

Spring 2018
Precalculus' Final Examination
nth Semester
Section 000

Objective: Assess understanding of function behavior and calculus readiness. This exam also aims to provide a comprehensive assessment on concepts and definitions that are necessary to be successful in further Mathematics courses.)

General Instructions: Read carefully each exercise. Fill in your *scantron* with a pencil and circle the correct answer on paper as well. Scratch paper is not allowed under any circumstances. All your work must be done in these pages.

- You have up to 120 minutes.
- Every item on the test awards 2 points for each correct answer, for a maximum possible score of 100 points.
- Non-graphing calculators are allowed. TI-84 or similar, including smart devices, are prohibited.
- One half-page formula sheet printed in black ink and showing the instructor's authorization may be used. Any other form of aid is not allowed.
- Mere suspicion of cheating, sharing calculators or using any unfair means of aid is enough to get your test withdrawn.
- When you are done, turn in the examination, your *scantron* and your formula sheet. Failure to do so will result in an automatic failing grade.

SIDE A

Part I. TRUE OR FALSE QUESTIONS.

1. Differential Calculus is the branch of mathematics dedicated to the study of rates of change.
A. True B. False
2. The domain of a function $f(x)$ is the set of all the values of x for which $f(x)$ is defined.
A. True B. False
3. The base of a natural logarithm, \ln , is e .
A. True B. False
4. $\sqrt{3}$ is an irrational number
A. True B. False
5. $f(x) = |x|$ is continuous for all real numbers.
A. True B. False
6. If $(x + a)$ is a factor of $p(x)$, then $x = a$ is a solution to $p(x)$
A. True B. False
7. If $f(x) = g^{-1}(x)$, then $f(x)$ has the same domain of $g(x)$ and $g^{-1}(x)$ has the same range of $f^{-1}(x)$
A. True B. False
8. All continuous functions are differentiable over their domain
A. True B. False
9. Polynomial functions are differentiable on $(-\infty, \infty)$.
A. True B. False
10. If $\lim_{x \rightarrow a} =$ does not exist, then $x = a$ must be undefined.
A. True B. False

Part II. MULTIPLE CHOICE QUESTIONS.

11. Simplify $\sqrt{500} - \sqrt{80}$
A. $5\sqrt{5}$
B. $6\sqrt{5}$
C. $10\sqrt{5}$
D. $16\sqrt{5}$
E. 25
12. The vertex of the parabola $y = (x - 2)^2$ is located at
A. $(-2, 0)$
B. $(2, 0)$
C. $(-2, -2)$
D. $(0, 0)$
E. none of the above

13. If a polynomial function of degree 2 has an absolute maximum, then this point must also be a
- A. tangent line
 - B. vertex
 - C. inflection point
 - D. absolute minimum
 - E. none of the above
14. Find the vertical asymptote of $f(x) = \frac{x}{-x^2 - 2x}$.
- A. $x = 1/2$
 - B. $x = -2$
 - C. $x = 4$
 - D. $x = -1/4$
 - E. $x = 2$
15. Which of the following has a vertical asymptote at $x = 0$?
- A. e^x
 - B. $\ln x$
 - C. $|x|$
 - D. $\sin x$
 - E. x
16. Which of the following best describes the function $f(x) = \frac{3x^2}{5x + 25}$
- A. polynomial
 - B. rational
 - C. quadratic
 - D. asymptotic
 - E. transcendental
17. Consider $f(x) = \frac{x}{x^2 + 8x + 16}$. Determine the values of x where $f(x)$ is discontinuous.
- A. $x = 0$
 - B. $x = 0, 4$
 - C. $x = 4$
 - D. $x = 3, 4$
 - E. $x = 1, 4$
18. The domain of $f(x) = \frac{1}{x^2}$ is:
- A. $(-\infty, 0) \cup (0, \infty)$
 - B. $[0, \infty)$
 - C. $(-\infty, 0]$
 - D. $(-\infty, 1]$
 - E. $\mathbb{R}, x \neq 0$
19. The domain of $f(x) = \frac{1}{1 + x^2}$ is:
- A. $(-\infty, \infty)$
 - B. $[0, \infty)$
 - C. $(-\infty, 0]$
 - D. $(-\infty, 1]$
 - E. $\mathbb{R}, x \neq 0$

