If n is a natural number, find

$$\binom{n}{1} + 2 \cdot \binom{n}{2} + 3 \cdot \binom{n}{3} + \cdots + n \cdot \binom{n}{n}$$

Solution:

$$\binom{n}{1} + 2 \cdot \binom{n}{2} + 3 \cdot \binom{n}{3} + \cdots + n \cdot \binom{n}{n} =$$

$$\left\langle Using\ the\ Binomial\ Coefficient\ Formula \middle| \binom{n}{k} = \frac{n!}{k! \cdot (n-k)!} \right\rangle$$

$$= \sum_{k=1}^{n} k \cdot \frac{n!}{k! \cdot (n-k)!} = \sum_{k=1}^{n} \frac{n!}{(k-1)! \cdot (n-k)!} = n \cdot \sum_{k=1}^{n} \frac{(n-1)!}{(k-1)! \cdot (n-1+1-k)!} =$$

$$= n \cdot \sum_{k=1}^{n} \frac{(n-1)!}{(k-1)! \cdot \big(n-1-(k-1)\big)!} =$$

$$= n \cdot \sum_{k=1}^{n} \binom{n-1}{k-1} = \langle let\ i := k-1 \rangle =$$

$$= n \cdot \sum_{i=0}^{n-1} \binom{n-1}{i} =$$

$$\left\langle Using\ the\ Binomial\ Theorem\ Formula: (x+y)^n = \sum_{k=0}^{n} \binom{n}{k} \cdot x^{n-k} \cdot y^k \middle| (1+1)^n = \sum_{k=0}^{n} \binom{n}{k} = 2^n \right\rangle$$

$$= n \cdot \sum_{i=0}^{n-1} \binom{n-1}{i} = n \cdot 2^{n-1}$$

Answer: $\boldsymbol{n \cdot 2^{n-1}}$