Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
C		2429491

The Ebb and Flow of Victory: Analyzing Momentum in Professional Tennis

Summary

”Momentum” in sports is an abstract concept referring to an intangible force or strength that a player acquires through a series of events during a match. Although widely discussed by players, commentators, and spectators, momentum is challenging to measure precisely. It is often perceived as a shift in a player’s psychological or physical state, providing them with an advantage over their opponent.

Firstly, we define momentum mathematically as a **function** of a player’s performance at a given point, considering both recent and imminent events. Four performance factors—winning a point, a game, a break point, or making an unforced error—are weighted differently to calculate a player’s momentum. The calculation window spans nine points, centered on the point of interest, and these momentum shifts are graphically represented for the Wimbledon final, highlighting the match’s dynamic nature. What is more, we apply a model which uses Markov chains to predict match outcomes, factoring in serving situations. The model’s predictions align with actual results, showcasing its consistency.

Secondly, we explore whether momentum changes in a match are random or exhibit a pattern. Using the **Wald–Wolfowitz runs test**, a statistical method, it’s determined that momentum shifts for Alcaraz in the Wimbledon final were not random, suggesting he may have been strategically adjusting his play or experiencing shifts in energy levels. Djokovic’s momentum changes seemed more random, indicating a consistent performance throughout. The Model of Markov chain also disprove this.

Thirdly, we describe the use of **Gradient Boosting Regressor** and **XGBRegressor** models to predict momentum swings. It discusses how these models are trained on match data to forecast the direction of momentum shifts, indicating a player’s likelihood of gaining an advantage. The models’ accuracy in predicting significant momentum swings, or turning points, in a match is evaluated. The analysis involves checking how closely the models’ predictions of swing points match the actual events. While the models are adept at identifying most swings, they also tend to predict swings that don’t occur. Then an analysis is presented on the impact of different match features on the model’s predictions using SHAP. It is shown that factors such as the distance run by the players and the game number have significant influence on the model’s output, while other features have varying levels of impact.

Fourthly, we suggest how players can use the **XGBoost model** to improve their in-match performance. By focusing on controllable aspects of the game, like serving aces or committing fewer errors, players can potentially influence the momentum in their favor. We apply a **genetic algorithm** approach to identify the optimal combination of actions for enhancing player performance, resulting in a strategy that maximizes the positive change in momentum.

Finally, we emphasize two key challenges for the generalizability of an model used for predicting tennis match outcomes. Firstly, the limited diversity in the training data, which consists of only 31 Wimbledon matches, may lead to overfitting. Secondly, factors like player gender, tournament types, court surfaces, and differences with other sports, such as table tennis, can hinder the model’s ability to generalize effectively.

Keywords: Wald–Wolfowitz runs test, Markov Chain, Gradient Boosting Regressor, Extreme Gradient Boosting, Trusted machine Learning

Contents

1 Introduction

1.1 Background

The concept of ”momentum” in sports refers to an intangible force or strength that a player gains due to motion or a series of events during a match. While it’s commonly spoken of by players, commentators, and spectators, momentum is challenging to quantify. It’s often perceived as a shift in the psychological or physical state that gives one competitor an edge over the other.

The swings in momentum can be dramatic, causing a player who seems to be in a dominant position to suddenly lose ground to their opponent, and vice versa. These shifts can happen over the course of several points or games and are part of the ebb and flow of a tennis match. They are attributed to a myriad of factors including player stamina, psychological resilience, crowd support, and critical points or errors in play.

The difficulty in measuring momentum lies in its subjective nature; it’s a phenomenon more felt and experienced than seen. It is not a physical measure like speed or distance that can be easily quantified. However, its effects are often observable in the performance shifts of players and the outcomes of matches. Momentum is considered a crucial element in sports that can significantly influence the result of a game. Despite its elusive character, players, teams, and coaches often seek to harness or combat momentum to gain a competitive advantage.

1.2 Our work

a) Analyzing Momentum in Professional Tennis

In this study, we aim to quantitatively define and analyze the concept of momentum in professional tennis. We develop a mathematical model to measure momentum during a match, considering factors such as point wins, game wins, break points, and unforced errors. This model enables us to examine the dynamics of a match and the significance of momentum shifts.

- **Mathematical Definition of Momentum**: We define momentum using a weighted function of performance factors, providing a quantitative measure of a player’s current state in the match.
- **Momentum Shift Analysis**: Using statistical tests, we examine whether shifts in momentum are random or exhibit patterns, providing insights into match dynamics.
- **Predictive Modeling of Momentum Swings**: We employ Gradient Boosting and XGBoost models to predict momentum shifts, evaluating their performance in forecasting critical match turning points.

b) Statistical and Predictive Analysis Using Markov Chains

We employ Markov Chains to model the sequence of scoring events in a tennis match, providing a statistical framework to analyze and predict match outcomes based on the current score and observed patterns.

- **Construction of Markov Transfer Matrix**: We detail the process of constructing the Markov transfer matrix for tennis scoring, enabling the analysis of match state transitions.

- **Analysis of Momentum and Score Dynamics**: We apply the Markov model to analyze the influence of momentum on scoring dynamics, offering a novel perspective on match flow.
- **Prediction of Match Outcomes**: We explore the predictive capabilities of the Markov model in forecasting match results based on current scorelines and momentum.

c) Machine Learning Models for Performance Improvement

Focusing on practical applications, we explore how machine learning models can be used to advise players and coaches on strategies to improve in-match performance.

- **Modeling with Gradient Boosting and XGBoost**: We elaborate on the use of Gradient Boosting and XGBoost models to analyze tennis match data and predict key performance indicators.
- **Strategies for Performance Enhancement**: Based on model insights, we propose actionable strategies for players to enhance their performance, focusing on serve placement, shot selection, and point construction.
- **Evaluation and Advice for Coaches**: We provide an evaluation of model predictions and offer advice to coaches on utilizing these insights for training and match preparation.

d) Generalizability of the Model

We conclude our study by addressing the generalizability of our models across different tennis environments, including variations by player gender, tournament type, and court surface.

- **Assessment of Model Robustness**: We assess the robustness of our models across various datasets, including matches from different levels of competition and surfaces.
- **Implications for Broader Sports Analytics**: We discuss the implications of our findings for sports analytics, highlighting the potential for cross-sport application of our methodologies.

