

ECE 541 Project 3
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1 Introduction

This project covers the topic of cylindrical and spherical scattering.

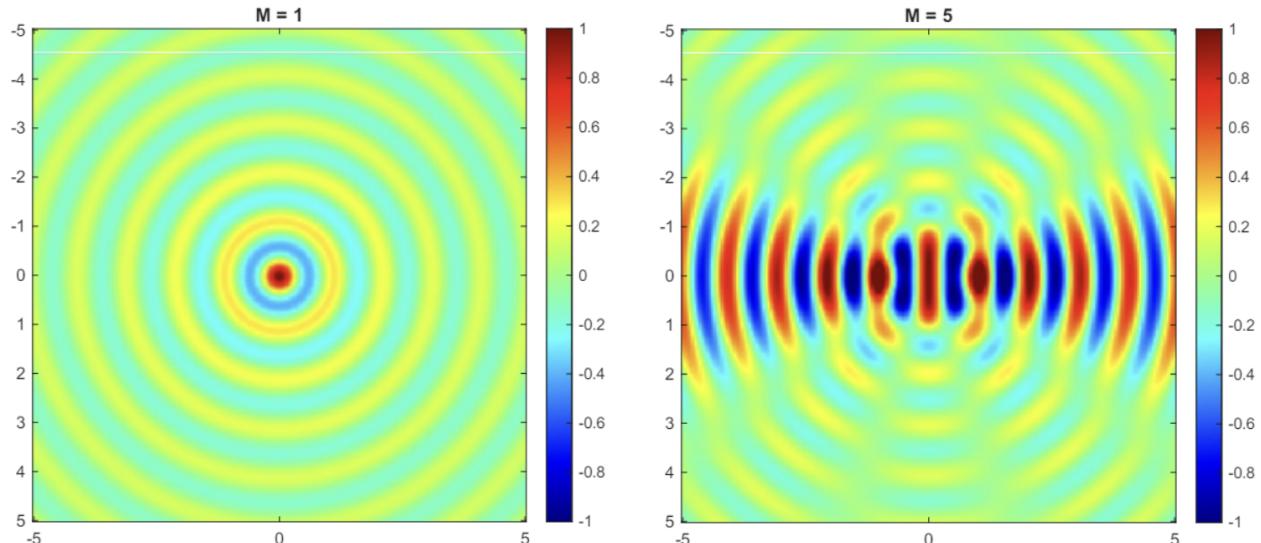
For the first part, this project will take an infinitely long cylinder and calculate the cylindrical wave transformation over different modes. The next part of this section of the project will plot the echo width of a circular conducting cylinder as a function of its normalized radius. Then, plots of the E_z^{sc} , H_z^{sc} , E_z^{tot} , and H_z^{tot} will be generated along with plots for the scattering width σ_{2D}/λ for both TE and TM cases.

The second part of this project will take an infinitely long dielectric cylinder and plot its E_z^{sc} and E_z^{tot} as well as the scattering width in the TM case.

The third part of the project

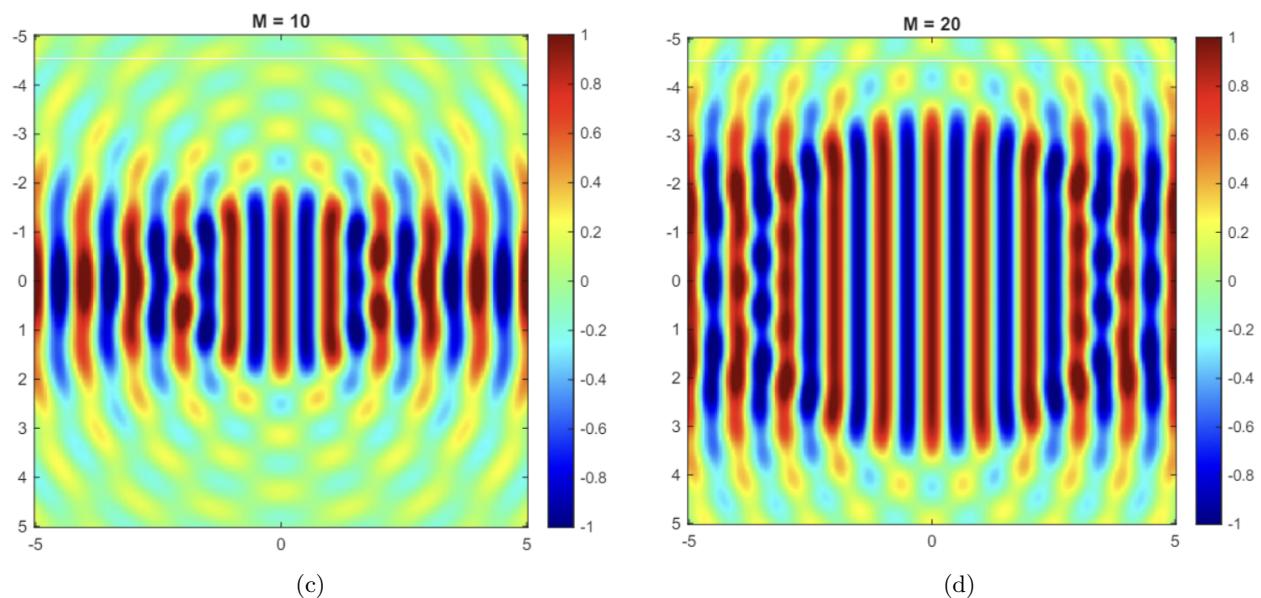
2 Infinitely Long Metallic (PEC) Cylinder

