# Survival Analysis of Atlantic Halibut

## Exploring the Influence of Tow Duration and Other Variables on the Time Until Death

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December 02, 2023

**Abstract**

Our study delves into the intricate survival dynamics of Atlantic halibut, emphasizing the impact of tow duration and various covariates on their survival rates. Notably, our analysis uncovers a significant difference in median survival times between two tow durations, specifically revealing that a 30-minute tow exhibits a statistically longer median time until death compared to a 100-minute tow. Building upon these insights, our investigation employs a comprehensive stratified Cox proportional hazards model to unravel the nuanced interplay of covariates on survival rates. The model considers factors such as the difference between maximum and minimum depth observed during tow (`DELDEPTH`), fork length (`LENGTH`), handling time (`HANDTIME`), the natural logarithm of total catch of fish in tow (`LOGCAT`), and their interactions, shedding light on key aspects. However, it is crucial to approach the interpretation of individual hazard ratios, particularly for variables like `DELDEPTH` and `LOGCAT`, with caution due to the introduced complexity of interaction terms. The model interpretations underscore that increasing `DELDEPTH` is associated with a reduced hazard, while longer fish lengths slightly decrease the hazard. Conversely, higher `HANDTIME` values correspond to a slightly elevated hazard, and the effect of `LOGCAT` indicates a substantially lower hazard with higher values. The interaction between `DELDEPTH` and `LOGCAT` suggests a modest increase in hazard, introducing a layer of complexity to the findings. These nuanced insights contribute to a more comprehensive understanding of the factors influencing the species' survival, offering a solid foundation for informed conservation and management strategies in the context of Atlantic halibut populations.

# Contents

# Introduction

Our objective is to investigate the determinants influencing the survival rate of Atlantic halibut. Our study incorporates data from 294 halibut, capturing their survival time and various variables linked to their time until death (some removed from the experiment before they die because of limited holding facility). The following presents a brief description of each variable considered in our study.

Table 1: Variable Description and Notation

| **Variable** | **Description** |
|---|---|
| *No* | Observation number (1,...,294) |
| *TIME* | Survival time of halibut (time until death) in hours |
| *CENSOR* | Censoring indicator. 1=uncensored observation; 0=censored observation |
| *TOWD* | Duration (in minutes) of time trawl net was towed on the bottom |
| *DELDEPTH* | Difference between maximum and minimum depth observed during tow (depth measured in meters) |
| *LENGTH* | Fork length of halibut in centimeters |
| *HANDTIME* | Handling time (in minutes) between net coming on board vessel and fish being placed in holding tanks |
| *LOGCAT* | Natural logarithm of total catch of fish in tow |

This study's primary aim is to construct a comprehensive framework accurately delineating the relationships between independent variables—specifically, `TOWD`, `DELDEPTH`, `LENGTH`, `HANDTIME`, and `LOGCAT` — and the survival variable (`TIME`, `CENSOR`) for the halibut under investigation. A key focus within this objective is the exploration of potential differences in survival rates associated with the two tow durations (`TOWD`). To accomplish these goals, we employ a range of methods, including initial analyses utilizing survival curves and log-rank tests, model selection and analysis, and diagnostic procedures. In conclusion, we present the outcomes of the selected model, offering interpretations pertinent to the research questions under consideration.

## Preliminary Analysis

We initiate this analysis with a preliminary examination of each covariate. This entails inspecting their survival functions and performing log-rank tests to discern potential differences between survival curves associated with each covariate. Through this process, we aim to gain a comprehensive understanding of the dataset.

