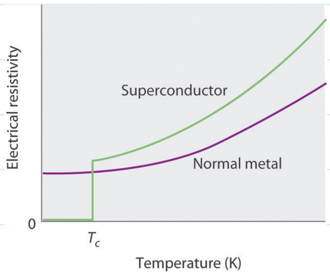
Homework 1-3

Sunday, May 10, 2020 5:31 PM

小レポート課題 No. 0 [課題]

超伝導の物理学的概念として重要な特徴（特徴的な現象、超伝導ギャップ、クーパー対など）につい ての解説を、A4 用紙 2 枚程度にまとめよ。必要に応じて、図を用いても良い。

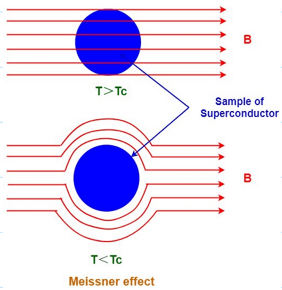
1. **Elementary properties of superconductors**



1) Zero electrical DC resistance and Electric field

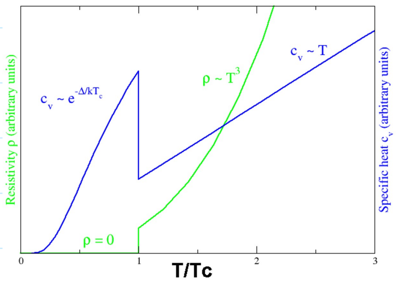
For normal metal, the resistance of the sample is given by Ohm's law as R = V / I. The resistance will decrease with the temperature drop, but it will never reach to 0. But for superconductors, their resistance will suddenly drop to 0 under the critical temperature(Tc). This means in the superconductors, current can travel without any degradation.

In a normal conductor, an electric current may be visualized as a fluid of electrons moving across a heavy ionic lattice. The electrons are constantly colliding with the ions in the lattice, and during each collision some of the energy carried by the current is absorbed by the lattice and converted into heat, which is essentially the vibrational kinetic energy of the lattice ions. As a result, the energy carried by the current is constantly being dissipated. This is the phenomenon of electrical resistance and Joule heating.



2) Meissner effect

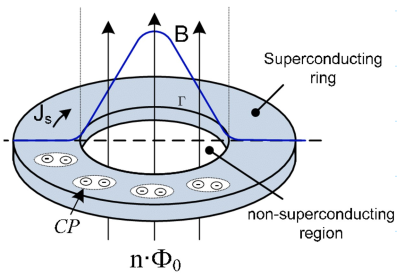
When a superconductor is placed in a weak external magnetic field H, and cooled below its transition temperature, the magnetic field is ejected. The Meissner effect does not cause the field to be completely ejected but instead the field penetrates the superconductor but only to a very small distance, characterized by a parameter λ, called the London penetration depth, decaying exponentially to zero within the bulk of the material. The Meissner effect is a defining characteristic of superconductivity. For most superconductors, the London penetration depth is on the order of 100 nm.



3)Phase transition

In superconducting materials, the characteristics of superconductivity appear when the temperature T is lowered below a critical temperature T_c. The value of this critical temperature varies from material to material. Conventional superconductors usually have critical temperatures ranging from around 20 K to less than 1 K. Solid mercury, for example, has a critical temperature of 4.2 K.

The onset of superconductivity is accompanied by abrupt changes in various physical properties, which is the hallmark of a phase transition. For example, the electronic heat capacity is proportional to the temperature in the normal (non-superconducting) regime. At the superconducting transition, it suffers a discontinuous jump and thereafter ceases to be linear. At low temperatures, it varies instead as $e^{-\alpha/T}$ for some constant, α. This exponential behavior is one of the pieces of evidence for the existence of the energy gap.

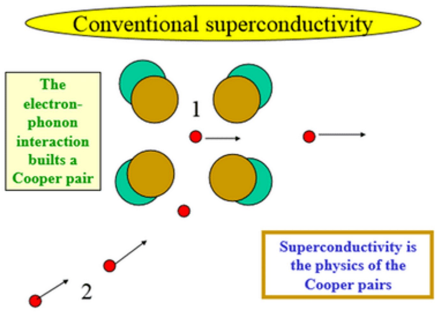


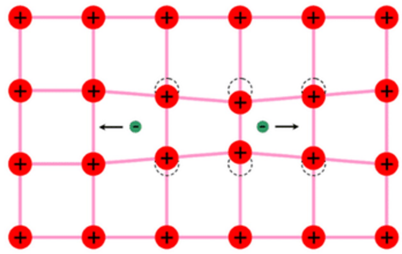
4)Magnetic Flux quantization

The magnetic flux, represented by the symbol Φ, threading some contour or loop is defined as the magnetic field B multiplied by the loop area S, i.e. Φ = B•S. Both B and S can be arbitrary and so is Φ. However, if one deals with the superconducting loop or a hole in a bulk superconductor, it turns out that the magnetic flux threading such a hole/loop is quantized. The (superconducting) magnetic flux quantum $\Phi_0 = h/(2e) \approx 2.067833848...\times10^{-15}$ Wb.

