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INVITED PAPER

LARGE N AS A CLASSICAL LIMIT ($1/N \approx \hbar$) OF MIXED VALENCE

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Earlier work has shown that the infinite U Anderson model for mixed valence is contained within a broader 'slave boson' field theory. For large spin degeneracy N, the slave boson theory behaves classically with $1/N$ playing the role of Planck's constant. Thermal equilibrium is studied in this classical limit. A 2nd order phase transition seperates weak coupling from a strong coupling regime where the boson field has a definite vacuum expectation value, forming a heavy band in the lattice. Quantum fluctuations at finite N convert this phase transition into a smooth crossover characterized by the development of power law correlations in the Bose field.

1. Introduction

Previous work on the infinite U Anderson model of a Mixed Valence (MV) impurity has shown the utility of large N expansions for the construction of variational groundstates [1] and the best resummations of perturbation series [2–4]. However, there is room for discomfort with the general application of this technique as it stands, for there is no overiding guidance as to when, or if the $1/N$ expansion is working and no convincing extension of the work to the lattice which explicitly incorporates translational invariance.

One aspect of large degeneracy with considerable bearing on these concerns has not previously been exploited. Instead of making an ad hoc expansion in the small quantity $1/N$, one can use a general result that in quantum systems with N equivalent degrees of freedom, large N corresponds to a quasiclassical limit [5,6] where $1/N$ plays the role of Planck's constant and operators become sharp, evolving according to simple differential equations. Such limits have already played an important role in the understanding of quantum spin systems and superconductivity [7,8]. Quantum fluctuations about this classical limit are a $1/N$ effect which provide the basis for a reliable $1/N$ expansion of static and dynamic properties at finite N.

The essential ingredient of a classical large N limit is a Hamiltonian which is extensive in N: $H(N) = NH_{\rm cl}(A_\alpha)$, so that as $N \to \infty$, the energy cost of fluctuations grows extensively, suppressing fluctuations $\langle \partial A_\alpha^2 \rangle = \mathcal{O}(1/N)$. In particular, if $H(N) = NH_{\rm cl}(A_\alpha)$ can be expressed as a function of a set of operators $\{A_\alpha\}$ which form a closed Lie algebra $[A_\alpha, A_\beta] = (1/N) f_{\alpha\beta\gamma} A_\gamma$ (c.f $[q_i, p_j] = \hbar(\mathrm{i}\delta_{ij})1$), then this quarantees the existence of a complete set of "coherent states" $|z\rangle$ [5], analagous to the wavepackets of the $\hbar \to 0$ limit. Expectation values in these states become sharp classical variables ($\langle AB \rangle = \langle A \rangle \langle B \rangle + \mathcal{O}(1/N)$) in the $N \to \infty$ limit whose time evolution is governed by a set of 1st order differential equations containing no N dependence. The challange is to find such a limit appropriate to MV.

This study takes advantage of a reformulation of the infinite U Anderson model in terms of a broader 'Slave Boson Field Theory' (SBFT) [9] to avoid technical difficulties of the standard representation. In the SBFT the singlet state of the MV ion is represented by a slave boson field b and the spinning valence state by a f electron field

$$b^\dagger |0\rangle = |f^n : j = 0\rangle, \tag{1}$$

$$f_m^\dagger |0\rangle = |f^{n+1} : jm\rangle. \tag{2}$$

A mixing term in the Hamiltonian

$$\hat{H}_{\rm mix}^{(i)} = \sum_{k,m=-j}^{m=j} \left\{ \frac{V(k)}{\sqrt{N}} c_{km}^{i\dagger} f_m^i b^{i\dagger} + \text{H.C.} \right\} \tag{3}$$

hybridizes the f-state at site i with a free electron partial wave state in the f channel $c_{km}^{i\dagger}|0\rangle = |\boldsymbol{R}^i : E(k),\ jm\rangle$. This term models valence fluctuations $f^{n+1} \rightleftharpoons f^n + \mathrm{e}^-$, conserving the 'charge' $\hat{Q} = \hat{n}_f^i + \hat{b}^{i\dagger} b^i$, where $\hat{n}_f^i$ is the f occupation at site i. The fluctuating Bose field describes the instantaneous availability of the MV ion for hybridization. The subspace of states relevant to MV is the eigenstates of the $\hat{Q}^i$ with quantum number $Q^i = 1$ at each site: $|\psi_{\rm MV}\rangle = |\psi : Q^1, Q^2 \ldots Q^r = 1\rangle$.