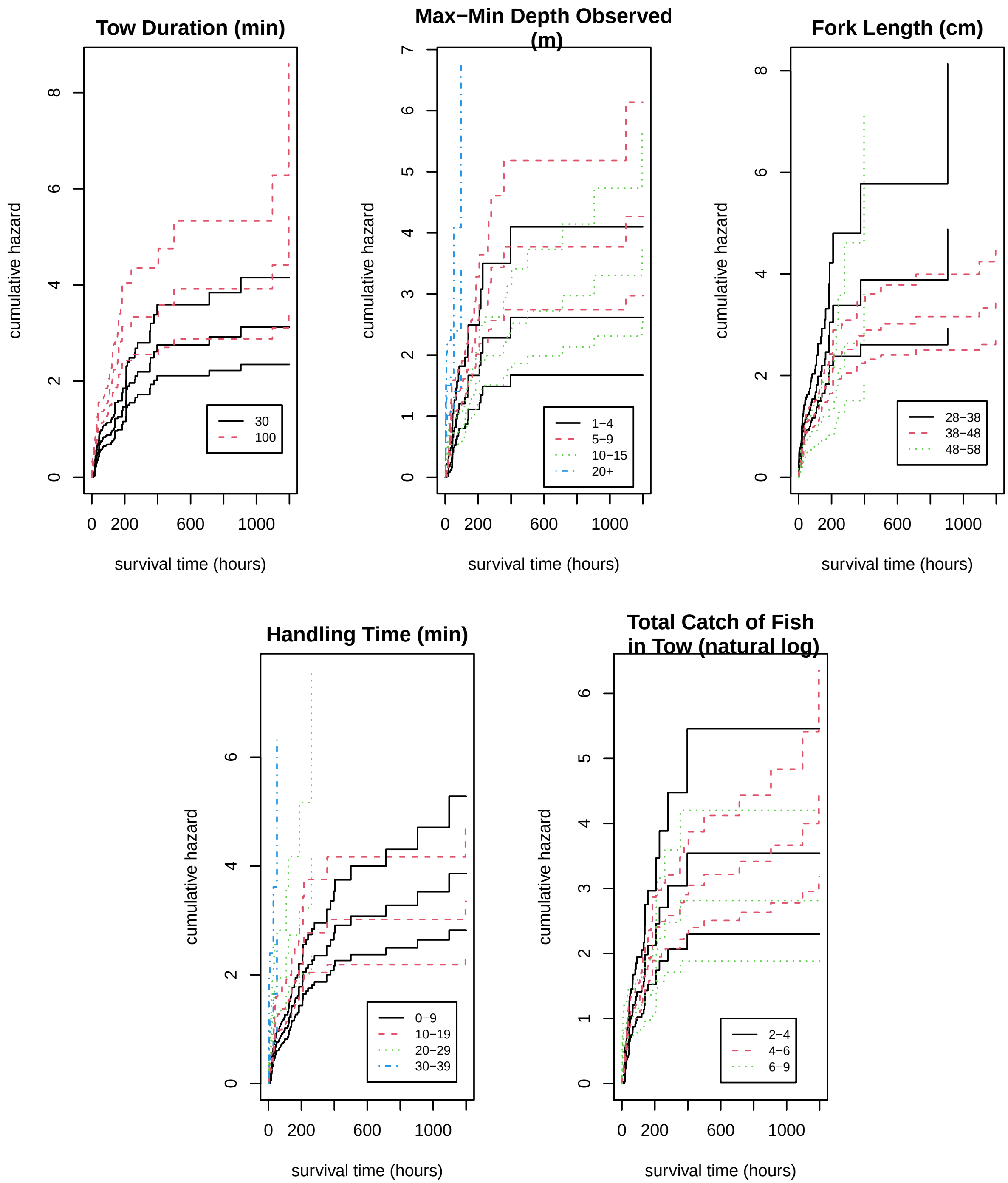
Presented below is a table featuring key statistics pertaining to the survival functions of each covariate, along with the corresponding p-values from their respective log-rank tests.

Table 2: Survival Functions - Covariate Statistics, Associated 95% Confidence Intervals of Survival Time, and Respective p-values from Log-Rank Test

| **Variable** | **Number of Events** (Uncensored) | **Median Survival Time** (Hours) | **95% Confidence Interval** | **$p$-value** |
|---|---|---|---|---|
| $TOWD = 30$ | 103 | 48.5 | (40.1, 117.0) | 4e-08 |
| $TOWD = 100$ | 170 | 23.2 | (17.2, 29.8) | |
| $DELDEPTH = 1-4$ | 34 | 50.3 | (44.1, 85.0) | 1e-11 |
| $DELDEPTH = 5-9$ | 124 | 29.8 | (29.5, 33.8) | |
| $DELDEPTH = 10-15$ | 81 | 30.75 | (17.3, 122.0) | |
| $DELDEPTH = 20+$ | 34 | 3.25 | (3.1, 7.9) | |
| $LENGTH = (28, 38]$ | 77 | 19.4 | (17.0, 29.8) | 0.002 |
| $LENGTH = (38, 48]$ | 162 | 30.6 | (29.5, 36.9) | |
| $LENGTH = (48, 58]$ | 34 | 42.1 | (29.5, 185.2) | |
| $HANDTIME = (0, 9]$ | 132 | 44.1 | (30.6, 77.9) | 1e-11 |
| $HANDTIME = (9, 19]$ | 86 | 30.6 | (29.8, 37.8) | |
| $HANDTIME = (19, 29]$ | 45 | 13.2 | (7.9, 20.2) | |
| $HANDTIME = (29, 39]$ | 10 | 2.8 | (2.6, NA) | |
| $LOGCAT = (2, 4]$ | 56 | 42.1 | (28.5, 55.7) | 0.8 |
| $LOGCAT = (4, 6]$ | 165 | 30.0 | (28.6, 36.7) | |
| $LOGCAT = (6, 9]$ | 52 | 7.9 | (4.0, 38.0) | |

Recognizing the tedious nature of parsing through the detailed survival data table for each variable, we instead can visually present cumulative hazard curves for each variable. This will offer a more accessible means to observe how the survival rate is influenced within each group of the respective variable.

