ECE541 Project 4

elim aschenberg

November 2025

1 2D Grounded Coplanar Waveguide Model

Introduction:
For this problem, a PDF is provided to follow. The purpose of this problem is to get the student comfortable and familiarized with Ansys.

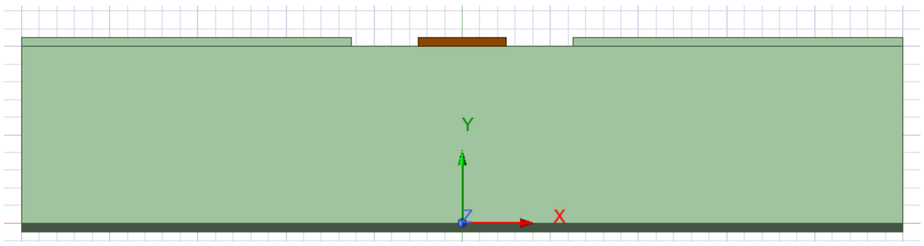

Figure 1: 2D Grounded Coplanar Waveguide Model

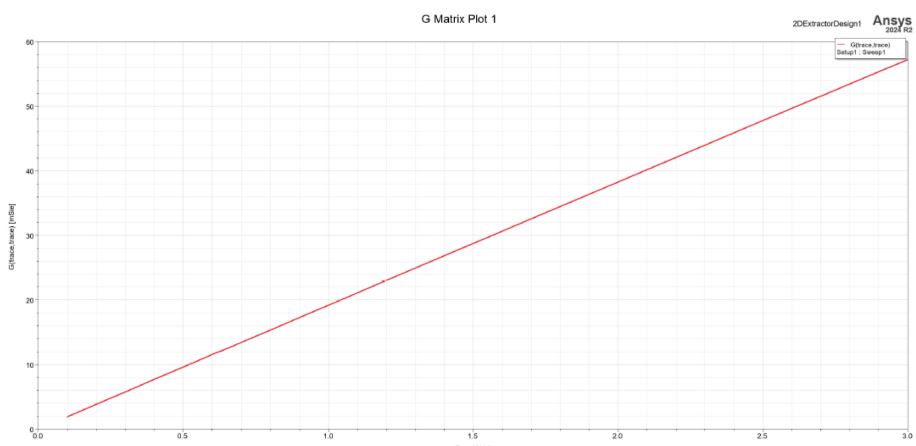

Figure 2: G matrix

G matrix comments:
The G matrix is linearly increasing with frequency. This indicates that as the frequency increases, so does the conductance. Furthermore, the conductance is proportional to the frequency.

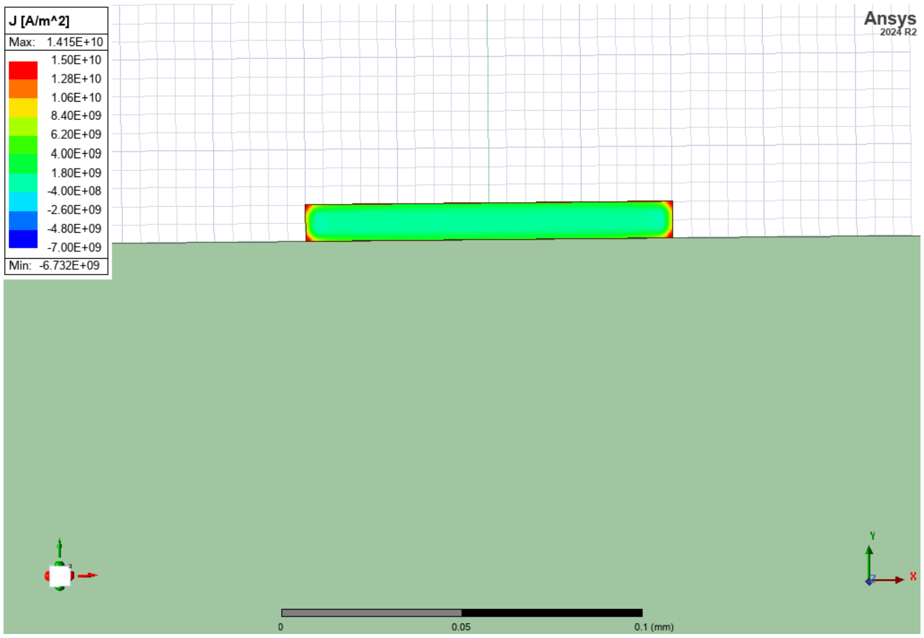

Figure 3: JrL field plot

JrL field comments:
The current density is most dense around the corners of the trace conductor. This makes sense because the electric field should be strongest at these points.

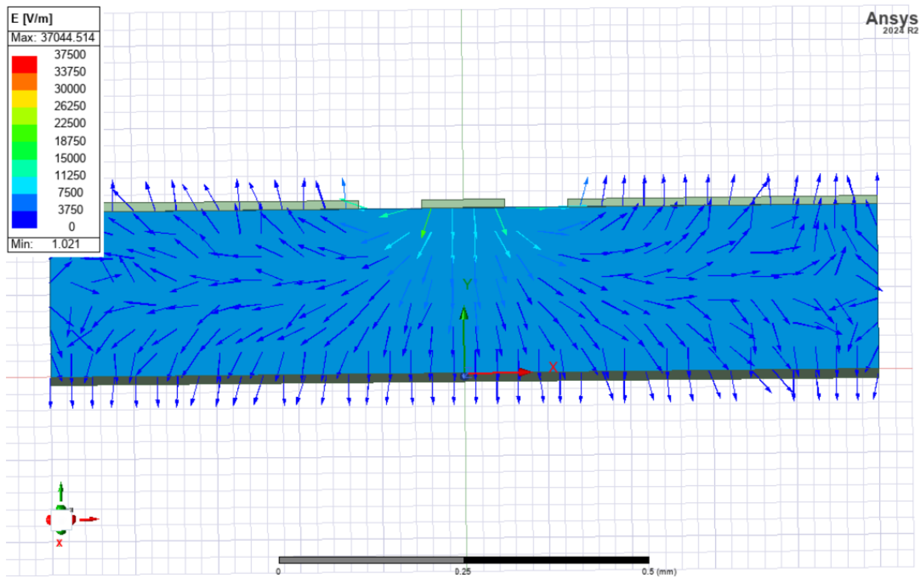

Figure 4: Electric field vector plot

Electric field vector plot comments: The electric fields radiate from the trance conductor as expected. They find their way to the reference ground as well as the floating ground. This is consistent with the theory that electric fields will always go to the lower potential, and since the ground has a potential of zero, it is the lowest.

