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Mathematical Modeling of Behavior
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Analysis of mode choice behavior in the London metropolitan area

Assignment 1

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Model 0 [2 points]

Start with a simple model specification. It should include:

1. alternative specific constants, and
2. cost and travel time of the different alternatives associated with generic parameters.

Report both the specification (i.e., the utility functions) and the estimates of the parameters. Comment on the estimation output (e.g., if the signs of the parameters match your expectations).

Solution

Assumption and Specification

In the base model, we assume that total travel time and travel cost are the only factors influencing the mode choice. We also assume that the coefficients of the explanatory variables are generic. Therefore, we define the deterministic parts of the utility functions by including the alternative specific constants (ASCs) and two attributes associated with generic parameters:

$$
\begin{aligned}
V_{\text{walk}} &= \text{ASC}_{\text{walk}} + \beta_{\text{time}} \cdot \text{time}_{\text{walk}} && (1)\\
V_{\text{cycle}} &= \text{ASC}_{\text{cycle}} + \beta_{\text{time}} \cdot \text{time}_{\text{cycle}} && (2)\\
V_{\text{pt}} &= \text{ASC}_{\text{pt}} + \beta_{\text{time}} \cdot \text{time}_{\text{pt}} + \beta_{\text{cost}} \cdot \text{cost}_{\text{pt}} && (3)\\
V_{\text{car}} &= \text{ASC}_{\text{car}} + \beta_{\text{time}} \cdot \text{time}_{\text{car}} + \beta_{\text{cost}} \cdot \text{cost}_{\text{car}} && (4)
\end{aligned}
$$

The parameters β_{cost} and β_{cost} are expressed in [GBP^{-1}] and [hour^{-1}] respectively.
Moreover, the alternative specific constant ASC_{walk} is arbitrarily chosen to be normalized to 0. This is because only the difference between the constants matters for the choice model. It will constitute the reference alternative for the ASCs and shall be used for the interpretation we will perform in the next paragraphs.

Estimation Outputs and Comment

In this section, we present the estimates of the 5 parameters (ASC_{walk} is fixed to 0) from the base model. As a result, the log-likelihood for the estimated parameters is given by:

$$\mathcal{L}_0 := \mathcal{L}(\hat{\beta}_{\text{model}\,0}) = -4638.610 \qquad (5)$$

Results are shown in Table 1. Firstly, we notice that the sign of coefficients for the travel time and the cost are both negative, indicating that utility perceived by the decision maker decreases with the increase of travel time or travel cost. Secondly, keeping in mind that $\text{ASC}_{\text{walk}} = 0$, we interpret the negative sign of every ASC as an intrinsic preference for the walk alternative when the other parts (parameter-dependent) of the utilities are equal. Especially, when $\text{time}_{\text{walk}} = \text{time}_{\text{cycle}}$, the model 0 predicts that the decision maker will rather choose the walk alternative than the bicycle one. In the same way, for identical travel time and cost between public transport and car alternatives, the model indicates a preference for public transport.
Finally, the value of time according to the model is given by

$$\text{VOT}_0 = \frac{\beta_{\text{time}}}{\beta_{\text{cost}}} = 30.05 \text{ GBP/hour} \tag{6}$$

and represents the price that a traveler is willing to pay to decrease the travel time in London.

	Value	Std err	t-test	p-value	Rob. Std err	Rob. t-test	Rob. p-value
ASC_{car}	-1.28	0.0721	-17.8	0.0	0.0817	-15.7	0.0
$\text{ASC}_{\text{cycle}}$	-3.82	0.101	-38.0	0.0	0.107	-35.7	0.0
ASC_{pt}	-0.533	0.0521	-10.2	0.0	0.0543	-9.81	0.0
β_{cost}	-0.182	0.0144	-12.6	0.0	0.0149	-12.2	0.0
β_{time}	-5.47	0.181	-30.1	0.0	0.216	-25.3	0.0

Table 1: Estimate of the parameters for Model 0

Model 1 [2 points]

Use Model 0 as the base model. Include alternative-specific parameters for at least one of the attributes of Model 0, and report both the specification and the estimates of the parameters. Answer to the following questions:

1. *What is the underlying assumption of defining alternative-specific parameters?*
2. *Comment on the estimation output.*
3. *Compare Model 0 and Model 1 with a statistical test. Which model is preferred and why? Denote the preferred model as Model* 1_{pref}.