20. Classify the discontinuity in $\frac{(x+1)(x-3)}{(x-4)}$
- A. infinite at $x = 3$, removable at $x = 4$
 - B. removable at $x = 3$ and $x = 4$
 - C. infinite at $x = 4$
 - D. removable at $x = 3$, infinite at $x = 1$
 - E. jump at $x = 4$ and $x = 1$
21. if $f = e^x$ and $g = x^2$, then $e^{x^2} =$
- A. $f \circ f$
 - B. $g \circ f$
 - C. $f \circ g$
 - D. f/g
 - E. fg
22. The inverse function of $f(x) = (x+3)^2$ is $f^{-1}(x) =$
- A. $\sqrt{x+3} + 2$
 - B. $\sqrt{x-3}$
 - C. $x^2 - 3$
 - D. $\sqrt{x} - 3$
 - E. $\sqrt{x-2} + 3$
23. All of the following are characteristic of non-polynomial functions, except:
- A. Minima and maxima
 - B. Asymptotes
 - C. Sharp turns
 - D. Jumps or gaps
 - E. none is correct
24. Find the derivative of a polynomial $p(x)$ with roots $x = 0, x = 1$ and $x = 3$; $p'(x) =$
- A. $3x^2 - 8x - 3$
 - B. $3x^3 + 8x$
 - C. $3x^2 - 8x + 3$
 - D. $x^2 - 3x$
 - E. $x^2 + x + 3$
25. Find $\lim_{x \rightarrow \infty} e^{-\sqrt{x}}$
- A. ∞
 - B. $-\infty$
 - C. 0
 - D. e
 - E. Does not exist
26. Find $\lim_{x \rightarrow 0} 3\sqrt{2x+3}$
- A. $3\sqrt{3}$
 - B. 9
 - C. $-\sqrt{3}$
 - D. 0
 - E. Does not exist
27. Find $\lim_{x \rightarrow 0} \frac{x^2 + 3x}{3x^2 + 4x}$
- A. $\frac{3}{4}$
 - B. $-\infty$
 - C. $\frac{1}{3}$
 - D. ∞
 - E. 1

Consider function f below:

