Light models

Scan-line rendering

As previously outlined, in scan-line rendering the scene is composed by a finite set of light sources. The contributions of all lights l are added together to compute the final color of the pixel.

Initially, we will ignore the possibility of objects to emit small amount of lights, further simplifying the equation.

$$L(x,\omega_r) = \cancel{L_e(x,\omega_r)} + \sum_l L_e\left(l,\overrightarrow{lx}\right) f_r\left(x,\overrightarrow{lx},\omega_r\right)$$

Scan-line rendering

Each term in the summation is the product of the *light model*, that computes the quantity and direction of the considered light source, and the *BRDF* which accounts how the surface reflects the light.

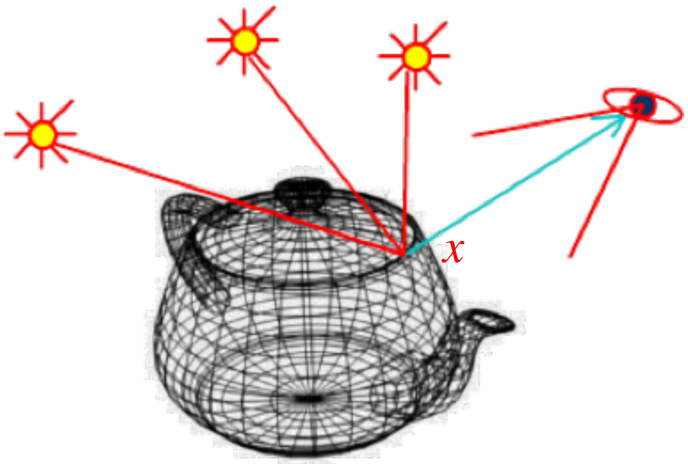
$$L(x, \omega_r) = \sum_{l} L\left(l, \overrightarrow{lx}\right) f_r\left(x, \overrightarrow{lx}, \omega_r\right)$$

Light model

BRDF

Light models

A light model describes how light is emitted in the different directions of the space. It takes as input the position of a point *x* of an object. It returns two elements: a vector that represents the *direction* of the light, and a color which accounts for the *intensity* of light received by point *x* for every wavelength.

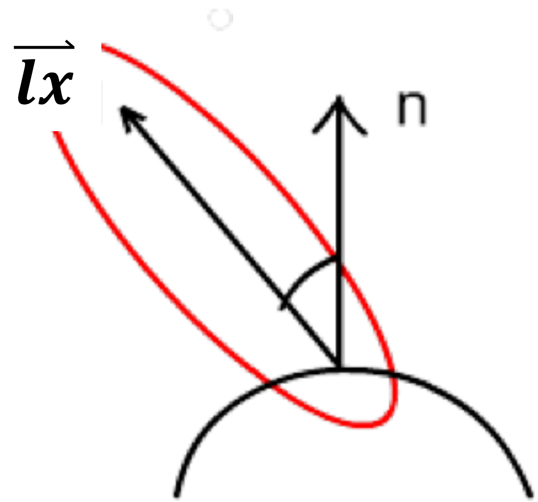


$$L(x, \omega_r) = \sum_l \underbrace{\boxed{L(l, \overrightarrow{lx})}}_{\text{intensity}} f_r(x, \underbrace{\boxed{\overrightarrow{lx}}}_{\text{direction}}, \omega_r)$$

Light direction

The light direction can then be specified with a vector $\overrightarrow{\boldsymbol{lx}} = (d_x, d_y, d_z)$: as a convention, the sign of the light direction is chosen to make the ray point toward the light source.

Moreover, the direction of the light is a unitary vector: $|\overrightarrow{\boldsymbol{lx}}| = 1$.



Light color

A vector $L(l, \overrightarrow{lx}) = (l_R, l_G, l_B)$ of RGB components defines the light intensity for each wavelength, specifying its color.

Components do not necessarily need to be in the 0 ~ 1 range: larger values can model stronger light sources.

Components, however, need to be non negative.

$L(l, \overrightarrow{lx}) = (0.3, 0.3, 0.3)$

$L(l, \overrightarrow{lx}) = (1, 1, 1)$

$L(l, \overrightarrow{lx}) = (10, 10, 10)$

Notation

As introduced in the previous lessons, the rendering equation must be solved for every color frequency considered (usually, the RGB colors).

Since light color $L\left(l, \overrightarrow{lx}\right)$ is encoded in a vector, the BRDF function $f_r(x, \overrightarrow{lx}, \omega_r)$ returns a color vector too.