2 Capturing the Flow of a Match

2.1 Defining Momentum

We defined momentum at a point number t_p to be the overall performance of a player around this point, which is $[t_p - t_w, t_p + t_w]$ where $t_w = 4$ is the window length. To capture the performance of a player, we considered four factors $\{x_1, x_2, x_3, x_4\}$ and computed their weighted sum as shown in the below table:

factor	explanation	value	weight
x_1	a player wins the point	1 if True; else 0	0.2 if the player serves; else 1
x_2	a player wins the game	1 if True; else 0	1.5
x_3	a player wins a break point	1 if True; else 0	1
x_4	a player makes an unforced error	1 if True; else 0	-1

Table 1: Parameters used in defining momentum

$\{x_1, x_2, x_3, x_4\}$ are the common factors considered in assessing a player's performance. For each point number t_p, the momentum of the player i is calculated using the equation:

$$M_i = \sum_{t=t_p-t_w}^{t_p+t_w} w_1x_1(t) + w_2x_2(t) + w_3x_3(t) + w_4x_4(t) \tag{1}$$

The momentum of the two players in the 2023 Wimbledon Gentlemen's final is shown in Figure1.

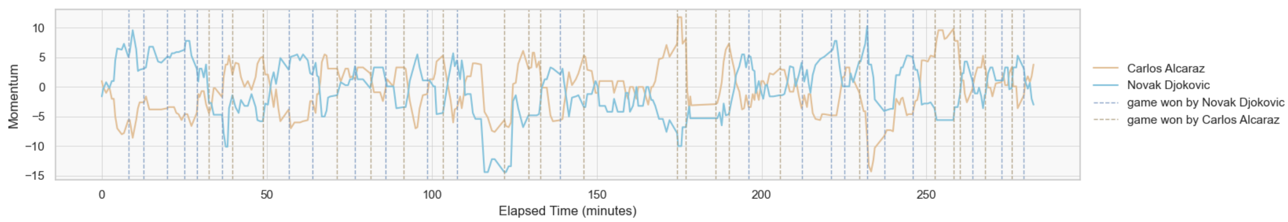

Figure 1: The momentum of the two players in the 2023 Wimbledon Gentlemen's final. Solid lines represent the momentum of the two player. Dashed lines indicate the winner of each game.

2.2 The Non-Randomness of Momentum

To test whether the change in the performance of a player is a random process, we adopted the **Wald–Wolfowitz runs test**[1] and checked whether the increase and decrease of momentum M_i occur in a random manner. In our model, a 'run' as a sequence of increasing/decreasing momentum values, which can also be interpreted as a **swing** in a match. If the momentum fluctuates randomly, which means that the change of M_i at different point number is mutually independent, the probability of having $M_i(t+1) > M_i(t)$ follows a binomial distribution.

Therefore we could generate the null hypothesis H_0 and the alternative hypothesis H_1:

H_0 : M changes in a random manner, $V(t) = \text{sign}(M(t+1) - M(t)) \sim i.i.d.\mathcal{B}(1, p), p \in (0, 1)$.

H_1 : Mdoes not change in a random manner.

As the number of data point is around 300 in each match, we could adopt the **normal approximation** and use the z-statistic:

$$Z = \frac{R - \bar{R}}{s_R} \tag{2}$$

where R is the observed number of runs, $\bar{R}$ and s_R are the expectation and standard deviation of the number of runs under null hypothesis, which can be computed as:

$$\bar{R} = \frac{2n_1n_2}{n_1 + n_2} + 1 \tag{3}$$

$$s_R^2 = \frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1 + n_2)^2(n_1 + n_2 - 1)} \tag{4}$$

in which n_1 and n_2 is the number of increases and decreases of M in consecutive time points.

Denoting the significance value as α, if $|Z| > Z_{1-\alpha/2}$, null hypothesis H_0 is rejected.The p-value of the test is $P(|Z_0| \geq |Z|) = 2(1 - \Phi(|Z|)$ in which Z_0 denotes the z-statistic of data generated from H_0, and $\Phi(x)$ is the cumulative probability density function of normal distribution.

By computing the p-values for the two players in the final, we find that the momentum of Alcaraz is of great non-randomness ($p = 0.01970$) while that of Djokovic is not ($p = 0.4393$). This implies that some players may actively adjust their strategy or be drown into a low-energy state, which results in a continuous increase/decrease in momentum over a period of time, while some have a more stable performance and their momentum fluctuate in a more random manner.

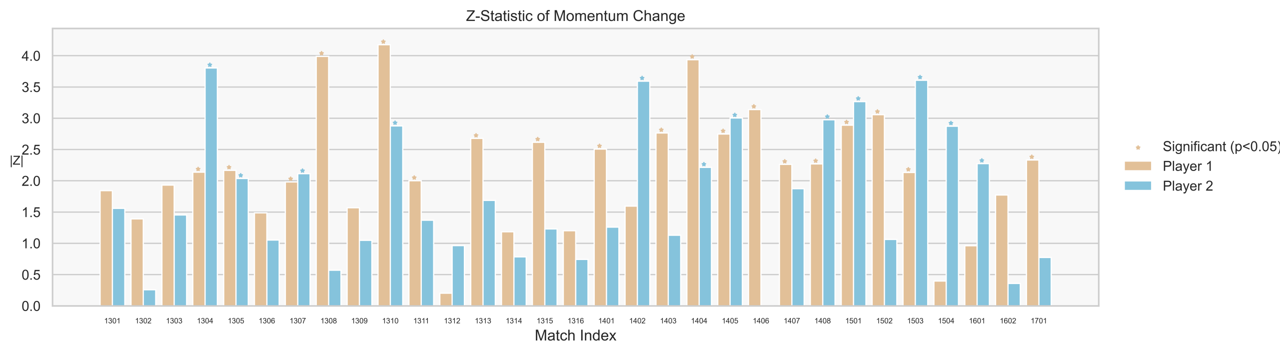

Figure 2: Z-statistic of the Runs Test

We then compute the z-statistic for all matches in Wimbledon 2023 men's matches after the first 2 rounds (see figure 2). Results show that half of players experienced significantly non-random changes in momentum, which indicates that **the swings in a tennis match(captured by a continuous increase or decrease in momentum) does not fully happen by chance**.

3 Figures and Prediction with Markov Chain

We can divide the game into different states by the current points and the monument(such as which player is at serving or the current situation). And we can assume these states as a Markov chain and simply this question into the calculation of the transfer probability among these states.