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In a real MV system, spin-half band electrons couple to spin-j f electrons introducing awkward Clebsch–Gordan coefficients. To study the classical limit this unnecessary complication is initially avoided by studying a model where all electrons have spin j. A realistic angular momentum structure is readily introduced into the equations at a later stage. For spin j electrons the SBFT Hamiltonian is written

$$\hat{H} = \sum_{km} E(k) c^{\dagger}_{km} c_{km} + \sum_{i,m} E_f f^{i\dagger}_m f^i_m + \sum_i \left[\hat{H}^i_{\rm mix} + \lambda^i (\hat{Q}^i - Q_0) \right], \tag{4}$$

where $c^{\dagger}_{km}$ creates a spin-j band electron of momentum $\boldsymbol{k}$ from which the partial wave states are constructed $c^{j\dagger}_{km} = \int d\Omega_{\hat{k}} \exp(-i\boldsymbol{k}\cdot\boldsymbol{R}^j) c^{\dagger}_{km}$. The λ^i are chemical potentials associated with the conserved charges $\hat{Q}^i$ and $Q_0 = 1$ for MV.

2. Classical behaviour of the SBFT

Classical variables of the SBFT are identified as the rescaled Bose fields

$$\tilde{b}^i = (1/\sqrt{N}) b^i, \quad \tilde{b}^{i\dagger} = (1/\sqrt{N}) b^{i\dagger}, \tag{5}$$

and the fermion bilinears

$$\hat{O}_{\alpha\beta} = \frac{1}{N} \sum_m \psi^{\alpha\dagger}_m \psi^{\beta}_m, \tag{6}$$

where $\psi^{\alpha}_m = \{c_{km}, f_m\}$ denote Fermi fields. These operators commute to order $(1/N)$, forming a closed Lie algebra as can be easily verified. The Hamiltonian is an extensive function of these operators $\hat{H}(N) = N H_{\rm cl}[\hat{O}_{\alpha\beta}, \boldsymbol{b}_{\alpha}, \boldsymbol{b}^{\dagger}_{\alpha}]$ where

$$\hat{H}_{\rm cl} = \sum_{\boldsymbol{k}} E(k) \hat{\mathcal{N}}_{\boldsymbol{k}} + \sum_i E_f \hat{\mathcal{N}}^f_i + \sum_{i,k} \left\{ \left[V(k) \hat{\mathcal{M}}_i(k) \tilde{b}^{i\dagger} + \text{H.C.} \right] + \lambda^i (\hat{q}^i - q_0) \right\}. \tag{7}$$

Here $\hat{\mathcal{N}}^f_i = (\hat{n}^{\dagger}_{fm})_m$, $\hat{\mathcal{N}}_{\boldsymbol{k}} = (\hat{n}_{\boldsymbol{k}m})_m$, $\hat{\mathcal{M}}_i(k) = (c^{i\dagger}_{km} f^i_m)_m$, where $(\hat{A}_m)_m = (1/N)\sum \hat{A}_m$ denotes an 'average' over spin indices. The $\hat{q}^i = \hat{Q}^i/N = \hat{\mathcal{N}}^f_i + \tilde{b}^{i\dagger}\tilde{b}^i$ are rescaled charges which must behave classically as $N \to \infty$.

