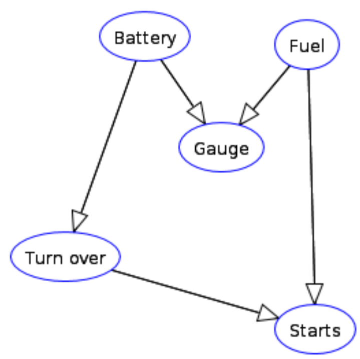
Probabilistic Graphical Models

1. From the textbook of C. Bishop: 8.3, 8.4
2. The Bayesian network shown in the figure captures the probability of a car engine starting.
   - The Battery (B) of the car can be either in *good* or *bad* condition.
   - The Fuel tank (F) can be either *empty* or *not empty*.
   - The Gauge or indicator (G) can also indicate *empty* or *not empty*.
   - The turn over (T) can be *false* or *true*.
   - The engine starts (S) can be *yes* or *no*.



   (a) Write down the factorization of the joint probability $p(B, F, G, T, S)$ induced by the network.

   (b) Argue whether the following conditional independences are satisfied or not.

      i. Is T independent of F if no evidence is provided?

      ii. Is T independent of F if we observe that the engine does not start?

      iii. Is B independent of F if we observe that the engine starts?

   (c) The following conditional probability tables fully define the model:

$$p(B = bad) = 0.02 \qquad p(F = empty) = 0.05$$
$$p(G = empty|B = good, F = notempty) = 0.04 \qquad p(G = empty|B = good, F = empty) = 0.97$$
$$p(G = empty|B = bad, F = notempty) = 0.1 \qquad p(G = empty|B = bad, F = empty) = 0.99$$
$$p(T = false|B = good) = 0.03 \qquad p(T = false|B = bad) = 0.98$$
$$p(S = no|T = true, F = notempty) = 0.01 \qquad p(S = no|T = true, F = empty) = 0.92$$
$$p(S = no|T = false, F = notempty) = 1 \qquad p(S = no|T = false, F = empty) = 0.99$$

   We observe the car does not start ($S = no$). Calculate the probability that the fuel tank is empty $p(F = empty|S = no)$. (Try to use the least possible number of terms in your calculations).

   You are encouraged to download and use the Java tool `http://aispace.org/bayes/` to check numerically that your results are correct.

   (d) Transform the model into an equivalent factor graph and describe it fully, e.g., write down all the involved factors and their values.

   (e) Write down the belief propagation messages from variables B, F to variable G (You can assume other required messages for these calculations equal to ones).