3.1 Markov Transfer Matrix

To introduce the Markov matrix, we set each situation as the rows and columns.

$$P = \begin{bmatrix} P_{1,1} & P_{1,2} & \dots & P_{1,j} & \dots & P_{1,S} \\ P_{2,1} & P_{2,2} & \dots & P_{2,j} & \dots & P_{2,S} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ P_{i,1} & P_{i,2} & \dots & P_{i,j} & \dots & P_{i,S} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ P_{S,1} & P_{S,2} & \dots & P_{S,j} & \dots & P_{S,S} \end{bmatrix}.$$

The total probability of each time-point should be 1, which is corresponding to following equation:

$$\sum_j P_{i,j} = 1$$

However, the probability is separated and almost only appeared to be 1 or 0. Thus, we tried to use the Gaussian Kernel to calculate the convolution.Utilizing Gaussian kernel-smoothed convolution, we aim to derive a comprehensive spatiotemporal matrix. By applying Gaussian kernel convolution, we achieve a smoothed representation of the scoring dynamics, enabling a more nuanced understanding of how a player's performance unfolds over time.

$$G(x) = \frac{1}{(2\pi)^{\frac{1}{2}}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

and then:

$$P_{ij}(t) = \int P_{ij}^0(\tau) \cdot G(t-\tau)\tau$$

Mention that we should separate the different server.

For example, we gain Alejandro Davidovich Fokina's server transfer matrix at 100 minutes as following matrix.

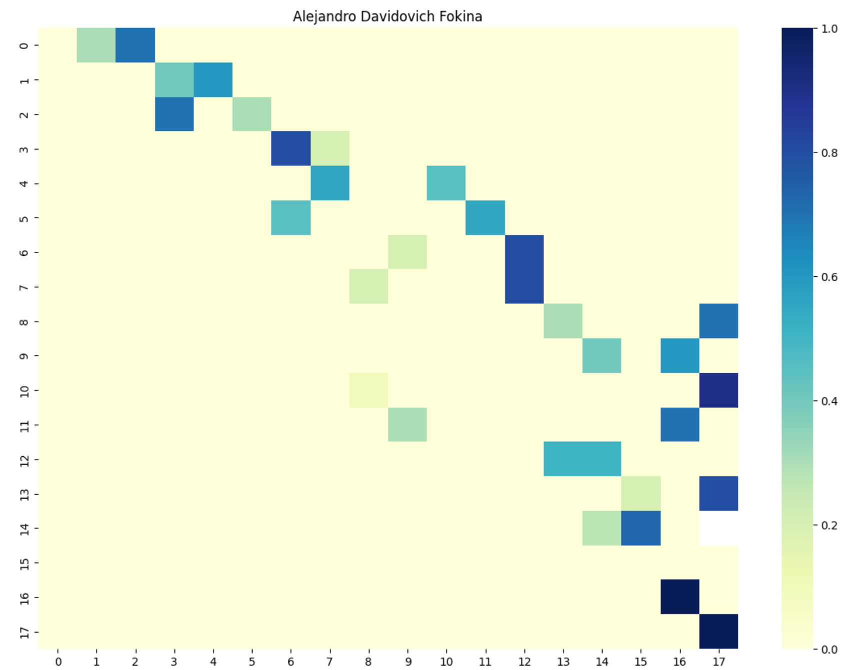

Figure 3: Alejandro Davidovich Fokina's server transfer matrix

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0,0	1,0	0,1	1,1	2,0	0,2	1,2	2,1	3,1	1,3	3,0	0,3	2,2	3,2	2,3	3,3	lose	win

Table 2: Corresponding

3.2 The Monument

As we continue our analysis, we will leverage the disparities in transition probabilities between losing and winning a particular point. This dissimilarity reflects, at a given moment in the match, the change in the likelihood of a player losing the current set relative to winning it. Such an analysis can offer profound insights into the dynamics of the game and the performance of the player.

Specifically, we can compute the transition probability differences for score changes after each set and further investigate which score situations pose greater challenges for the player. Additionally, we can examine whether the performance at critical moments aligns with the overall trends in the match.

To be continue, we apply the difference between transfer probability to lose this point and to win this point:

$$M = \sum_{total} (p_{\uparrow,getpoints} - p_{\downarrow,losepoints})$$

Visualizing these analytical results can enhance the understanding of a player's strengths and weaknesses during the game. Visualizations may include trend charts, probability distribution graphs, or other visual representations, providing a more comprehensive view of the match dynamics. We calculate the monument in the 2023-wimbledon-2023-wimbledon-1304 and campare to the real situation

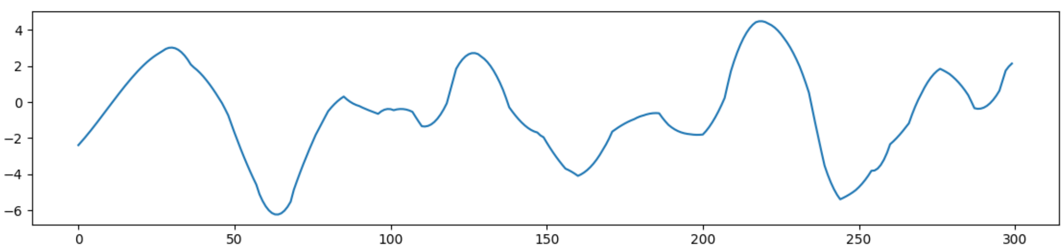

Figure 4: the prediction of 2023-wimbledon-2023-wimbledon-1304

Our model indicates a clear correlation between fitted data and the game's trajectory. This suggests discernible momentum in the match, both in current and past scores, which challenges the coach's notion of the game being almost random. The significant association highlights patterns and trends in scoring sequences, emphasizing the non-random nature of the game. These insights provide a nuanced understanding of strategic elements and underlying patterns shaping the match.

3.3 The Model Evaluation

3.3.1 Model Serialization

The model employs techniques from random calculus to effectively bridge the gap between discrete samples and a continuous framework. This transformation allows for a more nuanced and continuous representation of the data, enabling a more accurate analysis of the underlying dynamics.

Using the Kolmonglov-Feller Equation:

$$\frac{d}{dt}P(t) = \frac{P_{ij}(t+\Delta t) - P_{ij}(t)}{\Delta t} = \frac{\sum_k P_{ik}(t)P_{kj}(\Delta t) - P_{ij}(t)}{\Delta t} = P(t)Q$$

We can obtain the prediction of a game only need to define the server.