**Note:** In the preceding table, we presented the survival functions for each group's variable, including median hours of survival and other pertinent statistics. Below, we illustrate the *cumulative hazard* curves, distinct from the earlier survival functions. However, a clear relationship exists between the two; defining $S(t)$ as the survival function and $H(t)$ as the cumulative hazard function, we express this relationship as $S(t) = \exp(-H(t))$ or, equivalently, $H(t) = -\log S(t)$. The key takeaway is that as the survival function, $S(t)$, decreases over time, the cumulative hazard, $H(t)$, increases.



Derived from both the table and cumulative hazard curves, several crucial patterns within the dataset emerge:

- `TOWD`: Conducting the log-rank test, we observe a statistically significant distinction in median time until death between a tow duration of 30 minutes and one of 100 minutes. This discrepancy signifies a **notable difference in the survival curves for the two tow duration groups**.
- `DELDEPTH`: An apparent inverse relationship surfaces between the disparity in maximum and minimum tow depths (measured in meters) and the median time until death. Smaller depth variations correlate with lengthier median survival times.
- `LENGTH`: A direct relationship becomes evident between the fork length of halibut (in cm) and the median time until death, indicating that larger halibut exhibit lengthier median survival times.
- `HANDTIME`: An inverse relationship suggests a connection between handling time (measured in minutes) and median time until death. Shorter handling times correspond to lengthier median survival times, aligning with the plausible scenario where prolonged handling could limit access to water, potentially reducing survival time.
- `LOGCAT`: An inverse relationship surfaces between the natural logarithm of the total catch of fish in tow and median time until death. This implies that a smaller catch of fish associates with lengthier median survival times. However, it's crucial to note that the difference in total catch is not statistically significant. Consequently, cautious consideration is warranted when constructing the appropriate model, given the observed lack of significance in the catch difference.

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# Model Selection

Based on the preliminary data findings, our initial approach involved fitting a Cox Proportional Hazards (PH) regression model to explore the relationship between the survival variable and the independent variables. Notably, the variable `No` was excluded from this initial model due to its lack of substantive significance in the context of the data, serving primarily as an identifier for specific halibut observations. Additionally, we conducted an exploration of interactions through stepwise selection, incorporating two interaction terms into the initial model.

To gauge the adequacy of the model fit, we inspected the Proportional Hazards (PH) assumption. While the *global* PH assumption was satisfied, the variable `TOWD` and its interaction term deviated from this assumption. This discrepancy poses a challenge, as it is imperative for all variables in the model to exhibit hazards that remain constant over time.

Addressing the non-proportional hazards in the `TOWD` variable requires further modeling strategies. Potential avenues include stratification or exploring alternative models such as accelerated failure time (AFT) or frailty models to better capture the dynamics of this variable and enhance the overall model's predictive accuracy.

**Note**: Throughout the model selection process, diverse modeling approaches, including stratified Cox PH, Accelerated Failure Time (AFT), and frailty models, were employed to identify the most fitting model for the data. While AFT and frailty models demonstrate suitability for the data, they introduce additional intricacy to the model, potentially posing challenges in interpretation. Given this trade-off, we opt for the **final** selection of the Cox PH model, stratified by `TOWD`. This decision prioritizes interpretability over marginal potential gains in accuracy.

Additionally, to streamline the report, exhaustive steps for each model in the selection process will not be outlined. However, it is noteworthy that AFT and Frailty models were fitted and checked before the conclusive decision to adopt the stratified Cox PH model.

## Stratified Cox PH Regression Model

A concise theoretical overview of the Cox Proportional Hazard (PH) model and its equation is outlined below:

Let $h(t)$ be the hazard function for an individual at time $t$. The Cox PH model is expressed as:

$$h(t|X) = h_0(t) \times \exp(\beta_1 X_1 + \beta_2 X_2 + ... + \beta_p X_p)$$

where:

- $h(t|X)$ is the hazard function for an individual with predictor variables $X$ at time $t$.
- $h_0(t)$ is the baseline hazard function, representing the hazard in the absence of any predictors.
- $\beta_1, \beta_2, ..., \beta_p$ are the coefficients associated with the predictor variables $X_1, X_2, ..., X_p$, respectively.

The term $\exp(\beta_1 X_1 + \beta_2 X_2 + ... + \beta_p X_p)$ represents the multiplicative effect of the predictor variables on the hazard. If this term is equal to 1, the hazard is proportional, meaning that the relative hazard remains constant over time. If the term is greater than 1, it implies an increased hazard, and if less than 1, a decreased hazard.

In practice, the Cox PH model does not estimate the baseline hazard directly. Instead, it estimates the hazard ratios ($\exp(\beta)$) associated with each predictor variable. The model assumes that the hazard ratios are constant over time, making it a proportional hazards model.