2 Two wires above ground plane transmission line

Introduction:
This is the first problem of the project where the student is expected to calculate all the parameters of the transmission line. The two wires are above a ground plane that is significantly larger than the distance between the transmission lines, imitating an infinite ground.

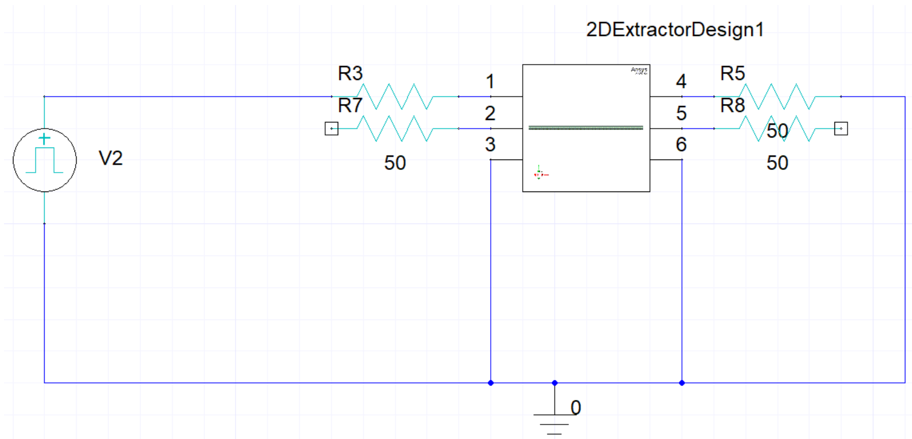

Figure 5: Two wires above ground plane transmission line

Comment on Figure 5: This is what the two wires above a ground plane transmission line look like in my Ansys modeler.

2.1 Compute [B'] and [L'] analytically, using potential coefficients for a system of wires above an infinite ground plane.

$$a_{ij} = \frac{1}{2\pi\epsilon} ln \frac{D}{d}$$

$$a_{ii} = \frac{1}{2\pi\epsilon} ln \frac{2h}{a}$$

$$\text{where } D = \sqrt{4h^2 + d^2}$$

This gives us a matrix of

$$a = \begin{bmatrix} 5.385 & 1.4465 \\ 1.4465 & 5.3850 \end{bmatrix} * 10^{10}$$

From this we can calculate the b matrix

$$[b] = [a]^{-1} = \begin{bmatrix} 5.385 & 1.4465 \\ 1.4465 & 5.3850 \end{bmatrix}^{-1} = \begin{bmatrix} 0.2001 & -0.0538 \\ -0.0538 & 0.2001 \end{bmatrix} * 10^{-10}$$

The inductance matrix can also be easily computed from the b matrix

$$[L'] = \epsilon\mu[b]^{-1} = \begin{bmatrix} 0.5991 & 0.1609 \\ 0.1609 & 0.5991 \end{bmatrix} \mu\text{H}$$

These are the ideal calculations for the b and L matrices. They appear to be reasonable.

2.2 Compute [B’] and [L’] at 0.1 GHz using Ansys 2D Extractor Design.

	Freq [MHz]	C(Conductor1,Conductor1) [pF] Setup1 : LastAdaptive	C(Conductor2,Conductor1) [pF] Setup1 : LastAdaptive	C(Conductor1,Conductor2) [pF] Setup1 : LastAdaptive	C(Conductor2,Conductor2) [pF] Setup1 : LastAdaptive
1	100.000000	20.060474	-5.404791	-5.404791	20.060296

Figure 6: Ansys B matrix

	Freq [MHz]	L(Conductor1,Conductor1) [nH] Setup1 : LastAdaptive	L(Conductor2,Conductor1) [nH] Setup1 : LastAdaptive	L(Conductor1,Conductor2) [nH] Setup1 : LastAdaptive	L(Conductor2,Conductor2) [nH] Setup1 : LastAdaptive
1	100.000000	598.837767	161.149314	161.149314	598.838501

Figure 7: Ansys L matrix

To put it into a more readable format,

$$[b] = \begin{bmatrix} 20.060474 & -5.404791 \\ -5.404791 & 20.060296 \end{bmatrix} * 10^{-12}$$

$$[L] = \begin{bmatrix} 598.837767 & 161.149314 \\ 161.149314 & 598.838501 \end{bmatrix} \text{nH}$$

These are the matrices for the b and L values from Ansys Electronic Solver. They are meant to be more accurate than the ideal calculations above.

2.3 Use the matrices, found by Ansys, as input to MATLAB to compute eigenvalues, eigenvectors, and the characteristic impedance matrix of the transmission line.

Starting with the equation

$$|\,[\tfrac{1}{c_m}[V] - [L'][b']]\,| = 0$$

we can simplify this expression to

$$|\begin{bmatrix} \frac{1}{c_m^2} - a_{11} & -a12 \\ -a_{21} & \frac{1}{c_m^2} - a_{22} \end{bmatrix}| = 0$$

where

$$[L'][b'] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 0.114 & 0 \\ 0 & 0.114 \end{bmatrix} * 10^{-16}$$

This makes our eigen equation look like

$$|\begin{bmatrix} \frac{1}{c_m^2} - 0.1114 \times 10^{-16} & 0 \\ 0 & \frac{1}{c_m^2} - 0.114 \times 10^{-16} \end{bmatrix}| = 0$$

We can solve this matrix to be

$$(\frac{1}{c_m^2})^2 - (a_{11}+a_{22})\frac{1}{c_m^2} + a_{11}a_{22} - a_{12}a_{21} = 0$$

which becomes

$$(\frac{1}{c_m^4}) - (2.2284\times 10^{-17})\frac{1}{c_m^2} + 2.2284\times 10^{-17} = 0$$

c_m comes out as having four solutions

$$c_m = \pm 2.9958\times 10^8, \pm 2.9959\times 10^8$$

The negative values can be disregarded since there cannot be negative speed. This gives our solution for the eigenvectors to be

$$\text{eigenvectors} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

We can find the characteristic impedance matrix by

$$[Z_0] = [S_V][S_I]^{-1}$$

where

$$[S_v] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} V_0$$

where $V_0 = \frac{1}{\sqrt{2}} = 0.707\text{v}$

$$[S_v] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \times 0.7071 = \begin{bmatrix} 0.7071 & 0.7071 \\ 0.7071 & -0.7071 \end{bmatrix} \text{v}$$

$$[S_I] = [L']^{-1}[S_v][\Lambda]$$

$$[\Lambda] = \begin{bmatrix} \frac{1}{c_1} & 0 \\ 0 & \frac{1}{c_2} \end{bmatrix} = \begin{bmatrix} 0.3338 & 0 \\ 0 & 0.3338 \end{bmatrix} \times 10^{-8}$$

