

Homework 2

(Math/CS 471)

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9/13/2020

1 Introduction

This report shows the convergence rate for Newton's Method across three different functions. This report will outline the basic method used, the raw results, and conclusions drawn.

2 Method

The method used to calculate these results was Newton's Method. This is an iterative method that will approximate the root of any given function. Newtons method is:

$$f(x_{n+1}) = x_n - \frac{f(x_n)}{f'(x_n)} \quad (1)$$

The three functions used to test Newton's method are listed below.

$$f(x) = x, \quad (2)$$

$$f(x) = x^2, \quad (3)$$

$$f(x) = \sin(x) + \cos(x^2). \quad (4)$$

The results are presented in Section 3, Results. These results use the raw output generated by running Newton's method, and performing calculations to generate the corresponding error and convergence ratios. The stopping criteria for the method used is an error tolerance of 10^{-15} . All calculations were computed using of Fortran code and Perl script.

3 Results

Below is a table showing the results of applying Newton's method to the three functions.

Table 1: Convergence of Newton's Method for 3 different functions

x	$(E_{abs})_{n+1}$	$\frac{(E_{abs})_{n+1}}{(E_{abs})_n}$	$\frac{(E_{abs})_{n+1}}{(E_{abs})_n^2}$
1	0	1.8636171971232487E-316	1.8636141339162444E-316
2	0	0	0

f(x) Iteration	$(E_{abs})_{n+1}$	$\frac{(E_{abs})_{n+1}}{(E_{abs})_n}$	$\frac{(E_{abs})_{n+1}}{(E_{abs})_n^2}$
x*x			
1	0	1.9532667919449512E-316	1.9532637287379470E-316
2	0.125	0.5	2
3	6.25E-02	0.5	4
4	3.13E-02	0.5	8
5	1.56E-02	0.5	16
6	7.81E-03	0.5	32
7	3.91E-03	0.5	64
8	1.95E-03	0.5	128
9	9.77E-04	0.5	256
10	4.88E-04	0.5	512
11	2.44E-04	0.5	1024
12	1.22E-04	0.5	2048
13	6.10E-05	0.5	4096
14	3.05E-05	0.5	8192
15	1.53E-05	0.5	16384
16	7.63E-06	0.5	32768
17	3.81E-06	0.5	65536
18	1.91E-06	0.5	131072
19	9.54E-07	0.5	262144
20	4.77E-07	0.5	524288
21	2.38E-07	0.5	1048576
22	1.19E-07	0.5	2097152
23	5.96E-08	0.5	4194304
24	2.98E-08	0.5	8388608
25	1.49E-08	0.5	16777216
26	7.45E-09	0.5	33554432
27	3.73E-09	0.5	67108864
28	1.86E-09	0.5	134217728
29	9.31E-10	0.5	268435456
30	4.66E-10	0.5	536870912
31	2.33E-10	0.5	1073741824

32	1.16E-10	0.5	2147483648
33	5.82E-11	0.5	4294967296
34	2.91E-11	0.5	8589934592
35	1.46E-11	0.5	17179869184
36	7.28E-12	0.5	34359738368
37	3.64E-12	0.5	68719476736
38	1.82E-12	0.5	1.37439E+11
39	9.09E-13	0.5	2.74878E+11
40	4.55E-13	0.5	5.49756E+11
41	2.27E-13	0.5	1.09951E+12
42	1.14E-13	0.5	2.19902E+12
43	5.68E-14	0.5	4.39805E+12
44	2.84E-14	0.5	8.79609E+12
45	1.42E-14	0.5	1.75922E+13
46	7.11E-15	0.5	3.51844E+13
47	3.55E-15	0.5	7.03687E+13
48	1.78E-15	0.5	1.40737E+14
49	8.88E-16	0.5	2.81475E+14
50	4.44E-16	0.5	5.6295E+14
51	2.22E-16	0.5	1.1259E+15
52	1.11E-16	0.5	2.2518E+15
53	5.55E-17	0.5	4.5036E+15
54	2.78E-17	0.5	9.0072E+15
55	1.39E-17	0.5	1.80144E+16
56	6.94E-18	0.5	3.60288E+16
57	3.47E-18	0.5	7.20576E+16
58	1.73E-18	0.5	1.44E+17
59	8.67E-19	0.5	2.88E+17
60	4.34E-19	0.5	5.76E+17
61	2.17E-19	0.5	1.15E+18
62	1.08E-19	0.5	2.31E+18
63	5.42E-20	0.5	4.61E+18
64	2.71E-20	0.5	9.22E+18
65	1.36E-20	0.5	1.84E+19
66	6.78E-21	0.5	3.69E+19
67	3.39E-21	0.5	7.38E+19
68	1.69E-21	0.5	1.48E+20
69	8.47E-22	0.5	2.95E+20