3.3.2 Momentum in Score

The model successfully captures the concept of momentum within the context of scores. However, a notable constraint arises as the model necessitates the segregation of different servers. This limitation stems from the model's requirement to treat servers as distinct entities, potentially overlooking certain nuances in momentum dynamics that involve both players.

However we may consider about a different weight of the situations:

$$M = \sum_{total} \alpha_{ij}(t)(p_{\uparrow,getpoints} - p_{\downarrow,losepoints})$$

3.3.3 Prediction of Match Trends

The model exhibits promising capabilities in predicting trends within the course of a match. By analyzing historical data, the model can provide valuable insights into the likely direction of the game, helping anticipate shifts in momentum and scoring patterns.

3.3.4 Limitations

- Limited Sample Size: The effectiveness of the model is hampered by a relatively small sample size. A larger dataset would enhance the model's robustness and reliability in capturing the intricacies of match dynamics.
- Factors Consideration: The analysis might be constrained by a lack of consideration for a comprehensive set of influencing factors. Expanding the scope to include additional variables could offer a more holistic understanding of the game dynamics.
- Markov Process: Relying solely on the Markov process introduces a limitation by potentially overlooking the impact of physical exertion on the game. Consideration of broader factors, such as player fitness and fatigue, could provide a more comprehensive perspective on match outcomes.

This detailed expansion provides a deeper insight into the nuances and considerations associated with each of the four aspects related to the model and its analysis of match dynamics.

4 Models and Advice for Coaches

4.1 Models show their ability to predict swings in the match

We apply GradientBoostingRegressor and XGBRegressor to learn features from a single match and predict the first order differential of momentum, which means the change of momentum during the process of the match, and their positive and negative signs symbolize the swing of the situation. In this case, we can use these model to directly predict the swing in the match.

4.1.1 Gradient Boosting Regressor

The core principle of Gradient Boosting involves iteratively adding weak learners to minimize the loss function as $L(y, F(x))$, where y is the true value and $F(x)$ is the prediction model. The Gradient Boosting algorithm updates the model by sequentially adding weak learners, $h(x)$, which are typically decision trees.[2]

1. **Loss Function**: The loss function measures the difference between the actual and predicted values, and here we define it as the Mean Squared Error (MSE), defined as:

$$L(y, F(x)) = \frac{1}{N}\sum_{i=1}^{N}(y_i - F(x_i))^2$$

   where N is the number of samples, y_i is the actual value, and $F(x_i)$ is the predicted value for the i-th sample.

2. **Gradient Descent**: At each step, the algorithm fits a new weak learner to the negative gradient of the loss function with respect to the prediction model. The update equation for the model at iteration t is:

$$F_t(x) = F_{t-1}(x) + \rho_t h_t(x)$$

   where $h_t(x)$ is the weak learner added at step t, and ρ_t is the learning rate.

3. **Additive Model**: The final model is an ensemble of weak learners added over T iterations:

$$F(x) = F_0(x) + \sum_{t=1}^{T}\rho_t h_t(x)$$

We use each dataset from matches provided to train the Gradient Boosting Regressor, setting the training and testing partition as 8:2, and then find that the our model behave well on all the dataset.

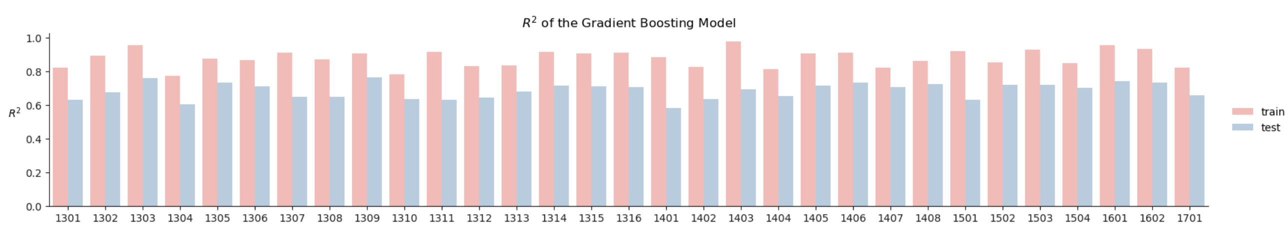

Figure 5: R^2 of Gradient Boosting Regressor on each match

We choose R^2 as the standard to assess the prediction of our model, as R^2 is a statistical measure that represents the proportion of the variance for a dependent variable that's explained by an independent variable or variables in a regression model. Mathematically, the R^2 score is defined as:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

Where:

1. y_i represents the actual values,
2. $\hat{y}_i$ represents the predicted values modeled by the regression,
3. $\bar{y}$ is the mean of the actual values,
4. n is the number of observations.

The R^2 score ranges from $-\infty$ to 1. A score of 1 indicates that the regression predictions perfectly fit the data. A score of 0 would mean that the model is no better at predicting than simply taking the mean of the actual values. Negative R^2 values indicate that the model performs worse than a horizontal line representing the mean of the actual values.

The figure 6 shows that the Gradient Boosting Regressor perform well on both training and testing data, which means that is can be used to predict the swings in the match.

4.1.2 XGBoost (Extreme Gradient Boosting)

XGBoost improves upon the gradient boosting framework by incorporating regularization terms in the objective function, handling missing values, and optimizing the computational efficiency.[**?**]

1. **Regularization**: The objective function in XGBoost includes both the loss function and regularization terms. It is given by:

$$\mathcal{L}(\phi) = \sum_{i=1}^{N} l(y_i, \hat{y}_i) + \sum_{k=1}^{K} \Omega(f_k)$$

   where l is the loss function, $\hat{y}_i$ is the predicted value for the i-th sample, f_k are the weak learners (trees), and Ω is the regularization term. The regularization term is defined as:

$$\Omega(f) = \gamma T + \frac{1}{2}\lambda \sum_{j=1}^{T} w_j^2$$

   where T is the number of leaves in the tree, w_j is the value of the j-th leaf, γ is the complexity control on the number of leaves, and λ is the L2 regularization term on the leaf weights.

2. **Sparsity Awareness and Tree Pruning**: XGBoost can handle sparse data and performs tree pruning using a depth-first approach. After reaching the maximum depth, it prunes the tree backwards, removing splits that provide no positive gain.