Classical behaviour of the SBFT is therefore approached by taking $N \to \infty$ with $Q^i/N = q_0$ fixed. As this limit is approached the rescaled groundstate energies of the subspaces of different Q, $E_0(N, q) = \langle 0|\hat{H}_{\rm cl}|0\rangle_{Q,N}$ will form a continuum. Thus to study the properties of a MV system of finite degeneracy, $j = \frac{5}{2}$ for instance, one must expand about the classical solution with $q_0 = 1/(2j+1) = \frac{1}{6}$ in this case. Thus

$$\langle \hat{H}_{\rm cl} \rangle_{N=6, Q=1} = E_{\rm cl}(q_0)|_{q0} = \tfrac{1}{6} + \mathcal{O}(1/N), \tag{8}$$

where $E_{\rm cl} = \langle \hat{H}_{\rm cl} \rangle |_{N=6}$.

q_0 is a physically important parameter describing the maximum available filling factor of the f-states. To preserve intersite couplings this parameter must remain finite. Old procedures took $N \to \infty$ with $Q = 1$, suppressing q_0 to zero, thereby losing intersite interactions and the important intersite quantum coherence of the f-states.

The classical $N \to \infty$ limit of the SBFT is interpreted physically as an electron gas moving in the potential of a classical Bose field. This time dependent scattering potential may be loosely associated with the effects of valence fluctuations. Time evolution of the Bose field is determined by the limiting quantum equations of motion

$$-i\frac{\partial b^i}{\partial t} = \lim_{N\to\infty} \langle N \left[\hat{H}_{\rm cl}, \tilde{b}^i \right] \rangle = \lambda^j b^j + \sum_{j,k} V(k) \mathcal{M}_j(k), \tag{9}$$

where $b^i = \langle \tilde{b}^i \rangle$, $\mathcal{M}_j(k) = \langle \hat{\mathcal{M}}_j(k) \rangle$.

Finite temperature properties are determined by the corresponding periodic solutions of imaginary time $\tau = it$ $(-i\partial/\partial t \to \partial/\partial\tau)$ with period $\Delta\tau = \beta$. The chemical potentials λ^i can then be adjusted to seek a static solution $b^i(\tau) =$ constant, determined by

$$\lambda^j b^i + \sum_{j,k} V(k) \mathcal{M}_j(k) = 0. \tag{10}$$

This solution must uniquely define thermal equilibrium, up to phase changes in the fields. The rigid Bose field then determines a renormalised Hamiltonian that governs the electron motions

$$\mathcal{H} = \sum_m \left\{ \sum_{\boldsymbol{k}} E(k) c^{\dagger}_{\boldsymbol{k}m} c_{\boldsymbol{k}m} + \sum_j \tilde{E}^j_f f^{j\dagger}_m f^j_m + \sum_{j,\boldsymbol{k}} \left[\tilde{V}^j(k) c^{\dagger}_{\boldsymbol{k}m} f^j_m e^{-i\boldsymbol{k}\cdot\boldsymbol{R}^j} + \text{H.C.} \right] \right\}, \tag{11}$$

where $\tilde{E}^j_f = E_f + \lambda^j$ and $\tilde{V}^j(k) = V(k) b^j$ are renormalized quantities. The effect of a magnetic field can be introduced in an ad hoc fashion at this point by replacing the $\tilde{E}_f$ by $\tilde{E}_f - (g/N) mB = \tilde{E}_{fm}$. From the one-particle thermal Green's functions of theis Hamiltonian one determines the bilinears $O_{\alpha\beta} = \langle \hat{O}_{\alpha\beta} \rangle$

$$0_{\alpha\beta} = \int_{-\infty}^{\infty} \frac{d\omega}{\pi} f(\omega) \, \text{Im} \langle \psi^{\alpha}_m(\omega - i\delta) \psi^{\beta\dagger}_m(\omega - i\delta) \rangle. \tag{12}$$

Finally, the constraint $Q/N = q_0$ is imposed directly, relying on the classical behaviour of $\hat{q}(\langle \mathrm{d}q^2 \rangle = \mathcal{O}(1/N))$ to ignore fluctuations.

$$\mathcal{N}_i^f + |b^i|^2 = q_0. \tag{13}$$

Eqs. ((10)–(13)) allow two distinct solutions, $b \neq 0$ or $b = 0$, corresponding to high and low temperature regimes of the theory. In the classical limit the Bose fields behave as order parameters associated with the conservation of Q. When $b = re^{i\theta} \neq 0$, with a definite phase, the $U(1)$ symmetry $b \to be^{i\theta}$, $f_m \to f_m e^{i\theta}$ associated with Q conservation is broken and the classical limit therefore contains two phases, separated by a 2nd order phase transition (fig. 1):

(i) $b = 0$, the weak coupling, high temperature phase where $H_{\mathrm{mix}} = 0$, so that f electrons are decoupled from the band electrons to form local moments.

