lecture4

May 7, 2019

1 Lecture 4

- Sampling
- Automatic gradient
- Parallel computing on GPUs
- Cython

```
In [2]: import numpy as np
        import numpy.random as rnd
        import matplotlib
        import matplotlib.pyplot as plt
        %matplotlib inline
```

1.1 Sampling

Draw random samples from a uniform distribution.

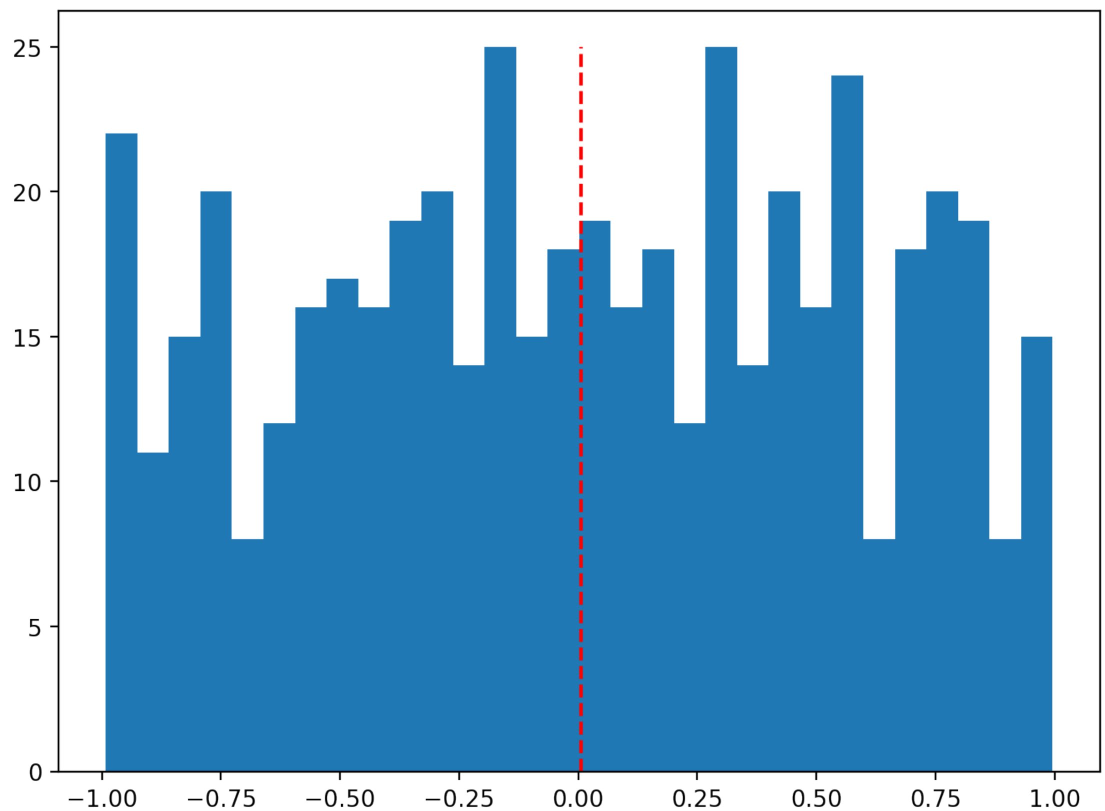
```
In [3]: rnd.uniform?

In [4]: U = rnd.uniform(-1,1, size = 500)
        print(U.shape)
        print("Mean: {:0.03f} var: {:0.02f}".format(U.mean(), U.var()))

(500,)
Mean: 0.006 var: 0.31
```

Draw random samples from a normal (Gaussian) distribution.

