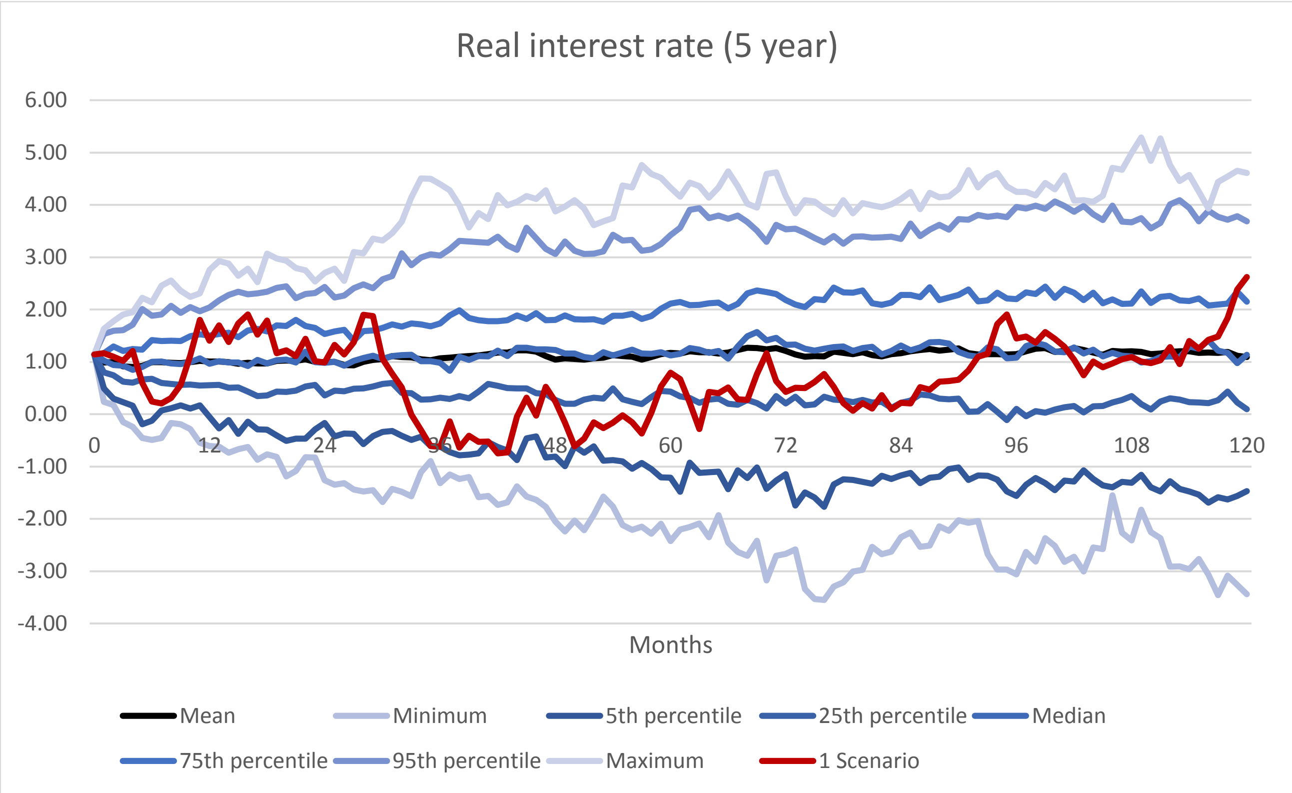


Figure 5.16 Generation of the 10 years Real Interest rate scenarios.