2.1 Wave Transformation



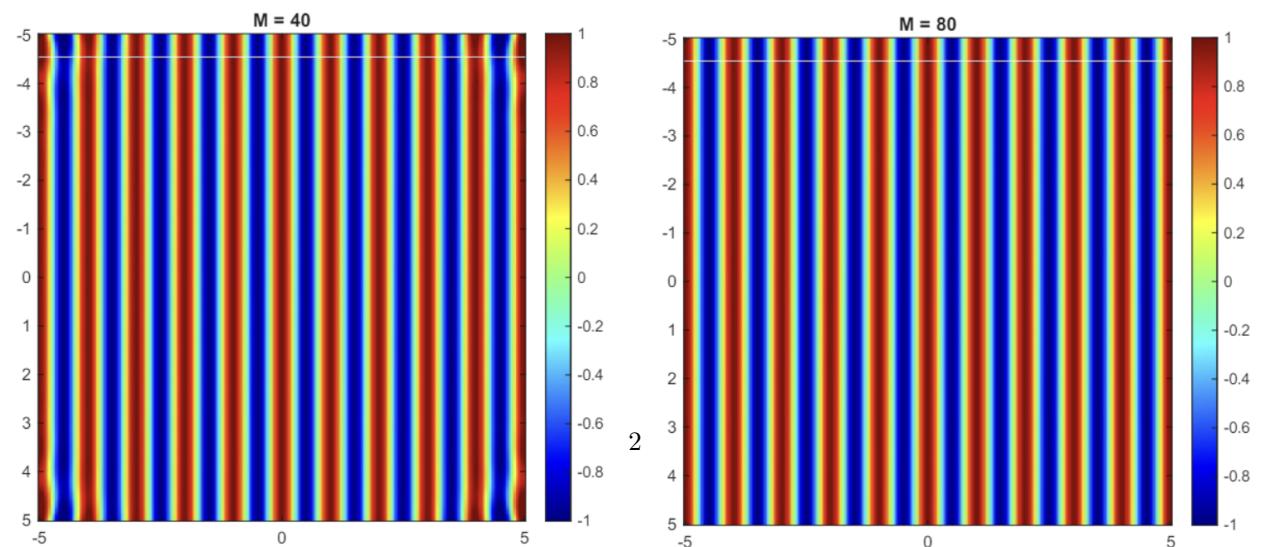
(a)

(b)



(c)

(d)



(e)

(f)

2.2 Echo Width

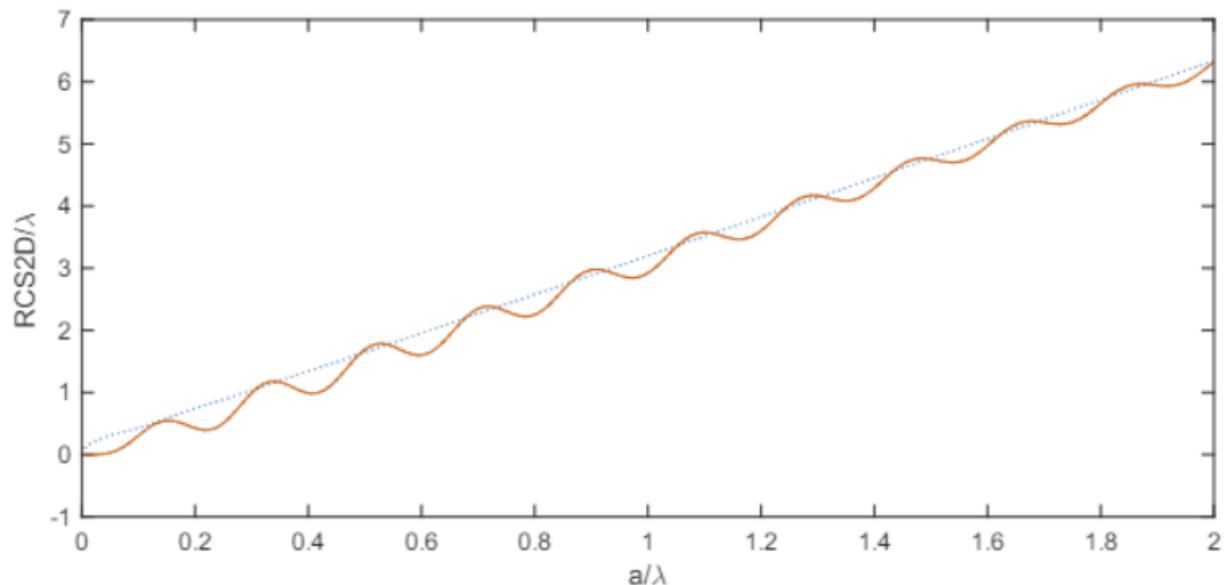


Figure 2: Echo width of a circular conducting cylinder as a function of its normalized radius

