Circle: $x^2 + y^2 = r^2$

Line: $y = mx + b$

Goal: $x^2 + (mx+b)^2 = r^2$

$\hookrightarrow \; ax^2 + bx + c = 0$

1. Given 2 points $(x_1, y_1), (x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$b = \quad y = mx + b$$

$$y_1 = mx_1 + b$$

$$y_1 - mx_1 = b$$

2. Given circle data (x_3, y_3) r

$$(x - x_3)^2 + ((mx + \underbrace{b) - y_3}_{z})^2 = r^2$$

$$x^2 - 2x_3x + x_3^2 + m^2x^2 + 2mzx + z^2 - r^2 = 0$$

$$(m^2+1)x^2 + (2mz - 2x_3)x + (x_3^2 + z^2 - r^2) = 0$$

$$\underset{\substack{\uparrow \\ a}}{(m^2+1)}x^2 + \underset{\substack{\uparrow \\ b}}{(2m(b-y_3) - 2x_3)}x + \underset{\substack{\uparrow \\ c}}{(x_3^2 + (b-y_3)^2 - r^2)} = 0$$

3. if $b^2 - 4ac$

$$\left.\begin{array}{l} < 0 = \text{no} \\ = 0 = 1 \\ > 0 = 2 \end{array}\right\} \text{intersection(s)}$$