M2 Biology

TD9b: Coding II

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https://github.com/Elieoriol/2021_UlmM2_ThNeuro/tree/master/TD9b

This last tutorial focuses on a model of unsupervised learning.

1 Bayesian inference

We consider a population of N neurons with various tuning curves $f_i(s)$: when a stimulus s is presented, the neuron i will fire on average $f_i(s)$ spikes in a given timebin. However, on each trial, the number of spikes emitted by neuron i is drawn from a Poisson process of mean $f_i(s)$. This variability is independent across neurons.

1. Suppose that a given stimulus s is presented, what is the probability of observing a given pattern of spikes $\{n_i\}$?
2. Supposing that a given pattern of spikes $\{n_i\}$ was observed, can you tell what stimulus was presented?
3. Suppose that the prior on the stimulus $p(s)$ is uniform. Suppose also that the tuning curves all have the same shape, a bell, with the preferred stimuli s_i of the various neurons evenly distributed across the stimulus range. Suppose that a given pattern of spikes $\{n_i\}$ was observed, give an estimate of the stimulus that was presented.
4. How does the accuracy of this estimate depend on the height f_0 and width σ of the tuning curves and on the number of neurons?
5. We will now be more quantitative: find the probability distribution of the stimulus using Baye's rule

   $$p(s|\{n_i\}) = \frac{p(\{n_i\}|s)p(s)}{p(\{n_i\})}$$

   Include all the terms which don't depend on s in a function $\Phi(\{n_i\})$.
6. Consider that the tuning curves are Gaussian:

   $$f_i(s) = f_0 \exp(-\frac{(s-s_i)^2}{2\sigma^2})$$

   Show that $p(s|\{n_i\})$ is also a Gaussian. What is its mean and variance? How does the variance depend on the various parameters? Under what conditions may the variance become infinitely small?
7. Suppose now that instead of responding to s, neurons respond to a jittered version of s, namely $\hat{s}$, where $p(s|\hat{s})$ is a Gaussian of variance σ_j^2. The number of spikes they fire is drawn from a Poisson distribution of mean $f_i(\hat{s})$.

   For a given stimulus s, is the variability still independent across neurons? Does the variance of $p(s|\{n_i\})$ still become infinitely small in the conditions considered previously?