2.3 Infinitely Long Dielectric Cylinder

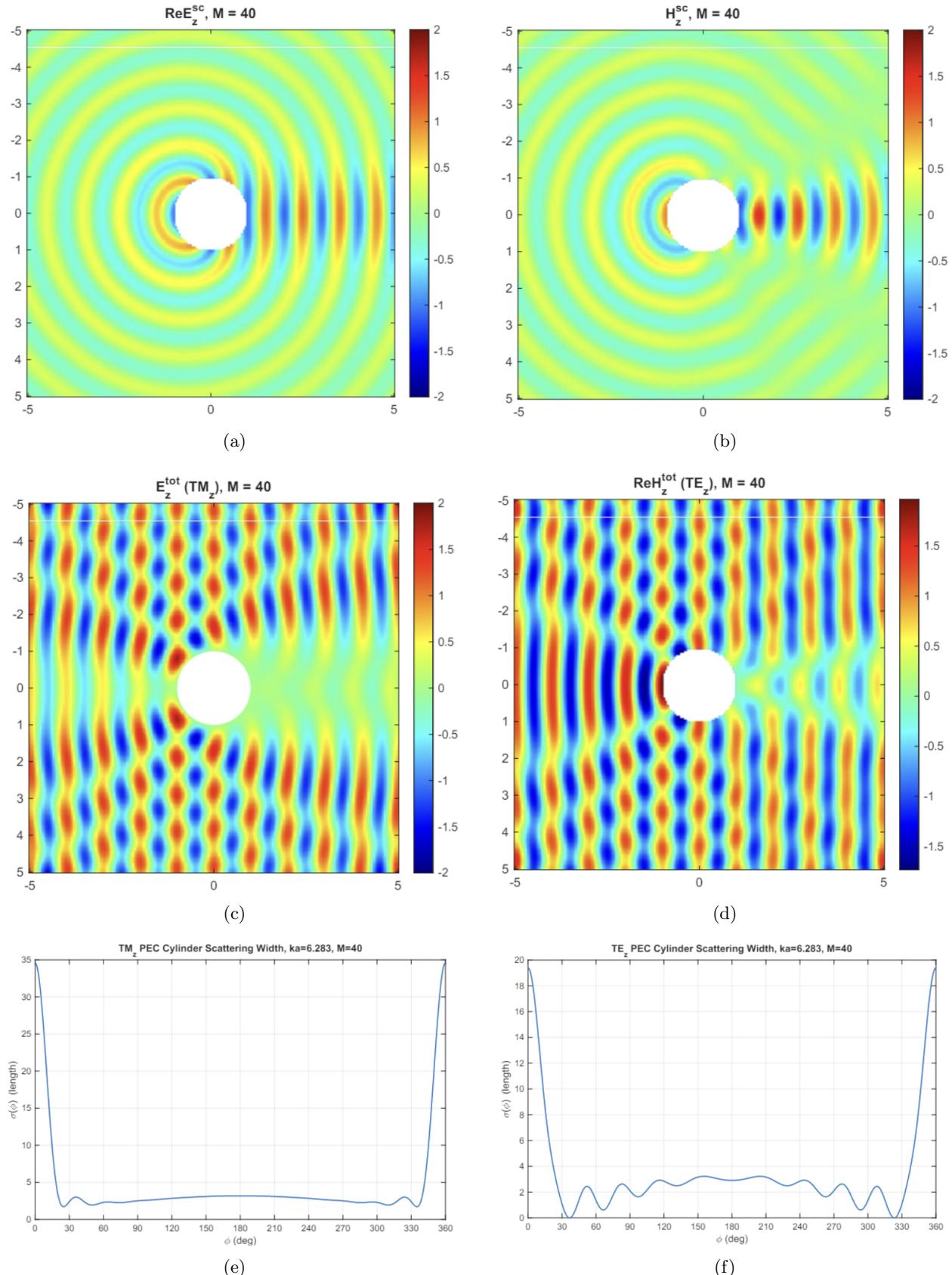


Figure 3: Enter⁴ overall caption

3 Infinitely Long Dielectric Cylinder

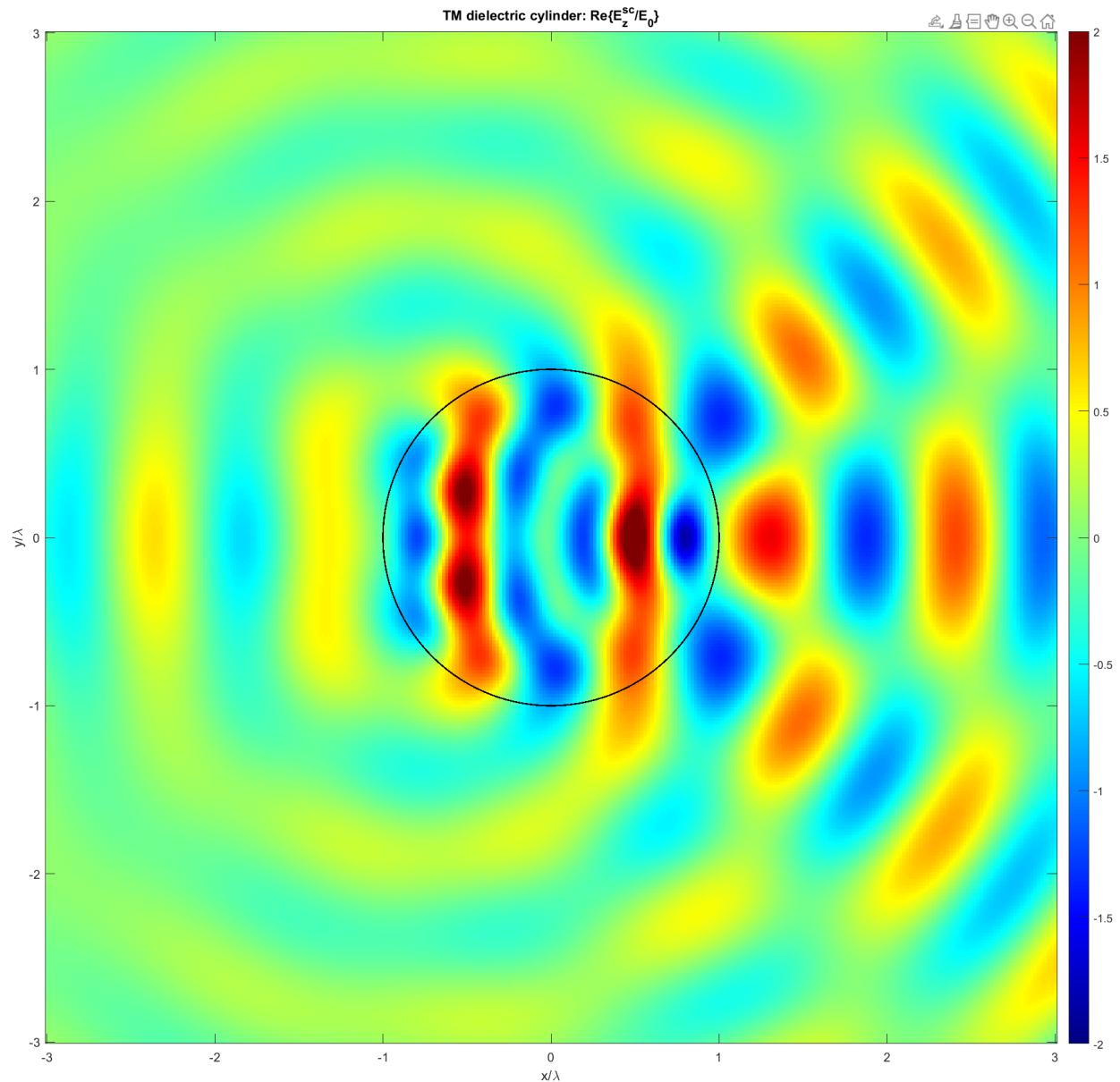


Figure 4: Scattered E field of a Dielectric Cylinder with $\epsilon_r = 4$

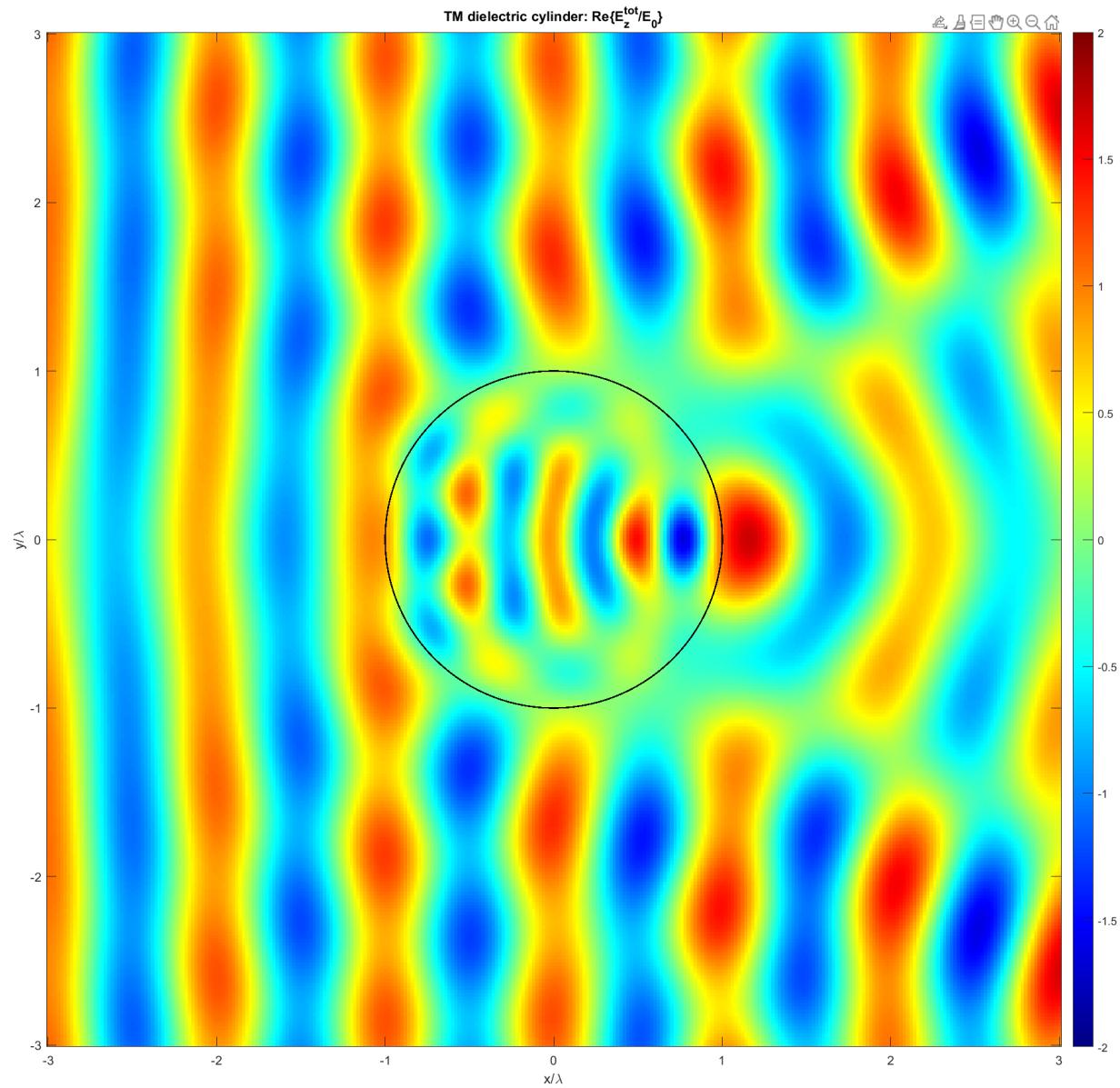


Figure 5: Total E field of a Dielectric Cylinder with $\epsilon_r = 4$

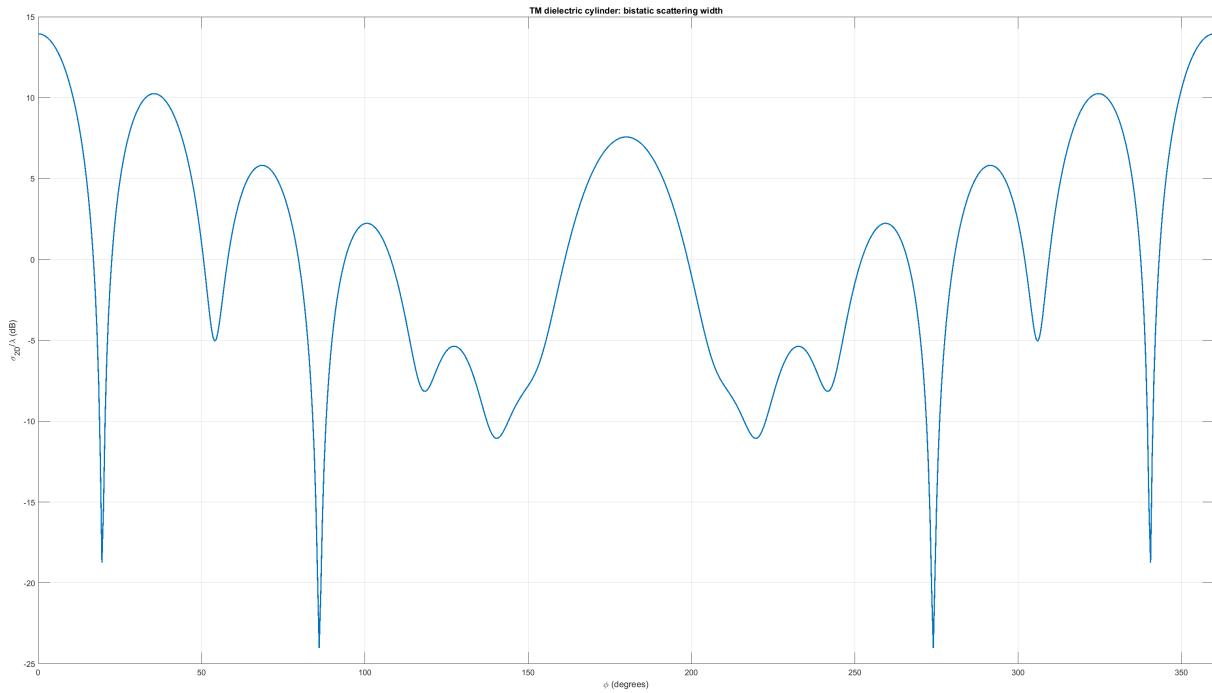


Figure 6: TM case of a Dielectric Cylinder with $\epsilon_r = 4$