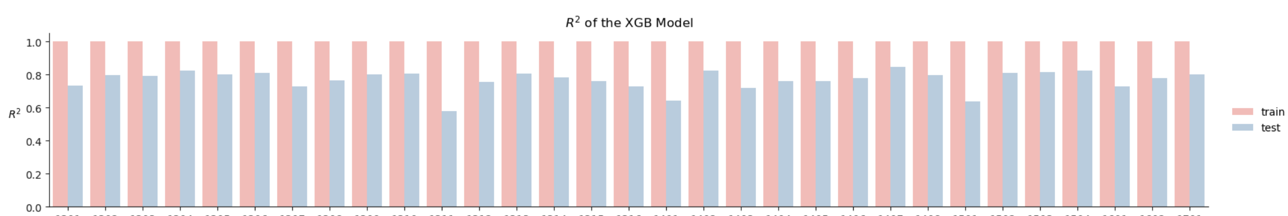

Figure 6: R^2 of XGBoost on each match

From figure 6, we find that the R^2 between the predicted data and the real ones appears better by XGBoost. Therefore, we continue the following analysis based on XGBoost model.

4.1.3 Predictions of the Swings

In order to examine our model's ability to predict the flow of a play, we test their the accuracy in detecting "**swing points**" in a match. Following the definition in section 2.2, we defined the swing point as the middle time point in a continuous increase/decrease of momentum, which could be interpreted as the time around which the performance of a player changes significantly and is closely related to the flow of a match.

We evaluated the consistency between the swing points identified in predicted and real sequences of momentum. If we find a predicted swing point close to a real swing point $|t - t_{\text{real}}| \leq 3$,we say that this real swing point is successfully captured by the model. Conversely, if there is not any real swing point close to a predicted swing point, the predicted swing point is identified as a false discovery of a swing.

We computed the false discovery rate(FDR) and the false negative rate(FNR) according to the following equations:

$$\text{FDR} = \frac{|\text{number of false discovery of swings}|}{|\text{total number of swings predicted}|} \tag{5}$$

$$\text{FNR} = \frac{|\text{number of real swings missed by prediction}|}{|\text{total number of real swings}|} \tag{6}$$

where FNR indicates the model's ability to identify swings in real matches, while FDR measures how likely it is to generate "false alarms" on swing points.

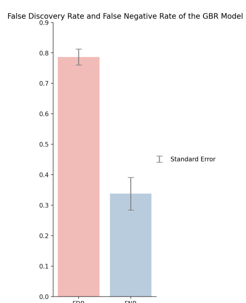

Figure 7: FDR&FNR of the GBR Model

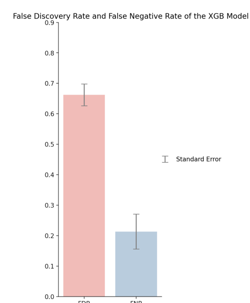

Figure 8: FDR&FNR of the XGB Model

The results in figure 7 and 8 shows our models' mean and standard error of FDR and FNR in the 31 matches. **Both the Gradient Boosting Regressor(GBR) and XGBoost(XGB) model successfully predicts the emergence of a swing in real match**, identifying approximately 80 percent of all swings. However, they also have a relatively high false discovery rate, which means that many of the swings they identifies do not exist.

As the prediction of our model is deterministic, we propose that the high FDR of our model may due to the fact that the change of momentum is a stochastic process instead. The fluctuation in the stochastic process can lead to a lower probability of experiencing a continuous increase/decrease in momentum, resulting in a high false discovery rate in the prediction of our models.

4.1.4 Factors Influencing the Model

This SHAP summary plot[3] shows the average impact of each feature on the model's output. Each bar's length represents the average magnitude of the feature's effect on the model's predictions, regardless of the direction of the effect.

we can make several observations:

1. p1_distance_run and p2_distance_run: These features have the highest average SHAP values, implying they have the strongest impact on the model's predictions. A longer bar indicates that the total distance run by player 1 or player 2 is very influential in predicting swings in the match.

2. game_no, p2_score, and p1_score: These are also important features but less so than the distance run. The game number (game_no) might indicate the progress of the match, and the scores (p2_score and p1_score) are direct indicators of the current state of the game.

3. Variation in Feature Impact: Some features have very short bars, suggesting that on average, they don't have a strong impact on the model's prediction. However, this doesn't mean they are always unimportant. In individual predictions, they could still have significant effects.

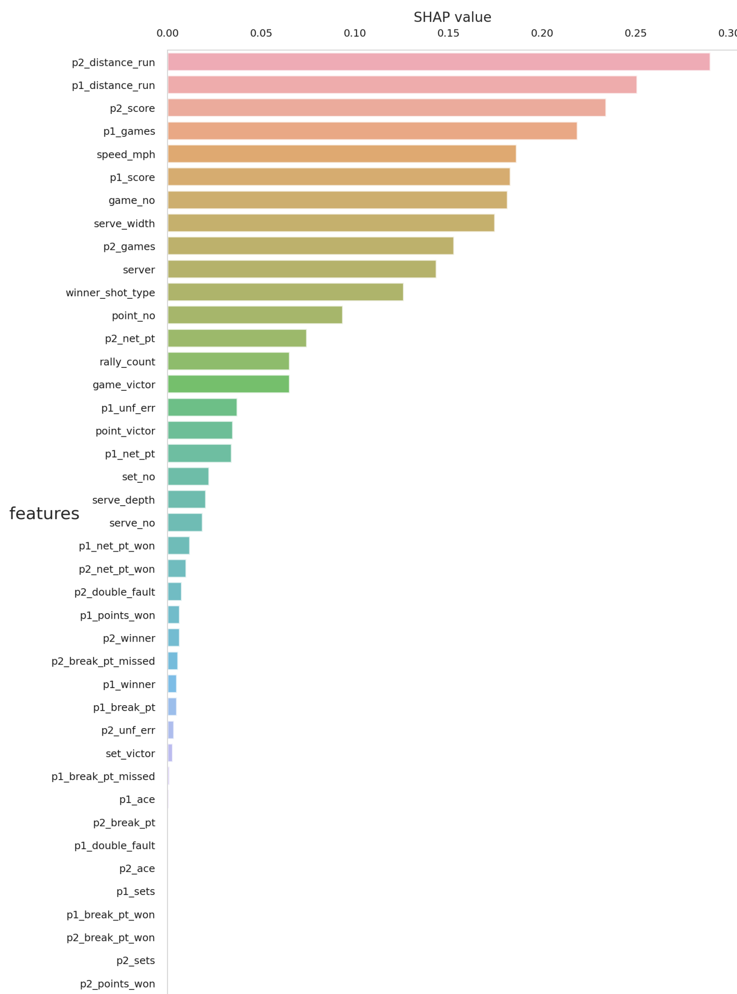

Figure 9: SHAP summary plot

In the SHAP force plot, which visualizes the contribution of each feature to a specific prediction (-2) made by the model. Each feature's impact on shifting the prediction away from the base value is represented by a colored segment pointing to the right (increasing the prediction) or to the left (decreasing the prediction).