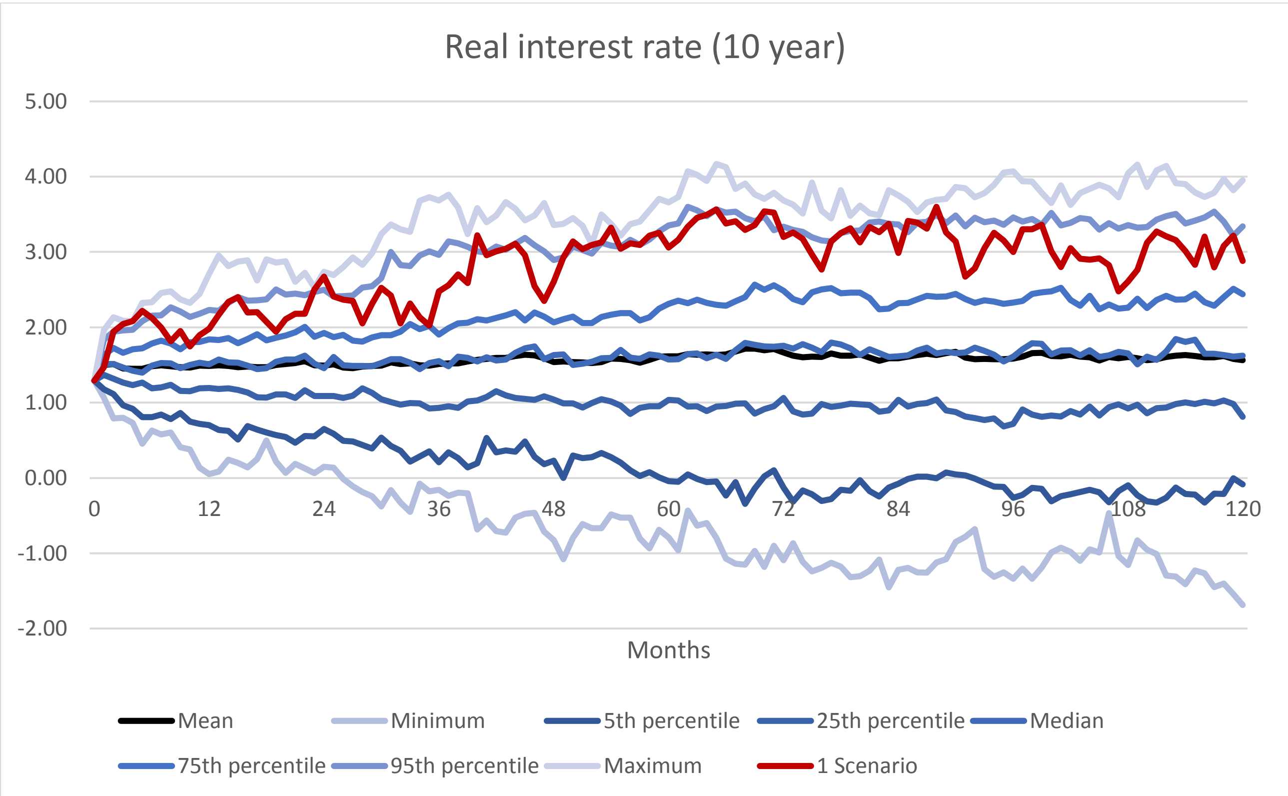


Figure 5.17 Generation of the 15 years Real Interest rate scenarios.