Using these we can solve for $[S_I]$

$$[s_I] = \begin{bmatrix} 598.837767\text{ nH} & 161.149314\text{ nH} \\ 161.149314\text{ nH} & 598.838501\text{ nH} \end{bmatrix}^{-1} \begin{bmatrix} 0.7071v & 0.7071v \\ 0.7071v & -0.7071v \end{bmatrix} \begin{bmatrix} 0.3338\times 10^{-8} & 0 \\ 0 & 0.3338\times 10^{-8} \end{bmatrix}$$

$$= \begin{bmatrix} 0.0031 & 0.0054 \\ 0.0031 & -0.0054 \end{bmatrix}$$

$$[Z_0] = \begin{bmatrix} 0.7071 & 0.7071 \\ 0.7071 & -0.7071 \end{bmatrix} \begin{bmatrix} 0.0031 & 0.0054 \\ 0.0031 & -0.0054 \end{bmatrix}^{-1} = \begin{bmatrix} 179.4027 & 48.2781 \\ 48.2782 & 179.4029 \end{bmatrix} \Omega$$

3 Two coupled microstrip lines

Introduction:
This part of the project is similar to part 1. The difference is that there is no trace conductor and that there are two conducting transmission lines.

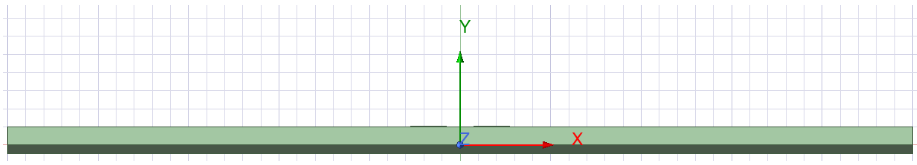

Figure 8: Two Coupled Microstrip Lines

	Freq [GHz]	C(Microstrip2,Microstrip2) [pF] Setup1 : LastAdaptive	C(Mircostrip1,Microstrip2) [pF] Setup1 : LastAdaptive	C(Microstrip2,Mircostrip1) [pF] Setup1 : LastAdaptive	C(Mircostrip1,Mircostrip1) [pF] Setup1 : LastAdaptive
1	1.000000	117.764573	-6.156154	-6.156154	118.022033

Figure 9: B' matrix

	Freq [GHz]	L(Microstrip2,Microstrip2) [nH] Setup1 : LastAdaptive	L(Mircostrip1,Microstrip2) [nH] Setup1 : LastAdaptive	L(Microstrip2,Mircostrip1) [nH] Setup1 : LastAdaptive	L(Mircostrip1,Mircostrip1) [nH] Setup1 : LastAdaptive
1	1.000000	292.681243	35.968671	35.968671	292.681725

Figure 10: L' matrix

	Freq [GHz]	Z0(Microstrip2,Microstrip2) Setup1 : LastAdaptive	Z0(Mircostrip1,Microstrip2) Setup1 : LastAdaptive	Z0(Microstrip2,Mircostrip1) Setup1 : LastAdaptive	Z0(Mircostrip1,Mircostrip1) Setup1 : LastAdaptive
1	1.000000	49.887347 + 0.128623i	4.370521 + 0.041602i	4.370521 + 0.041602i	49.833111 + 0.128639i

Figure 11: Z0' matrix

Comments: These are the screenshots of the matrices obtained from Ansys analysis.

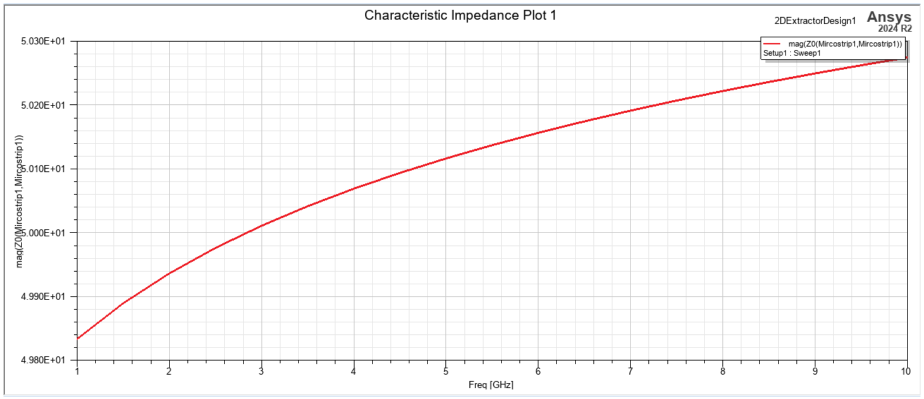

Figure 12: Z0'(1,1) over frequency sweep

Comment on Figure 12:
The impedance graph shows a decaying growth as frequency increases.

To make them more readable:

$$[B'] = \begin{bmatrix} 118.022033 & -6.156154 \\ -6.156154 & 117.764573 \end{bmatrix} \text{pF}$$

$$[L'] = \begin{bmatrix} 292.681725 & 35.968671 \\ 35.968671 & 292.681243 \end{bmatrix} \text{nH}$$

$$[Z_0] = \begin{bmatrix} 49.887347 + 0.128623\,\mathrm{i} & 4.370521 + 0.041602\,\mathrm{i} \\ 4.370521 + 0.041602\,\mathrm{i} & 49.833111 + 0.128639\,\mathrm{i} \end{bmatrix}$$

Using the exact same method as in problem 2, we can compute the eigenvalues and vectors to be

$$\text{eigenvalues} = \pm 1.7088 \times 10^8, \pm 1.7069 \times 10^8$$

$$\text{eigenvectors} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

4 Circuit Analysis of two coupled mincrostrip lines

Introduction:
Using the two coupled microstrip lines from the previous problem, we can create a circuit and perform analyses from it. Comment: This is for the third part of the problem, where we disconnect the second microstrip on both ends.