70	4.24E-22	0.5	5.90E+20
71	2.12E-22	0.5	1.18E+21
72	1.06E-22	0.5	2.36E+21
73	5.29E-23	0.5	4.72E+21
74	2.65E-23	0.5	9.44E+21
75	1.32E-23	0.5	1.89E+22
76	6.62E-24	0.5	3.78E+22
77	3.31E-24	0.5	7.56E+22
78	1.65E-24	0.5	1.51E+23
79	8.27E-25	0.5	3.02E+23
80	4.14E-25	0.5	6.04E+23
81	2.07E-25	0.5	1.21E+24
82	1.03E-25	0.5	2.42E+24
83	5.17E-26	0.5	4.84E+24
84	2.58E-26	0.5	9.67E+24
85	1.29E-26	0.5	1.93E+25
86	6.46E-27	0.5	3.87E+25
87	3.23E-27	0.5	7.74E+25
88	1.62E-27	0.5	1.55E+26
89	8.08E-28	0.5	3.09E+26
90	4.04E-28	0.5	6.19E+26
91	2.02E-28	0.5	1.24E+27
92	1.01E-28	0.5	2.48E+27
93	5.05E-29	0.5	4.95E+27
94	2.52E-29	0.5	9.90E+27
95	1.26E-29	0.5	1.98E+28
96	6.31E-30	0.5	3.96E+28
97	3.16E-30	0.5	7.92E+28
98	1.58E-30	0.5	1.58E+29
99	7.89E-31	0.5	3.17E+29
100	3.94E-31	0.5	6.34E+29
101	1.97E-31	0.5	1.27E+30
102	9.86E-32	0.5	2.54E+30
103	4.93E-32	0.5	5.07E+30
104	2.47E-32	0.5	1.01E+31
105	1.23E-32	0.5	2.03E+31
106	6.16E-33	0.5	4.06E+31
107	3.08E-33	0.5	8.11E+31

108	1.54E-33	0.5	1.62E+32
109	7.70E-34	0.5	3.25E+32
110	3.85E-34	0.5	6.49E+32
111	1.93E-34	0.5	1.30E+33
112	9.63E-35	0.5	2.60E+33
113	4.81E-35	0.5	5.19E+33
114	2.41E-35	0.5	1.04E+34
115	1.20E-35	0.5	2.08E+34
116	6.02E-36	0.5	4.15E+34
117	3.01E-36	0.5	8.31E+34
118	1.50E-36	0.5	1.66E+35
119	7.52E-37	0.5	3.32E+35
120	3.76E-37	0.5	6.65E+35
121	1.88E-37	0.5	1.33E+36
122	9.40E-38	0.5	2.66E+36
123	4.70E-38	0.5	5.32E+36
124	2.35E-38	0.5	1.06E+37
125	1.18E-38	0.5	2.13E+37
126	5.88E-39	0.5	4.25E+37
127	2.94E-39	0.5	8.51E+37
128	1.47E-39	0.5	1.70E+38
129	7.35E-40	0.5	3.40E+38
130	3.67E-40	0.5	6.81E+38
131	1.84E-40	0.5	1.36E+39
132	9.18E-41	0.5	2.72E+39
133	4.59E-41	0.5	5.44E+39
134	2.30E-41	0.5	1.09E+40
135	1.15E-41	0.5	2.18E+40
136	5.74E-42	0.5	4.36E+40
137	2.87E-42	0.5	8.71E+40
138	1.43E-42	0.5	1.74E+41
139	7.17E-43	0.5	3.48E+41
140	3.59E-43	0.5	6.97E+41
141	1.79E-43	0.5	1.39E+42
142	8.97E-44	0.5	2.79E+42
143	4.48E-44	0.5	5.58E+42
144	2.24E-44	0.5	1.12E+43
145	1.12E-44	0.5	2.23E+43