28. For what values of x is $f(x)$ discontinuous?

- A. 0, 2, 3
- B. 0, 2, 4
- C. -2, 2, 3
- D. -2, 2
- E. 3, 5

29. Find $f(2)$

- A. 2
- B. 1
- C. -1
- D. 0
- E. Undefined

30. Find the $\lim_{x \rightarrow 3} f(x)$

- A. 3
- B. 2
- C. ∞
- D. 0
- E. undetermined

31. Find $\lim_{x \rightarrow 2^+} f(x)$

- A. 0
- B. 1
- C. 1
- D. ∞
- E. 2

32. Find $\lim_{x \rightarrow 2} f(x)$

- A. 0
- B. 1
- C. Does not exist
- D. ∞
- E. 2

33. A right triangle has a hypotenuse of length 6 and one leg of length $\sqrt{5}$. What is the length of the other leg?

- A. $\sqrt{31}$
- B. 21
- C. $\sqrt{41}$
- D. 31
- E. $\sqrt{10}$

34. Which of the following angles is coterminal to 325°

- A. -45°
- B. -25°

- C. -35°
 - D. 75°
 - E. 125°
35. A triangle in the unit circle has a terminal point $(\frac{1}{2}, \frac{\sqrt{3}}{2})$. What is θ ?
- A. 30°
 - B. 120°
 - C. 45°
 - D. 60°
 - E. 210°
36. Where does the terminal side of the angle -320° lie, if drawn in standard position?
- A. Quadrant I
 - B. Quadrant II
 - C. Quadrant III
 - D. Quadrant IV
 - E. Standard position angles can't be negative
37. Find the length of the arc, in cm, formed by an angle of $\theta = 45^\circ$ if the distance from the terminal point to the center is $r = 2 \text{ cm}$
- A. π B. $\frac{\pi}{3}$ C. $\frac{4\pi}{3}$ D. $\frac{2\pi}{3}$ E. $\frac{3\pi}{2}$
38. Find the area of a sector of a circle with radius 10 centimeters formed by the arc subtended from the angle $\theta = \frac{\pi}{5}$ rad.
- A. $5\pi \text{ cm}^2$
 - B. $50\pi \text{ cm}^2$
 - C. $25\pi \text{ cm}^2$
 - D. $10\pi \text{ cm}^2$
 - E. $12.5\pi \text{ cm}^2$

39. Which of the following best describes what a derivative is?
- A. The slope of the tangent line or instantaneous rate of change
 - B. The slope of the secant line or average rate of change
 - C. The equation of a tangent line at a point
 - D. The equation of the tangent line anywhere on the curve
 - E. none of the above
40. Find the derivative of $f(x) = x \ln x$
- A. $x + x \ln x$
 - B. $\ln x + 1$
 - C. $\ln x$
 - D. $\ln x + \frac{1}{x}$
 - E. $1 + \frac{1}{x}$
41. Find the derivative of $f(x) = x - \frac{1}{x^2}$
- A. $1 + x^3$
 - B. $\frac{1}{x^3}$
 - C. $1 + \frac{1}{x^3}$
 - D. $\frac{1}{x^{-3}}$
 - E. $1 - \frac{1}{2x}$
42. Differentiate $f(x) = 2\sqrt{x}$
- A. $\frac{1}{2\sqrt{x}}$
 - B. $-\frac{\sqrt{x}}{2x}$
 - C. $\frac{1}{\sqrt{x}}$
 - D. $1/2x$
 - E. $2\sqrt{x}$
43. Find the slope of the line tangent to $y = -x^{-3}$ at $x = 1$
- A. $m = 3/2$
 - B. $m = -3$
 - C. $m = 3/4$
 - D. $m = -3/4$
 - E. $m = 3$
44. Find the equation of the line which at the point $(1, 6)$ is tangent to the graph of $y = -4x^3 + 7x^2 - 9x + 12$
- A. $y = 5x + 1$
 - B. $y = -12x - 9$
 - C. $y = -7x + 13$
 - D. $y = -4x + 12$
 - E. $y = -24x - 14$

45. Find $f'(2)$ given

$$f(x) = 3x^3 - 2x - 5$$

- A. 64
- B. 34
- C. 3
- D. -3
- E. 72

46. The trajectory of a body in motion is modeled by

$$p(t) = 3t^3 - 2t^2 - 18$$

Find the position of the body after 2 seconds.

- A. $-9m$
- B. $-18 m$
- C. $-2 m$
- D. $12 m$
- E. $18 m$

47. Find the total displacement of this particle from the moment motion begins and $t = 3s$

- A. $41 m$
- B. $39 m$
- C. $28 m$
- D. $43 m$
- E. $45 m$

48. Find an equation for the velocity of the body

- A. $6t^2 - 8t - 18 m/s$
- B. $3t^2 - 4t m/s$
- C. $9t^2 - 4t m/s$
- D. $9t^2 + 4t - 18 m/s$
- E. $9t^2 - 4$

49. Find the exact time at which the body's acceleration is $0 m/s^2$

- A. $\frac{1}{2} s$
- B. $\frac{2}{3} s$
- C. $\frac{9}{2} s$
- D. $\frac{1}{9} s$
- E. $\frac{5}{3} s$

50. What is the best team in the world?

- A. Atletico Madrid
- B. Real Madrid
- C. Barcelona
- D. Tigres
- E. none of the above