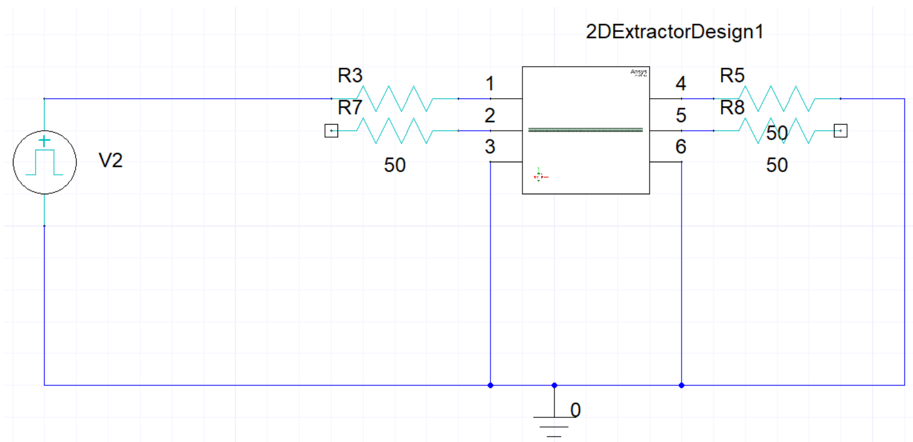

Figure 13: Circuit Design of two coupled microstrips

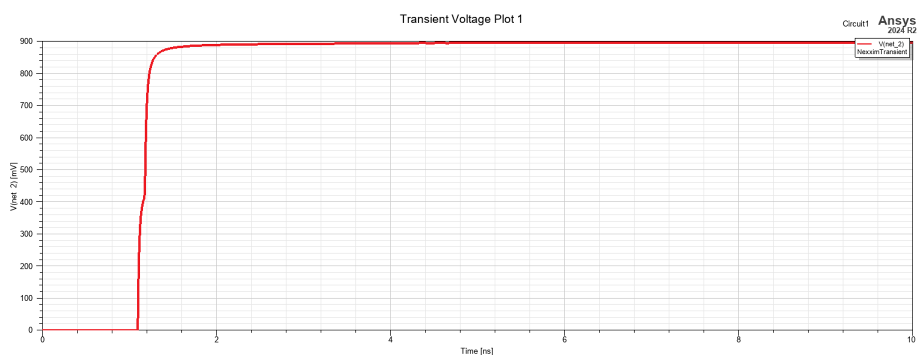

Figure 14: Time response voltage

Comment: Right around 1 second, the voltage seen by the end of the microstrip is around 900 mV. It is almost an immediate rise.

Figure 15: Rise time of 0.2 ns

Comment: The rise time 0.2 ns smooths out the plot and delays the time that the end of the microstrip sees the voltage.

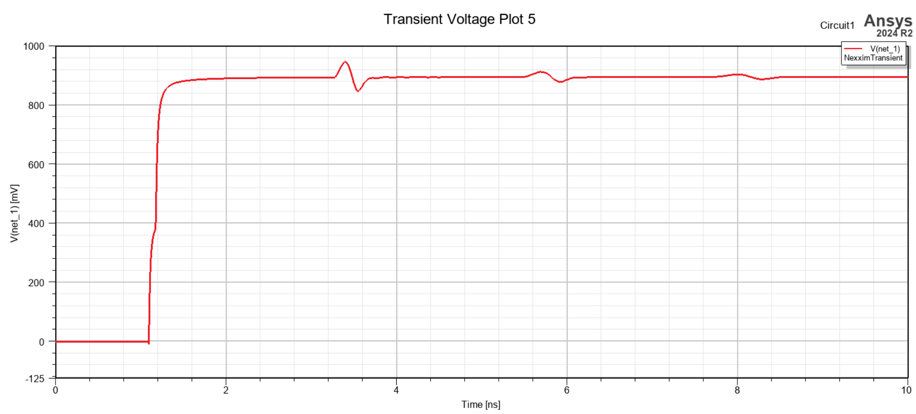

Figure 16: Enter Caption

Comment: With the other microstrip open on both ends, this induces some crosstalk between the microstrips. Initially, the impact creates some noise, but as time increases, the effects decrease.

5 Five coupled microstrip lines

Introduction:
This example is analogous to problem 3, except that in this problem there are five microstrips instead of 2.

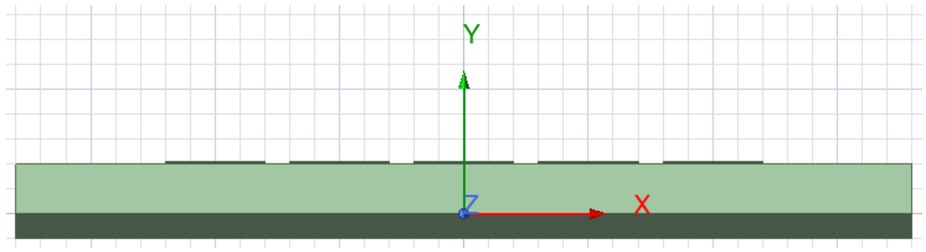

Figure 17: Five coupled microstrip lines

	Freq [GHz]	C(Mircostrip1,Mircostrip1) [pF] Setup1 : LastAdaptive	C(Mircostrip2,Mircostrip1) [pF] Setup1 : LastAdaptive	C(Mircostrip3,Mircostrip1) [fF] Setup1 : LastAdaptive	C(Mircostrip4,Mircostrip1) [fF] Setup1 : LastAdaptive	C(Mircostrip5,Mircostrip1) [fF] Setup1 : LastAdaptive	C(Mircostrip1,Mircostrip2) [pF] Setup1 : LastAdaptive	C(Mircostrip... Setup1 : La...
1	10.000000	131.854213	-20.277006	-920.521244	-429.782664	-322.466550	-20.277006	137.481716

Figure 18: B Matrix

	Freq [GHz]	L(Mircostrip1,Mircostrip1) [nH] Setup1 : LastAdaptive	L(Mircostrip2,Mircostrip1) [nH] Setup1 : LastAdaptive	L(Mircostrip3,Mircostrip1) [nH] Setup1 : LastAdaptive	L(Mircostrip4,Mircostrip1) [nH] Setup1 : LastAdaptive	L(Mircostrip5,Mircostrip1) [nH] Setup1 : LastAdaptive	L(Mircostrip1,Mircostrip2) [nH] Setup1 : LastAdaptive	L(Mircostrip2,... Setup1 : LastA...
1	10.000000	282.329061	69.710847	23.404702	10.101452	5.438991	69.710847	274.452251

Figure 19: L matrix

Comments: The matrices are too large to be captured in a single screenshot. Consequently, I have made the matrices below for easier viewage.Additionally, once this was realized, I did not take a screenshot of the modal voltages since it would not encapsulate the entire data set. I exported the data sets as CSV files and then used those to create the matrices seen below.