For our model, we can express it as

$$\begin{aligned} h_{\text{TOWD = 30}}(t|Z) = h_{0,\text{TOWD = 30}} \times \exp(&\beta_1 \cdot \text{DELDEPTH} + \beta_2 \cdot \text{LENGTH} \\ &+\beta_3 \cdot \text{HANDTIME} + \beta_4 \cdot \text{LOGCAT} + \beta_5 \cdot \text{DELDEPTH x LOGCAT}) \end{aligned}$$

$$\begin{aligned} h_{\text{TOWD = 100}}(t|Z) = h_{0,\text{TOWD = 100}} \times \exp(&\beta_1 \cdot \text{DELDEPTH} + \beta_2 \cdot \text{LENGTH} \\ &+\beta_3 \cdot \text{HANDTIME} + \beta_4 \cdot \text{LOGCAT} + \beta_5 \cdot \text{DELDEPTH x LOGCAT}) \end{aligned}$$

In this context, we present two equations, reflecting the nature of our final model, which is a **stratified** Cox proportional hazards model. In other words, the baseline hazard is stratified based on the variable `TOWD`. This implies that the model accommodates distinct baseline hazards for varying levels or categories of the `TOWD` variable (specifically, 30 minutes and 100 minutes, respectively). The additional terms in the formula represent the effects of the specified predictor variables on the hazard, including any interactions considered in the model.

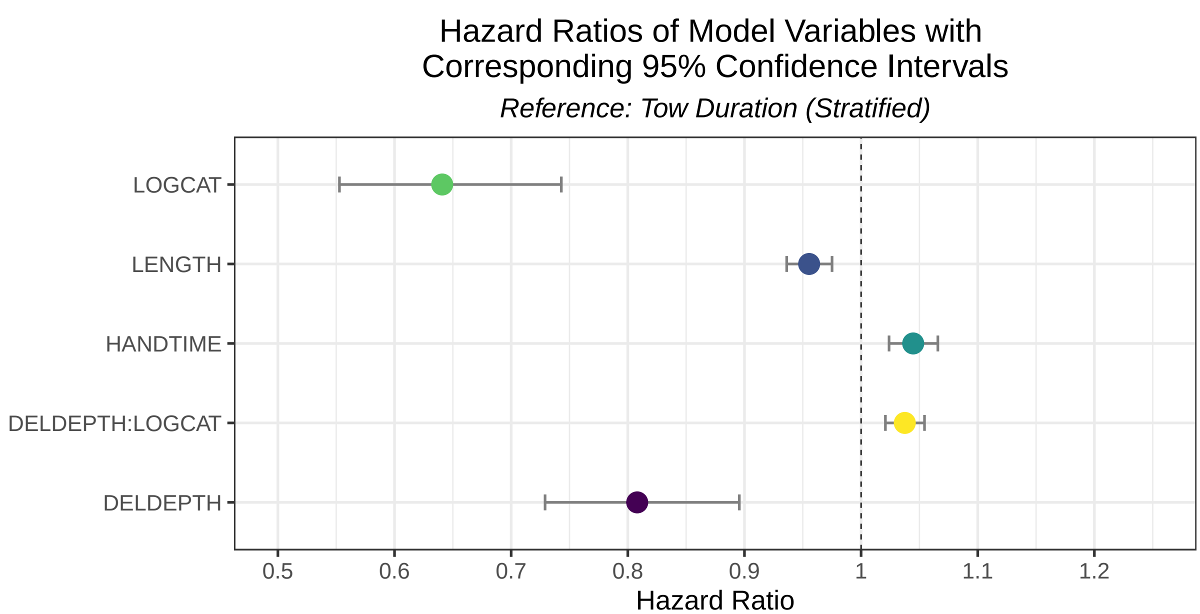
Below, we provide a tabulation of the estimated coefficients of the model along with their corresponding 95% confidence intervals:

Table 3: Stratified Cox PH Regression Model: Estimated Coefficients, Associated 95% Confidence Intervals, and Respective p-values

| Parameter | exp(Estimate) | 95% Confidence Interval | *p*-value |
|---|---|---|---|
| $\beta_1$ | 0.8081 | (0.7291, 0.8956) | 4.89e-05 |
| $\beta_2$ | 0.9555 | (0.9362, 0.9751) | 1.12e-05 |
| $\beta_3$ | 1.0447 | (1.0240, 1.0658) | 1.89e-05 |
| $\beta_4$ | 0.6409 | (0.5528, 0.7430) | 3.70e-09 |
| $\beta_5$ | 1.0375 | (1.0208, 1.0544) | 8.06e-06 |

## Model Summary and Interpretations

Utilizing the coefficient estimates and their associated 95% confidence intervals from the aforementioned table, we will proceed to interpret the hazard ratios. To enhance clarity, we intend to generate plots depicting the hazard ratios along with their corresponding 95% confidence intervals. These visualizations will facilitate a more intuitive understanding of the comparisons between different variables relative to the reference group (`TOWD`). Despite the stratification of `TOWD`, the hazard ratios are applicable to **both** groups (30 minutes and 100 minutes). By examining these plots, we aim to extract meaningful interpretations of the hazard ratios and their implications for the survival time of the 294 halibut under study.