For this specific prediction, it seems that features like "p1_distance_run", "serve_width_W", and "serve_depth_CTL" are contributing towards a higher prediction value, while "p2_score_30", "p1_games", and "game_no" are contributing towards a lower prediction value. The net effect of all these feature contributions leads to the final prediction, which is different from the base value.

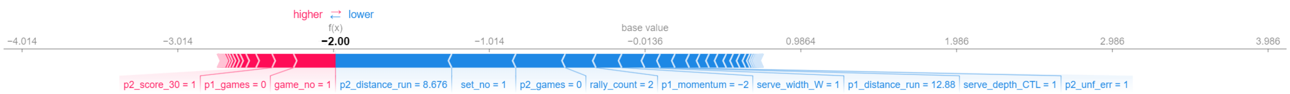

Figure 10: SHAP Force plot

However, how each feature influence the prediction varies across the dataset. The Figure 11 shows the variance of feature impacts across different instances, highlighting the complexity of the model's decision-making process.

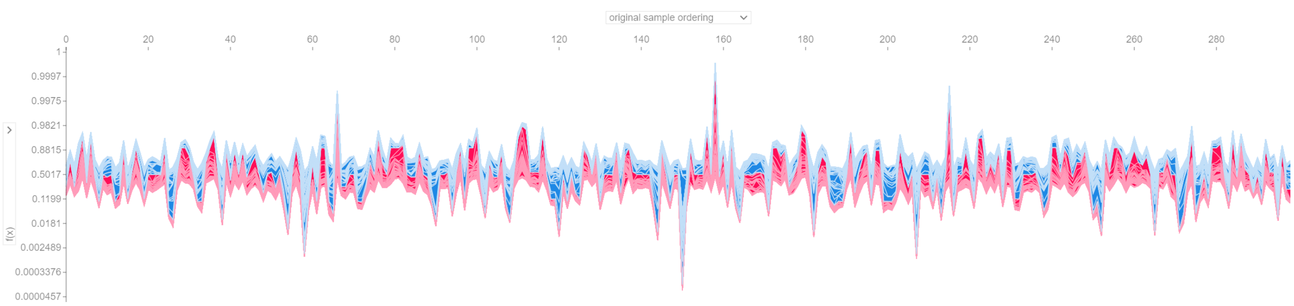

Figure 11: The continuous change of SHAP value

Figure 12 is the SHAP beeswarm plot, which shows the distribution of the SHAP values of each feature across many data points.

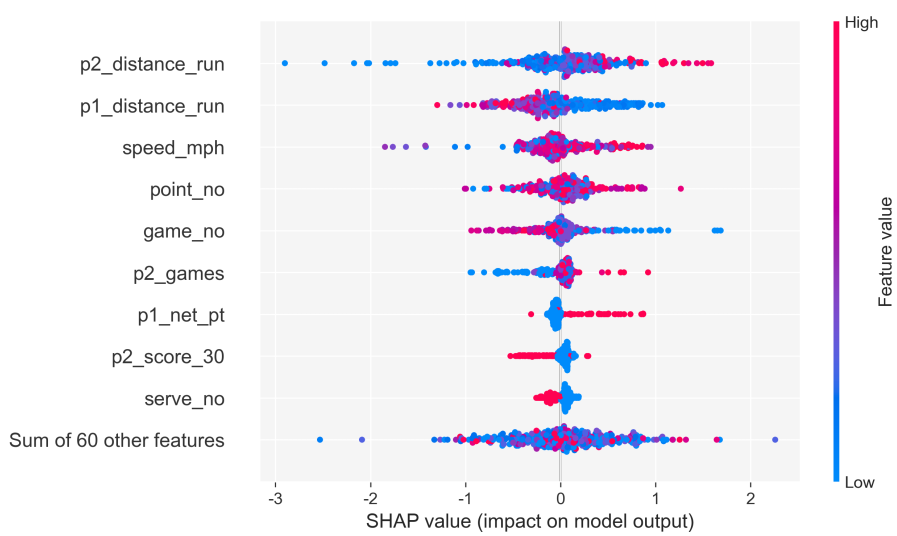

Figure 12: SHAP beeswarm plot

We can infer which features are the most important in XGBoost model and how the value of each feature affects the prediction from it. For instance, 'p2_distance_run' has a high impact on the model output with generally positive SHAP values, indicating it likely increases the prediction value when it is high (red points). On the other hand, 'p1_net_pt' has a mix of positive and negative impacts, suggesting a more complex relationship with the prediction outcome.

4.2 Use Models to Improve the Performance of a Player

In these part, we will demonstrate how to use XGBoost model to improve the performance of player1. From the statistical data provided, we can see for each data point, many features are given, and some of them are can not be changed by conscious activity. For features like 'p1_games' and 'p1_sets' are objective condition, and all features about player2 are apparently not the features that can be control by player1.

Here are the variables that player1 can control to influence the swing of his momentum:

1. p1_ace: Indicates whether Player 1 served an ace, which is a serve that is unreturnable by the opponent and scores a point directly. Player 1 can influence this variable by executing serves that are powerful and accurately placed, making it difficult for the opponent to reach or return the ball.

2. p1_winner: Represents whether Player 1 has hit a winner, which is a shot that lands in the opponent's court and is not returnable, resulting in a point. Player 1 can impact this variable by producing shots that are both precise and powerful, aiming to place the ball where the opponent cannot defend.

3. p1_double_fault: Signifies whether Player 1 has committed a double fault, which occurs when both serve attempts fail to land in the correct service box, leading to the loss of a point. Player 1 can affect this variable by improving serve reliability, focusing on consistency over risk, especially under pressure.

4. p1_unf_err: Indicates whether Player 1 has made unforced errors, which are mistakes made under no pressure from the opponent, leading to the loss of points. Player 1 can influence this variable by playing more cautiously, reducing risky shots, and maintaining focus to avoid unnecessary mistakes.

5. p1_net_pt: Shows whether Player 1 successfully approaches the net. Player 1 can impact this variable by increasing the frequency of net approaches, utilizing strategy and timing to control the net play and apply pressure on the opponent.