In the following, we will use the * symbol to denote the component-wise product, and a dot · symbol to express the standard scalar product (dot product) of two vectors.

$$\boldsymbol{a} * \boldsymbol{b} = (a.R * b.R,\ a.G * b.G,\ a.B * b.B)$$
$$\boldsymbol{v} \cdot \boldsymbol{u} = v.x * u.x + v.y * u.y + v.z * u.z$$

In GLSL, component wise product is also denoted with symbol * , while dot product is computed with the `dot()` function.

Light models

In this course we will present the three basic direct light models for real time graphics:

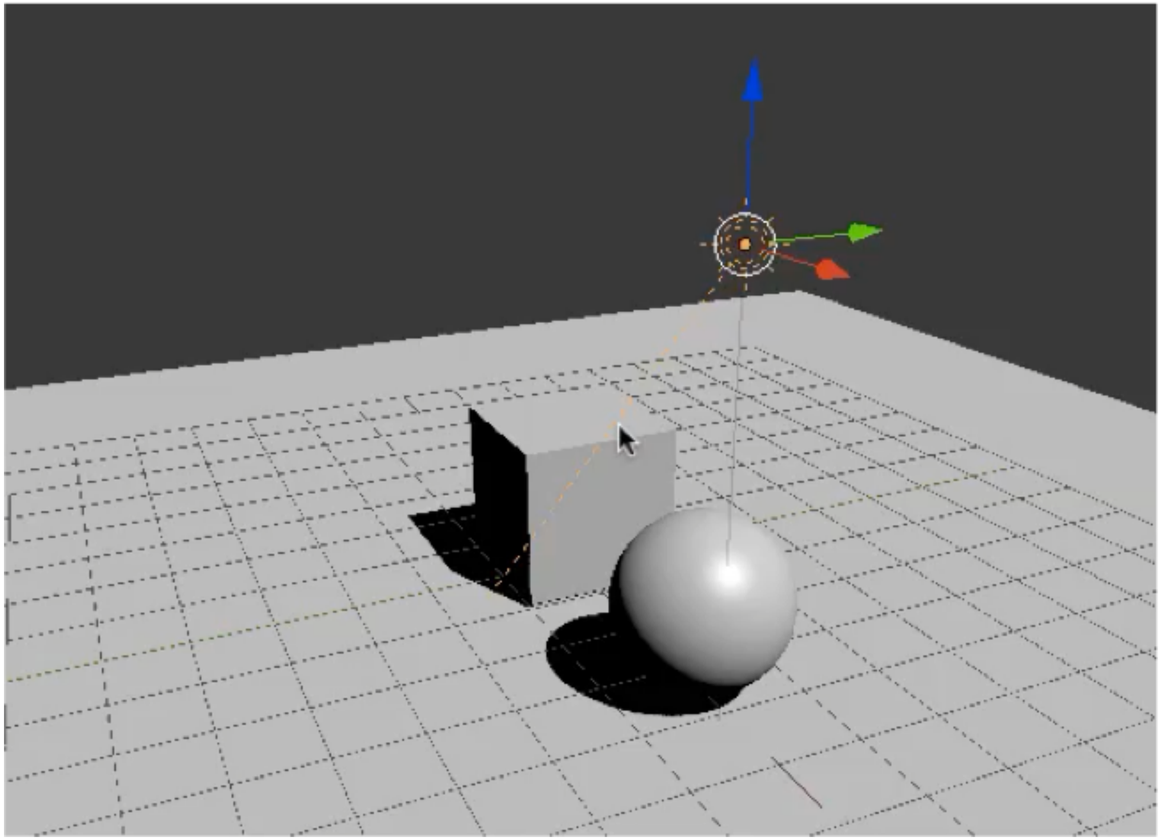
- Direct light
- Point light
- Spot light

Direct light models

Directional lights are used to model distant sources such as sunlight.