To make these matrices easier to read and to encapsulate their full information, they are written out.

$$[B] = \begin{bmatrix} 131.854 & -20.277 & -920.521 & -429.783 & -322.467 \\ -20.277 & 137.482 & -19.919 & -837.054 & -429.820 \\ -920.521 & -19.919 & 137.342 & -19.938 & -920.724 \\ -429.783 & -837.054 & -19.938 & 137.508 & -20.269 \\ -322.467 & -429.820 & -920.724 & -20.269 & 131.951 \end{bmatrix} \text{pF}$$

$$[L'] = \begin{bmatrix} 284.516 & 71.334 & 24.785 & 11.447 & 6.946 \\ 71.334 & 275.714 & 68.875 & 23.981 & 11.447 \\ 24.785 & 68.875 & 275.054 & 68.876 & 24.785 \\ 11.447 & 23.981 & 68.876 & 275.714 & 71.334 \\ 6.946 & 11.447 & 24.785 & 71.334 & 284.514 \end{bmatrix} \text{nH}$$

$$[V_{\text{modal}}] = \begin{bmatrix} -0.407730 & 0.383913 & 0.567308 & -0.206584 & -0.542714 \\ 0.576321 & 0.473687 & 0.422538 & 0.512583 & 0.141271 \\ 0.002771 & 0.504325 & 0.002276 & -0.628314 & 0.610179 \\ -0.579616 & 0.474559 & -0.420244 & 0.508269 & 0.145193 \\ 0.406998 & 0.385592 & -0.568343 & -0.203635 & -0.540467 \end{bmatrix}$$

Using the exact same methods as the ones used in problem two, the eigenvectors and eigenvalues can be easily calculated.

$$\text{eigenvalues} = \pm 3.1475 \times 10^8, \pm 4.0511 \times 10^8$$

again, velocity cannot be zero, so the eigenvector will be

$$eigenvector = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

6 Circuit design of 5 coupled microstrip lines

Introduction:
Similar to problem 4, taking the 5 coupled microstrip lines instead of 2. I will then analyze the results obtained.

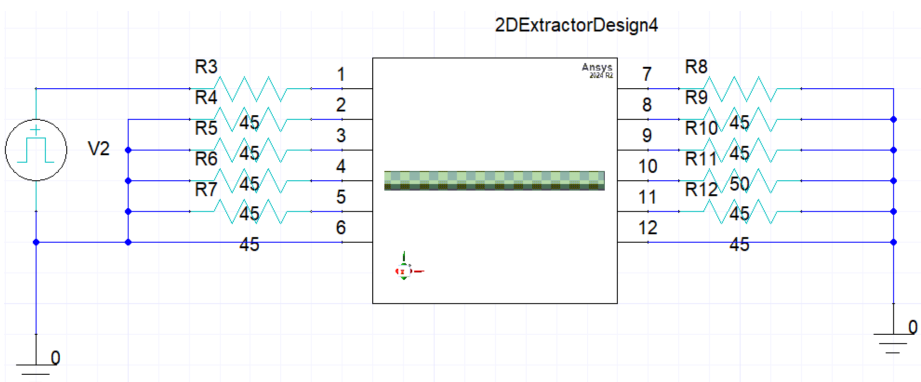

Figure 20: Circuit design of 5 coupled microstrip lines

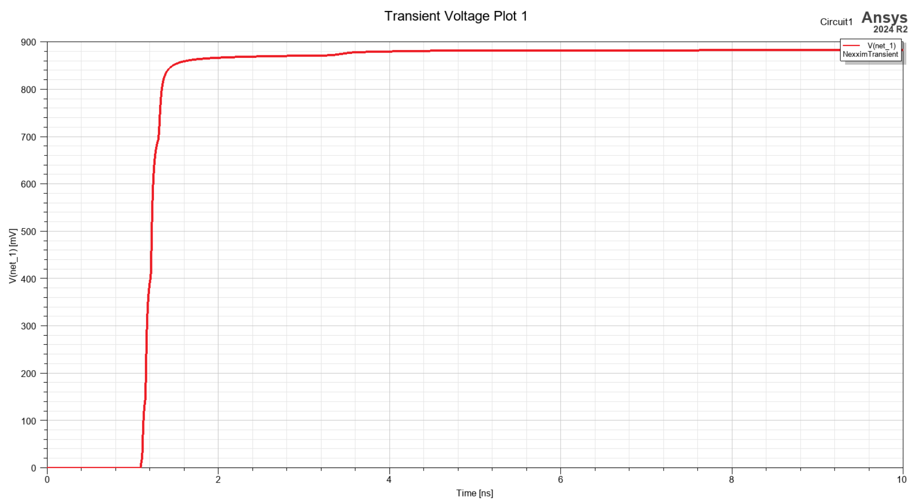

Figure 21: Time response of the voltage at the end of the excited transmission line

Comments: This is very similar to the response seen with the two microstrips, however, there is more variation along the path. Most notably, along the rise path were there are larger variations around 700mV and 400 mV. The differences should be more aggressive, but despite attempting to achieve different responses, I was unable to obtain such a result. Consequently, this is going to be the response reported.