6. p1_net_pt_won: Indicates whether Player 1 has won points at the net. Player 1 can influence this variable by refining volleying skills, improving net presence, and making strategic decisions on when to approach the net to maximize the chances of winning points.

7. p1_break_pt: Represents whether Player 1 has opportunities to break the opponent's serve. Player 1 can affect this variable by putting pressure on the opponent's serve, creating opportunities through aggressive return plays and capitalizing on any weaknesses in the opponent's service games.

8. p1_break_pt_won: Signifies whether Player 1 has successfully broken the opponent's serve. Player 1 can influence this variable by executing effective return strategies, maintaining composure during critical points, and exploiting opportunities to take control of the game.

9. p1_break_pt_missed: Indicates whether Player 1 has missed opportunities to break the opponent's serve. Player 1 can impact this variable by improving mental toughness, focusing during key moments to convert break points, and adjusting tactics to better challenge the opponent's serve.

Each of these variables captures specific aspects of a tennis player1's performance during a match, reflecting his strategic choices, skill execution, and psychological resilience and they can be represented as a Boolean value. By optimizing these areas, Player 1 can significantly improve his momentum.

For instance,for data in "2023-wimbledon-1301", the change of the momentum on 51th data point is 0.199 and his strategy at that point is [0,0,0,0,0,0,0,0,0], and each of the elements stands for whether or not the condition is achieved as is mentioned above. Then we use genetic algorithm[4] to find the combination to find the highest first order differential of momentum.

1. **Chromosome Representation**

   Each chromosome is represented as a binary string of length 9, where each bit represents one of the binary options. Mathematically, a chromosome c can be expressed as:

   $$c = \{c_1, c_2, \ldots, c_9\}$$

   where $c_i \in \{0, 1\}$ for $i = 1, 2, \ldots, 9$.

2. **Fitness Function**

   The fitness function $f(c)$ calculates the "first order differential of momentum" for a given chromosome c. Assuming a simple model where each option directly influences the momentum, the function could be a weighted sum:

   $$f(c) = \sum_{i=1}^{9} w_i \cdot c_i$$

   Here, w_i represents the weight or influence of the ith option on the momentum differential.

3. **Selection**

   This process selects chromosomes for reproduction based on their fitness. For example, in tournament selection, a subset of chromosomes is randomly chosen, and the one with the highest fitness is selected.

4. **Crossover**

   Crossover combines parts of two parent chromosomes to create offspring. For a single-point crossover at point k, the offspring o_1 and o_2 are produced as follows:

   $$o_1 = \{c_{1_1}, c_{1_2}, \ldots, c_{1_k}, c_{2_{k+1}}, \ldots, c_{2_9}\}$$

   $$o_2 = \{c_{2_1}, c_{2_2}, \ldots, c_{2_k}, c_{1_{k+1}}, \ldots, c_{1_9}\}$$

   where c_1 and c_2 are the parent chromosomes.

5. **Mutation**

   Mutation introduces genetic diversity by randomly flipping genes in a chromosome. For chromosome c and a mutation rate m, each gene c_i has a probability m of being mutated:

   $$c'_i = \begin{cases} 1 - c_i & \text{with probability } m \\ c_i & \text{with probability } 1 - m \end{cases}$$

6. **New Generation Creation**

   The new generation is formed by combining the selected parents and offspring, potentially incorporating elitism to ensure the retention of the best solutions.

The final result is that if we use [1, 1, 1, 1, 0, 0, 1, 0, 0] (as we discussed above), and then we can get the highest improvement of momentum 0.5619, which means that if the coach use the algorithm to guide the player 1, then his momentum can reach to its highest given the object condition.

4.2.1 Advice to the coach

By summarizing the above discovery, we can provide several practical suggestions for the coach and player:

1. **Vary the shot placement to increase the running distance of the opponent.** It is shown in figure 12 the distance run by the opponent is the most significant factor affecting the change in momentum, and a player's momentum is likely to increase after his opponent travels a long distance in the last few shots. This could be achieved by playing with a more varied shot placement, which forces the opponent to move and consumes their energy.

2. **Speed is also important, but less than variability.** We find that although a high speed of serve may increase the momentum of a player, its impact is less significant compared to the running distance of players. Therefore, a player could try to serve at a higher speed to, but this should not be done at the cost of variability in shot placement.

3. **Net points have special effects in boosting morale.** We discover that when a player make it to the net, his momentum is likely to increase in the future. Running to the net can help a player to gain confidence and alertness, which may in turn increase the rate of winning.

4. **Failure in first-serve affects the future.** From figure 12 we can see that if a player used the opportunity of the second serve, his momentum will probably get lower. Therefore, although failures in the first serve do not affect the current point, it has significant impact on the future. In order to maintain a favorable flow of the play, one should ensure the success rate of his first serve.

5 Generalizability of Model

5.1 Factors Needed For Future Models

Based on previous sports news report and personal experience, we propose several factors that should be considered in the future:

1. **The spin of the ball.** Spin in tennis significantly impacts match outcomes. When a player imparts topspin, the ball rotates forward, causing it to dive over the net and bounce higher. Topspin shots are effective for aggressive play and are commonly used on clay courts. Conversely, backspin (slice) makes the ball float and skid, altering its trajectory and frustrating opponents.Therefore, spin influences the game's dynamics and should be considered when predicting the flow of a match.

2. **Special pauses in the match.** In tennis matches, some athletes ask for medical timeouts to adjust their physical and mental states, which helps them recover and may cause a change of flow in the match. In addition, as players are allowed to pause for a while before they make a serve, many of them utilize this period of time to calm down and rethink about their strategies.

3. **Challenges in the match.** When players ask for a challenge for a line call, Hawk-Eye provides an immediate and accurate verdict on whether the ball was in or out. A successful challenge could boost the confidence of the player, and an unsuccessful one brings pressure. Moreover, the time spent in watching the video recall gives player an opportunity to recover and may in turn affect the flow of the play.

5.2 Data Distribution and Diversity

As the training data may not diverse enough (31 matches about Wimbledon matches) and can not represent the population of possible matches, the model may overfit to the specific patterns in dataset and fail to generalize. It should be enlarged by adding data collected from matches of different levels as well as abundant conditions.

5.3 Feature Relevance Across Domains

Player Gender: Men's and women's tennis matches can have different dynamics due to physiological differences, playing styles, and match formats (e.g., Grand Slam men's matches are best-of-five sets, women's are best-of-three). If our model has learned patterns specific to men's physiology or playstyle, it might not transfer well to women's matches.