**Interpretations**

- **DELDEPTH**: For a one-unit increase in `DELDEPTH`, the hazard of the event is reduced by approximately 19.19% (100% - 80.81%), holding other variables constant. This suggests that higher values of `DELDEPTH` are associated with a lower hazard.
- **LENGTH**: For a one-unit increase in `LENGTH`, the hazard of the event is reduced by approximately 4.45%, holding other variables constant. This implies that longer values of LENGTH are associated with a slightly lower hazard.
- **HANDTIME**: For a one-unit increase in `HANDTIME`, the hazard of the event increases by approximately 4.47%, holding other variables constant. This suggests that higher values of HANDTIME are associated with a slightly higher hazard.
- **LOGCAT**: For a one-unit increase in `LOGCAT`, the hazard of the event is reduced by approximately 35.91%, holding other variables constant. This indicates that higher values of LOGCAT are associated with a substantially lower hazard.
- **DELDEPTH:LOGCAT**: The interaction effect between `DELDEPTH` and `LOGCAT` results in a hazard ratio of 1.0374745. This suggests that the combined effect of DELDEPTH and LOGCAT increases the hazard by approximately 3.75%, holding other variables constant.

In summary, these hazard ratios provide valuable insights into the direction and magnitude of the effects associated with each predictor variable and their interactions on the hazard of the event. Positive hazard ratios signify an increased hazard, whereas values less than 1 indicate a decreased hazard.

However, careful consideration is needed when analyzing the hazard ratios for `DELDEPTH` and `LOGCAT` individually. The introduction of the interaction term complicates the interpretation, as it represents a combined effect that is not easily discernible when examining the individual hazard ratios. In simpler terms, the impact of `DELDEPTH` on the hazard rate is not constant across different levels of `LOGCAT`, and vice versa. The interaction term `DELDEPTH:LOGCAT` accounts for the joint influence of both `DELDEPTH` and `LOGCAT` on the hazard.

Consequently, interpreting the hazard ratio for `DELDEPTH` or `LOGCAT` individually may not fully capture the nuanced relationship between these variables due to their interaction. The combined effect, as represented by the interaction term, can modify the individual hazard ratios. To gain a more comprehensive understanding of the relationships, especially in the presence of interaction terms, it is valuable to consider the joint effects of variables and visualize the relationship in the forthcoming model analysis.