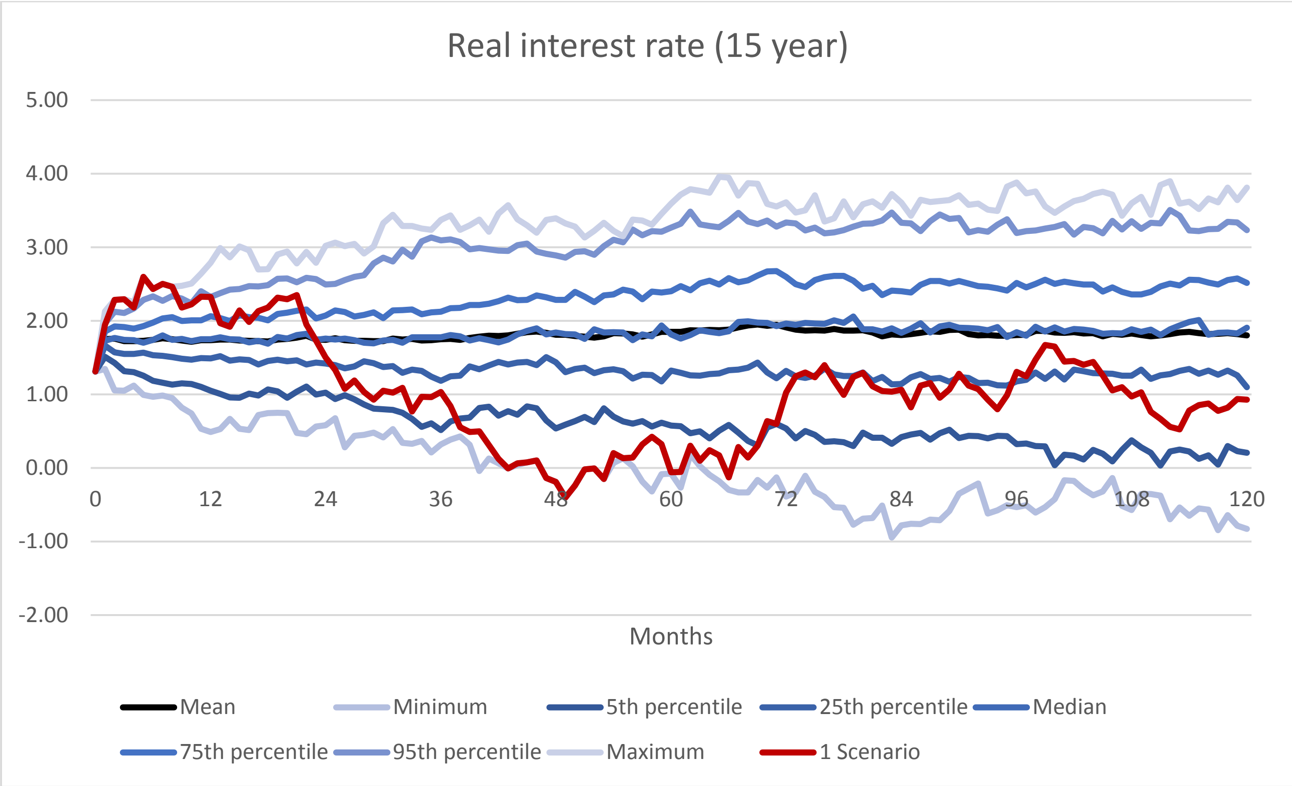
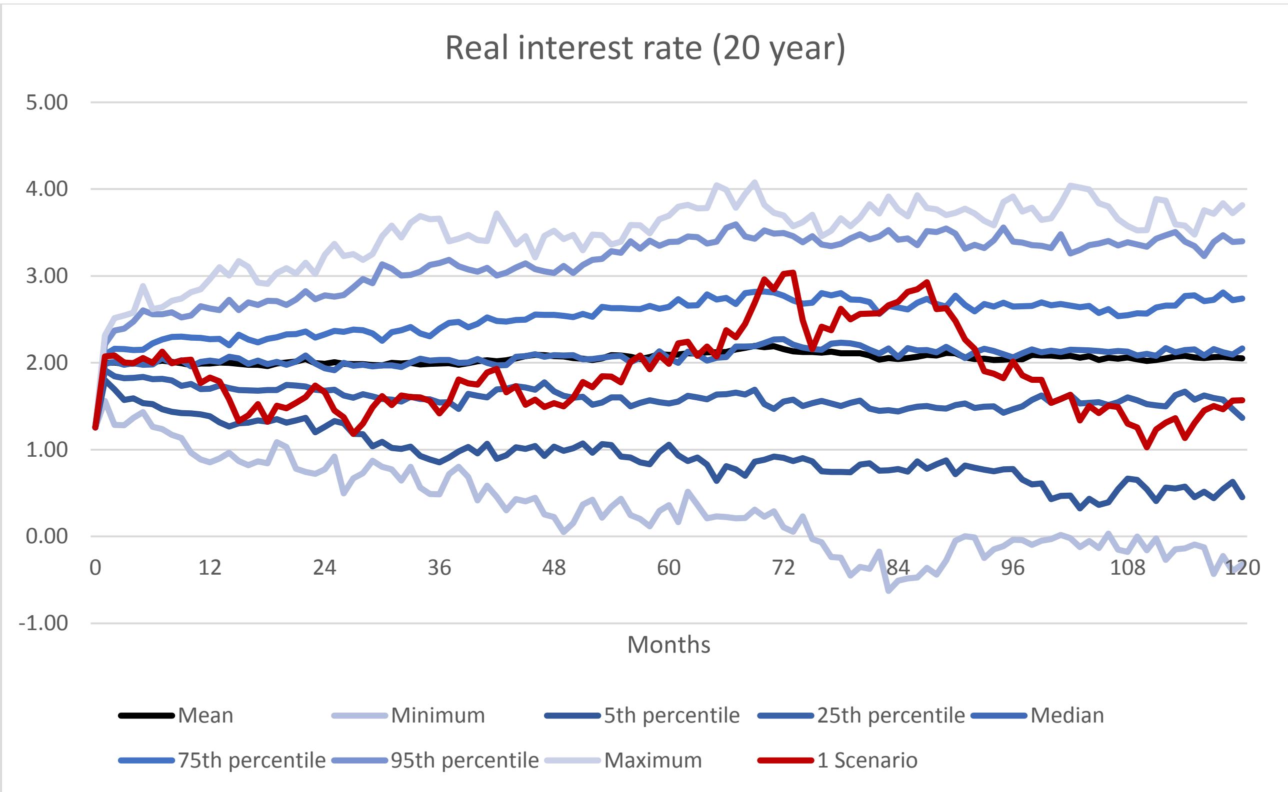