146	5.61E-45	0.5	4.46E+43
147	2.80E-45	0.5	8.92E+43
148	1.40E-45	0.5	1.78E+44
149	7.01E-46	0.5	3.57E+44
150	3.50E-46	0.5	7.14E+44
151	1.75E-46	0.5	1.43E+45
152	8.76E-47	0.5	2.85E+45
153	4.38E-47	0.5	5.71E+45
154	2.19E-47	0.5	1.14E+46
155	1.09E-47	0.5	2.28E+46
156	5.47E-48	0.5	4.57E+46
157	2.74E-48	0.5	9.13E+46
158	1.37E-48	0.5	1.83E+47
159	6.84E-49	0.5	3.65E+47
160	3.42E-49	0.5	7.31E+47
161	1.71E-49	0.5	1.46E+48
162	8.55E-50	0.5	2.92E+48
163	4.28E-50	0.5	5.85E+48
164	2.14E-50	0.5	1.17E+49
165	1.07E-50	0.5	2.34E+49
166	5.35E-51	0.5	4.68E+49
167	2.67E-51	0.5	9.35E+49
168	1.34E-51	0.5	1.87E+50
169	6.68E-52	0.5	3.74E+50
170	3.34E-52	0.5	7.48E+50
171	1.67E-52	0.5	1.50E+51
172	8.35E-53	0.5	2.99E+51
173	4.18E-53	0.5	5.99E+51
174	2.09E-53	0.5	1.20E+52
175	1.04E-53	0.5	2.39E+52
176	5.22E-54	0.5	4.79E+52
177	2.61E-54	0.5	9.58E+52
178	1.31E-54	0.5	1.92E+53
179	6.53E-55	0.5	3.83E+53
180	3.26E-55	0.5	7.66E+53
181	1.63E-55	0.5	1.53E+54
182	8.16E-56	0.5	3.06E+54
183	4.08E-56	0.5	6.13E+54

184	2.04E-56	0.5	1.23E+55
185	1.02E-56	0.5	2.45E+55
186	5.10E-57	0.5	4.90E+55
187	2.55E-57	0.5	9.81E+55
188	1.27E-57	0.5	1.96E+56
189	6.37E-58	0.5	3.92E+56
190	3.19E-58	0.5	7.85E+56
191	1.59E-58	0.5	1.57E+57
192	7.97E-59	0.5	3.14E+57
193	3.98E-59	0.5	6.28E+57
194	1.99E-59	0.5	1.26E+58
195	9.96E-60	0.5	2.51E+58
196	4.98E-60	0.5	5.02E+58
197	2.49E-60	0.5	1.00E+59
198	1.24E-60	0.5	2.01E+59
199	6.22E-61	0.5	4.02E+59
200	3.11E-61	0.5	8.03E+59
201	1.56E-61	0.5	1.61E+60
202	7.78E-62	0.5	3.21E+60
203	3.89E-62	0.5	6.43E+60
204	1.94E-62	0.5	1.29E+61
205	9.72E-63	0.5	2.57E+61
206	4.86E-63	0.5	5.14E+61
207	2.43E-63	0.5	1.03E+62
208	1.22E-63	0.5	2.06E+62
209	6.08E-64	0.5	4.11E+62
210	3.04E-64	0.5	8.23E+62
211	1.52E-64	0.5	1.65E+63
212	7.60E-65	0.5	3.29E+63
213	3.80E-65	0.5	6.58E+63
214	1.90E-65	0.5	1.32E+64
215	9.50E-66	0.5	2.63E+64
216	4.75E-66	0.5	5.27E+64
217	2.37E-66	0.5	1.05E+65
218	1.19E-66	0.5	2.11E+65
219	5.93E-67	0.5	4.21E+65
220	2.97E-67	0.5	8.42E+65
221	1.48E-67	0.5	1.68E+66