Solution

Assumption and Specification

In the model 1, we will use alternative specific coefficients for the travel time. This is based on the assumption that people perceive a minute spent in one mode to be different from a minute spent in other mode. Therefore, we introduce different travel time coefficients for four different modes. We would expect that if the decision maker prefers spending one minute in mode 1 than in mode 2 (because of comfort, space, ...), then $\beta_{\text{time},1} > \beta_{\text{time},2}$.

Therefore, the utility functions for this model are given by:

$$V_{\text{walk}} = \text{ASC}_{\text{walk}} + \beta_{\text{time, walk}} \cdot \text{time}_{\text{walk}} \tag{7}$$
$$V_{\text{cycle}} = \text{ASC}_{\text{cycle}} + \beta_{\text{time, cycle}} \cdot \text{time}_{\text{cycle}} \tag{8}$$
$$V_{\text{pt}} = \text{ASC}_{\text{pt}} + \beta_{\text{time, pt}} \cdot \text{time}_{\text{pt}} + \beta_{\text{cost}} \cdot \text{cost}_{\text{pt}} \tag{9}$$
$$V_{\text{car}} = \text{ASC}_{\text{car}} + \beta_{\text{time, car}} \cdot \text{time}_{\text{car}} + \beta_{\text{cost}} \cdot \text{cost}_{\text{car}} \tag{10}$$

Estimation Output and Comment

The estimation output is shown in Table 2. The log-likelihood for the estimated parameters in model 1 is:

$$\mathcal{L}_1 := \mathcal{L}(\hat{\beta}_{\text{model 1}}) = -4360.112 \tag{11}$$

Here again, the coefficients for the explanatory variables have negative sign and the specification of the model does not contradict with the interpretation performed in model 0. However, comparing the β_{time} between alternatives allows us to identify some tendencies in the mode choice. Indeed, $\beta_{\text{time, walk}} = -8.14\text{h}^{-1}$ is the most negative coefficient and seems to indicate that the willingness to choose the walk alternative decreases even more with travel time than for other alternatives. Moreover, it is interesting to observe that $\beta_{\text{time, car}} < \beta_{\text{time, cycle}} < \beta_{\text{time, pt}}$ which demonstrates that one minute spent in the car in London is perceived to be less accommodating than one spent in other alternatives such as bicycle and public transport. We will investigate such perceptions when the driving traffic data is taken into account in model 2.

	Value	Std err	t-test	p-value	Rob. Std err	Rob. t-test	Rob. p-value
ASC_{car}	-1.9	0.104	-18.3	0.0	0.149	-12.8	0.0
$\text{ASC}_{\text{cycle}}$	-4.74	0.167	-28.4	0.0	0.199	-23.8	0.0
ASC_{pt}	-2.25	0.111	-20.2	0.0	0.149	-15.1	0.0
β_{cost}	-0.165	0.0158	-10.5	0.0	0.0167	-9.9	0.0
$\beta_{\text{time, car}}$	-6.28	0.332	-18.9	0.0	0.378	-16.6	0.0
$\beta_{\text{time, cycle}}$	-5.12	0.401	-12.8	0.0	0.404	-12.7	0.0
$\beta_{\text{time, pt}}$	-3.52	0.234	-15.0	0.0	0.247	-14.3	0.0
$\beta_{\text{time, walk}}$	-8.14	0.288	-28.2	0.0	0.456	-17.8	0.0

Table 2: Estimate of the parameters for Model 1

Statistical Test of Model 0 and Model 1

In this section, we aim to compare model 0 and 1 with a statistical test: the likelihood ratio test. Indeed, the definition of alternative specific parameters certainly improves the likelihood, but the test comes to evaluate whether this improvement is statistically significant. Keeping in mind the trade-off between the model complexity and its accuracy, we perform a statistical comparison between model 0 and model 1.
The null hypothesis is that the definition of alternative specific parameters for travel time is meaningless:

$$H_0 : \beta_{\text{time, walk}} = \beta_{\text{time, cycle}} = \beta_{\text{time, pt}} = \beta_{\text{time, car}} := \beta_{\text{time}}$$

Labelling the model 0 as the restricted (R) model and the model 1 as the unrestricted (U) one, it can be shown that under H_0, $-2(\mathcal{L}_R - \mathcal{L}_U) \sim \chi^2_{(K_U - K_R)}$ where K denotes the number of parameters in a model. Therefore, according to the likelihood ratio test, we will reject H_0 with level of confidence $(1-\alpha)$ if:

$$-2(\mathcal{L}_R - \mathcal{L}_U) > \chi_{(1-\alpha,\, df)} \tag{12}$$

where $df = K_U - K_R$.
Here, we obtain $-2(\mathcal{L}_R - \mathcal{L}_U) = 557$ and for $df = 3$, the critical value for $\chi_{(0.999,\,3)}$ is 16.27[1]. Therefore, we reject H_0 (and thus the generic parameter hypothesis) with a level of confidence of 0.999. We conclude that the use of alternative specific coefficients for travel time significantly improves the model: we keep model 1.

Other possible specifications (rejected)

Moreover, another model specification would have been the definition of alternative specific coefficients for travel cost β_{cost}. Indeed, an estimation of the parameters have been made for this hypothetical model (where β_{time} is set to be generic) and we found:

$$\beta_{\text{cost, car}} = -0.234, \quad \beta_{\text{cost, pt}} = 0.0122$$

Although the log likelihood obtained showed a good improvement ($-2(\mathcal{L}_R - \mathcal{L}_U) = 88.8 \gg \chi_{(0.999,\,1)}$ where R is model 0), we decided not to keep this specification for the following reasons:

- The positive sign of parameter $\beta_{\text{cost, pt}}$ does not behaviorally make sense. It indicates that utility for pt perceived by the decision maker increases with its cost.
- Even more than time, cost is considered as a resource and we assume that it should not matter in which mode you spend it.
- Also, as the cost variable only appears for the car and pt alternatives, this specification does not provide a global improvement for the mode choice. It increases the complexity without adding more information for the walk and bicycle utilities.
- Finally, keeping in mind that the data set provides other explanatory variables such as socioeconomic parameters, we aim to limit the complexity of the first model, which will be the starting point of models 2 and 3.

Model 2 [3 points]

Use Model 1_{pref} as the base model. Include at least an additional attribute and at least one interaction of a socioeconomic variable with either the ASCs or one of the attributes. Report both the specification and the estimates of the parameters.
Answer to the following questions:

1. *What is the underlying assumption of the included attribute(s) and interaction(s)?*
2. *Comment on the estimation output.*
3. *Compare Model 1_{pref} and Model 2 with a statistical test. Which model is preferred and why? Denote the preferred model as Model 2_{pref}.*

[1] www.di-mgt.com.au/chisquare-table.html

Solution

Assumption and Specification

In model 2, we consider the driving traffic percent (dtp) as our additional attribute in the drive alternative utility. The underlying assumption is that the traffic variability has a negative impact on the willingness to drive for the decision maker. Moreover, we will consider age as an explanatory socioeconomic variable, because people in different age groups may make different mode choices. As an example, when the decision maker gets older, we expect a decrease in utilities for modes involving physical activities (walk, bicycle). As usual, a normalization is done for parameters which value does not vary between alternatives such as the age (normalization with respect to walk mode). Thus, we modify our utility functions as follows.

$$
\begin{aligned}
V_{\text{walk}} &= \text{ASC}_{\text{walk}} + \beta_{\text{time, walk}} \cdot \text{time}_{\text{walk}} && (13)\\
V_{\text{cycle}} &= \text{ASC}_{\text{cycle}} + \beta_{\text{time, cycle}} \cdot \text{time}_{\text{cycle}} + \beta_{\text{age}} \cdot \text{age} && (14)\\
V_{\text{pt}} &= \text{ASC}_{\text{pt}} + \beta_{\text{time, pt}} \cdot \text{time}_{\text{pt}} + \beta_{\text{cost}} \cdot \text{cost}_{\text{pt}} \\
&\quad + \beta_{\text{age}} \cdot \text{age} && (15)\\
V_{\text{car}} &= \text{ASC}_{\text{car}} + \beta_{\text{time, car}} \cdot \text{time}_{\text{car}} + \beta_{\text{cost}} \cdot \text{cost}_{\text{car}} \\
&\quad + \beta_{\text{dtp}} \cdot \text{dtp} + \beta_{\text{age}} \cdot \text{age} && (16)
\end{aligned}
$$

Estimation Output and Comment

The estimation output is shown in Table 3. The log-likelihood for the estimated parameters in model 2 is:

$$\mathcal{L}_2 := \mathcal{L}(\hat{\beta}_{\text{model 2}}) = -4258.636 \quad (17)$$

The coefficient of age variable β_{age} is positive which reflects a preference of older individuals for other alternatives with respect to walking. Concerning driving traffic percentage, the negative sign means that traffic variability is negatively perceived by decision makers when considering the car alternative. This is consistent with our behavioral expectations.

	Value	Std err	t-test	p-value	Rob. Std err	Rob. t-test	Rob. p-value
ASC_{car}	-1.784	0.147	-12.120	0.0	0.185	-9.644	0.0
$\text{ASC}_{\text{cycle}}$	-5.169	0.194	-26.590	0.0	0.228	-22.644	0.0
ASC_{pt}	-2.806	0.150	-18.670	0.0	0.184	-15.227	0.0
β_{cost}	-0.130	0.017	-7.727	0.0	0.018	-6.996	0.0
$\beta_{\text{time, car}}$	-4.209	0.345	-12.188	0.0	0.374	-11.268	0.0
$\beta_{\text{time, cycle}}$	-4.576	0.402	-11.390	0.0	0.402	-11.384	0.0
$\beta_{\text{time, pt}}$	-2.961	0.234	-12.672	0.0	0.240	-12.352	0.0
$\beta_{\text{time, walk}}$	-8.104	0.291	-27.831	0.0	0.464	-17.451	0.0
β_{dtp}	-2.989	0.226	-13.226	0.0	0.231	-12.914	0.0
β_{age}	0.010	0.002	4.201	0.0	0.002	4.374	0.0

Table 3: Estimate of the parameters for Model 2

Statistical Test of Model 1 and Model 2

In this part, we use likelihood ratio test to compare model 1 and model 2. The null hypothesis is that:

$$H_0 : \beta_{\text{age}} = \beta_{\text{dtp}} = 0$$

As usual, $-2(\mathcal{L}_1 - \mathcal{L}_2) \sim \chi^2_{(K_2-K_1)}$. The degree of freedom is $K_2 - K_1 = 2$. We set the level of confidence to be 0.999. In this case, we compute that

$$-2(-4360.112 + 4258.636) = 202.952 > \chi^2_{(0.999,2)} = 13.816 \tag{18}$$

Based on the test, we reject H_0 with a level of confidence 0.999. Therefore, model 2 is better than model 1 and we shall continue with model 2 as 2_{pref} .

Other possible specifications (rejected)

In this section, we consider another possible specification for the model, which is the use of the distance as an explanatory variable. This variable is defined as the straight line distance between trip origin and destination in metres. To do so, we added from model 1 a term $\beta_{\text{dist}} \cdot \text{dist}$ to the utility functions except for the walk alternative for which it is normalized to 0. Indeed, since the distance does not vary between the alternatives and only differences in utilities matter, we need to normalize one alternative to zero.
As a result, we obtained $\beta_{\text{dist}} = 0.382\text{km}^{-1}$ and a log-likelihood of $\mathcal{L}(\hat{\beta}) = -4359.033$. The sign of the coefficient makes sense since it is normalized with respect to the walk alternative: when the distance increases, all alternatives but the walk one are increasing their utility. However, when it comes to compare this model with model 1 (Eq.11), we obtain the following statistic:

$$-2(-4360.112 + 4359.033) = 2.158 < \chi^2_{(0.999,1)} = 10.83$$

Therefore, we cannot reject the hypothesis that $\beta_{\text{dist}} = 0$ with the level of significance we chose (99.9%). That is why the distance parameter does not appear in the model 2 we constructed.

Model 3 [3 points]

Use Model 2_{pref} as the base model. Include at least one appropriate non-linear specification for one of the variables, and report both the specification and the estimates of the parameters. Answer to the following questions:

1. *What is the underlying assumption of the included non-linear specification(s)?*
2. *Comment on the estimation output.*
3. *Compare Model 2_{pref} and Model 3 with a statistical test. Which model is preferred and why?*

Solution

Assumption and Specification

In this part, non-linear specifications for the explanatory variables are tested in order to improve the model. These specifications were performed and tested independently and those of which results were good enough were kept for defining model 3. As a result, we found three appropriate non linear specifications:

- a Piecewise specification over the walking time,
- a Piecewise specification over the driving time,
- a Box-Cox transform performed over the public-transport cost.

In addition a Box-Cox transform which did not sufficiently improve the model is studied at the end of this part.

We define both piecewise thresholds to be the expectation value of transport time (respectively walking and driving time) provided the associated mode was chosen. This means that we compute the mean of the walking time variable restricted to the individuals who chose the walk alternative. As a result, we found as transport time means: $\hat{w} = 0.1832$ h , $\hat{d} = 0.436$ h for walking and driving alternatives respectively (which is very different from 1.13h and 0.28h given in the data set description). In the piecewise specification, the non-linearity comes from the fact that multiple β's are defined and act on different domains of the variable. The utility functions are based on model 2 but include different definitions for $\beta_{\text{time,walk}}$ and $\beta_{\text{time,drive}}$:

$$\beta_{\text{time,walk}} = \begin{cases} \beta_{\text{time,walk}>0.1832} & \text{if} \quad \text{walk\_time} > 0.1832 \\ \beta_{\text{time,walk}<0.1832} & \text{if} \quad \text{walk\_time} < 0.1832 \end{cases} \tag{19}$$

$$\beta_{\text{time,car}} = \begin{cases} \beta_{\text{time,car}>0.436} & \text{if} \quad \text{car\_time} > 0.436 \\ \beta_{\text{time,car}<0.436} & \text{if} \quad \text{car\_time} < 0.436 \end{cases} \tag{20}$$

These specifications are based on the assumption that people perceive a minute spent in a mode differently if they are travelling for a long time or if their travel only lasts a few minutes.

Estimation Output and Comment

The result for the walking time (Eq.(19) specification) is given in Table 4 and the corresponding likelihood is:

$$\mathcal{L}_{3,1} := \mathcal{L}(\hat{\beta}_{\text{model}\,3,1}) = -4239.868 \tag{21}$$

The likelihood for the driving time (Eq.(20) specification) is given by:

$$\mathcal{L}_{3,2} := \mathcal{L}(\hat{\beta}_{\text{model}\,3,2}) = -4243.796 \tag{22}$$

As one can see, there is an improvement in comparison to the value of the log-likelihood for model 2 (Eq.(17)). Moreover, the piecewise specification doesn't change the sign of the coefficient but only their amplitudes. As expected, adding a few minutes to the travel time is perceived to be less a disadvantage when the travel is already long. That is why we obtain a lower amplitude in β_{time} when the travel time threshold is already reached.

However this difference between amplitudes must depend on the transport because the comfort is not the same, as well as the threshold for the travel time. By defining the mean gap (MG) as the ratio of the β's before and after the threshold, we can compute this quantity for the drive and the walk alternatives and obtain $\text{MG}_{\text{walk}} = 3.42$, $\text{MG}_{\text{car}} = 2.09$. We notice that $\text{MG}_{\text{walk}} > \text{MG}_{\text{car}}$. This result can be interpreted as an implicit demonstration of the comfort parameter. Indeed, the negative contribution of time (in utility) increases more rapidly for the walk alternative as soon as the threshold is reached.

	Value	Std err	t-test	p-value	Rob. Std err	Rob. t-test	Rob. p-value
$\mathrm{ASC}_{\mathrm{cycle}}$	-8.02	0.577	-13.9	0.0	0.602	-13.3	0.0
$\mathrm{ASC}_{\mathrm{car}}$	-4.64	0.563	-8.24	2.22e-16	0.588	-7.89	2.89e-15
$\mathrm{ASC}_{\mathrm{pt}}$	-5.65	0.562	-10.0	0.0	0.585	-9.66	0.0
β_{age}	0.00928	0.0023	4.03	5.57e-05	0.00223	4.16	3.22e-05
β_{cost}	-0.129	0.0168	-7.65	2.02e-14	0.0185	-6.95	3.69e-12
β_{dtp}	-2.96	0.227	-13.1	0.0	0.233	-12.7	0.0
$\beta_{\mathrm{time,\,cycle}}$	-4.59	0.402	-11.4	0.0	0.403	-11.4	0.0
$\beta_{\mathrm{time,\,car}}$	-4.25	0.346	-12.3	0.0	0.375	-11.3	0.0
$\beta_{\mathrm{time,\,pt}}$	-2.99	0.235	-12.8	0.0	0.241	-12.4	0.0
$\beta_{\mathrm{time,\,walk}>0.1832}$	-7.31	0.301	-24.3	0.0	0.479	-15.3	0.0
$\beta_{\mathrm{time,\,walk}<0.1832}$	-25.0	3.16	-7.91	2.44e-15	3.4	-7.35	1.95e-13