4 Metallic (PEC) Sphere in Free Space Illuminated by a Uniform Plane Wave Propagating in the Positive z Direction

5 Simulate Section 2 in WIPL-D and HFSS

abs(E) [V/m] Z=0

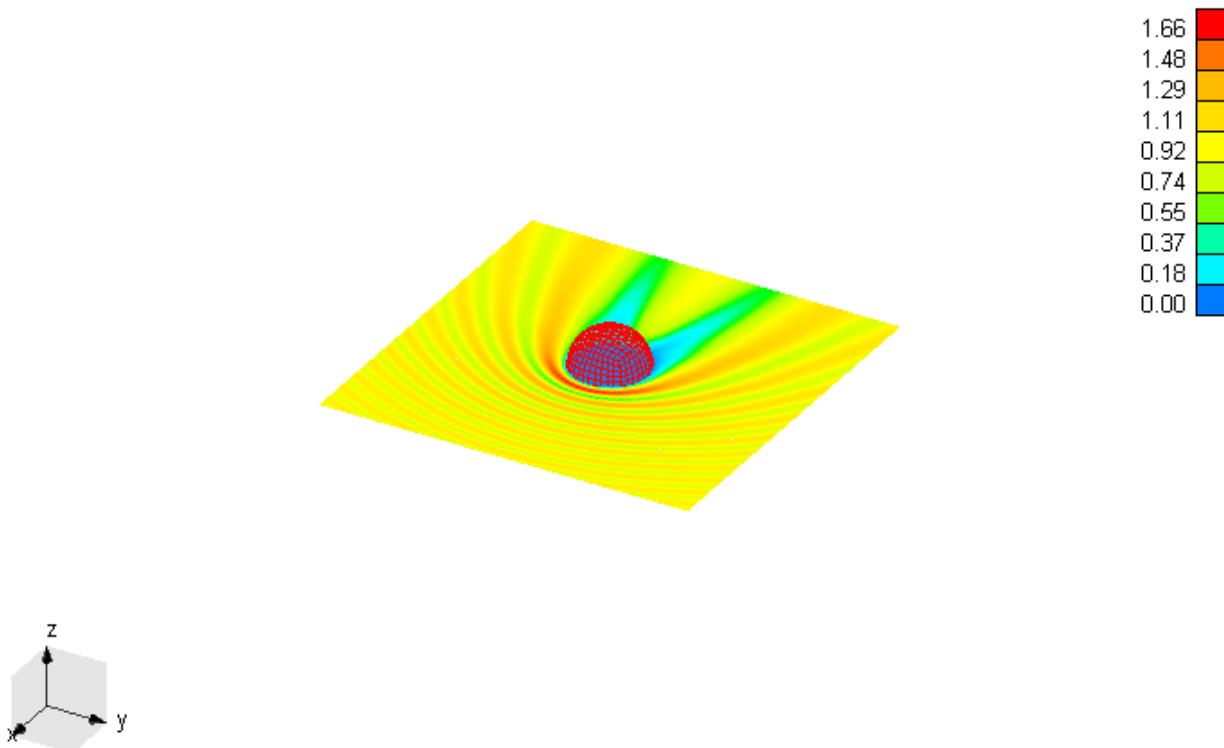


Figure 7: WIPL-D simulation of $|E|$

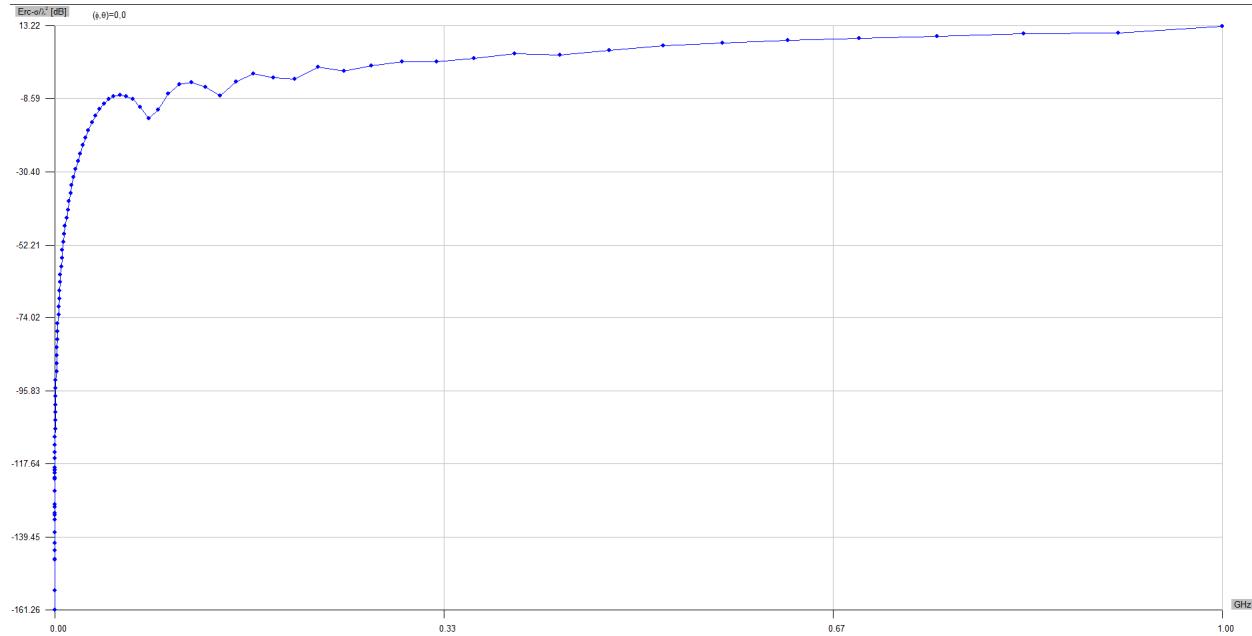


Figure 8: WIPL-D RCS

6 Code

ECE 541 Fall 2025 Project 6

Problem 1(a)

```

1 % ECE 541 Fall 2025 Project 6
2 % problem 1(a)
3
4 clc
5 clear
6
7 j = sqrt(-1);
8 lambda = 1;
9 k = 2 * pi / lambda;
10 M = [1,5,10,20,40,80];
11 maxM = max(M);
12 x = -5*lambda:0.05*lambda:5*lambda;
13 y = x;
14 [X,Y] = meshgrid(x,y);
15 [PHI,RHO] = cart2pol(X,Y);
16 func = zeros(2*maxM+1,length(y),length(x));
17 for n = -maxM:maxM
18     i = n + maxM + 1 ;
19     func(i,:,:,:) = j^(-n).*besselj(n,k*RHO).*exp(j*n*PHI);
20 end
21 for i = 1 : length(M)
22     J = real(sum(func(maxM+1-M(i)):maxM+1+M(i),:,:),1));
23     J1(:,:,i) = J(1,:,:);
24     figure(i);
25     imagesc(x,y,J1);
26     axis equal tight;
27     caxis([-1 1]);

```

```

28 title(['M = ', num2str(M(i))]);
29 colormap(jet)
30 colorbar;
31 clear J J1
32 end