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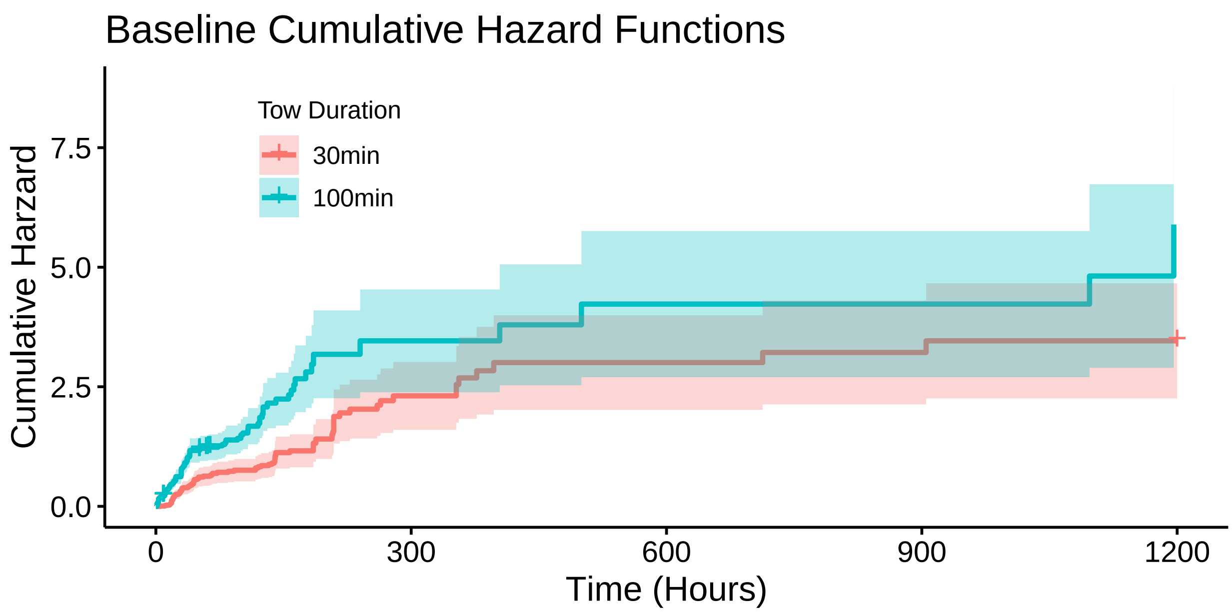
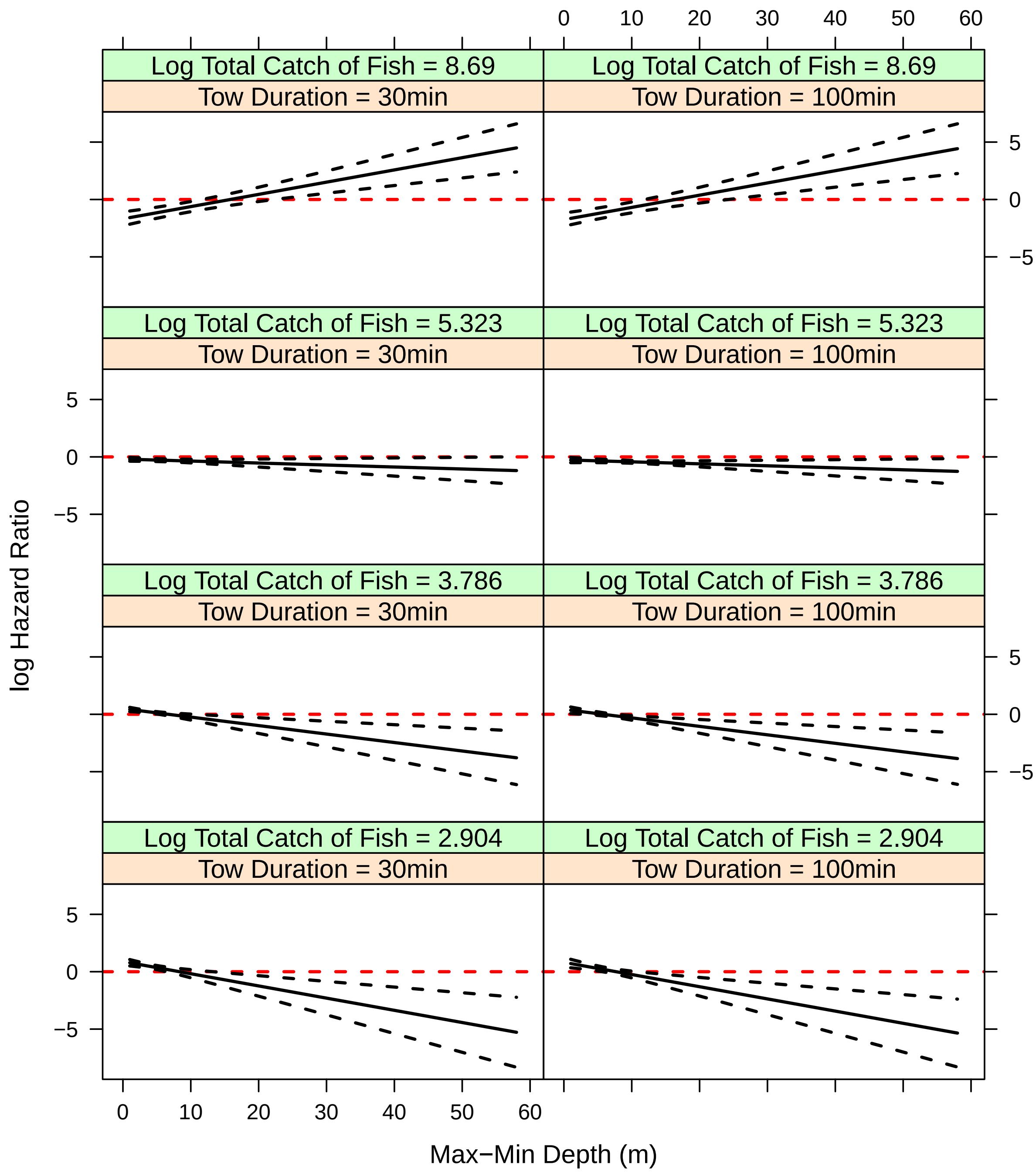
## Model Analysis/Predictions

Here, we affirm the appropriateness of a stratified Cox PH model through the plot of baseline cumulative hazard functions. It is noteworthy that at each level of tow duration (30 minutes and 100 minutes), the cumulative hazard distinctly exhibits a different baseline:




Moreover, we can enhance the understanding of our model by generating model predictions through the creation of effect plots. As previously mentioned, given the introduction of an interaction term in our final model, the increased complexity necessitates the use of effect plots to facilitate interpretation of the intricate relationships between variables.

The illustration below depicts that for each tow duration group, at lower levels of the log total catch of fish, an increase in `DELDEPTH` levels (indicating a larger distance of depth observed during tow) is associated with an expected decrease in the log hazard ratio. Interestingly, for a log total catch of fish just above 5, the log hazard ratio remains constant across different levels of observed depth. Notably, when the catch of fish becomes substantial (around 8), the pattern shifts, indicating that increased depth levels observed during the tow are associated with an elevated log hazard ratio. These predictions illuminate the interactions among the three variables — `TOWD`, `DELDEPTH`, and `LOGCAT` — within the stratified Cox PH model.