222	7.42E-68	0.5	3.37E+66
223	3.71E-68	0.5	6.74E+66
224	1.85E-68	0.5	1.35E+67
225	9.27E-69	0.5	2.70E+67
226	4.64E-69	0.5	5.39E+67
227	2.32E-69	0.5	1.08E+68
228	1.16E-69	0.5	2.16E+68
229	5.80E-70	0.5	4.31E+68
230	2.90E-70	0.5	8.63E+68
231	1.45E-70	0.5	1.73E+69
232	7.24E-71	0.5	3.45E+69
233	3.62E-71	0.5	6.90E+69
234	1.81E-71	0.5	1.38E+70
235	9.06E-72	0.5	2.76E+70
236	4.53E-72	0.5	5.52E+70
237	2.26E-72	0.5	1.10E+71
238	1.13E-72	0.5	2.21E+71
239	5.66E-73	0.5	4.42E+71
240	2.83E-73	0.5	8.83E+71
241	1.41E-73	0.5	1.77E+72
242	7.07E-74	0.5	3.53E+72
243	3.54E-74	0.5	7.07E+72
244	1.77E-74	0.5	1.41E+73
245	8.84E-75	0.5	2.83E+73
246	4.42E-75	0.5	5.65E+73
247	2.21E-75	0.5	1.13E+74
248	1.11E-75	0.5	2.26E+74
249	5.53E-76	0.5	4.52E+74
250	2.76E-76	0.5	9.05E+74
251	1.38E-76	0.5	1.81E+75
252	6.91E-77	0.5	3.62E+75
253	3.45E-77	0.5	7.24E+75
254	1.73E-77	0.5	1.45E+76
255	8.64E-78	0.5	2.89E+76
256	4.32E-78	0.5	5.79E+76
257	2.16E-78	0.5	1.16E+77
258	1.08E-78	0.5	2.32E+77
259	5.40E-79	0.5	4.63E+77

260	2.70E-79	0.5	9.26E+77
261	1.35E-79	0.5	1.85E+78
262	6.75E-80	0.5	3.71E+78
263	3.37E-80	0.5	7.41E+78
264	1.69E-80	0.5	1.48E+79
265	8.43E-81	0.5	2.96E+79
266	4.22E-81	0.5	5.93E+79
267	2.11E-81	0.5	1.19E+80
268	1.05E-81	0.5	2.37E+80
269	5.27E-82	0.5	4.74E+80
270	2.64E-82	0.5	9.49E+80
271	1.32E-82	0.5	1.90E+81
272	6.59E-83	0.5	3.79E+81
273	3.29E-83	0.5	7.59E+81
274	1.65E-83	0.5	1.52E+82
275	8.24E-84	0.5	3.04E+82
276	4.12E-84	0.5	6.07E+82
277	2.06E-84	0.5	1.21E+83
278	1.03E-84	0.5	2.43E+83
279	5.15E-85	0.5	4.86E+83
280	2.57E-85	0.5	9.71E+83
281	1.29E-85	0.5	1.94E+84
282	6.43E-86	0.5	3.89E+84
283	3.22E-86	0.5	7.77E+84
284	1.61E-86	0.5	1.55E+85
285	8.04E-87	0.5	3.11E+85
286	4.02E-87	0.5	6.22E+85
287	2.01E-87	0.5	1.24E+86
288	1.01E-87	0.5	2.49E+86
289	5.03E-88	0.5	4.97E+86
290	2.51E-88	0.5	9.95E+86
291	1.26E-88	0.5	1.99E+87
292	6.28E-89	0.5	3.98E+87
293	3.14E-89	0.5	7.96E+87
294	1.57E-89	0.5	1.59E+88
295	7.85E-90	0.5	3.18E+88
296	3.93E-90	0.5	6.37E+88
297	1.96E-90	0.5	1.27E+89

298	9.82E-91	0.5	2.55E+89
299	4.91E-91	0.5	5.09E+89
300	2.45E-91	0.5	1.02E+90
301	1.23E-91	0.5	2.04E+90
302	6.14E-92	0.5	4.07E+90
303	3.07E-92	0.5	8.15E+90
304	1.53E-92	0.5	1.63E+91
305	7.67E-93	0.5	3.26E+91
306	3.84E-93	0.5	6.52E+91
307	1.92E-93	0.5	1.30E+92
308	9.59E-94	0.5	2.61E+92
309	4.79E-94	0.5	5.21E+92
310	2.40E-94	0.5	1.04E+93
311	1.20E-94	0.5	2.09E+93
312	5.99E-95	0.5	4.17E+93
313	3.00E-95	0.5	8.34E+93
314	1.50E-95	0.5	1.67E+94
315	7.49E-96	0.5	3.34E+94
316	3.75E-96	0.5	6.67E+94
317	1.87E-96	0.5	1.33E+95
318	9.36E-97	0.5	2.67E+95
319	4.68E-97	0.5	5.34E+95
320	2.34E-97	0.5	1.07E+96
321	1.17E-97	0.5	2.14E+96
322	5.85E-98	0.5	4.27E+96
323	2.93E-98	0.5	8.54E+96
324	1.46E-98	0.5	1.71E+97
325	7.32E-99	0.5	3.42E+97
326	3.66E-99	0.5	6.84E+97
327	1.83E-99	0.5	1.37E+98
328	9.14E-100	0.5	2.73E+98
329	4.57E-100	0.5	5.47E+98
330	2.29E-100	0.5	1.09E+99
331	1.14E-100	0.5	2.19E+99
332	5.71E-101	0.5	4.37E+99
333	2.86E-101	0.5	8.75E+99
334	1.43E-101	0.5	1.75E+100
335	7.14E-102	0.5	3.50E+100