```

Problem 1(b)

```

1 % ECE 541 Fall 2025 Project 6
2 % Problem 1(b)
3
4 clc
5 clear
6
7 j = sqrt(-1);
8 lambda = 1;
9 k = 2*pi/lambda;
10 M = 80;
11 PHI = pi;
12 a = 0.005*lambda:0.005*lambda:2*lambda;
13 RHO = 100*a;
14 funcTM = zeros(2*M+1,length(a));
15 funcTE = zeros(2*M+1,length(a));
16
17 for i = 1:2*M+1
18     n = i-M-1;
19     coeffa = j^(-n)*besselj(n,k*a)./besselh(n,2,k*a);
20     jprim = 1/2*(besselj(n-1,k*a)-besselj(n+1,k*a));
21     hprim = 1/2*(besselh(n-1,2,k*a)-besselh(n+1,2,k*a));
22     coeffb = -j^(-n)*jprim./hprim;
23     funcTM(i,:) = coeffa.*besselh(n,2,k*RHO).*exp(j*n*PHI);
24     funcTE(i,:) = coeffb.*besselh(n,2,k*RHO).*exp(j*n*PHI);
25 end
26
27 RCS2DTM = 2*pi*RHO.*abs(sum(funcTM,1)).^2;
28 RCS2DTE = 2*pi*RHO.*abs(sum(funcTE,1)).^2;
29 plot(a,RCS2DTM/lambda,:,:,a,RCS2DTE/lambda,:);
30 legend('TM wave','TE wave','Location','best');
31 xlabel('a/\lambda');
32 ylabel('RCS2D/\lambda');
33 ylim([-1,7]);

```

Problem 1(c) — Scattered E

```

1 % ECE 541 Fall 2025 Project 6
2 % problem 1(c) - Scattered E
3
4 clc
5 clear
6
7 j = 1i;
8 lambda = 1;
9 a = 1*lambda;
10 k = 2*pi/lambda;
11
12 M = 40;

```

```

13
14 x = -5*lambda:0.05*lambda:5*lambda;
15 y = x;
16 [X,Y] = meshgrid(x,y);
17 [PHI,RHO] = cart2pol(X,Y);
18
19 Ez_sc = zeros(2*M+1, length(y), length(x));
20
21 ka = k*a;
22
23 for n = -M:M
24     idx = n + M + 1;
25
26     Jn_ka = besselj(n, ka);
27     Hn_ka = bessellh(n, 2, ka);
28
29     a_n = - (j^(-n)) * (Jn_ka / Hn_ka);
30
31     Ez_sc(idx,:,:) = a_n .* bessellh(n, 2, k*RHO) .* exp(j*n*PHI);
32 end
33
34 S = sum(Ez_sc, 1);
35 Ez_sc = squeeze(S(1,:,:));
36
37 mask = (RHO < a);
38 Ez_plot = real(Ez_sc);
39 Ez_plot(mask) = NaN;
40
41 figure;
42 h = imagesc(x, y, Ez_plot);
43 set(gcf, 'Renderer', 'opengl');
44 set(h, 'AlphaData', ~isnan(Ez_plot));
45 set(gca, 'Color', 'w');
46
47 axis equal tight;
48 caxis([-2 2]);
49 title(sprintf('Re{E_z^{sc}}}, M = %d', M));
50 colormap(jet);
51 colorbar;

```

Problem 1(c) — Scattered H

```

1 % ECE 541 Fall 2025 Project 6
2 % problem 1(c) - Scattered H
3
4 clc
5 clear
6
7 j = 1i;
8 lambda = 1;
9 a = 1*lambda;
10 k = 2*pi/lambda;
11
12 M = 40;
13
14 x = -5*lambda:0.05*lambda:5*lambda;
15 y = x;

```

```

16 [X,Y] = meshgrid(x,y);
17 [PHI,RHO] = cart2pol(X,Y);
18
19 Hz_sc = zeros(2*M+1, length(y), length(x));
20
21 ka = k*a;
22
23 for n = -M:M
24
25     idx = n + M + 1;
26
27     dJdx = 0.5*( besselj(n-1,ka) - besselj(n+1,ka) );
28     dHdx = 0.5*( besselh(n-1,2,ka) - besselh(n+1,2,ka) );
29
30     a_n = - (j^(-n)) * (dJdx / dHdx);
31
32     Hz_sc(idx,:,:) = a_n .* besselh(n,2,k*RHO) .* exp(j*n*PHI);
33 end
34
35 S = sum(Hz_sc, 1);
36 Hz_sc = squeeze(S(1,:,:));
37
38 mask = (RHO < a);
39
40 Hz_plot = real(Hz_sc);
41 Hz_plot(mask) = NaN;
42
43 figure;
44
45 h = imagesc(x, y, Hz_plot);
46 caxis([-2 2]);
47 set(gcf, 'Renderer', 'opengl');
48 set(h, 'AlphaData', ~isnan(Hz_plot));
49 set(gca, 'Color', 'w');
50
51 axis equal tight;
52 title(sprintf('H_z^{sc}, M = %d', M));
53 colormap(jet);
54 colorbar;

```

Problem 1(c) — Total E

```

1 % ECE 541 Fall 2025 Project 6
2 % problem 1(c) - Total E
3
4 clc
5 clear
6
7 j = 1i;
8
9 lambda = 1;
10 a = 1*lambda;
11 k = 2*pi/lambda;
12 ka = k*a;
13
14 M = 40;
15

```

```

16 x = -5*lambda:0.05*lambda:5*lambda;
17 y = x;
18 [X,Y] = meshgrid(x,y);
19 [PHI,RHO] = cart2pol(X,Y);
20
21 Ez_tot = zeros(size(RHO));
22
23 for n = -M:M
24     c_n = j^(-n);
25
26     b_n = -c_n * ( besselj(n,ka) / besselh(n,2,ka) );
27
28     Ez_inc_n = c_n .* besselj(n, k*RHO) .* exp(j*n*PHI);
29     Ez_sc_n = b_n .* besselh(n, 2, k*RHO) .* exp(j*n*PHI);
30
31     Ez_tot = Ez_tot + (Ez_inc_n + Ez_sc_n);
32 end
33
34 figure;
35 imagesc(x, y, real(Ez_tot));
36 axis equal tight;
37 caxis([-2 2]);
38 title(sprintf('E_z^{tot} (TM_z), M = %d', M));
39 colormap(jet)
40 colorbar;
41
42 hold on
43 th = linspace(0,2*pi,500);
44 fill(a*cos(th), a*sin(th), 'w', 'EdgeColor','none');
45 hold off