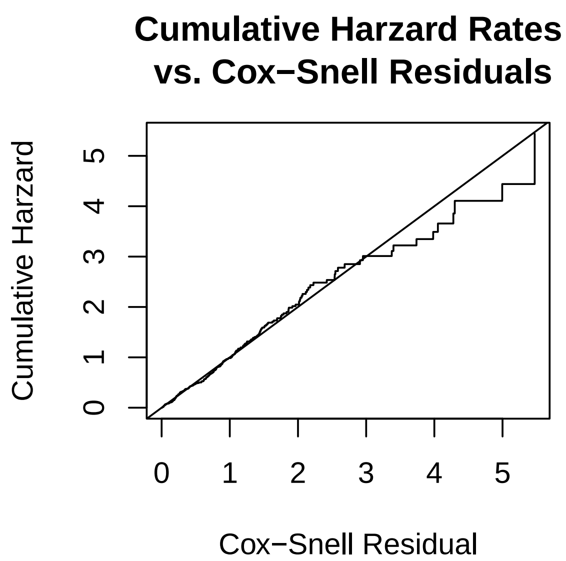


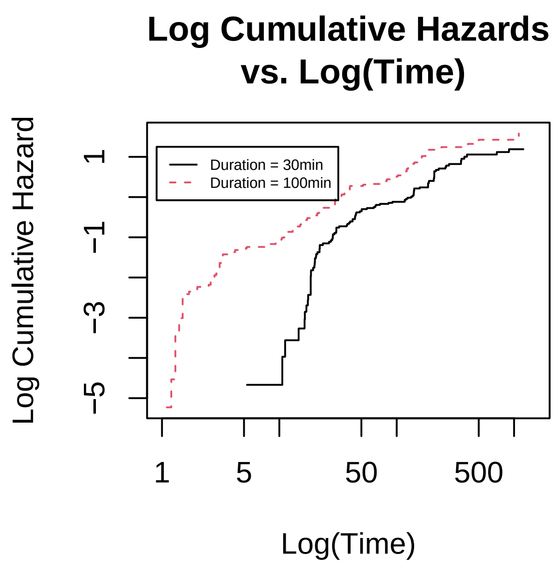
Log Total Catch of Fish = 8.69
Tow Duration = 30min
Log Total Catch of Fish = 8.69
Tow Duration = 100min
Log Total Catch of Fish = 5.323
Tow Duration = 30min
Log Total Catch of Fish = 5.323
Tow Duration = 100min
Log Total Catch of Fish = 3.786
Tow Duration = 30min
Log Total Catch of Fish = 3.786
Tow Duration = 100min
Log Total Catch of Fish = 2.904
Tow Duration = 30min
Log Total Catch of Fish = 2.904
Tow Duration = 100min
log Hazard Ratio
Max−Min Depth (m)
0
10
20
30
40
50
60
5
0
−5

# Diagnostics

In this section, we investigate several diagnostic plots to ensure the adequacy of our stratified Cox PH regression model in fitting the data.

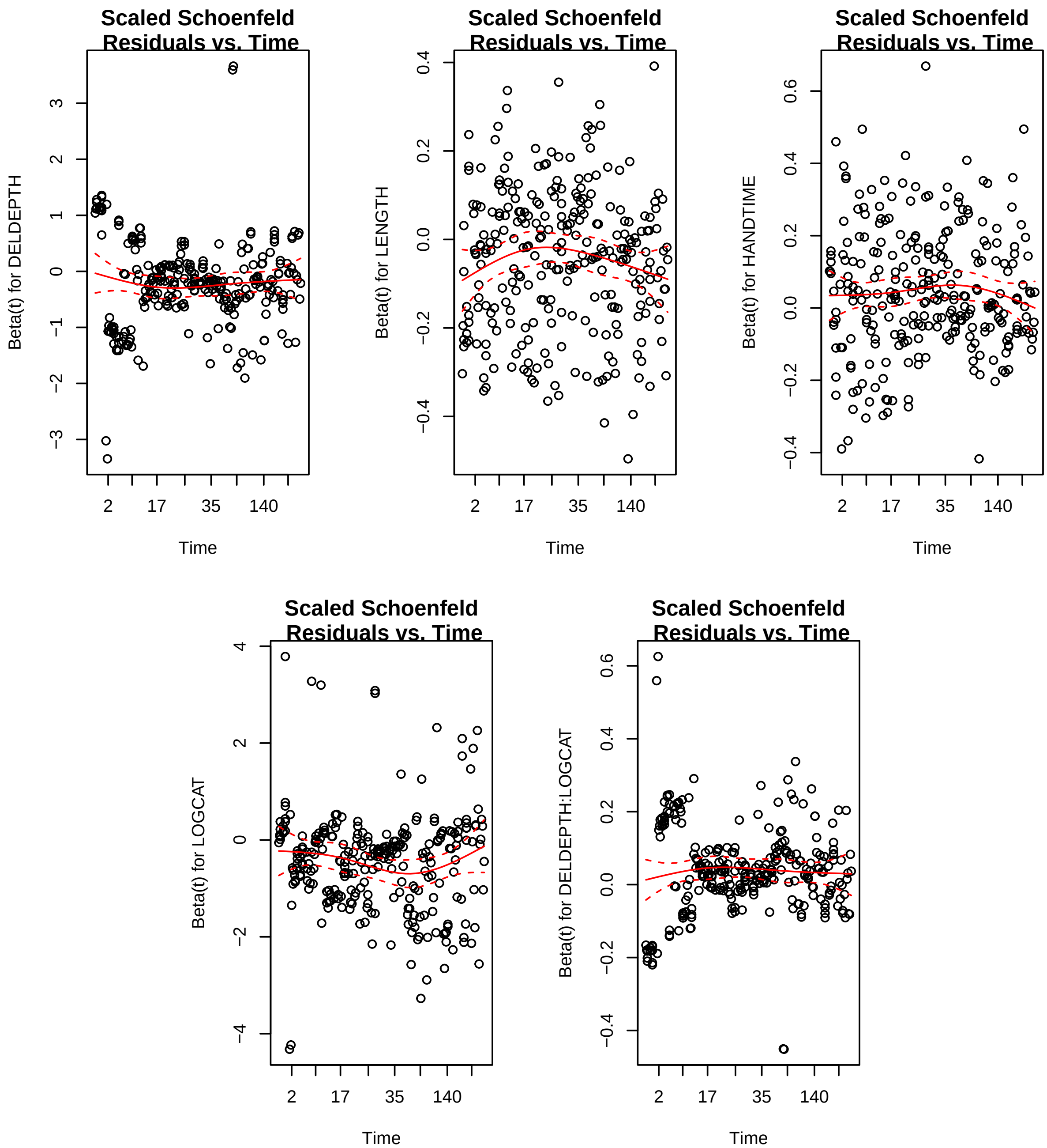
**Note**: While a comprehensive diagnostic process is omitted for brevity in this report, it is important to note that all diagnostics were thoroughly reviewed and confirmed.




Initially, we evaluate the fit of our model by inspecting the plot of estimated cumulative hazard rates against Cox-Snell residuals. The nearly straight line observed in the plot suggests a robust model fit.

Furthermore, given the challenges encountered with the Proportional Hazards (PH) assumption during model selection, we employ additional diagnostic checks. A plot depicting log cumulative hazards versus log time is included above. If the PH assumption is satisfied, the lines in this plot should run parallel. Indeed, our observation indicates that both lines maintain a fairly parallel trajectory.




To perform a more in-depth examination of the Proportional Hazards (PH) assumption, we generate plots of the scaled Schoenfeld residuals against time for each covariate. Notably, the residuals display a relatively random scatter. Moreover, across all covariates, there is a noticeable absence of discernible upward or downward trends in the Schoenfeld residuals over time. This observation suggests that the hazard ratios between groups remain roughly constant over time. Furthermore, the Schoenfeld residuals consistently hover around the zero line without displaying persistent departures. This pattern suggests that the proportional hazards assumption holds, signifying that the effect of covariates on the hazard remains constant over time.

## Conclusions

In conclusion, our analysis of Atlantic halibut survival dynamics has provided critical insights into the influence of tow duration and various covariates, effectively addressing the key questions posed in our study. The identified statistically significant difference in median survival times between the 30-minute and 100-minute tow durations underscores the importance of considering the duration of fishing activities in fisheries management. Notably, our stratified Cox proportional hazards model has brought to light the nuanced interplay of covariates on survival rates. It reveals that increased differences in tow depth (`DELDEPTH`) contribute to a reduced hazard, while longer fish lengths (`LENGTH`) marginally decrease the hazard. However, the interpretation of hazard ratios for individual variables, such as `DELDEPTH` and `LOGCAT`, requires caution due to the complicating influence of the introduced interaction term. Nevertheless, the observed association between longer handling times (`HANDTIME`) and a higher natural logarithm of total catch of fish in tow (`LOGCAT`) with survival rates adds valuable dimensions to our understanding. The interaction between `DELDEPTH` and `LOGCAT` suggests a modest increase in hazard, highlighting the complexity introduced by this interaction.

These findings emphasize the multifaceted nature of factors influencing halibut survival, underlining the need for tailored management strategies that account for both tow duration and individual covariate effects. Moreover, extending this research could involve investigating the broader ecological implications of these findings and exploring adaptive management strategies that consider the complex interactions identified in our model. Ultimately, our study lays the groundwork for a more holistic approach to fisheries management, promoting sustainable practices that safeguard the survival of Atlantic halibut populations in the face of diverse environmental and human-related pressures.