(ii) $b \neq 0$, the strong coupling, low temperature phase where the f electrons are delocalized, forming an array of resonant levels hybridizing with the band to produce a renormalized (singlet) Fermi liquid. At infinite N, the quasiparticles experience a rigid scattering potential through which they move without interaction. The size of b determines the degree of renormalization and hence the band width. When $\mathcal{N} \approx q$, b is small, and there is little capacity for hybridization leading to very narrow band formation ('Kondo regime').

The phase boundary is determined by setting $b = 0^+$ in ((10)–(13)). To lowest order in b the Green's functions in (12) are those of an impurity f state, so the shape of the $N = \infty$ phase boundary is independent of crystal structure. Along the phase boundary, the mean field equations are

$$\lambda = \lim_{N\to\infty} \frac{1}{N} \sum_m \int_{-D}^{0} \frac{\mathrm{d}\epsilon}{\pi} f(\epsilon) \frac{\Delta}{\omega - \tilde{E}_{fm}} + \mathcal{O}\left(\frac{\Delta b^2}{D}\right) \tag{14}$$

and

$$\mathcal{N}^f = \lim_{N\to\infty} \frac{1}{N} \sum_m f(\tilde{E}_{fm}) = \frac{1}{\beta g B} \ln\left\{ \frac{1 + e^{-\beta[\tilde{E}_f - gB/2]}}{1 + e^{-\beta[\tilde{E}_f + gB/2]}} \right\}. \tag{15}$$

Here $\Delta = \pi |V(k_F)|^2 \rho(\mu)$ is the unrenormalized hybridization width of an f level, where $\rho(\mu)$ is the band electron density of states at the Fermi energy μ. D is the half bandwith. The transition temperature at $B = 0$ is given simply by eq. (15) $f(\tilde{E}_f) = q_0$ or

$$\tilde{E}_f - T_c \ln(q_0^{-1} - 1) = 0, \tag{16}$$

where $\tilde{E}_f = E_f + (\Delta/\pi) \ln(D/\tilde{E}_f) + \mathcal{O}(T/\tilde{E}_f)$ by (14). $T_c = \tilde{E}_f / \ln(q_0^{-1} - 1)$ has no analytic expansion about $q_0 = 0$, highlighting a severe problem with the old large N limiting procedure. Setting $q_0 = 1/N_0$ gives $\tilde{E}_f - T_c \ln N + \mathcal{O}(1/N) = 0$, which indicates that $T_c \approx \tilde{E}_f / \ln N$ is supressed to zero by the conventional method of taking $N \to \infty$, so previous high temperature expansions do not access the strong coupling regime. The 'upper critical field' can also be shown to have no analytic expansion about $q = 0$.

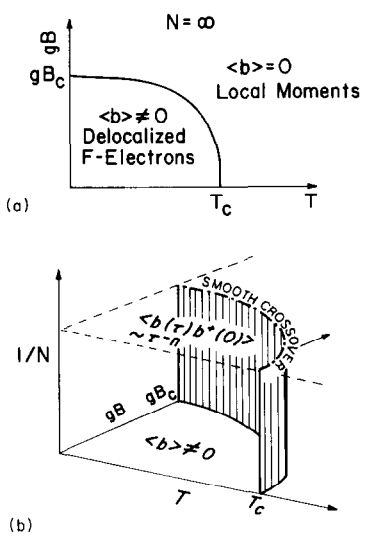

Fig. 1. (a) Phase transition between strong and weak coupling in classical limit; (b) evolution of crossover and development of power law fluctuations at finite degeneracy.

