M2 Biology	22/10/18

TD3: Models of Neurons III

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https://github.com/Elieoriol/2021_UlmM2_ThNeuro/tree/master/TD3

In the first tutorial, we have developed the most basic model of a spiking neuron: the LIF neuron. In the second one, we have underlined the inability of this model to account for a fundamental property of real neurons that is post-spiking refractoriness, and have developed possible theoretical mechanisms to address this problem. We have also introduced the QIF model, richer in behavior than the LIF one.

In this tutorial we will be interested in the modeling of two more features of biological neurons: adaptation and shunting inhibition.

1 Firing rate adaptation

A well-known property of neurons is adaptation. For instance, driven by an injected current, a decrease in time of the firing rate of a neuron to a steady-state value can be observed.

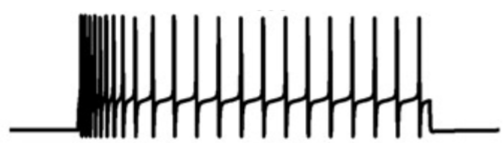

Figure 1: Example of firing rate adaptation in response to an injected current.

We are going to model this phenomenon by considering the effect of ion channels which open whenever a neuron fires a spike and let in negative current, such that:

$$\tau_m \frac{\mathrm{d}V}{\mathrm{d}t} = -V - W + I \tag{1}$$

where, after each spike occurring at V_{th}, W is increased by W_R and V is reset to 0. Between spikes, W decays back to zero with time constant τ_w:

$$\tau_w \frac{\mathrm{d}W}{\mathrm{d}t} = -W \tag{2}$$

1.1 First approximation

We first make the approximation that W is constant between spikes. A constant current $I_{syn} > V_{th}$ is injected into the neuron.

1. Discuss qualitatively what happens after the first spike.

2. At which value of W does the model stop spiking? Show that the total number of spikes emitted is roughly $(I_{syn} - V_{th})/W_R$.

3. Compute the duration of an interspike interval (ISI) as a function of W in that interval.

1.2 General firing rate

It is no longer possible to ignore the decay of W if the ISI becomes comparable to the time constant of the decay τ_w.

4. Can you explain why? Is it possible for the neuron to stop spiking?

We therefore consider that the system has reached its equilibrium firing rate and fires spikes with a period T.

5. Compute the time course of W between two successive spikes, assuming that immediately after the first of the two spikes $W(t = 0) = W_0$.

6. Show that W_0 is given by:

$$W_0 = \frac{W_R}{1 - \exp(-T/\tau_w)}. \tag{3}$$

7. We assume that $T \ll \tau_w$, such that W can be approximated by its average value during the whole interspike interval. Show that the period T of spike emission is given by:

$$T = \tau_m \cdot \log\left(\frac{I - W_R\tau_w/T}{I - V_{th} - W_R\tau_w/T}\right). \tag{4}$$

8. Show that, as the injected current increases, the neuron firing rate $r(I)$ behaves as:

$$r(I) \sim aI \tag{5}$$

   with $a = [\tau_w W_R + \tau_m V_{th}]^{-1}$.

9. How does this compare to an integrate-and-fire neuron without firing rate adaptation?

2 Conductance-based synapses and shunting inhibition

Another interesting phenomenon is shunting inhibition. Experimentally, one can observe that the effects of inhibition and excitation do not necessarily sum in a linear fashion, such that inhibition can "shunt" the effect of excitation. This effect particularly applies with excitatory synapses spanning the dendritic tree and inhibitory synapses closer to the soma.

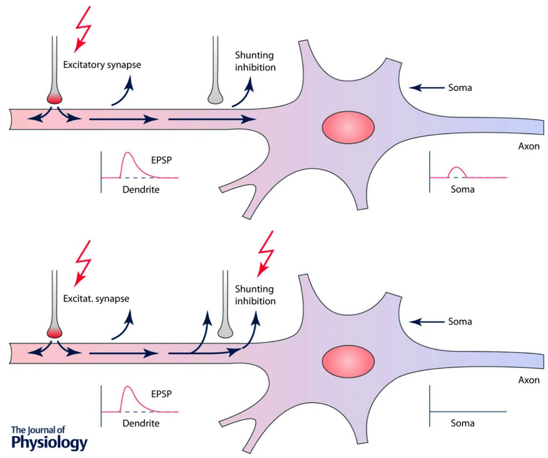

Figure 2: Shunting inhibition.

We consider a neuron well below the threshold for action potential initiation. The neuron is described by its membrane potential $V(t)$ that obeys the equation:

$$C_m \frac{\mathrm{d}V}{\mathrm{d}t} = -g_L(V - V_L) - g_E(t)(V - V_E) - g_I(t)(V - V_I) \tag{6}$$

with:

- C_m the membrane capacitance
- g_L the leak conductance and V_L the resting membrane potential
- $g_E(t)$, $g_I(t)$ the time-dependent excitatory/inhibitory synaptic conductances
- V_E, V_I the excitatory/inhibitory reversal potentials

In real neurons, $V_L \sim -70$mV, $V_I \sim -70$mV or lower, $V_E \sim 0$ mV. For the sake of simplicity we set here the resting potential to be $V_L = 0$mV, and we set $V_I = V_L$.

We consider the situation in which the voltage is $V = 0$ at $t = 0$, and investigate the effects of various combinations of excitatory and inhibitory conductances on the voltage response.

1. Rewrite (6) in terms of the membrane time constant $\tau_m = C_m/g_L$, rescaled conductances $\tilde{g}_E(t) = g_E(t)/g_L$, $\tilde{g}_I(t) = g_I(t)/g_L$, and V_E.

2. Show that (6) can be rewritten as:

$$\tau_{eff}(t)\frac{\mathrm{d}V}{\mathrm{d}t} = -V + V_{eff}(t) \tag{7}$$

   where $\tau_{eff}(t)$ and $V_{eff}(t)$ are functions of τ_m, $\tilde{g}_E(t)$, $\tilde{g}_I(t)$, and V_E.

2.1 Excitation only

We consider a situation in which there is no inhibition. The (rescaled) excitatory conductance opens abruptly at $t = 0$, and closes abruptly at $t = \tau_E$,

$$\tilde{g}_E(t) = \begin{cases} 0 & t < 0 \\ g_E & t \in [0, \tau_E] \\ 0 & t > \tau_E \end{cases} \tag{8}$$

3. Compute the response of the voltage (EPSP, excitatory post-synaptic potential) in both intervals $t \in [0, \tau_E]$ and $t > \tau_E$. Sketch qualitatively the shape of the EPSP.
4. What is the amplitude of the peak of the EPSP? Discuss qualitatively how it depends on g_E, V_E and the ratio of time constants τ_E/τ_m.

2.2 Inhibition only

We consider the reverse situation in which there is no excitation, and the (rescaled) inhibitory conductance opens abruptly at $t = 0$, and closes abruptly at $t = \tau_I$,

$$\tilde{g}_I(t) = \begin{cases} 0 & t < 0 \\ g_I & t \in [0, \tau_I] \\ 0 & t > \tau_I \end{cases} \tag{9}$$

5. Compute the response of the voltage (IPSP, excitatory post-synaptic potential). What does it look like?

2.3 Both

We now consider the situation in which there is a tonic inhibitory conductance, $g_I(t) = g_I$. The excitatory conductance again opens abruptly at $t = 0$ and closes abruptly at $t = \tau_E$.

6. Repeat the steps of section 1. Compare what happens with and without inhibition. Does the system sum linearly excitatory and inhibitory inputs?