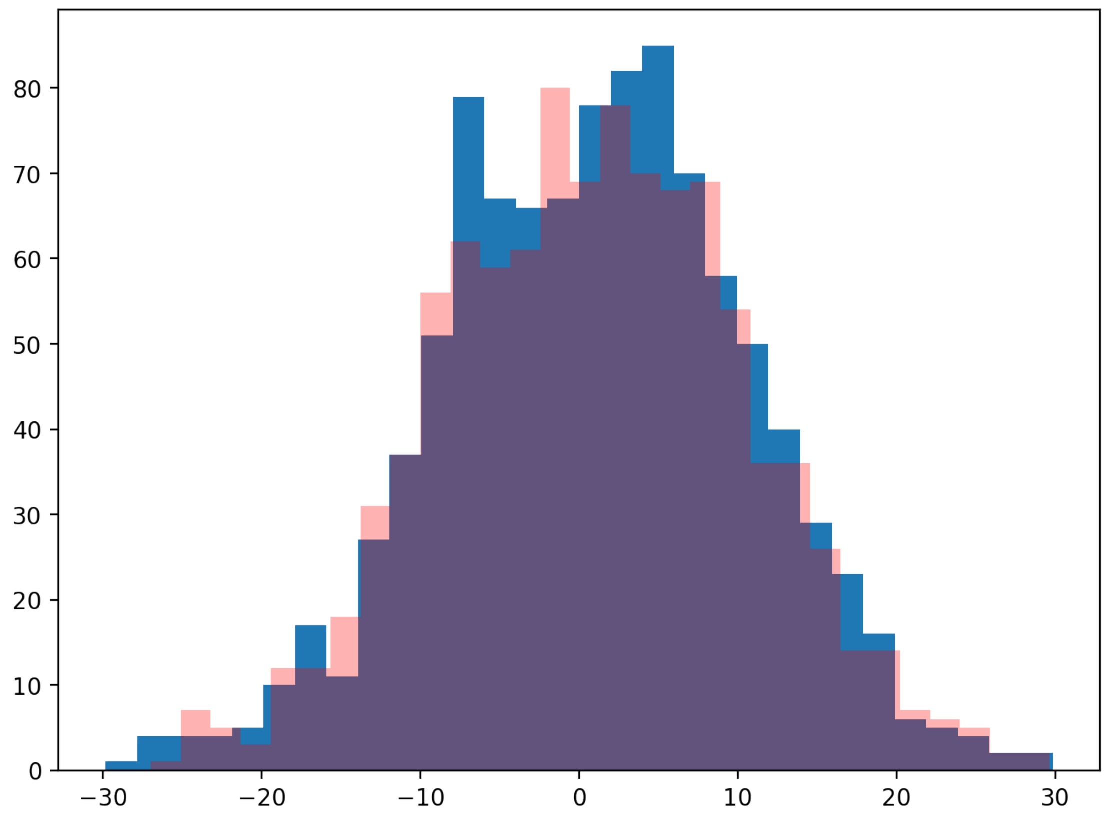
```
In [5]: rnd.normal?

In [6]: G = rnd.normal(loc = 1, scale = 10, size = [1000,2])
        print(G.shape)
        print("Mean: [{:0.03f},{:0.03f}]".format(*np.mean(G,0)))

(1000, 2)
Mean: [0.950,0.889]
```

In [7]:
```python
num_bins = 30

nums,ranges = np.histogram(U, bins = num_bins)

print(nums)
print(ranges)

plt.figure(figsize=(8,6), dpi=200)
_ = plt.hist(U, bins=num_bins)

_ = plt.plot([U.mean()]*2,[0, np.max(nums)], '--', color ='r') # plot dashed mean line
```

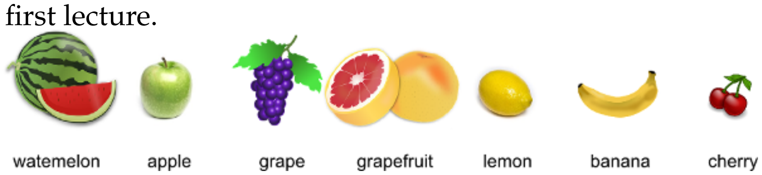
```
[22 11 15 20  8 12 16 17 16 19 20 14 25 15 18 19 16 18 12 25 14 20 16 24
  8 18 20 19  8 15]
[-0.99190391 -0.92563863 -0.85937334 -0.79310806 -0.72684277 -0.66057749
 -0.5943122  -0.52804692 -0.46178164 -0.39551635 -0.32925107 -0.26298578
 -0.1967205  -0.13045521 -0.06418993  0.00207535  0.06834064  0.13460592
  0.20087121  0.26713649  0.33340178  0.39966706  0.46593234  0.53219763
  0.59846291  0.6647282   0.73099348  0.79725877  0.86352405  0.92978934
  0.99605462]
```



```
In [8]: plt.figure(figsize=(8,6), dpi=200)
        _ = plt.hist(G[:,0], bins=30)
        _ = plt.hist(G[:,1], bins=30, alpha=0.3, color = 'r')
```



Let's draw some random samples from a multinomial distribution. We'll use our fruits from the first lecture.




```
In [9]: fruits = np.array([
            'watermelon',
            'apple',
            'grape',
            'grapefruit',
            'lemon',
            'banana',
            'cherry'
        ])
```

```
In [10]: n = 5 # number of samples
         p = [1/len(fruits)]*len(fruits)
```

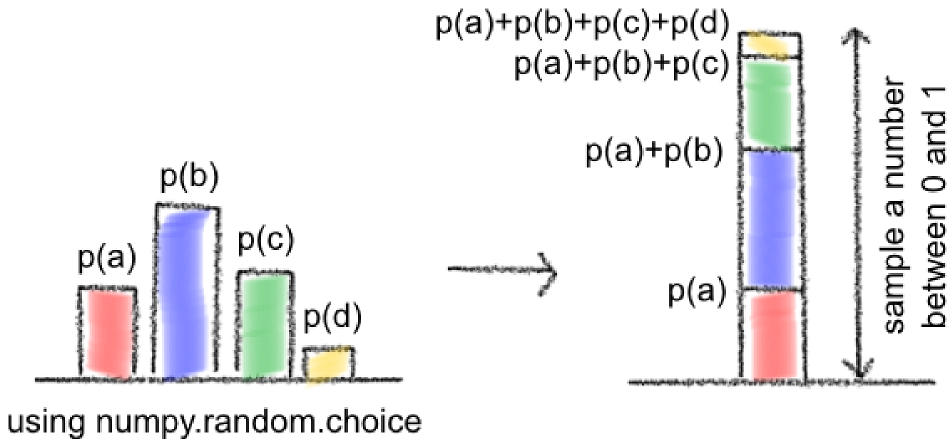
```
        repeat = np.tile(fruits, (n,1)) # repeat multiple (5) times
        print(repeat)

        mlt = rnd.multinomial(1, p, size=(5)) # draw multinomial samples 5 times with an equa
        print(mlt)

        samples =repeat[mlt.astype(bool)] # show drown samples
        print(samples)
```

```
[['watermelon' 'apple' 'grape' 'grapefruit' 'lemon' 'banana' 'cherry']
 ['watermelon' 'apple' 'grape' 'grapefruit' 'lemon' 'banana' 'cherry']
 ['watermelon' 'apple' 'grape' 'grapefruit' 'lemon' 'banana' 'cherry']
 ['watermelon' 'apple' 'grape' 'grapefruit' 'lemon' 'banana' 'cherry']
 ['watermelon' 'apple' 'grape' 'grapefruit' 'lemon' 'banana' 'cherry']]
[[0 0 0 0 1 0 0]
 [0 0 0 0 0 1 0]
 [0 0 0 0 0 1 0]
 [0 0 1 0 0 0 0]
 [0 0 0 0 0 1 0]]
['lemon' 'banana' 'banana' 'grape' 'banana']
```

```
In [11]: p = [0.05, 0.70, 0.05, 0.05, 0.05, 0.05, 0.05] # adjust probailities

         mlt = rnd.multinomial(1, p, size=(5)) # draw multinomial samples 5 times with given p
         print(mlt)

         samples =repeat[mlt.astype(bool)] # show drown samples
         print(samples)
```

```
[[0 1 0 0 0 0 0]
 [0 1 0 0 0 0 0]
 [0 1 0 0 0 0 0]
 [0 1 0 0 0 0 0]
 [0 1 0 0 0 0 0]]
['apple' 'apple' 'apple' 'apple' 'apple']
```