3. Example: 1d chain of ions.

A chain of ions, separated by a uniform distance a, provides a suitable example of the application of the classical equations of motion. The strong coupling solution must be translationally invariant, so the $\lambda^i = a$ constant λ. Defining plane wave f states $f_{mk}^\dagger = (1/\sqrt{n_{sites}}) \Sigma_R f_m^\dagger(R) \exp(-ikR)$ then the classical limit is a renormalised band structure defined by

$$\hat{H} = \sum_{km} \tilde{E}_f f_{km}^\dagger f_{km} + \sum_{kKm} \left\{ E(k+K) \hat{n}_{k+Km} + \left[\tilde{V}(k) c_{k+Km}^\dagger f_{km} + \mathrm{H.C.} \right] \right\}, \tag{17}$$

where $\tilde{V}(k) = V(k)b$ and the $K = 2\pi m/a$ are the reciprocal lattice vectors. Relevant Green's functions are

$$\langle f_{km}(\omega) f_{km}^\dagger(\omega) \rangle = \left(\omega - \tilde{E}_{fm} - \sum_K \frac{|\tilde{V}(k+K)|^2}{(\omega - E(k+K))} \right)^{-1},$$

$$\langle f_{km}(\omega)c^{\dagger}_{km}(\omega)\rangle = \frac{\tilde{V}(k)}{(\omega - E(k))}\langle f_{km}(\omega)f^{\dagger}_{km}(\omega)\rangle. \tag{18}$$

Provided that the renormalized f-level does not intersect the free electron band near a Brillouin zone boundary, then sums over K can be restricted to the first zone without significant loss of accuracy and in this approximation $\epsilon^{\pm}_k = (E(k) + \tilde{E}_f)/2 \pm \sqrt{\left[(E(k) - \tilde{E}_f)/2\right]^2 + |\tilde{V}(k)|^2}$ are the eigenstate energies of the renormalized band which determine the mean field equations

$$\lambda = \sum_k |V(k)|^2 \frac{f(\epsilon^+_k) - f(\epsilon^-_k)}{\epsilon^+_k - \epsilon^-_k},$$

$$\mathcal{N}^f = \sum_k \left[f(\epsilon^+_k) + f(\epsilon^-_k) - f(E(k))\right] + n_p. \tag{19}$$

$n_p = \Sigma_k \langle n_{km}\rangle_{b\neq 0} - \langle n_{km}\rangle|_{b=0} = \mathcal{O}(\Delta b^2/D)$ is the polarization of the band electrons by the MV ions, which can be ignored for large D. The first term is simply the increase in the Fermi surface 'volume' resulting from hybridisation, so the constraint $\mathcal{N}^f = q_0 - |b|^2$ naturally incorporates Luttinger's Fermi surface sum rule [10].

By ignoring small variations in $\rho(\epsilon)$ over the narrow region of the renormalized f band, the mean field expressions can be integrated at zero temperature [12], yielding the following self consistent equations

$$\lambda = \Delta/\pi \ln(D/\tilde{E}_f),$$

$$|b|^2 + \frac{\Delta|b|^2}{\pi \tilde{E}_f} = q_0. \tag{20}$$

To achieve partial occupation of the f state, the renormalized f level $\tilde{E}_f$ must lie above the Fermi energy. Very similar equations, with $q_0 = 1/N$ have been derived for the impurity problem by Read and Newns [11] using path integrals, emphasising the close relationship between the lattice and impurity problems sofar as energy scales are concerned.

The groundstate energy per spin degree of freedom, per ion can be calculated in the classical limit as

$$E_{cl} = \langle \hat{H}_{cl}\rangle_{N=\infty,T=0} = \lambda|b|^2 - \lambda q_0 + \sum_k \theta(-\epsilon^-_k)\epsilon^-_k$$

$$= \int_{-D}^{0} dE\rho(E)E - \lambda q_0. \tag{21}$$

Putting $q_0 = 1/N_0$, the total energy per ion is $N(\hat{H}_{cl}) = E_{band} + E_f - \tilde{E}_f$, so $\tilde{E}_f$ is the reduction in energy per site due to correlated f-band formation. $\mathcal{O}(1/N)$ corrections must of course be added for a system of finite degeneracy.

