<u>Modèle</u> : $\hat{y} = \beta_0 + \beta_1 x$ <u>Dataset</u> : $\{(x_i, y_i) : i = 1, \dots, N\}$

On dérive $\mathrm{RSS}(\beta_0, \beta_1) = \sum_{i=1}^{N} (\hat{y}_i - y_i)^2$

$$= \sum_{i=1}^{N} (\beta_0 + \beta_1 x_i - y_i)^2$$

p.r. à β_0 et β_1.

$$\frac{\partial \mathrm{RSS}}{\partial \beta_0} = \frac{\partial}{\partial \beta_0} \sum (\beta_0 + \beta_1 x_i - y_i)^2$$

$$= \frac{\partial}{\partial \beta_0} \sum \Big(\beta_0 - \underbrace{(y_i - \beta_1 x_i)}_{:= z_i}\Big)^2$$

$$= \frac{\partial}{\partial \beta_0} \sum (\beta_0^2 - 2\beta_0 z_i + z_i^2)$$

$$= \sum (2\beta_0 - 2 z_i)$$

$$= 2N\beta_0 - 2\sum z_i = 0$$

ssi: $\hat{\beta}_0 = \frac{1}{N} \sum z_i = \frac{1}{N} \sum (y_i - \beta_1 x_i)$

$$= \frac{1}{N} (N\bar{y} - N\beta_1 \bar{x})$$

$$= \bar{y} - \hat{\beta}_1 \bar{x} \qquad (1)$$

$$\frac{\partial RSS}{\partial \beta_1} = \frac{\partial}{\partial \beta_1} \sum \left(\beta_0 + \beta_1 x_i - y_i \right)^2$$

$$= \frac{\partial}{\partial \beta_1} \sum \left(\beta_1 x_i - \underbrace{(y_i - \beta_0)}_{:= z_i} \right)^2$$

$$= \frac{\partial}{\partial \beta_1} \sum \left(\beta_1^2 x_i^2 - 2 \beta_1 x_i z_i + z_i^2 \right)$$

$$= \sum \left(2 \beta_1 x_i^2 - 2 x_i z_i \right)$$

$$= 2 \sum \left(\beta_1 x_i^2 - x_i (y_i - \beta_0) \right)$$

$$\overset{①}{=} 2 \sum \left(\beta_1 x_i^2 - x_i (y_i - \bar{y} + \beta_1 \bar{x}) \right)$$

$$= 2 \sum \left(\beta_1 (x_i^2 - x_i \bar{x}) - x_i (y_i - \bar{y}) \right)$$

$$= 2 \sum \beta_1 x_i (x_i - \bar{x}) - 2 \sum x_i (y_i - \bar{y})$$

$$= 0 \quad \text{ssi}$$

$$\beta_1 = \frac{\sum_{i=1}^{N} x_i (y_i - \bar{y})}{\sum_{i=1}^{N} x_i (x_i - \bar{x})} \qquad ②$$

Remarques :

$$\sum x_i(y_i - \bar{y}) = \sum(x_i y_i - x_i \bar{y})$$

$$= \sum(x_i y_i - x_i \bar{y}) \underbrace{- N\bar{x}\bar{y} + N\bar{x}\bar{y}}_{=0}$$

$$= \sum(x_i y_i - x_i \bar{y} - \bar{x} y_i - \bar{x}\bar{y})$$

$$= \sum (x_i - \bar{x})(y_i - \bar{y}) = S_{xy}$$

$$\sum x_i(x_i - \bar{x}) = \sum(x_i^2 - x_i \bar{x})$$

$$= \sum(x_i^2 - 2x_i\bar{x} + x_i\bar{x})$$

$$= \sum(x_i^2 - 2x_i\bar{x} + \bar{x}^2)$$

$$= \sum (x_i - \bar{x})^2 = S_{xx}$$