336	3.57E-102	0.5	7.00E+100
337	1.79E-102	0.5	1.40E+101
338	8.93E-103	0.5	2.80E+101
339	4.46E-103	0.5	5.60E+101
340	2.23E-103	0.5	1.12E+102
341	1.12E-103	0.5	2.24E+102
342	5.58E-104	0.5	4.48E+102
343	2.79E-104	0.5	8.96E+102
344	1.40E-104	0.5	1.79E+103
345	6.98E-105	0.5	3.58E+103
346	3.49E-105	0.5	7.17E+103
347	1.74E-105	0.5	1.43E+104
348	8.72E-106	0.5	2.87E+104
349	4.36E-106	0.5	5.73E+104
350	2.18E-106	0.5	1.15E+105
351	1.09E-106	0.5	2.29E+105
352	5.45E-107	0.5	4.59E+105
353	2.73E-107	0.5	9.17E+105
354	1.36E-107	0.5	1.83E+106
355	6.81E-108	0.5	3.67E+106
356	3.41E-108	0.5	7.34E+106
357	1.70E-108	0.5	1.47E+107
358	8.52E-109	0.5	2.94E+107
359	4.26E-109	0.5	5.87E+107
360	2.13E-109	0.5	1.17E+108
361	1.06E-109	0.5	2.35E+108
362	5.32E-110	0.5	4.70E+108
363	2.66E-110	0.5	9.39E+108
364	1.33E-110	0.5	1.88E+109
365	6.65E-111	0.5	3.76E+109
366	3.33E-111	0.5	7.52E+109
367	1.66E-111	0.5	1.50E+110
368	8.32E-112	0.5	3.01E+110
369	4.16E-112	0.5	6.01E+110
370	2.08E-112	0.5	1.20E+111
371	1.04E-112	0.5	2.40E+111
372	5.20E-113	0.5	4.81E+111
373	2.60E-113	0.5	9.62E+111

374	1.30E-113	0.5	1.92E+112
375	6.50E-114	0.5	3.85E+112
376	3.25E-114	0.5	7.70E+112
377	1.62E-114	0.5	1.54E+113
378	8.12E-115	0.5	3.08E+113
379	4.06E-115	0.5	6.16E+113
380	2.03E-115	0.5	1.23E+114
381	1.02E-115	0.5	2.46E+114
382	5.08E-116	0.5	4.93E+114
383	2.54E-116	0.5	9.85E+114
384	1.27E-116	0.5	1.97E+115
385	6.34E-117	0.5	3.94E+115
386	3.17E-117	0.5	7.88E+115
387	1.59E-117	0.5	1.58E+116
388	7.93E-118	0.5	3.15E+116
389	3.97E-118	0.5	6.30E+116
390	1.98E-118	0.5	1.26E+117
391	9.91E-119	0.5	2.52E+117
392	4.96E-119	0.5	5.04E+117
393	2.48E-119	0.5	1.01E+118
394	1.24E-119	0.5	2.02E+118
395	6.20E-120	0.5	4.03E+118
396	3.10E-120	0.5	8.07E+118
397	1.55E-120	0.5	1.61E+119
398	7.75E-121	0.5	3.23E+119
399	3.87E-121	0.5	6.46E+119
400	1.94E-121	0.5	1.29E+120
401	9.68E-122	0.5	2.58E+120
402	4.84E-122	0.5	5.16E+120
403	2.42E-122	0.5	1.03E+121
404	1.21E-122	0.5	2.07E+121
405	6.05E-123	0.5	4.13E+121
406	3.03E-123	0.5	8.26E+121
407	1.51E-123	0.5	1.65E+122
408	7.56E-124	0.5	3.31E+122
409	3.78E-124	0.5	6.61E+122
410	1.89E-124	0.5	1.32E+123
411	9.45E-125	0.5	2.64E+123