The crucial respect in which a lattice differs from the impurity is in the dynamic properties of the electrons in strong coupling, for now the propagator for band electrons is

$$\langle c_{km}(\omega)c^{\dagger}_{km}(\omega)\rangle = \left(\omega - E(k) - \frac{|V(k)|^2|b|^2}{(\omega - \tilde{E}_f)}\right)^{-1}. \tag{22}$$

which explicitly displays the effect of coherent scattering off the array of renormalized resonant levels (each having weight $Z = |b|^2$). Heavy band formation enters naturally into the mean field equations of the classical limit.

4. $1/N$ fluctuations

The mean field equations of the classical limit provide a starting point for controlled $1/N$ expansion which already incorporates both the long range coherence of the wave function and crucial non perturbative renormalization effects responsible for narrow band formation. At finite N the rigidity of the classical limit is lost and the Bose field undergoes 'small' ($\mathcal{O}(1/N)$) fluctuations about its static classical value. Zero point corrections to the Free energy must be added to the leading 'classical value'. Furthermore, the Bose field now responds to the motion of quasiparticles, generating weak $\mathcal{O}(1/N)$ interactions between them. Calculation of these effects is vital to determine dynamic response functions and for studying the magnetic or superconducting instabilities of a real MV system.

Path integrals are the natural framework for incorporating $1/N$ fluctuations. Within this framework the classical time evolution is the saddle point path which dominates the large N behaviour (saturating the path integral in this limit). The procedure is to place $N =$ finite in the classical mean field equations and then to evaluate the leading $1/N$ corrections by carrying out the Gaussian integrals about the mean field solution. The different equilibrium positions of the Bose field in strong and weak coupling demand two different sorts of expansions in the fluctuations. Within this approach coherence in strong coupling is regarded as the result of an almost broken symmetry and one treats fluctuations about an almost rigid order parameter to determine the properties of a finite degeneracy system.

Valuable work on the strong coupling regime has already been carried out by Read and Newns [11], and

their work stands on firmer footing by the isolation of a classical large N limit. Crucial aspects of the path integral approach are sketched here and details are given in ref. [12]. Newns and Read have introduced a free energy functional for the SBFT

$$\beta F = \int_0^\beta d\tau \left[\sum_\alpha \bar{\psi}^\alpha \frac{\partial}{\partial \tau} \psi^\alpha + \sum_i \bar{b}^i \frac{\partial}{\partial \tau} b^i + H(\bar{\psi}, \psi, \bar{b}, b, Q_0) \right] \tag{23}$$

which determines the partition function for MV as

$$Z_{MV} = \int_{\lambda_{cl} - i\pi/\beta}^{\lambda_{cl} + i\pi/\beta} \prod_j \frac{\beta d\lambda^j}{2\pi i} \int D[\psi, \bar{\psi}, b, \bar{b}] e^{-\beta F} |_{Q_0 = 1}. \tag{24}$$

The integral over the λ^j projects out the subspace $\{Q^j = Q_0 = 1\}$ (for MV). If one considers the classical limit (maintaining a finite f filling factor), then a rescaling the fields $\psi^\alpha_{cl} = \psi^\alpha/\sqrt{N}$, $b_{cl} = b/\sqrt{N}$, the free energy functional is recognized as an extensive quantity in N, $\beta F = N \beta F_{cl}$ $[\psi_{cl}, \bar{\psi}_{cl}, b_{cl}, \bar{b}_{cl}]$ enabling fluctuations to be treated as Gaussian in a first $\mathcal{O}(1/N)$ approximation. With some work the classical meanfield equations can be shown to be the saddle point of this path integral.