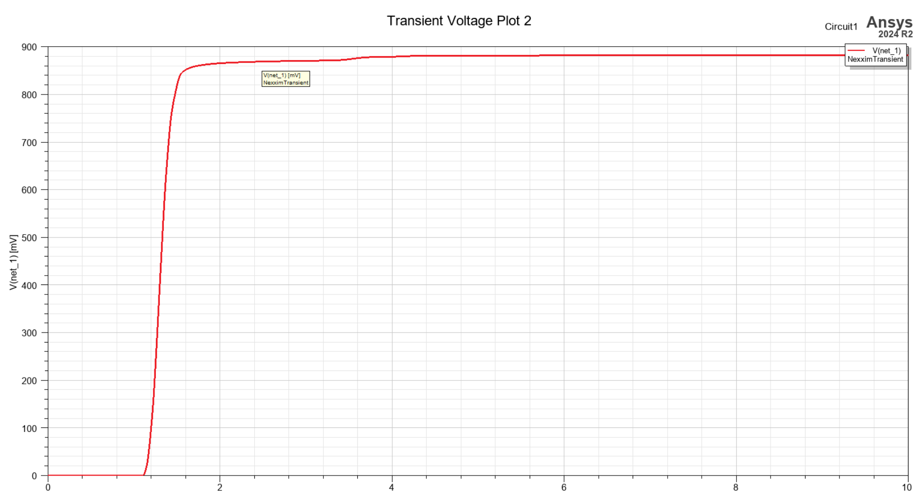

Figure 22: Rise time of 0.2 ns

Comments: Again, there should be a larger effect from the other microstrips on the first microstrip, however, despite trying, I was unsuccessful at achieving such a result. Despite this, the rise time of 0.2 ns smooths out the response and delays the time it takes for the voltage to reach the end of the transmission line.

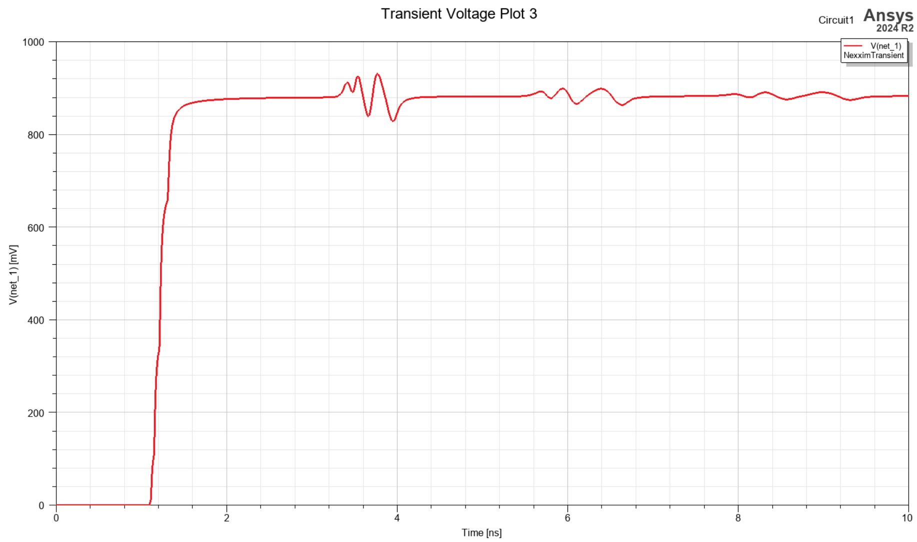

Figure 23: All other transmission lines are open on both ends

Comments: When the other microstrips are disconnected on both ends, the crosstalk is evident. There is significantly more than when there was only one other microstrip, but the response is similar nonetheless.

7 Circuit design of 5 coupled microstrip lines with variable resistors

Introduction:
This part of the project analyzes the five coupled microstrips with variable resistors. Additionally, some transmission lines are connected to others, some are connected to resistors, while others are connected to capacitors or inductors. This drastically changes the response of the system.

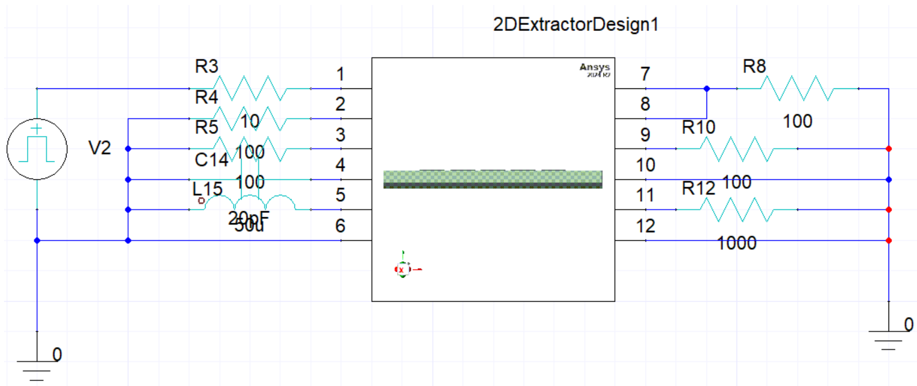

Figure 24: Circuit design of 5 coupled microstrip lines with variable resistors

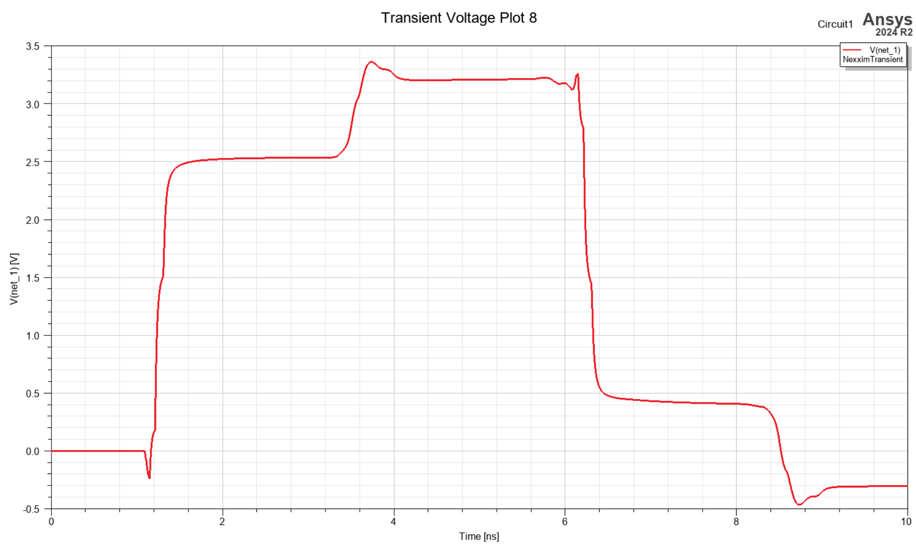

Figure 25: Voltage response at the end of the first microstrip

Comments: This response is aggressive and vastly different from the previous response of the original five coupled microstrips circuit design. The voltage takes a dip just after 1 ns before rising to 2.5 volts with slight variation in its rise. It then rises again to roughly 3.2 V just before 4 ns. It retains this value before dropping back down to about 0.3V just after 6 ns. Finally, it drops for a final time to a negative voltage around 9 ns. I can only assume that this response is because of the unmatched circuit and the use of an inductor and a capacitor. Additionally, some lines are connected directly to ground or other transmission lines. I have to contribute these changes to the massive change in the response.