412	4.73E-125	0.5	5.29E+123
413	2.36E-125	0.5	1.06E+124
414	1.18E-125	0.5	2.12E+124
415	5.91E-126	0.5	4.23E+124
416	2.95E-126	0.5	8.46E+124
417	1.48E-126	0.5	1.69E+125
418	7.39E-127	0.5	3.38E+125
419	3.69E-127	0.5	6.77E+125
420	1.85E-127	0.5	1.35E+126
421	9.23E-128	0.5	2.71E+126
422	4.62E-128	0.5	5.42E+126
423	2.31E-128	0.5	1.08E+127
424	1.15E-128	0.5	2.17E+127
425	5.77E-129	0.5	4.33E+127
426	2.89E-129	0.5	8.66E+127
427	1.44E-129	0.5	1.73E+128
428	7.21E-130	0.5	3.47E+128
429	3.61E-130	0.5	6.93E+128
430	1.80E-130	0.5	1.39E+129
431	9.02E-131	0.5	2.77E+129
432	4.51E-131	0.5	5.55E+129
433	2.25E-131	0.5	1.11E+130
434	1.13E-131	0.5	2.22E+130
435	5.64E-132	0.5	4.44E+130
436	2.82E-132	0.5	8.87E+130
437	1.41E-132	0.5	1.77E+131
438	7.04E-133	0.5	3.55E+131
439	3.52E-133	0.5	7.10E+131
440	1.76E-133	0.5	1.42E+132
441	8.81E-134	0.5	2.84E+132
442	4.40E-134	0.5	5.68E+132
443	2.20E-134	0.5	1.14E+133
444	1.10E-134	0.5	2.27E+133
445	5.50E-135	0.5	4.54E+133
446	2.75E-135	0.5	9.09E+133
447	1.38E-135	0.5	1.82E+134
448	6.88E-136	0.5	3.63E+134
449	3.44E-136	0.5	7.27E+134

450	1.72E-136	0.5	1.45E+135
451	8.60E-137	0.5	2.91E+135
452	4.30E-137	0.5	5.81E+135
453	2.15E-137	0.5	1.16E+136
454	1.07E-137	0.5	2.33E+136
455	5.37E-138	0.5	4.65E+136
456	2.69E-138	0.5	9.30E+136
457	1.34E-138	0.5	1.86E+137
458	6.72E-139	0.5	3.72E+137
459	3.36E-139	0.5	7.44E+137
460	1.68E-139	0.5	1.49E+138
461	8.40E-140	0.5	2.98E+138
462	4.20E-140	0.5	5.95E+138
463	2.10E-140	0.5	1.19E+139
464	1.05E-140	0.5	2.38E+139
465	5.25E-141	0.5	4.76E+139
466	2.62E-141	0.5	9.53E+139
467	1.31E-141	0.5	1.91E+140
468	6.56E-142	0.5	3.81E+140
469	3.28E-142	0.5	7.62E+140
470	1.64E-142	0.5	1.52E+141
471	8.20E-143	0.5	3.05E+141
472	4.10E-143	0.5	6.10E+141
473	2.05E-143	0.5	1.22E+142
474	1.03E-143	0.5	2.44E+142
475	5.13E-144	0.5	4.88E+142
476	2.56E-144	0.5	9.76E+142
477	1.28E-144	0.5	1.95E+143
478	6.41E-145	0.5	3.90E+143
479	3.20E-145	0.5	7.80E+143
480	1.60E-145	0.5	1.56E+144
481	8.01E-146	0.5	3.12E+144
482	4.00E-146	0.5	6.24E+144
483	2.00E-146	0.5	1.25E+145
484	1.00E-146	0.5	2.50E+145
485	5.01E-147	0.5	4.99E+145
486	2.50E-147	0.5	9.99E+145
487	1.25E-147	0.5	2.00E+146