```

Problem 1(c) — Total H

```

1 % ECE 541 Fall 2025 Project 6
2 % problem 1(c) - Total H
3
4 clc
5 clear
6
7 j = 1i;
8 lambda = 1;
9 a = 1*lambda;
10 k = 2*pi/lambda;
11 ka = k*a;
12
13 M = 40;
14
15 x = -5*lambda:0.05*lambda:5*lambda;
16 y = x;
17 [X,Y] = meshgrid(x,y);
18 [PHI,RHO] = cart2pol(X,Y);
19
20 Hz_tot = zeros(size(RHO));
21
22 for n = -M:M
23     J_ka_prime = 0.5*( besselj(n-1,ka) - besselj(n+1,ka) );

```

```

25 H_ka_prime = 0.5*( besselh(n-1,2,ka) - besselh(n+1,2,ka) );
26 b_n = -j^(-n) * (J_ka_prime / H_ka_prime);
27
28 Hz_inc_n = j^(-n) .* besselj(n, k*RHO) .* exp(j*n*PHI);
29 Hz_sc_n = b_n .* besselh(n,2,k*RHO) .* exp(j*n*PHI);
30
31 Hz_tot = Hz_tot + (Hz_inc_n + Hz_sc_n);
32
33 end
34
35 mask = (RHO < a);
36 Hz_plot = real(Hz_tot);
37 Hz_plot(mask) = NaN;
38
39 figure;
40 h = imagesc(x, y, Hz_plot);
41 set(gcf,'Renderer','opengl');
42 set(h,'AlphaData', ~isnan(Hz_plot));
43 set(gca,'Color','w');
44
45 axis equal tight;
46 title(sprintf('Re{H_z^{tot}} (TE_z), M = %d', M));
47 colormap(jet);
48 colorbar;
49 caxis([-2 2]);

```

Problem 1(c) — TM Scattering Width

```

1 % ECE 541 Fall 2025 Project 6
2 % problem 1(c) - TM scattering width
3
4 clc
5 clear
6
7 j = 1i;
8 lambda = 1;
9 a = 1*lambda;
10 k = 2*pi/lambda;
11 ka = k*a;
12
13 M = 40;
14 n = (-M:M).';
15
16 c_n = j.^(-n);
17 b_n = -c_n .* ( besselj(n,ka) ./ besselh(n,2,ka) );
18
19 phi = linspace(0,2*pi,2000);
20 F = ( (b_n .* (j.^n)).' * exp(1i*n*phi) );
21
22 sigma = (4/k) * abs(F).^2;
23
24 Sigma_total = trapz(phi, sigma);
25
26 % Plot
27 figure;
28 plot(phi*180/pi, sigma, 'LineWidth', 1.2);
29 xlim([0 360]);

```

```

30 xticks(0:30:360);
31 grid on;
32 xlabel('phi (deg)');
33 ylabel('sigma(phi) (length)');
34 title(sprintf('TM_z PEC Cylinder Scattering Width, ka=%.3f, M=%d', ka, M));
35 fprintf('Backscatter (phi=180 deg): sigma_b = %.6g\n', sigma(round(end/2)));
36 fprintf('Forward scatter (phi=0 deg): sigma_f = %.6g\n', sigma(1));
37 fprintf('Total scattering width (integrated): Sigma = %.6g\n', Sigma_total);

```

Problem 1(c) — TE Scattering Width

```

1 % ECE 541 Fall 2025 Project 6
2 % problem 1(c) - TE scattering width
3
4 clc
5 clear
6
7 j = 1i;
8 lambda = 1;
9 a = 1*lambda;
10 k = 2*pi/lambda;
11 ka = k*a;
12
13 M = 40;
14 n = (-M:M).';
15
16 c_n = j.^(-n);
17
18 J_ka_prime = 0.5*( besselj(n-1,ka) - besselj(n+1,ka) );
19 H_ka_prime = 0.5*( bessellh(n-1,2,ka) - bessellh(n+1,2,ka) );
20
21 b_n = -c_n .* ( J_ka_prime ./ H_ka_prime );
22
23 phi = linspace(0, 2*pi, 2000);
24 F = ( (b_n .* (j.^n)).' * exp(1i*n*phi) );
25
26 sigma = (4/k) * abs(F).^2;
27
28 Sigma_total = trapz(phi, sigma);
29
30 % Plot
31 figure;
32 plot(phi*180/pi, sigma, 'LineWidth', 1.2);
33 grid on;
34 xlim([0 360]);
35 xticks(0:30:360);
36 xlabel('phi (deg)');
37 ylabel('sigma(phi) (length)');
38 title(sprintf('TE_z PEC Cylinder Scattering Width, ka=%.3f, M=%d', ka, M));
39 [~,i0] = min(abs(phi - 0));
40 [~,i180] = min(abs(phi - pi));
41 fprintf('Forward scatter (phi=0 deg): sigma_f = %.6g\n', sigma(i0));
42 fprintf('Backscatter (phi=180 deg): sigma_b = %.6g\n', sigma(i180));
43 fprintf('Total scattering width: Sigma = %.6g\n', Sigma_total);

```

Problem 2 — Scattered E

```

1 % ECE 541 Fall 2025 Project 6
2 % problem 2 - Scattered E
3
4 clc
5 clear
6
7 lambda = 1;
8 a = 1*lambda;
9 eps_r = 4.0;
10 mu_r = 1.0;
11
12 k = 2*pi/lambda;
13 kd = k*sqrt(eps_r*mu_r);
14
15 M = 40;
16
17 x = -3*lambda : 0.02*lambda : 3*lambda;
18 y = x;
19 [X,Y] = meshgrid(x,y);
20 [PHI,RHO] = cart2pol(X,Y);
21
22 Einc = exp(-1j*k*X);
23
24
25 n = (-M:M).';
26 jneg = (1j).^( -n );
27
28 ka = k*a;
29 kda = kd*a;
30
31 Jn_ka = besselj(n, ka);
32 Jn_kda = besselj(n, kda);
33
34 Jp_ka = 0.5*(besselj(n-1,ka) - besselj(n+1,ka));
35 Jp_kda = 0.5*(besselj(n-1,kda) - besselj(n+1,kda));
36
37 Hn_ka = bessellh(n,2,ka);
38 Hp_ka = 0.5*(bessellh(n-1,2,ka) - bessellh(n+1,2,ka));
39
40 den = sqrt(mu_r).*Hp_ka.*Jn_kda - sqrt(eps_r).*Hn_ka.*Jp_kda;
41
42 an = -jneg .* ( sqrt(mu_r).*Jp_ka.*Jn_kda - sqrt(eps_r).*Jn_ka.*Jp_kda ) ./ den;
43
44 cn = (1j).^( -(n+1) ) .* (2*sqrt(mu_r)) ./ (pi*ka .* den);
45
46 Esc_out = zeros(size(RHO));
47 Eint = zeros(size(RHO));
48
49 for ii = 1: numel(n)
50     ni = n(ii);
51     phase = exp(1j*ni*PHI);
52
53     Esc_out = Esc_out + an(ii) .* bessellh(ni,2,k*RHO) .* phase;
54
55     Eint = Eint + cn(ii) .* besselj(ni, kd*RHO) .* phase;
56 end
57
58 Esc = Esc_out;