1.1.1 Another way to make discrete choices

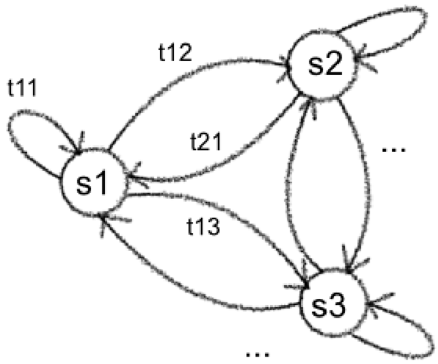



```
In [12]: p = [0.05, 0.70, 0.05, 0.05, 0.05, 0.05, 0.05]

         # Cumulate them
         l = np.cumsum([0] + p[:-1]) # lower-bounds
         h = np.cumsum(p)            # upper-bounds

         print(l)
         print(h)

         # Draw a number between 0 and 1
         u = np.random.uniform(0, 1)

         # Find which basket it belongs to
         s = np.logical_and(u > l, u < h)
         print(s)

         # retrieve the label
         fruits[np.argmax(s)]
```

```
[0.   0.05 0.75 0.8  0.85 0.9  0.95]
[0.05 0.75 0.8  0.85 0.9  0.95 1.  ]
[False  True False False False False False]
```

```
Out[12]: 'apple'
```

1.2 Markov Chain



A Markov chain transits between a set of states, where the transition between pairs of states is associated with a fixed probability. The set of probabilities can be stored in a transition matrix.

```
In [13]: # Transition matrix
         T = np.array([
             [0.9,0.1,0.0], # transiting from state 1 to state 1,2,3
             [0.0,0.9,0.1], # transiting from state 2 to state 1,2,3
             [1.0,0.0,0.0], # transiting from state 3 to state 1,2,3
         ])
```

```
In [14]: # Add empty state to transition matrix
         pad_shape = ((0, 0), (1, 0))  # ((before_1, after_1), (before_2, after_2))
         P = np.pad(T, pad_shape, mode='constant')
         print(P)
```

```
[[0.  0.9 0.1 0. ]
 [0.  0.  0.9 0.1]
 [0.  1.  0.  0. ]]
```

```
In [15]: def mcstep(X, P):
             Xp = np.dot(X, P)
             Xc = np.cumsum(Xp, axis=1)
             L,H = Xc[:, :-1], Xc[:, 1:]
             R = np.random.uniform(0, 1, (len(Xp), 1))
             states = np.logical_and((R > L), (R < H))
             #print(states.astype('int32'))
             return states.astype('int32')
```