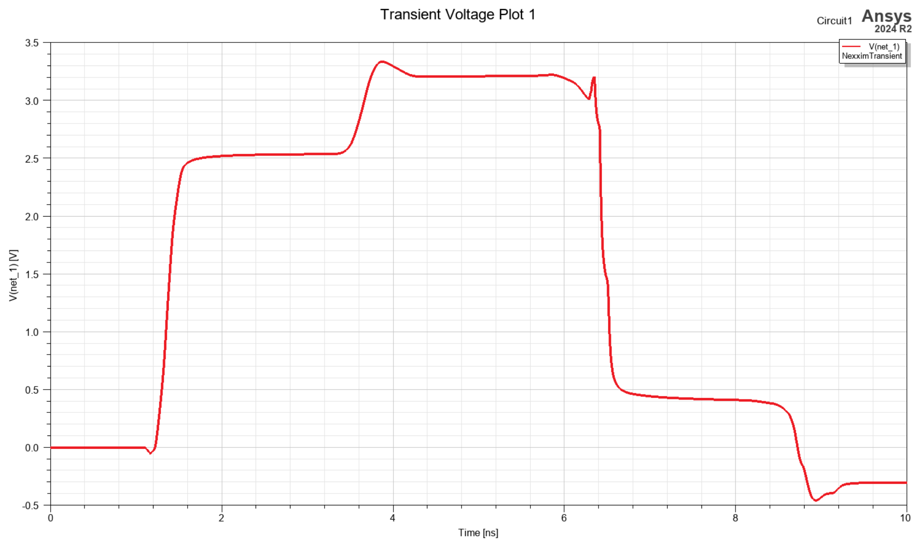

Figure 26: rise time of 0.2 ns

Comment: Similar to all previous responses with a 0.2 ns rise time, the response stays similar to its original, but has a smoother and is slightly delayed.

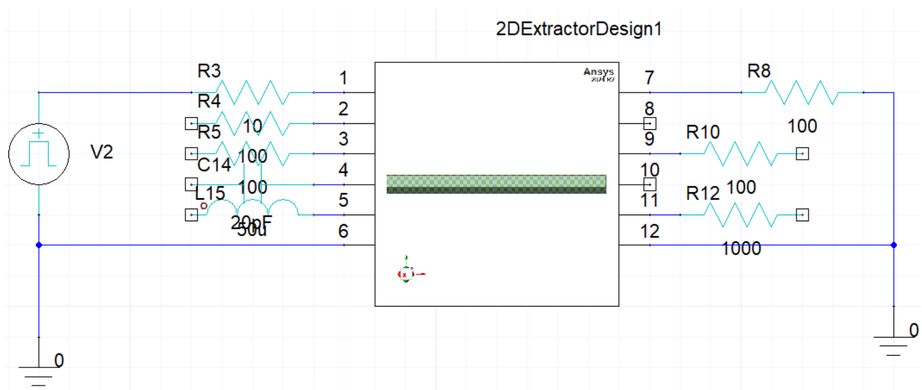

Figure 27: Circuit design for all other microstrips being disconnected

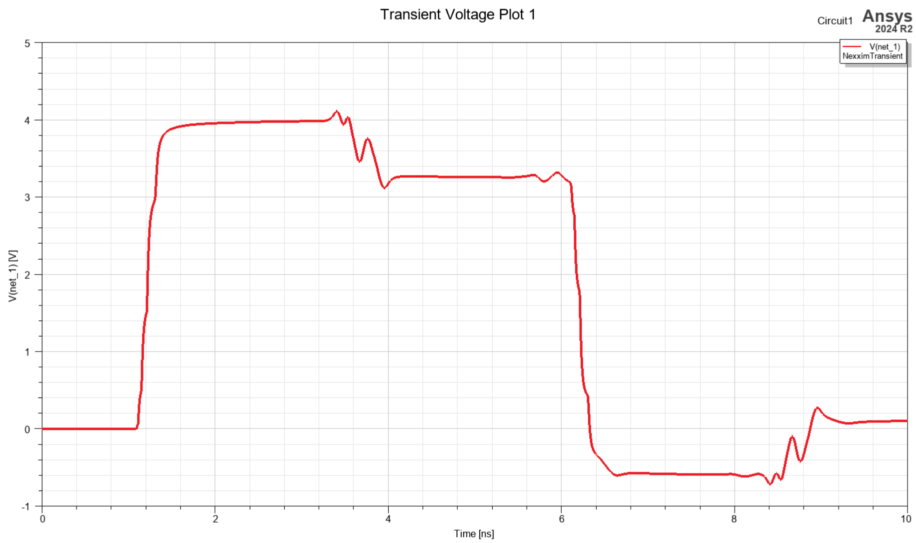

Figure 28: Voltage response at the end of first microstrip when all other microstrips are left open on both ends

Comment: This retains a similar shape to the original, but is shifted left, meaning that the excitations are experienced earlier. Additionally, sudden changes in direction before either increasing or decreasing are no longer present. However, the noise is similar to the other responses.

8 Hand Calculations for two coupled microstrip lines

Introduction: This part of the project takes the two coupled microstrips from problem 3 and applies the analysis of the two wires above a ground plane in problem 2.