A great facility of the path integral approach arises from the way it treats the constraint $q^j = q_0$. Rather than attempting to enforce this constraint exactly, it is only imposed to the accuracy required by the $1/N$ expansion, treating the integral over λ^j as Gaussian in the leading approximation, thereby enforcing the constraint to $\mathcal{O}(1/N^2)$, $\langle (\delta \hat{q}_i)^2 \rangle = \mathcal{O}(1/N^2)$. This certainly means that the final answer contains small contributions from the $Q = 0$ and $Q = 2, \ldots$, etc subspaces, but these only effect the accuracy to $\mathcal{O}(1/N^2)$, which doesn't matter in a calculation of the leading fluctuations. Attempts to impose the constraint $Q^j = Q_0$ exactly gain no increased accuracy in the $1/N$ expansion and pay a severe price of losing the facility of a coherent state representation of the Bose field. The situation is ramarkably analogous to superconducting fluctuations in finite sized systems, where 1/(volume) plays the analogous role to $1/N$.

The path integral approach is not dependent on an explicit construction of a classical limit, and having found such a limit for the naive spin j electron system, one can proceed to derive the saddle point equations for a system with realistic angular momentum structure. The renormalized bands now acquire the full double group symmetry of the lattice.

A specific calculation proceeds by first eliminating the Fermion degrees of freedom to yield an effective action in the Bose fields

$$\beta F_{cl}^{eff} = \int_0^\beta d\tau \left\{ \sum_i \left[\bar{b}^i_{cl} \left(\frac{\partial}{\partial \tau} + \lambda^i \right) b_{cl} - \lambda^i q_0 \right] + \beta F_\psi [\bar{b}^i_{cl}, b^i_{cl}, \lambda^i] \right\}, \tag{25}$$

where F_ψ is the trace over the Fermion degrees of freedom. This expression is then expanded to leading Gaussian order, which corresponds to calculating the RPA electron-hole self energies of the Bose field. These self energies then determine the Bose propagators which are used to calculate the zero point corrections and the dynamic correlation functions.

In weak coupling the f-electrons are localized, so the leading RPA diagrams involve no intersite propagation and the leading zero point correction to the free energy energy contains no intersite corrections. The results of the sum over the Gaussian fluctuations are

$$F_{MV}(T, N) = E_f - k_B T \ln N - \int \frac{d\nu}{\pi} n(\nu) \tan^{-1} \left[\frac{\Pi_I(\nu)}{\lambda + \Pi_R(\nu) - \nu} \right] + \mathcal{O}\left(\frac{1}{N} \right) \tag{26}$$

per RE ion, where

$$\Pi(\nu) = \Pi_R(\nu) + i\Pi_I(\nu) = \frac{\Delta}{\pi} \int_{-D}^{D} d\epsilon \frac{f(\epsilon) - q_0}{\nu - \lambda_{cl} - E_f} \tag{27}$$

is the self-energy of the Bose field [12]. This novel expression for the free energy is invariant under Haldane scaling [12,13]. Breakdown of weak coupling is signalled by the appearance of a zero frequency pole at T_c in the Bose excitation spectrum $\tan^{-1}(\Pi_I(\nu)/(\lambda + \Pi_R(\nu) - \nu))$.

In strong coupling, calculations are facilated by treating the Bose fluctuations in polar coordinates. In the strong coupling lattice the RPA diagrams involve intersite propagation and the Bose excitations are extended. The zero point fluctuations involve sums over the frequencies and momenta of the bosons, leading to important intersite corrections to the ground state energy [12]. Fluctuations about the 'almost rigid' mean-field value of the Bose field are determined by a matrix propagator relating fluctuations in phase and magnitude δr.

$$\frac{1}{N} \mathcal{R}(\kappa) = \begin{pmatrix} \langle \delta r(\kappa) \delta r^*(\kappa) \rangle & i\langle \delta r(\kappa) \dot{\theta}^*(\kappa) \rangle \\ -i\langle \dot{\theta}(\kappa) \delta r^*(\kappa) \rangle & -\langle \dot{\theta}(\kappa) \dot{\theta}^*(\kappa) \rangle \end{pmatrix}, \quad (\kappa = (i\nu_n, k)). \tag{28}$$

$\mathscr{R}(\kappa)$ has been calculated for the impurity model, and one of its most important features is a linear excitation spectrum at the Fermi energy. This guarantees the preservation of Fermi liquid relations when the effect of fluctuations on the quasiparticles are included. $\mathscr{R}(\kappa)$ determines the quasiparticle interactions and in the lattice offers the possibility of studying extended spin excitations and instabilities.

