

MACHINE LEARNING HOMEWORK 2

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a) Why do we normalize f ?

We want to see the effect of noise, N and target complexity(Q_f) on overfit measure. Since y values depending on f and noise level, we normalize the f to get more compact data points, only varied by noise.

b) How can we obtain g_2, g_{10} ?

We can think this problem like a linear regression. To generate a higher order equation we can add powers of the original features as new features. This will still considered as a linear model as the coefficients associated with the features are still linear. After that, applying linear regression on the new features will give us the expected curve.

c) How can we compute E_{out} for a given g_{10} ?

Basically we try polynomial regression. With the help of `PolynomialFeatures` method from `sklearn` library we can select the degree of polynomial as 10. Then using pipeline we can combine the degree with linear regression to create our pipeline frame. After that using r^2 scoring we can calculate the E_{out} for a given g_{10} .

d) Since trying all the Q_f , N and noise level values would take too much time and effort, I pick 6 values from each set to try over. The selected values are:

```
n_samples_array=[20,30,50,80,95,120]
qf_array=[3,18,30,43,88,92]
var_array = [0,0.3,0.5,1.2,1.7,2]
```

In total 216 experiments are run.

During each experiment out of sample error for H_2 and H_{10} are calculated, the difference between them is assigned to `overfit_measure` variable. Each iteration the minimum and maximum H_2 error and H_{10} error are hold, also the N, Q_f and variance values that leads to these minimum and maximum errors are kept. A sum of H_2 and H_{10} errors are calculated. After each experiment is completed, the average H_2 error, H_{10} error, the minimum overfit measure, the maximum overfit measure and the $N, Q_f, \text{variance}$ values that lead to them are printed.

From the observations we can come up to these conclusions:

Increase in the number of data points(N) decreases the overfitting.

Increase in the variance(noise level) increases the overfitting.

Increase in the target complexity(Q_f) increases the overfitting.

The output is given as:

```
Average out of error for H10 637101.6832350367
Average out of error for H2 1294.8303817816009
```

Minimum overfit measure -121660.69520884981

Maximum overfit measure 66479136.46022499

N,QF,noise values that making overfit measure minimum 30 3 0.3

N,QF,noise values that making overfit measure maximum 30 30 0.3

e) Why do we take the average over many experiments?

As we select the data points and training and testing datasets randomly, it is always better to try different experiments to average over. Especially, in our case we want to see if H2 or H10 model fits the data, trying with different parameters such as N size, different noise level and different $f(x)$, then calculate the out of sample errors for each different circumstances for both model and getting the average, will give us a very clear picture on which model to use. If we try not enough experiment then we could get wrong assumptions, such as H10 model fitting better, on the other hand after a satisfactory number of experiments, continuing to do so would just be a waste of time. The balance over number of experiments conducted needs to be found.

To select the acceptable number of experiments, we look into bias-variance trade-off. A high bias results in underfitting while a high variance results in overfitting. As the model complexity increases, the bias decreases and the variance increases.

Notes:

A sample model plot for $N=30$, $Qf=50$, noise=0.5 can be given as:

