



SECI1013: DISCRETE STRUCTURES

SESSION 2023/2024 – SEMESTER 1

ASSIGNMENT 2 (CHAPTER 2)

INSTRUCTIONS:

- a. This assignment must be conducted in a group. Please clearly write the group members' names & matric numbers on the front page of the submission.
- b. Solutions for each question must be readable and neatly written on plain A4 paper. Every step or calculation should be properly shown. Failure to do so will result in the rejection of the submission of the assignment.
- c. This assignment has 13 questions (100 marks), contributing 5% of overall course marks.

STRUCTURES:

1. Chapter 2 Part 1: Relation [50 Marks]
2. Chapter 2 Part 2: Function [30 Marks]
3. Chapter 2 Part 3: Recurrence Relation [20 Marks]

Q1. Relation

1. Given $A = \{2, 3, 4, 5, 6, 7, 8\}$ and R a relation over A . Draw the directed graph of R after realising that xRy iff $x-y = 3n$ for some $n \in \mathbb{Z}$. Find all possible equivalence relations for R .

(5 marks)

2. Let $A = \{1, 2, 3\}$ and $B = \{9, 8, 7\}$.

Let $R: A$ to B . For all $(a, b) \in A \times B$, and given $a R b \Leftrightarrow a+b$ is an even number,

- Determine R and R^{-1} .
- Draw arrow diagrams for both.
- Describe R^{-1} in words.

(10 marks)

3. Let $A = \{1, 2, 3, 4, 5\}$, and let R be the relation on A that has the matrix (given below)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Construct the digraph of R , and list in-degrees and out-degrees of all vertices.

(6 marks)

4. Given $A = \{0, 1, 2, 3, 4\}$, and

$R = \{(0, 0), (0, 1), (0, 3), (0, 4), (1, 0), (1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 0), (3, 2), (3, 3), (3, 4), (4, 0), (4, 3), (4, 4)\}$. Draw the relation graph and find if R is reflexive, symmetric, or transitive?

(12 marks)

5. Relation R in the set $A = \{1, 2, 3, \dots, 13, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$, Determine whether the relation is
- Reflexive
 - Symmetric
 - Transitive

Support your answer with the reason.

(9 marks)

6. Suppose that the given is a relation matrix for R and S ,

$$R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Using Boolean Arithmetic, Find

- RS
- SR

(8 marks)

Q2. Function

7. What is the different between Relation and Function?

(2 Marks)

8. If $A = \{2, 3, 4, 5\}$, then write whether each of the following relations on set A is a function or not. Give reasons also.

- $\{(2, 3), (3, 4), (4, 5), (5, 2)\}$
- $\{(2, 4), (3, 4), (5, 4), (4, 4)\}$
- $\{(2, 3), (2, 4), (5, 4)\}$
- $\{(2, 3), (3, 5), (4, 5)\}$
- $\{(2, 2), (2, 3), (4, 4), (4, 5)\}$

(8 marks)

9. Given the relation of $R = \{(x, y) | y = x + 5, x \text{ is } \mathbb{Z}^+ \text{ less than } 6\}$. Depict this relationship using roster form. Write down the domain and the range.

(3 marks)

10. In the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(v) $f = R \rightarrow R, f(x) = 1 - 2x$

(vi) $f = R \rightarrow R, f(x) = 5x^2 - 1$

(vii) $f = R \rightarrow R, f(x) = x^4$

(viii) $f = R \rightarrow R, f(x) = \left(\frac{x-2}{x-3}\right)$

(8 marks)

11. Given the following functions, find the function $f(g(x))$ and find the value of the function if $x = \{0, 1, 2, 3\}$

(ix) $f(x) = 3x - 1 ; g(x) = x^2 - 1$

(x) $f(x) = x^2 ; g(x) = 5x - 6$

(xi) $f(x) = x - 1 ; g(x) = x^3 + 1$

(9 marks)

Q3. Recurrence Relation

12. Solve the recurrence relation given;

(xii) $a_n = 6a_{n-1} - 9a_{n-2} ;$ initial conditions $a_0 = 1$ and $a_1 = 6$

(xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3} ;$
initial conditions $a_0 = 2, a_1 = 5$ and $a_2 = 15$

(xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$
initial conditions $a_0 = 1, a_1 = -2$ and $a_2 = -1$

(12 marks)

13. A sequence $a_1, a_2, a_3, a_4, \dots$ is given by

$$a_{n+1} = 5a_n - 3 ; a_1 = k$$

where k is a non-zero constant.

(i) Find the value of a_4 in terms of k .

(ii) Given that $a_4 = 7$, determine the value of k .

(8 marks)