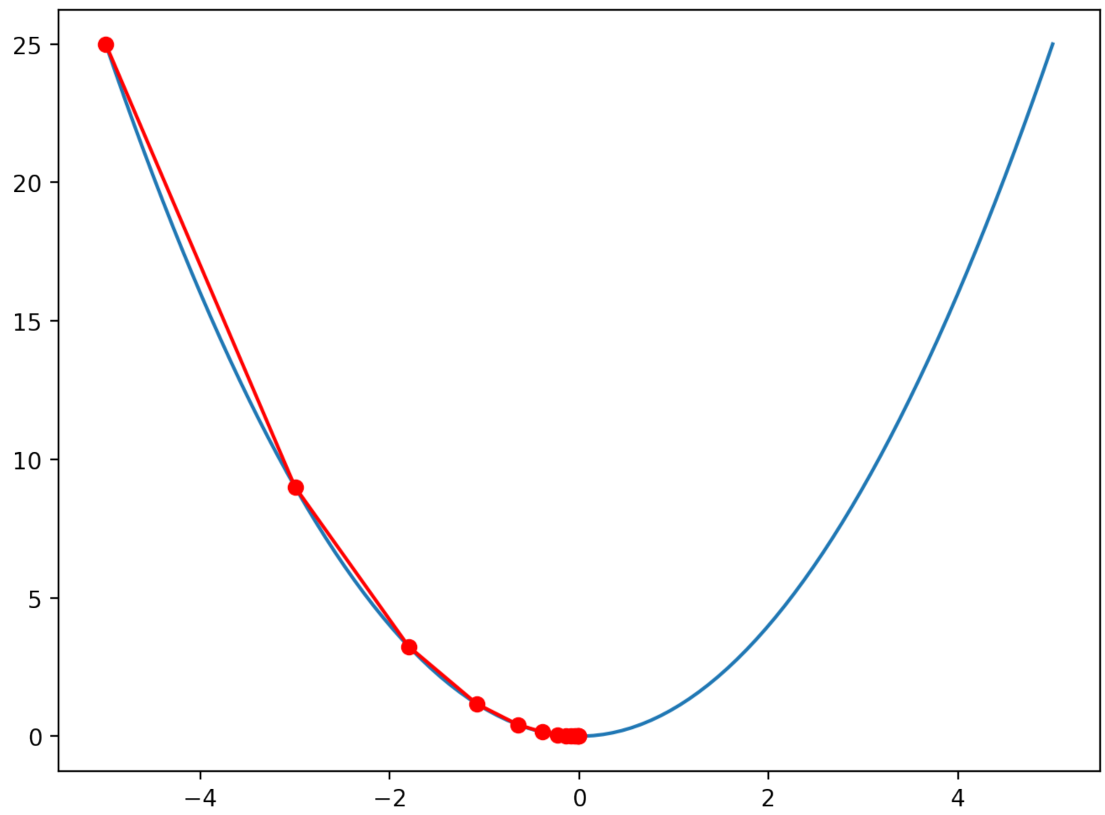
```
In [17]: A = np.tile([1.0,0,0], (5,1))
         # or
         A = np.outer(np.ones([5]),[1.0,0,0]) # (5,1) x (1,3) -> (5,3)

         num_steps = 10
         for i in range(num_steps):
             A = mcstep(A, P)
```

```
         A.mean(axis=0)

Out[17]: array([0.6, 0.4, 0. ])
```

1.3 Autograd

(https://github.com/HIPS/autograd)

```
In [21]: import autograd.numpy as ag_np
         from autograd import grad
         x = ag_np.ones(1)
         y = lambda x: 3 * x**2 + 2
         print(grad(y)(x))

float64
[6.]

In [22]: x1 = ag_np.ones(1)
         x2 = ag_np.ones(1)

         y = lambda x1,x2 : 3*x1**3 + 4*2**x2

         print(grad(y,0)(x1,x2))
         print(grad(y,1)(x1,x2)) # 4*2**x2*ln(x2)

[9.]
[5.54517744]

In [26]: y = lambda x: x**2
         step_size = 0.2
         xs = np.array([-5.0])
         while abs(grad(y)(xs[-1])) > 1e-2:
             curr_val = xs[-1]
             next_val = curr_val - step_size*grad(y)(curr_val)
             xs = np.append(xs, next_val)

In [27]: x = np.linspace(-5, 5, 100)
         plt.figure(figsize=(8,6), dpi=200)
         _ = plt.plot(x,y(x))
         _ = plt.plot(xs, y(xs), "-o", c="r")
```