```

```

59 inside = (RHO < a);
60 Esc(inside) = Eint(inside) - Einc(inside);
61
62 % ----- Plot -----
63 figure('Color','w');
64 imagesc(x/lambda, y/lambda, real(Esc));
65 axis equal tight;
66 set(gca,'YDir','normal');
67 xlabel('x/\lambda'); ylabel('y/\lambda');
68 caxis([-2 2]);
69 title('TM dielectric cylinder: Re{E_z^{sc}}/E_0');
70 colormap(jet)
71 colorbar;
72
73 % Cylinder boundary
74 hold on;
75 th = linspace(0,2*pi,600);
76 plot((a/lambda)*cos(th),(a/lambda)*sin(th),'k','LineWidth',1.2);
77 hold off;

```

Problem 2 — Total E

```

1 % ECE 541 Fall 2025 Project 6
2 % problem 2 - Total E
3
4 clc
5 clear
6
7 lambda = 1;
8 a = 1*lambda;
9 eps_r = 4.0;
10 mu_r = 1.0;
11
12 k = 2*pi/lambda;
13 kd = k*sqrt(eps_r*mu_r);
14
15 M = 40;
16
17 x = -3*lambda : 0.02*lambda : 3*lambda;
18 y = x;
19 [X,Y] = meshgrid(x,y);
20 [PHI,RHO] = cart2pol(X,Y);
21
22 Einc = exp(-1j*k*X);
23
24 n = (-M:M).';
25 jneg = (1j).^( -n);
26
27 ka = k*a;
28 kda = kd*a;
29
30 Jn_ka = besselj(n, ka);
31 Jn_kda = besselj(n, kda);
32
33 Jp_ka = 0.5*(besselj(n-1,ka) - besselj(n+1,ka));
34 Jp_kda = 0.5*(besselj(n-1,kda) - besselj(n+1,kda));
35

```

```

36 Hn_ka = besselh(n,2,ka);
37 Hp_ka = 0.5*(besselh(n-1,2,ka) - besselh(n+1,2,ka));
38
39 den = sqrt(mu_r).*Hp_ka.*Jn_kda - sqrt(eps_r).*Hn_ka.*Jp_kda;
40
41 an = -jneg .* (sqrt(mu_r).*Jp_ka.*Jn_kda - sqrt(eps_r).*Jn_ka.*Jp_kda) ./ den;
42 cn = (1j).^(-(n+1)) .* (2*sqrt(mu_r)) ./ (pi*ka .* den);
43
44 Esc_out = zeros(size(RHO));
45 Eint = zeros(size(RHO));
46
47 for ii = 1:numel(n)
48     ni = n(ii);
49     phase = exp(1j*ni*PHI);
50
51     Esc_out = Esc_out + an(ii) .* besselh(ni,2,k*RHO) .* phase;
52     Eint = Eint + cn(ii) .* besselj(ni, kd*RHO) .* phase;
53 end
54
55 Etotal = Einc + Esc_out;
56 inside = (RHO < a);
57 Etotal(inside) = Eint(inside);
58
59 figure('Color','w');
60 imagesc(x/lambda, y/lambda, real(Etotal));
61 axis equal tight;
62 set(gca,'YDir','normal');
63 xlabel('x/\lambda'); ylabel('y/\lambda');
64 caxis([-2 2]);
65 title('TM dielectric cylinder: Re{E_z^{tot}/E_0}');
66 colormap(jet)
67 colorbar;
68
69 hold on;
70 th = linspace(0,2*pi,600);
71 plot((a/lambda)*cos(th),(a/lambda)*sin(th),'k','LineWidth',1.2);
72 hold off;

```

Problem 2 — TM Case

```

1 % ECE 541 Fall 2025 Project 6
2 % problem 2 - TM Case
3
4 clc
5 clear
6
7 lambda = 1;
8 a = 1*lambda;
9 eps_r = 4.0;
10 mu_r = 1.0;
11
12 k = 2*pi/lambda;
13 kd = k*sqrt(eps_r*mu_r);
14
15 M = ceil(k*a + 15);
16
17 n = (-M:M).';

```

```

18 jneg = (1j).^( -n );
19
20 ka = k*a;
21 kda = kd*a;
22
23 Jn_ka = besselj(n, ka);
24 Jn_kda = besselj(n, kda);
25
26 Jp_ka = 0.5*(besselj(n-1,ka) - besselj(n+1,ka));
27 Jp_kda = 0.5*(besselj(n-1,kda) - besselj(n+1,kda));
28
29 Hn_ka = bessellh(n,2,ka);
30 Hp_ka = 0.5*(bessellh(n-1,2,ka) - bessellh(n+1,2,ka));
31
32 den = sqrt(mu_r).*Hp_ka.*Jn_kda - sqrt(eps_r).*Hn_ka.*Jp_kda;
33
34 an = -jneg .* (sqrt(mu_r).*Jp_ka.*Jn_kda - sqrt(eps_r).*Jn_ka.*Jp_kda) ./ den;
35
36 phi = linspace(0, 2*pi, 721);
37 ang = phi + pi/2;
38
39 E = exp(1j*(n*ang));
40 F = (an.' * E);
41
42 sigma2D = (4/k) * abs(F).^2;
43 sigma2D_norm = sigma2D / lambda;
44 sigma2D_dB = 10*log10(sigma2D_norm);
45
46 figure('Color','w');
47 plot(rad2deg(phi), sigma2D_dB, 'LineWidth', 1.6);
48 grid on;
49 xlim([0 360]);
50 xlabel('\phi (degrees)');
51 ylabel('\sigma_{2D}/\lambda (dB)');
52 title('TM dielectric cylinder: bistatic scattering width');

```