Tournaments: Different tournaments can have varying levels of player fields, prestige, and playing conditions. A model trained on Grand Slam data might not perform well on ATP 250 series matches due to different player motivations and competition levels.

Court Surfaces: The court surface (clay, grass, hard, carpet) significantly affects play. Patterns learned on one surface may not apply to others. For instance, clay courts slow down the ball and produce a high bounce, favoring baseline players, while grass courts favor fast serves and volleying.

Other Sports: Generalizing to other sports like table tennis is even more challenging because the dynamics of the game, scoring, and duration are entirely different. The factors that predict success in tennis may not be relevant in table tennis.

Our Discovery and Advice for Coaches

In light of our study on the impact of momentum in professional tennis, we have unearthed pivotal insights that could significantly enhance our coaching strategies and player performances. Our comprehensive analysis, grounded in mathematical modeling, statistical testing, and advanced machine learning techniques, sheds light on the elusive concept of momentum—its quantification, implications, and strategies for harnessing its power during matches. This letter aims to distill our findings into actionable advice, focusing on preparing players to effectively respond to the dynamic ebb and flow of play.

Understanding Momentum

Momentum in tennis is a multifaceted phenomenon, manifesting as a perceived shift in the game's flow favoring one player. It is influenced by a series of events, such as winning critical points, making or capitalizing on errors, and psychological factors. Our analysis confirms that momentum is not purely psychological; it can be quantified and leveraged.

Key Insights for Coaches

Quantifying Momentum: We defined momentum as a function of recent performance, incorporating factors such as point wins, game wins, break points, and unforced errors. This quantification allows us to visualize momentum shifts and understand their impact on match outcomes.

Randomness of Momentum Shifts: Our findings reveal that momentum shifts are not entirely random. Certain patterns emerge, indicating that players can indeed influence the game's flow through strategic play and mental resilience.

Predictive Modeling of Momentum Swings: By employing machine learning models, we can predict potential momentum shifts within a match. These predictions can guide players to capitalize on opportunities to gain or regain momentum.

Advice for Coaches

Based on our research, here are strategic recommendations to help players harness momentum in their favor:

Focus on Psychological Training: Since momentum is closely tied to a player's psychological state, incorporating mental resilience training into your coaching regimen is crucial. Teach players to recognize the signs of a momentum shift and respond calmly and strategically.

Strategic Play During Critical Moments: Our models highlight the importance of critical points (e.g., break points) in influencing momentum. Train players to elevate their focus and execution during these moments. Emphasize the significance of game and break points and develop tailored strategies for these high-pressure situations.

Customized Pre-Match Strategies: Utilize predictive models to anticipate possible scenarios where momentum could shift. Prepare players with strategies tailored to specific opponents, focusing on their weaknesses that can be exploited during pivotal moments.

Adaptability and In-Match Strategy Adjustment: Teach players to be adaptable, recognizing when momentum shifts against them and having the flexibility to adjust their strategy mid-match. This includes altering serve patterns, adjusting baseline or net play, and changing shot selection to disrupt the opponent's rhythm.

Physical Conditioning and Recovery: Our analysis indicates that physical factors, such as the total distance run, significantly impact momentum. Ensure players are in peak physical condition to maintain high performance throughout the match. Additionally, focus on recovery strategies to prevent fatigue, which can adversely affect momentum.

Enhance Shot Selection and Error Management: Encourage players to make smart shot selections, minimizing unforced errors, and strategically placing shots to challenge opponents. Effective shot selection can shift momentum by putting the opponent under pressure and creating scoring opportunities.

Implementing the Advice

Implementing these strategies requires a holistic coaching approach, combining physical training, mental conditioning, and tactical gameplay. Regularly simulate match scenarios in practice sessions that mimic the conditions where momentum shifts are likely to occur, preparing players to navigate these situations confidently.

In conclusion, our study underscores the tangible impact of momentum in tennis and provides a framework for coaches and players to understand, predict, and influence the flow of play. By integrating these insights into your coaching philosophy, you can equip your players with the tools to master the art of momentum, turning it into a formidable ally on the court.

Reference

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Report on Use of AI

1. OpenAI ChatGPT (Feb 4, 2024 version, ChatGPT-3.5)
    - Query1: Please introduce the rule of tennis in detail.
    - Output:

Introduction

Tennis is a sport where either two players (singles) or two teams of two players (doubles) use rackets to hit a ball over a net to the opponent's side of the court. The main objective is to hit the ball in such a way that the opponent is not able to play a valid return.

The Court

- The game is played on a rectangular flat surface, typically grass, clay, or a hard court of concrete and/or asphalt.
- The court is 78 feet (23.77 m) long. Its width is 27 feet (8.2 m) for singles matches and 36 feet (10.97 m) for doubles matches.
- A net divides the court into two equal halves and is stretched across the full width of the court, parallel to the baselines, dividing it into two equal ends.

The Match Play

- A coin toss or a racket spin is generally used to determine which player or team will serve first.
- Players stand on opposite sides of the net. One player is designated the server, and the opposing player is the receiver.
- The serve is initiated from behind the baseline, alternating from the left and the right sides of the court.
- The match is typically played as best-of-three or best-of-five sets.

Scoring

- Games are scored starting at ”love” (zero), with points called ”15”, ”30”, ”40”, and then ”game”.
- If both players reach 40, the score is called ”deuce”. After deuce, a player must win by a two-point margin.
- A set is won by the first player to win six games with a two-game margin.
- If the score reaches 6–6 in a set, a tiebreaker is played, which is scored point-by-point.

Serving and Returning

- The server must serve diagonally across the net to the opponent's service box.
- If the server fails to serve correctly, it is called a ”fault”. Two consecutive faults result in the opponent winning the point, known as a ”double fault”.
- The receiver must allow the ball to bounce once before returning it.

In Play

- Once the ball is in play, players may hit it before it bounces (a volley) or after its first bounce (a groundstroke).
- The ball must land within the court boundaries on each shot.
- Points are won by hitting the ball in such a way that the opponent fails to return it within the court.

Faults and Fouls

- A ball not landing within the correct court boundaries.
- Not reaching the ball before it bounces twice.
- Touching the net with racket or body during play.
- Hitting the ball before it has crossed the net.

Conclusion

Understanding these rules is essential for both playing the game and appreciating it as a spectator. Tennis is a sport of physical prowess, strategy, and mental endurance, and these rules help to ensure fair and competitive play.