1.4 GPUs

(https://github.com/cupy/cupy)

```
In [36]: import numpy as np
         from time import time
         n = 1000
         X = np.random.normal(size = (n,n))
         Y = np.random.normal(size = (n,n))
         %timeit X.dot(Y)
```

```
5.64 ms ś 108 ţs per loop (mean ś std. dev. of 7 runs, 100 loops each)
```

```
In [37]: import cupy as cp
         X = cp.array(X)
         Y = cp.array(Y)
         %timeit X.dot(Y)
```

```
571 ţs ś 346 ns per loop (mean ś std. dev. of 7 runs, 10000 loops each)
```

1.5 Cython

- Create new file *hello.pyx*. See the example file (hello.pyx) in the same Folder. The file contains a custom implementation of the matrix product with loops

Cython hello.pyx file cimport cython import numpy as np cimport openmp from cython.parallel cimport prange

@cython.boundscheck(False) @cython.wraparound(False)

cpdef dot(float[:,:] X, float[:,:] Y): cdef: int n,i,j,k float[:,:] Z n = X.shape[0] Z = np.zeros((n,n), dtype = 'float') n = len(X) for i in prange(n, nogil = True): for j in range(n): for k in range(n): Z[i,j] += X[i, k] * Y[k, j] return Z

- Create *setup.py* with compiler commands in order to build a new python package. See example file in the same folder

setup.py file from distutils.core import setup from Cython.Build import cythonize from distutils.extension import Extension from Cython.Distutils import build_ext

setup(name = "hello", cmdclass = {"build_ext": build_ext}, ext_modules = [Extension("hello", ["hello.pyx"], extra_compile_args = ["-O0", "-fopenmp"], extra_link_args=['-fopenmp'])])

Compile the *hello.pyx* file with the following command from terminal. * *python setup.py build_ext –inplace*

After your module is compiled you can import it into your notebook as usual

You can profile your cython code in order to know, either your computations are made efficiently. For the reason you may create an HTML file highlighting the line with the bad performance.

Create file profiling snapshot by calling the following command in your terminal * *cython -a hello.pyx*

Then open a new generated *hello.html* file in a browser. The lines colored yellow still need some python interactions and therefore slow, so you can still find a way to optimize them. But for now it's enought to have no yellow lines within the loops.

```
Generated by Cython 0.29.7

Yellow lines hint at Python interaction.
Click on a line that starts with a "+" to see the C code that Cython generated for it.

Raw output: hello.c

+01: cimport cython
+02: import numpy as np
 03: cimport openmp
 04: from cython.parallel cimport prange
 05:
 06: @cython.boundscheck(False)
 07: @cython.wraparound(False)
 08:
+09: cpdef dot(float[:,:] X, float[:,:] Y):
 10:     cdef:
 11:         int n,i,j,k
 12:         float[:,:] Z
+13:     n = X.shape[0]
+14:     Z = np.zeros((n,n), dtype = 'float')
+15:     n = len(X)
+16:     for i in prange(n, nogil = True):
+17:         for j in range(n):
+18:             for k in range(n):
+19:                 Z[i,j] += X[i, k] * Y[k, j]
+20:     return Z
```

Import your brand new module as usual

```
In [50]: import hello
```

```
In [ ]: Test the performance. It
```

```
In [51]: from time import time
         X = np.array(X.tolist()).astype('float32')
         Y = np.array(X.tolist()).astype('float32')
         %timeit hello.dot(X,Y)
```

```
783 ms ś 43.9 ms per loop (mean ś std. dev. of 7 runs, 1 loop each)
```

As you already know normal python loops will take much more time to finish all the computations…

```
In [47]: st = time()
         Z = np.zeros((n,n))
         for i in range(n):
             for j in range(n):
                 for k in range(n):
                     Z[i,j] += X[i, k] * Y[k, j]
         print(time() - st)
         print(np.sum(Z))
```

```
995.5045590400696
6907.071765647259
```