Using the values gathered in part 3, we begin solving this problem.

$$[B'] = \begin{bmatrix} 118.022033 & -6.156154 \\ -6.156154 & 117.764573 \end{bmatrix} \text{pF}$$

$$[L'] = \begin{bmatrix} 292.681725 & 35.968671 \\ 35.968671 & 292.681243 \end{bmatrix} \text{nH}$$

Using these values, following the exact same method as in part 2, which I explicitly covered, we can solve for the eigenvalues.

$$C_m = \pm 1.7088 \times 10^8, \pm 1.7069 \times 10^8$$

Plugging this value into

$$[S] = \begin{bmatrix} \frac{1}{c_m^2} - a_{11} & -a12 \\ -a_{21} & \frac{1}{c_m^2} - a_{22} \end{bmatrix}$$

For this data, my a matrix gives the wrong values so my eigen solutions are off. I get:

$$[a] = \begin{bmatrix} 0.3432 & 0.0243 \\ 0.0244 & 0.3425 \end{bmatrix} \times 10^{-16}$$

$$\mathbf{S}_1 = 1.0 \times 10^{-17} \begin{bmatrix} -0.0000 & -0.2434 \\ -0.2443 & 0.0075 \end{bmatrix}$$

$$\mathbf{S}_2 = 1.0 \times 10^{-17} \begin{bmatrix} -0.0075 & -0.2434 \\ -0.2443 & 0.0000 \end{bmatrix}$$

However, I know that the v matrix should be

$$[V] = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} * V_0$$

So I will be using this for problem 9.

9 Hand Calculations for two coupled microstrip lines voltage waveforms

We want to start by calculating the time it takes for the pulse to reach the end of the transmission line.

$$\tau_1 = \frac{L}{c_{m1}} = \frac{200\text{mm}}{1.7069 \times 10^8 \text{ m/s}} = 1.172 \text{ ns}$$

$$\tau_2 = \frac{L}{c_{m2}} = \frac{200\text{mm}}{1.7088 \times 10^8 \text{ m/s}} = 1.170 \text{ ns}$$

This can be put into a matrix of its own

$$[\tau] = \begin{bmatrix} 1.172 \\ 1.170 \end{bmatrix} \text{ns}$$

From the problem, it is known that the generator voltage is a step (Heavyside) signal of step 2V. To solve for the voltages at the beginning and end of the transmission lines, we can use the equations

$$V_1(z = 0, t) = \frac{V_{g0}}{2} h(t) = V_1^{(1)}(0, t) + V_1^{(2)}(0, t)$$

$$V_2(z = 0, t) = \frac{V_{g0}}{2} h(t) = V_2^{(1)}(0, t) + V_2^{(2)}(0, t)$$

$$V_1(z = L, t) = V_1^{(1)}(0, t - \tau_1) + V_1^{(2)}(0, t - \tau_2)$$
$$V_2(z = L, t) = V_2^{(1)}(0, t - \tau_1) + V_2^{(2)}(0, t - \tau_2)$$

We can simplify these expressions by saying

$$A = \frac{V_{g0}}{4} h(t) =$$

making

$$V_1(z = 0, t) = 2A \text{ and } V_2(z = 0, t) = 0$$

Assuming that h(t) is 1, we can compute the voltages to be

$$[V_m] = \frac{[V] * 2}{4} = \begin{bmatrix} 0.5 & 0.5 \\ -0.5 & 0.5 \end{bmatrix}$$

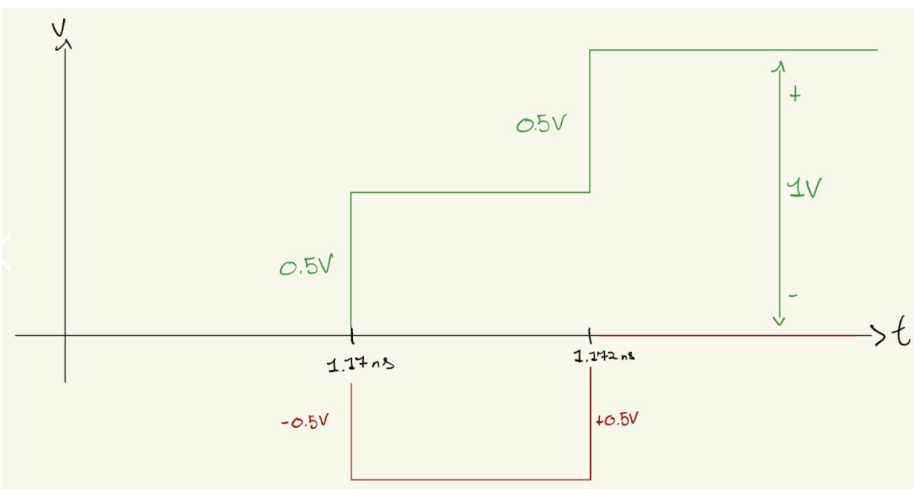

Figure 29: Voltage Response of two coupled microstrips

Comments: This is the hand-calculated response of the two coupled microstrips. The two pulses start at zero volts, but at 1.17 ns the first microstrip rises by 0.5v while the other falls by the same value. This decrease in voltage in the second microstrip causes a delay in the rise time of the second microstrip. I calculated this difference in rise time to be 0.002 ns. The crosstalk can be seen when the voltage of the first miscrostrip does not immediately go to its steady state voltage, but instead only goes to half of that.

10 Matlab Code

The main matlab code that I used was as follows.

```
clc
clear

d = 10e-3;
h = d;
D = sqrt(d^2+4*h^2);
a = 1e-3;
epsilon = 8.854E-12;
mu = 4*pi*10^-7;

a11 = 1/(2*pi*epsilon) * log(2*h/a);
a22 = a11;
a12 = 1/(2*pi*epsilon) * log(D/d);
a21 = a12;

a_matrix = [a11 a12;
            a21 a22];
b_maxtrix = a_matrix^-1;
L_matrix = epsilon * mu * b_maxtrix^-1;

b_matrix_ansys=[20.060474 -5.404791;
                -5.404791 20.060296]*10^-12;
L_matrix_ansys=[598.837767 161.149314;
                161.149314 598.838501]*10^-9;

a_matrix_ansys = L_matrix_ansys * b_matrix_ansys;

a11 = a_matrix_ansys(1,1);
a22 = a_matrix_ansys(2,2);
a_sum = a11+a22;

syms c_m

eqn = (1/(c_m^2))^2 - (a11+a22)*(1/(c_m^2))+a11*a22 == 0;
cm_sol = solve(eqn, c_m);
cm_num = double(cm_sol);
M = reshape(cm_num, 2, 2);

V0 = 1/sqrt(2);
matrix = [1 1; 1 -1];
S_v_matrix = matrix * V0;
Lambda_matrix = [1/M(2,2) 0;
                 0 1/M(2,1)];
S_I_matrix = L_matrix_ansys^-1*S_v_matrix*Lambda_matrix;
Z0_matrix = S_v_matrix*S_I_matrix^-1;
```

This is the MATLAB code for problem 2, but it was largely used in all other problems, with slight modifications to the values depending upon the problem