488	6.26E-148	0.5	4.00E+146
489	3.13E-148	0.5	7.99E+146
490	1.56E-148	0.5	1.60E+147
491	7.82E-149	0.5	3.20E+147
492	3.91E-149	0.5	6.39E+147
493	1.96E-149	0.5	1.28E+148
494	9.78E-150	0.5	2.56E+148
495	4.89E-150	0.5	5.11E+148
496	2.44E-150	0.5	1.02E+149
497	1.22E-150	0.5	2.05E+149
498	6.11E-151	0.5	4.09E+149
499	3.05E-151	0.5	8.18E+149
500	1.53E-151	0.5	1.64E+150
501	7.64E-152	0.5	3.27E+150
502	3.82E-152	0.5	6.55E+150
503	1.91E-152	0.5	1.31E+151
504	9.55E-153	0.5	2.62E+151
505	4.77E-153	0.5	5.24E+151
506	2.39E-153	0.5	1.05E+152
507	1.19E-153	0.5	2.09E+152
508	5.97E-154	0.5	4.19E+152
509	2.98E-154	0.5	8.38E+152
510	1.49E-154	0.5	1.68E+153
511	7.46E-155	0.5	3.35E+153
512	3.73E-155	0.5	6.70E+153
513	1.86E-155	0.5	1.34E+154
514	9.32E-156	0.5	2.68E+154
515	4.66E-156	0.5	5.36E+154
516	2.33E-156	0.5	1.07E+155
517	1.17E-156	0.5	2.15E+155
518	5.83E-157	0.5	4.29E+155
519	2.91E-157	0.5	8.58E+155
520	1.46E-157	0.5	1.72E+156
521	7.28E-158	0.5	3.43E+156
522	3.64E-158	0.5	6.86E+156
523	1.82E-158	0.5	1.37E+157
524	9.10E-159	0.5	2.75E+157
525	4.55E-159	0.5	5.49E+157

526	2.28E-159	0.5	1.10E+158
527	1.14E-159	0.5	2.20E+158
528	5.69E-160	0.5	4.39E+158
529	2.85E-160	0.5	8.79E+158
530	1.42E-160	0.5	1.76E+159
531	7.11E-161	0.5	3.51E+159
532	3.56E-161	0.5	7.03E+159
533	1.78E-161	0.5	1.41E+160
534	8.89E-162	0.5	2.81E+160
535	4.45E-162	0.5	5.62E+160
536	2.22E-162	0.5	1.12E+161
537	1.11E-162	0.5	2.25E+161
538	0	0	NaN
f(x) Iteration	$(E_{abs})_{n+1}$	$\frac{(E_{abs})_{n+1}}{(E_{abs})_n}$	$\frac{(E_{abs})_{n+1}}{(E_{abs})_n^2}$
sin(x)+cos(x*x)			
1	0	1.4113218372440045E-316	1.4113187740370002E-316
2	8.05E-02	0.184928076	0.425019751
3	5.25E-03	6.53E-02	0.811123336
4	2.13E-05	4.05E-03	0.771382573
5	3.47E-10	1.63E-05	0.767750705
6	1.11E-16	3.20E-07	919.7077587
7	0	0	0

4 Conclusion

When Newton's method (1) is applied to the function (2) we see rapid convergence to the zero. Because this converged after a single iteration, we cannot classify the convergence rate to be linear or quadratic when applied to (2). When applying (1) to (3) we see a clear linear convergence of Newton's method. This is because the linear convergence ratio $\frac{(E_{abs})_{n+1}}{(E_{abs})_n}$ converges to a constant (in this case 0.5). This shows us that the next error depends linearly

on the current error. Newton's method has the ability to converge quadratically, so it is evident that the method is performing less than adequately. This is because the true root of (3) is at $x = 0$. Since $f'(0) = 0$, we have an issue with Newton's method because it contains the term $\frac{f(x_n)}{f'(x_n)}$ which, when approaching the root, would be close to $\frac{0}{0}$. An alternative method that could be used to regain quadratic convergence would be modified Newton's method $f(x_{n+1}) = x_n - 2\frac{f(x_n)}{f'(x_n)}$. When applying (1) to (4) we can see that the convergence of Newton's method is quadratic. This is because the quadratic convergence ratio $\frac{(E_{abs})_{n+1}}{(E_{abs})_n^2}$ is closer to remaining constant compared to linear convergence ratio which we see rapidly decreasing, suggesting a quadratic and not linear relationship between the current error and next error.