1 Exercise 1

Let $\alpha = 1 + i$, $\beta = 2 + 3i$. Calculate $-\alpha + 2\beta$.
$-\alpha + 2\beta = -(1 + i) + 2 \times (2 + 3i) = (-1 - i) + (4 + 6i) = 4 - 1 + 6i - i = 3 + 5i$

2 Exercise 2

Let $\alpha = -1 + 2i$, $\beta = \frac{1}{2} + i$. Calculate $\alpha\beta$.
$\alpha\beta = (-1 + 2i)(\frac{1}{2} + i) = -1 \times \frac{1}{2} + (-1 \times i) + \frac{1}{2} \times 2i + 2i \times i = -\frac{1}{2} - i + i + 2i^2 = -\frac{1}{2} + 2 \times (-1) = -\frac{5}{2}$

3 Exercise 3

Let $\alpha = 1 + 1.5i$, $\beta = 3 + 2i$. Calculate $\frac{\alpha}{\beta}$.
$\frac{\alpha}{\beta} = \frac{1+1.5i}{3+2i} \times \frac{3-2i}{3-2i} = \frac{(1+1.5i)(3-2i)}{(3+2i)(3-2i)} = \frac{3-2i+4.5i-3i^2}{9-6i+6i-4i^2} = \frac{6+2.5i}{13} = \frac{6}{13} + \frac{2.5i}{13}$

4 Exercise 4

Let $\alpha = a + bi$, $\beta = c + di$. Calculate $(\alpha\beta)^*$ and $\alpha^*\beta^*$. Are they equal?

- $(\alpha\beta)^*$
  $(\alpha\beta)^* = ((a + bi)(c + di))^* = (ac + aid + bic + bdi^2)^* = ((ac - bd) + i(ad + bc)^* = (ac - bd) - i(ad + bc)$
- $\alpha^*\beta^*$
  $\alpha^*\beta^* = (a + bi)^*(c + di)^* = (a - bi)(c - di) = ac - adi - bic + bdi^2 = (ac - bd) - i(ad + bc)$

$(\alpha\beta)^*$ and $\alpha^*\beta^*$ are equal.

5 Exercise 5

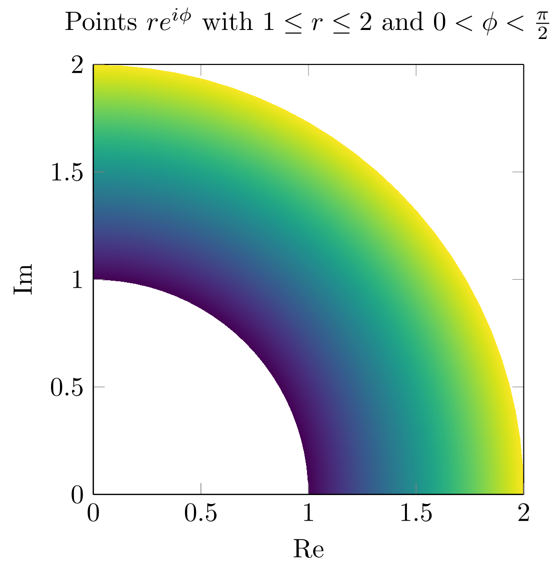
Let $\alpha = -4 + 4i$. Write α in polar coordinates, i.e. calculate r and ϕ.

- Finding r
  $r = \sqrt{a^2 + b^2} = \sqrt{(-4)^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$
- Finding ϕ
  $\phi = \arctan\left(\frac{b}{a}\right) + \pi = \arctan\left(\frac{4}{-4}\right) + \pi = \arctan(-1) + \pi = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}$

Polar coordinates $\alpha = 4\sqrt{2}e^{\frac{3\pi}{4}i}$

6 Exercise 6

Draw on the complex plane all the points $re^{i\phi}$ with $1 \leq r \leq 2$, $0 < \phi < \frac{\pi}{2}$.



7 Exercise 7

Let $\alpha = 3 + 3\sqrt{3}i$, $\beta = \sqrt{3} + i$. Calculate $\frac{\alpha}{\beta}$ and α^2 using polar coordinates.

- Write α in polar coordinates.
  $r = \sqrt{3^2 + (3\sqrt{3})^2} = \sqrt{9 + 9 \times 3} = \sqrt{36} = 6$ $\phi = \arctan\left(\frac{3\sqrt{3}}{3}\right) = \frac{\pi}{3}$
  Polar coordinates $\alpha = 6e^{\frac{\pi}{3}i}$
- Write β in polar coordinates.
  $s = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$ $\gamma = \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$
  Polar coordinates $\beta = 2e^{\frac{\pi}{6}i}$
- $\frac{\alpha}{\beta}$
  $\frac{\alpha}{\beta} = \frac{6e^{\frac{\pi}{3}i}}{2e^{\frac{\pi}{6}i}} = \frac{6}{2}e^{i(\frac{\pi}{3} - \frac{\pi}{6})} = 3e^{i\frac{\pi}{6}}$
- α^2
  $\alpha^2 = 6e^{\frac{\pi}{3}i} \times 6e^{\frac{\pi}{3}i} = 6 \times 6e^{i(\frac{\pi}{3} + \frac{\pi}{3})} = 36e^{i\frac{2\pi}{3}}$

8 Exercise 8

For arbitrary vectors $|a\rangle$, $|b\rangle$ and operator A are inner products $(A\,|a\rangle)^\dagger\,|b\rangle$ and $\langle a|\,(A^\dagger\,|b\rangle)$ equal?

- $(A\,|a\rangle)^\dagger\,|b\rangle = |a\rangle^\dagger\,A^\dagger\,|b\rangle = \langle a|\,A^\dagger\,|b\rangle = \langle a|\,(A^*)^T\,|b\rangle$
- $\langle a|\,(A^\dagger\,|b\rangle) = \langle a|\,((A^*)^T\,|b\rangle) = |a\rangle\,(A^*)^T\,|b\rangle$

We can see, that $(A\,|a\rangle)^\dagger\,|b\rangle$ and $\langle a|\,(A^\dagger\,|b\rangle)$ are equal.