Table 4: Estimate of the parameters in model 3 using piecewise method for walk time

	Value	Std err	t-test	p-value	Rob. Std err	Rob. t-test	Rob. p-value
$\mathrm{ASC}_{\mathrm{cycle}}$	-5.3	0.2	-26.5	0.0	0.239	-22.1	0.0
$\mathrm{ASC}_{\mathrm{car}}$	-1.72	0.148	-11.7	0.0	0.184	-9.35	0.0
$\mathrm{ASC}_{\mathrm{pt}}$	-2.89	0.152	-19.0	0.0	0.189	-15.3	0.0
β_{age}	0.00946	0.00227	4.16	3.15e-05	0.00218	4.33	1.47e-05
β_{cost}	-0.13	0.0168	-7.71	1.27e-14	0.0185	-7.02	2.2e-12
β_{dtp}	-2.57	0.235	-10.9	0.0	0.236	-10.9	0.0
$\beta_{\mathrm{time,\,cycle}}$	-5.19	0.439	-11.8	0.0	0.456	-11.4	0.0
$\beta_{\mathrm{time,\,walk}}$	-8.52	0.303	-28.1	0.0	0.485	-17.6	0.0
$\beta_{\mathrm{time,\,pt}}$	-3.48	0.254	-13.7	0.0	0.26	-13.4	0.0
$\beta_{\mathrm{time,\,car}>0.436}$	-3.17	0.37	-8.58	0.0	0.377	-8.43	0.0
$\beta_{\mathrm{time,\,car}<0.436}$	-6.63	0.565	-11.7	0.0	0.578	-11.5	0.0

Table 5: Estimate of the parameters in model 3 using piecewise method for drive time

Box-Cox for public transport cost

Next, we make the assumption that a non linear specification for the public transport cost is a relevant because if the ticket price is low, an increase on its price will have a big consequence. In contrary, if the travel cost is high, only few peoples would afford it and an increase would less affect the willingness to pay. Therefore, a Box-Cox transform is performed to estimate the non-linearity of the public transport cost. We base the model on model 2 except for the utility of the public transport which is given by:

$$V_{\mathrm{pt}} = \mathrm{ASC}_{\mathrm{pt}} + \beta_{\mathrm{time,pt}} \cdot \mathrm{time}_{\mathrm{pt}} + \beta_{\mathrm{cost}} \cdot \left(\frac{\mathrm{cost}^{\lambda} - 1}{\lambda} \right) + \beta_{\mathrm{age}} \cdot \mathrm{age} + \beta_{\mathrm{dtp}} \cdot \mathrm{dtp} \tag{23}$$

The results are given in Table 6, and the log-likelihood is:

$$\mathcal{L}_{3,3} = -4235.71 \tag{24}$$

Here again, we obtain an improvement compared to the likelihood of model 2. One can notice that $\lambda < 1$ meaning the transformation is concave, which corresponds to a significant variation for little value but almost no variation when the cost increases. This is consistent with our expectation about the individual's behavior relative to a change in ticket price.

	Value	Std err	t-test	p-value	Rob. Std err	Rob. t-test	Rob. p-value
$\text{ASC}_{\text{cycle}}$	-5.19	0.196	-26.5	0.0	0.231	-22.5	0.0
ASC_{car}	-1.75	0.148	-11.8	0.0	0.186	-9.44	0.0
ASC_{pt}	-3.16	0.155	-20.3	0.0	0.188	-16.8	0.0
β_{age}	0.00931	0.00229	4.06	4.89e-05	0.00221	4.21	2.6e-05
β_{cost}	-0.0932	0.0174	-5.35	9.02e-08	0.0173	-5.39	7.16e-08
β_{dtp}	-3.19	0.23	-13.9	0.0	0.236	-13.5	0.0
$\beta_{\text{time, cycle}}$	-4.65	0.409	-11.3	0.0	0.413	-11.3	0.0
$\beta_{\text{time, car}}$	-4.39	0.359	-12.2	0.0	0.388	-11.3	0.0
$\beta_{\text{time, pt}}$	-3.08	0.236	-13.1	0.0	0.244	-12.6	0.0
$\beta_{\text{time, walk}}$	-8.18	0.293	-28.0	0.0	0.466	-17.5	0.0
λ	0.147	0.0342	4.3	1.7e-05	0.0332	4.43	9.47e-06