Quantum fluctuations in the phase of the Bose field destroy the broken symmetry at finite N. However, the crossover to strong coupling remains a vestage of the infinite N phase transition and the 'almost broken symmetry' is signalled by a change in the long time correlation of the Bose field. In weak coupling the Bose fields are exponentially correlated $\langle b^i(t)b^{i\dagger}(0)\rangle \sim \delta_{ij}e^{-\lambda t}$. At low temperatures the nature of the long time correlations is determined from the Gaussian fluctuations in phase of the 'almost rigid' order parameter

$$\langle \tilde{b}^j(t)\tilde{b}^{j\dagger}(0)\rangle \approx r_{\rm cl}^2\langle e^{i[\theta^i(t)-\theta^i(0)]}\rangle = \delta_{ji}r_{\rm cl}^2\, e^{-\frac{1}{2}\langle\delta\theta^i(t)^2\rangle}. \tag{29}$$

The linear excitation spectrum of $\mathscr{R}(\kappa)$ at low energies leads to a logarithmic divergence in the variance of the phase at long times $\frac{1}{2}\langle\delta\theta^j(t)^2\rangle \sim (\alpha/N\pi)\ln t$ [12], so that

$$\langle \hat{b}^i(t)\hat{b}^{j\dagger}(0)\rangle \sim 1/t^{\alpha/N\pi}, \quad (0 \ll t \ll \hbar/k_{\rm B}T). \tag{30}$$

Thermal interference re-introduces an exponential correlation at times $t \gg \hbar/k_{\rm B}T$. Power law correlations are a known symptom of the relaxation of a Fermi liquid in response to a suddenly changing local potential [14], and their appearance signals Fermi liquid formation. In this case fluctuations in the Bose field cause the power law relaxation. Read has related [15] $\alpha/N\pi = n_f^2/N + \mathcal{O}(1/N^2)$ for the single impurity to the exact exponent $\alpha/N\pi = N\delta_f^2/\pi^2 = n_f^2/N$ predicted from the X-ray theory of Nozieres and de Dominicis [14], for relaxation of a Fermi liquid in the presence of an f channel scattering potential with phase shift $\delta_f = \pi n_f/N$ (determined by the Friedel sum rule). This change in correlations that accompanies the crossover to strong coupling supports a picture which regards strong coupling as a as a regime of almost broken symmetry.

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References

[1] O. Gunnarson and K. Schonhammer, Phys. Rev. Lett. 50 (1983) 604.
[2] H. Kojima, Y. Kuromoto and M. Tachiki, Z. Phys. B 54 (1984) 293.
[3] F.C. Zhang and T.K. Lee, Phys. Rev. B28 (1983) 33.
[4] J. Rasul and A.C. Hewson, J. Phys. C 14 (1984) 2555.
[5] L.G. Yaffe, Rev. Mod. Phys. 54 (1982) 407.
[6] A. Jevicki and N. Papnicaloau, Nucl. Phys. B 171 (1980) 362.
[7] M.J. Klein and R.J. Smith, Phys. Rev. 80 (1951) 111.
[8] P.W. Anderson, Phys. Rev. 112 (1958) 1900.
[9] P. Coleman, Phys. Rev. B29 (1984) 3035.
[10] R.M. Martin, Phys. Rev. Lett. 48 (1982) 362.
[11] N. Read and D.M. Newns, J. Phys. C 16 (1982) 3273.
[12] P. Coleman, Ph.D. Thesis, Princeton University (University Microfilms International, 1984) p. 59.
[13] F.D.M. Haldane, Phys. Rev. Lett. 40 (1978) 416.
[14] P. Nozieres and C. De Dominicis, Phys. Rev. 178 (1968) 1097.
[15] N. Read, 4th Intern. Conf. on Valence Fluctuations (North-Holland, Amsterdam, 1984).