Figure 5.18 Generation of the 20 years Real Interest rate scenarios.



Appendix 6 – Correlation Tables

Table 6.1 Historical Correlations

	HF	PPT	CM	CPI	RPI	World	JPN	UK	EMU	NAm	PACFIC	EM	NOM.1	NOM.5	NOM.10	NOM.15	NOM.20	NOM.25	REAL.5	REAL.10	REAL.15	REAL.20
Hedge funds	1.00	0.29	0.53	0.01	0.03	0.81	0.64	0.68	0.75	0.76	0.74	0.85	0.05	0.07	0.04	0.01	-0.03	-0.03	0.00	0.01	0.04	0.06
Property	0.29	1.00	0.22	0.05	0.28	0.29	0.27	0.22	0.26	0.27	0.30	0.27	0.09	0.10	0.09	0.03	-0.02	-0.03	-0.07	0.01	0.08	0.13
Commodities	0.53	0.22	1.00	0.06	0.13	0.38	0.33	0.34	0.28	0.38	0.40	0.50	0.09	0.10	0.10	0.07	0.05	0.05	0.03	0.05	0.08	0.10
CPI	0.01	0.05	0.06	1.00	0.87	0.08	0.11	0.15	0.06	0.06	0.10	0.05	-0.09	-0.10	-0.07	-0.03	0.00	0.03	-0.15	-0.15	-0.15	-0.14
RPI	0.03	0.28	0.13	0.87	1.00	0.07	0.12	0.09	0.04	0.06	0.11	0.05	0.02	-0.01	0.01	0.02	0.02	0.04	-0.14	-0.12	-0.10	-0.08
World Equity	0.81	0.29	0.38	0.08	0.07	1.00	0.71	0.91	0.93	0.98	0.80	0.79	-0.09	-0.10	-0.13	-0.15	-0.16	-0.15	-0.17	-0.16	-0.14	-0.12
Japanese Equity	0.64	0.27	0.33	0.11	0.12	0.71	1.00	0.57	0.60	0.61	0.98	0.62	-0.08	-0.10	-0.13	-0.15	-0.16	-0.14	-0.16	-0.16	-0.14	-0.10
UK Equity	0.68	0.22	0.34	0.15	0.09	0.91	0.57	1.00	0.87	0.87	0.67	0.70	-0.08	-0.09	-0.11	-0.13	-0.13	-0.12	-0.12	-0.12	-0.11	-0.11
European Monetary Union Equity	0.75	0.26	0.28	0.06	0.04	0.93	0.60	0.87	1.00	0.86	0.70	0.75	-0.03	-0.04	-0.07	-0.11	-0.14	-0.14	-0.10	-0.10	-0.09	-0.07
North American Equity	0.76	0.27	0.38	0.06	0.06	0.98	0.61	0.87	0.86	1.00	0.71	0.74	-0.11	-0.12	-0.14	-0.15	-0.16	-0.14	-0.19	-0.17	-0.15	-0.13
Pacific (Ex Japan) Equity	0.74	0.30	0.40	0.10	0.11	0.80	0.98	0.67	0.70	0.71	1.00	0.71	-0.07	-0.09	-0.12	-0.15	-0.15	-0.13	-0.15	-0.15	-0.12	-0.09
Emerging Markets Equity	0.85	0.27	0.50	0.05	0.05	0.79	0.62	0.70	0.75	0.74	0.71	1.00	0.01	0.02	-0.01	-0.04	-0.06	-0.06	-0.04	-0.02	0.01	0.03
Nominal interest rate (1 year)	0.05	0.09	0.09	-0.09	0.02	-0.09	-0.08	-0.08	-0.03	-0.11	-0.07	0.01	1.00	0.96	0.88	0.75	0.60	0.46	0.88	0.83	0.79	0.75
Nominal interest rate (5 year)	0.07	0.10	0.10	-0.10	-0.01	-0.10	-0.10	-0.09	-0.04	-0.12	-0.09	0.02	0.96	1.00	0.97	0.88	0.76	0.65	0.94	0.93	0.90	0.86
Nominal interest rate (10 year)	0.04	0.09	0.10	-0.07	0.01	-0.13	-0.13	-0.11	-0.07	-0.14	-0.12	-0.01	0.88	0.97	1.00	0.97	0.88	0.79	0.92	0.94	0.91	0.87
Nominal interest rate (15 year)	0.01	0.03	0.07	-0.03	0.02	-0.15	-0.15	-0.13	-0.11	-0.15	-0.15	-0.04	0.75	0.88	0.97	1.00	0.97	0.91	0.87	0.91	0.89	0.85
Nominal interest rate (20 year)	-0.03	-0.02	0.05	0.00	0.02	-0.16	-0.16	-0.13	-0.14	-0.16	-0.15	-0.06	0.60	0.76	0.88	0.97	1.00	0.98	0.78	0.84	0.83	0.80