Table 6: Estimate of the parameters in model 3 using Box Cox for the public transport cost

Statistical Test of Model 2 and Model 3

Finally, we define our last model to be composed of the 3 non-linear specification we studied. This is model 3. As usual, a likelihood ratio test is performed in order to evaluate the relevance of model 3 compared to model 2. The results of the model are shown in Table 7. The log-likelihood for model 3 is:

$$\mathcal{L}_3 = -4197.897 \tag{25}$$

Thus, according to the likelihood ratio test (Eq(26)), the null hypothesis $\{H_0$: model 2 is a more accurate model$\}$ can be rejected with a level of confidence 0.999. Therefore, model 3 is a better than model 2 and the added changes are statistically significant.

$$-2(-4258.636 + 4197.897) = 121.478 \gg \chi_{(0.999,3)} = 16,2662 \tag{26}$$

	Value	Std err	t-test	p-value	Rob. Std err	Rob. t-test	Rob. p-value
$\mathrm{ASC}_{\mathrm{cycle}}$	-8.18	0.581	-14.1	0.0	0.609	-13.4	0.0
$\mathrm{ASC}_{\mathrm{car}}$	-4.52	0.565	-8.01	1.11e-15	0.592	-7.65	2.07e-14
$\mathrm{ASC}_{\mathrm{pt}}$	-6.1	0.566	-10.8	0.0	0.588	-10.4	0.0
β_{age}	0.00897	0.00233	3.85	0.00012	0.00228	3.94	8.3e-05
β_{cost}	-0.0934	0.0174	-5.38	7.43e-08	0.017	-5.49	4.01e-08
β_{dtp}	-2.68	0.239	-11.2	0.0	0.241	-11.1	0.0
$\beta_{\mathrm{time,\,cycle}}$	-5.41	0.452	-12.0	0.0	0.478	-11.3	0.0
$\beta_{\mathrm{time,\,pt}}$	-3.71	0.257	-14.4	0.0	0.264	-14.0	0.0
λ	0.141	0.0322	4.37	1.24e-05	0.0311	4.53	5.96e-06
$\beta_{\mathrm{time,\,car}>0.436}$	-3.22	0.379	-8.48	0.0	0.381	-8.45	0.0
$\beta_{\mathrm{time,\,car}<0.436}$	-7.27	0.578	-12.6	0.0	0.592	-12.3	0.0
$\beta_{\mathrm{time,\,walk}>0.1832}$	-7.88	0.315	-25.0	0.0	0.504	-15.6	0.0
$\beta_{\mathrm{time,\,walk}<0.1832}$	-25.5	3.17	-8.03	8.88e-16	3.41	-7.46	8.37e-14

Table 7: Estimate of the parameters in model 3

Other possible specifications (rejected)

In this section, a case where the null hypothesis of linear specification cannot be rejected is presented. We proceed to a Box-Cox transform over the drive traffic percentage parameter Thus the utilities are based on model 2 despite the one of the car which is given by:

$$V_{\mathrm{car}} = \mathrm{ASC}_{\mathrm{car}} + \beta_{\mathrm{time,\,car}} \cdot \mathrm{time}_{\mathrm{car}} + \beta_{\mathrm{cost}} \cdot \mathrm{cost}_{\mathrm{car}} + \beta_{\mathrm{age}} \cdot \mathrm{age} + \beta_{\mathrm{dtp}} \cdot \left(\frac{\mathrm{dtp}^{\lambda} - 1}{\lambda} \right) \tag{27}$$

The null hypothesis is H_0: λ=1. Under this hypothesis, one have a linear specifications in the driving traffic percentage. Therefore the likelihood is the one of model 2, that is $\mathcal{L}_R = -4258.636$. Using the Box-Cox transform, we obtain $\lambda = 1.26$ and $\mathcal{L}_U = -4257.612$. We perform a likelihood ratio test with $df = 1$ (because only the λ parameter is added) and obtain the result given Eq(28). As a result, we cannot reject the hypothesis λ=1 with the level of significance chosen (90%). This seems relevant because the unrestricted model finds a λ close to 1, so if linearity is not the exact relation, it remains a good approximation for the choice model.

$$-2(\mathcal{L}_R - \mathcal{L}_U) = 2.048 < 2.7055 = \chi^2_{(0.9,1)} \tag{28}$$