1. (20 points) Examine the propositions below and determine whether they are true or false. You do not need to provide an explanation, just mark your choice. (Each is 4 points)

True — False \int The set $\{0\}$, with the usual operations of addition and scalar multiplication, forms a vector space.

True — False \checkmark The set of column vectors of a 5×7 matrix A must be linearly dependent.

True — False

A change-of-basis matrix is always a square matrix.

True — False \bigvee / If A is a 7 × 9 matrix with nullity(A) = 2, then $rowspace(A) = \mathbb{R}^7$.

True — False (If $T: \mathbb{R}^8 \to \mathbb{R}^3$ is an onto linear transformation, then Ker(T) is five-dimensional.

dim [Zan(7)=W da (Ran (7)) > 3

2. Prove the following statements

(a) (10 points) Let $T:V\to W$ be a linear transformation, and assume that V and W are both finite dimensional. If T is one-to-one, then

$$dim(V) \le dim(W)$$
.

(b) (10 points) Let $T: V \to W$ be a linear transformation. Then



$$Ran(T) = \{T(\mathbf{v}) \mid \mathbf{v} \in V\}$$

is a subspace of W.

Addition:
$$\vec{W} + \vec{z} = T(\vec{w}) + T(\vec{v}) = T(\vec{w} + \vec{v}) \in \mathbb{R}$$
an(7)



3. Consider the following set of polynomials

$$S_1 = \{2x + 5x^2, 1 + x\}$$

$$S_2 = \{1 - x + x^2, -1 + 2x - x^2, 2x + x^2\}$$

$$S_3 = \{2, 4x, 5x^2, 8x + x^2\}$$

(a) (5 points) Which one can be a basis for $P_2(\mathbb{R})$?

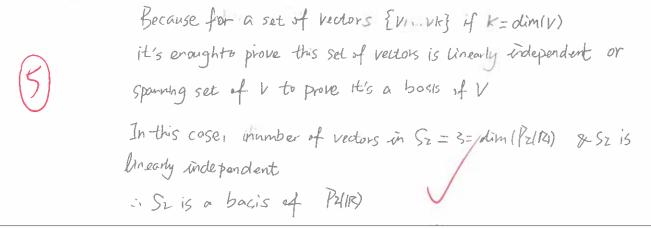
dim (Pz(1R)) = 3

(b) (10 points) For your guess in part (a), use Wronskian method to show the set is linearly independent.

$$W[f_{1}f_{2}f_{3}](\pi) = der \begin{pmatrix} 1-x+x^{2} & -1+2x-x^{2} & 2x+x^{2} \\ -1+2x & 2-2x & 2+2x \end{pmatrix}$$
when $x=0$ WIf $f_{1}f_{2}f_{3}](0) = det \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 2 \\ 2 & -2 & 2 \end{pmatrix} = 4-4+4-2 = 2 \neq 0$

$$\therefore S_{2} \text{ is linearly independent}$$

(c) (5 points) Why does part (b) finish to prove that your guess is indeed a basis? Explain.



4. Consider the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ defined by

$$T(a, b, c) = (a - b, 3c).$$

(a) (7 points) Find the matrix representation of T.

$$IR^{3} = Span \{(1,0,0)(0,1,0)(0,0,1)\}$$

$$T(1,0,0) = (1,0)$$

$$T(0,1,0) = (-1,0)$$

$$T(0,0,1) = (0,3)$$

$$Azx3 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(b) (8 points) Find Ker(T) and Ran(T).

(c) (5 points) Determine whether T is one-to-one and/or onto?

$$\frac{1}{2} \frac{(Ker(T) = Span \{(1,1,0)\} + \{(0,0,0)\}}{Tisn't one to one}$$

$$\frac{1}{2} \frac{(Im(Ran(T)) = Z Ran(T) \subseteq IR^2}{Ran(T) = IR^2}$$

$$Tisonto$$

5. Let
$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 4 \\ 3 & 1 & 8 \end{bmatrix}$$
.

(a) (4 points) Define rowspace(A) and colspace(A). definition

V Q

(b) (6 points) Reduce A into the reduced row-echelon form.

$$\begin{bmatrix}
1 & 0 & 2 \\
1 & 1 & 4
\end{bmatrix}
\underbrace{A12(-1)}_{A13(-2)}
\begin{bmatrix}
1 & 0 & 2 \\
0 & 1 & 2
\end{bmatrix}
\underbrace{A21(-1)}_{A23(-1)}
\begin{bmatrix}
0 & 1 & 2 \\
0 & 1 & 2
\end{bmatrix}
\underbrace{A23(-1)}_{0}
\begin{bmatrix}
1 & 0 & 2 \\
0 & 1 & 2
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\underbrace{A23(-1)}_{0}
\begin{bmatrix}
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\end{bmatrix}
\underbrace{A23(-1)}_{0}
\begin{bmatrix}
1 & 0 & 2 \\
0 & 1 & 2
\end{bmatrix}
\underbrace{A23(-1)}_{0}
\underbrace$$

(c) (10 points) Find the bases for rowspace(A) and colspace(A).

Powspace(A) =
$$span \{ (1,0,2) (0,1,2) \}$$
 basis is $\{ (1,0,2) (0,1,2) \}$
Colspace (A) = $span \{ (1,1,3) (0,1,1) \}$ basis is $\{ (1,1,3) (0,1,1) \}$



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If you would like work on this page scored, then clearly indicate to which question the work belongs and indicate on the page containing the original question that there is work on this page to score.