Nominal interest rate (25 year)	-0.03	-0.03	0.05	0.03	0.04	-0.15	-0.14	-0.12	-0.14	-0.14	-0.13	-0.06	0.46	0.65	0.79	0.91	0.98	1.00	0.67	0.76	0.77	0.75
Real interest rate (5 year)	0.00	-0.07	0.03	-0.15	-0.14	-0.17	-0.16	-0.12	-0.10	-0.19	-0.15	-0.04	0.88	0.94	0.92	0.87	0.78	0.67	1.00	0.98	0.94	0.89
Real interest rate (10 year)	0.01	0.01	0.05	-0.15	-0.12	-0.16	-0.16	-0.12	-0.10	-0.17	-0.15	-0.02	0.83	0.93	0.94	0.91	0.84	0.76	0.98	1.00	0.99	0.95
Real interest rate (15 year)	0.04	0.08	0.08	-0.15	-0.10	-0.14	-0.14	-0.11	-0.09	-0.15	-0.12	0.01	0.79	0.90	0.91	0.89	0.83	0.77	0.94	0.99	1.00	0.99
Real interest rate (20 year)	0.06	0.13	0.10	-0.14	-0.08	-0.12	-0.10	-0.11	-0.07	-0.13	-0.09	0.03	0.75	0.86	0.87	0.85	0.80	0.75	0.89	0.95	0.99	1.00

Appendix 6 – Correlation Tables

Table 6.2 Generated Correlations

	HF	PPT	CM	CPI	RPI	World	JPN	UK	EMU	NAm	PACFIC	EM	NOM.1	NOM.5	NOM.10	NOM.15	NOM.20	NOM.25	REAL.5	REAL.10	REAL.15	REAL.20
Hedge funds	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
Property	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
Commodities	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
CPI	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
RPI	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
World Equity	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
Japanese Equity	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
UK Equity	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
European Monetary Union Equity	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
North American Equity	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
Pacific (Ex Japan) Equity	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
Emerging Markets Equity	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
Nominal interest rate (1 year)	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
Nominal interest rate (5 year)	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
Nominal interest rate (10 year)	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
Nominal interest rate (15 year)	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
Nominal interest rate (20 year)	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
Nominal interest rate (25 year)	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
Real interest rate (5 year)	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
Real interest rate (10 year)	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
Real interest rate (15 year)	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13
Real interest rate (20 year)	1.00	1.00	1.00	0.16	0.57	-0.25	-0.22	0.84	0.61	0.72	0.78	0.81	0.73	0.83	0.03	0.18	0.25	0.19	0.13	0.13	-0.20	-0.13