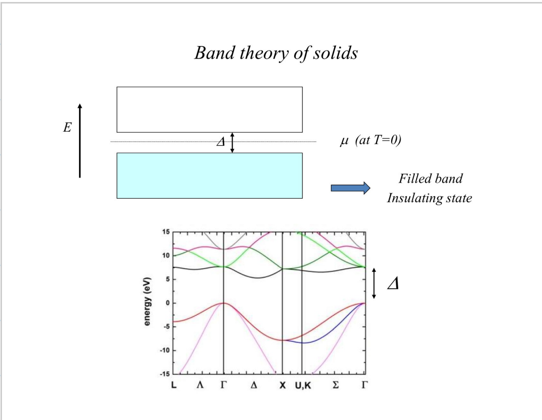
2.Superconducting energy gap and Cooper pair

In condensed matter physics, a Cooper pair or BCS pair is a pair of electrons (or other fermions) bound together at low temperatures in a certain manner. Cooper showed that an arbitrarily small attraction between electrons in a metal can cause a paired state of electrons to have a lower energy than the Fermi energy, which implies that the pair is bound. In conventional superconductors, this attraction is due to the electron–phonon interaction. The Cooper pair state is responsible for superconductivity.





Although Cooper pairing is a quantum effect, the reason for the pairing can be seen from a simplified classical explanation. An electron in a metal normally behaves as a free particle. The electron is repelled from other electrons due to their negative charge, but it also attracts the positive ions that make up the rigid lattice of the metal. This attraction distorts the ion lattice, moving the ions slightly toward the electron, increasing the positive charge density of the lattice in the vicinity. This positive charge can attract other electrons. At long distances, this attraction between electrons due to the displaced ions can overcome the electrons' repulsion due to their negative charge, and cause them to pair up. The rigorous quantum mechanical explanation shows that the effect is due to electron–phonon interactions, with the phonon being the collective motion of the positively-charged lattice.



The energy of the pairing interaction is quite weak, of the order of 10^{-3} eV, and thermal energy can easily break the pairs. So only at low temperatures, in metal and other substrates, are a significant number of the electrons in Cooper pairs.

For superconductors the energy gap is a region of suppressed density of states around the Fermi energy, with the size of the energy gap much smaller than the energy scale of the band structure. The superconducting energy gap is a key aspect in the theoretical description of superconductivity and thus features prominently in BCS theory. Here, the size of the energy gap indicates the energy gain for two electrons upon formation of a Cooper pair.

3.Explanation of Superconductivity

1) London equations

London and London derived a phenomenological theory of superconductivity which correctly describes the Meissner effect.

2) BCS Theory

At sufficiently low temperatures, electrons near the Fermi surface become unstable against the formation of Cooper pairs. Cooper showed such binding will occur in the presence of an attractive potential, no matter how weak. In conventional superconductors, an attraction is generally attributed to an electron-lattice interaction. The BCS theory, however, requires only that the potential be attractive, regardless of its origin. In the BCS framework, superconductivity is a macroscopic effect which results from the condensation of Cooper pairs. These have some bosonic properties, and bosons, at sufficiently low temperature, can form a large Bose–Einstein condensate. Superconductivity was simultaneously explained by Nikolay Bogolyubov, by means of the Bogolyubov transformations.

In many superconductors, the attractive interaction between electrons (necessary for pairing) is brought about indirectly by the interaction between the electrons and the vibrating crystal lattice (the phonons). Roughly speaking the picture is the following:

An electron moving through a conductor will attract nearby positive charges in the lattice. This deformation of the lattice causes another electron, with opposite spin, to move into the region of higher positive charge density. The two electrons then become correlated. Because there are a lot of such electron pairs in a superconductor, these pairs overlap very strongly and form a highly collective condensate. In this "condensed" state, the breaking of one pair will change the energy of the entire condensate - not just a single electron, or a single pair. Thus, the energy required to break any single pair is related to the energy required to break all of the pairs (or more than just two electrons). Because the pairing increases this energy barrier, kicks from oscillating atoms in the conductor (which are small at sufficiently low temperatures) are not enough to affect the condensate as a whole, or any individual "member pair" within the condensate. Thus the electrons stay paired together and resist all kicks, and the electron flow as a whole (the current through the superconductor) will not experience resistance. Thus, the collective behavior of the condensate is a crucial ingredient necessary for superconductivity.