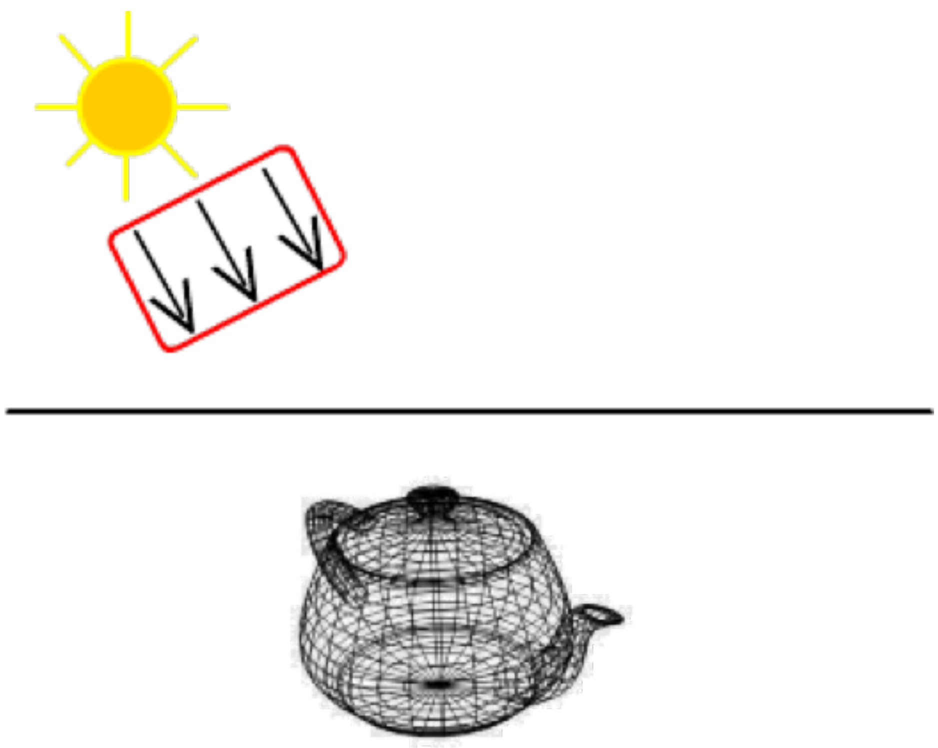
Direct light models

They are sources that are very far away from the objects, so that they uniformly influence the entire scene.



Direct light models

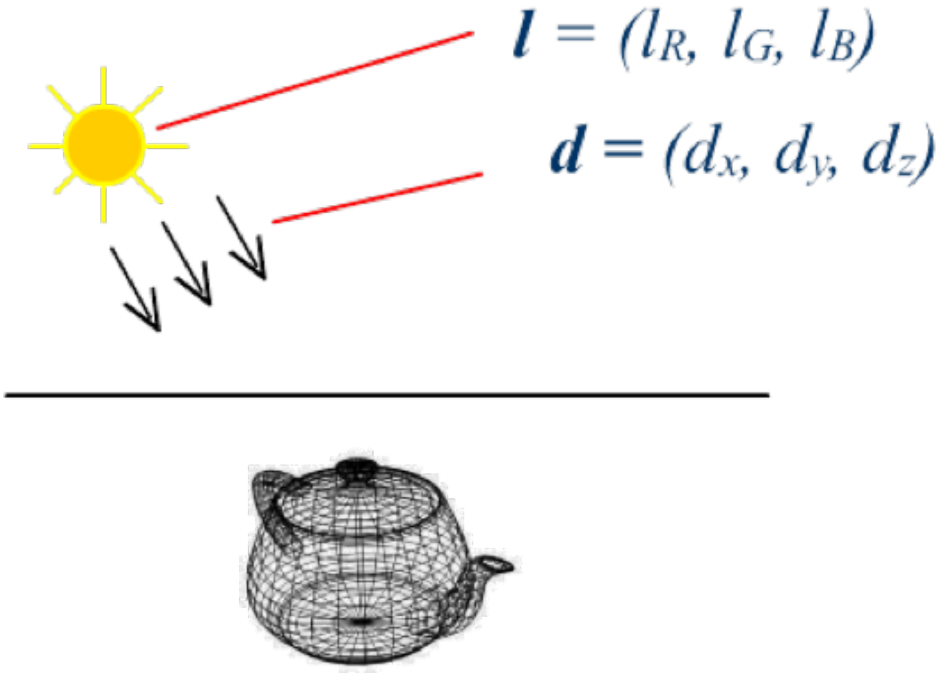
Due to the distance of the source, rays are parallel to each other in all the positions of the space, and constant in color and intensity.



Direct light models

The light direction can then be specified with a constant vector $\boldsymbol{d} = (d_x, d_y, d_z)$ that is independent of the position *x* on the object.

Light color is also specified by a constant vector $\boldsymbol{l} = (l_R, l_G, l_B)$



Direct light models

For every point of an object, the direction of the light and its color used in the rendering equations are expressed with these two constant values $\boldsymbol{d}$ and $\boldsymbol{l}$:

$$L(x, \omega_r) = \sum_l L(l, \overrightarrow{lx}) * f_r(x, \overrightarrow{lx}, \omega_r)$$

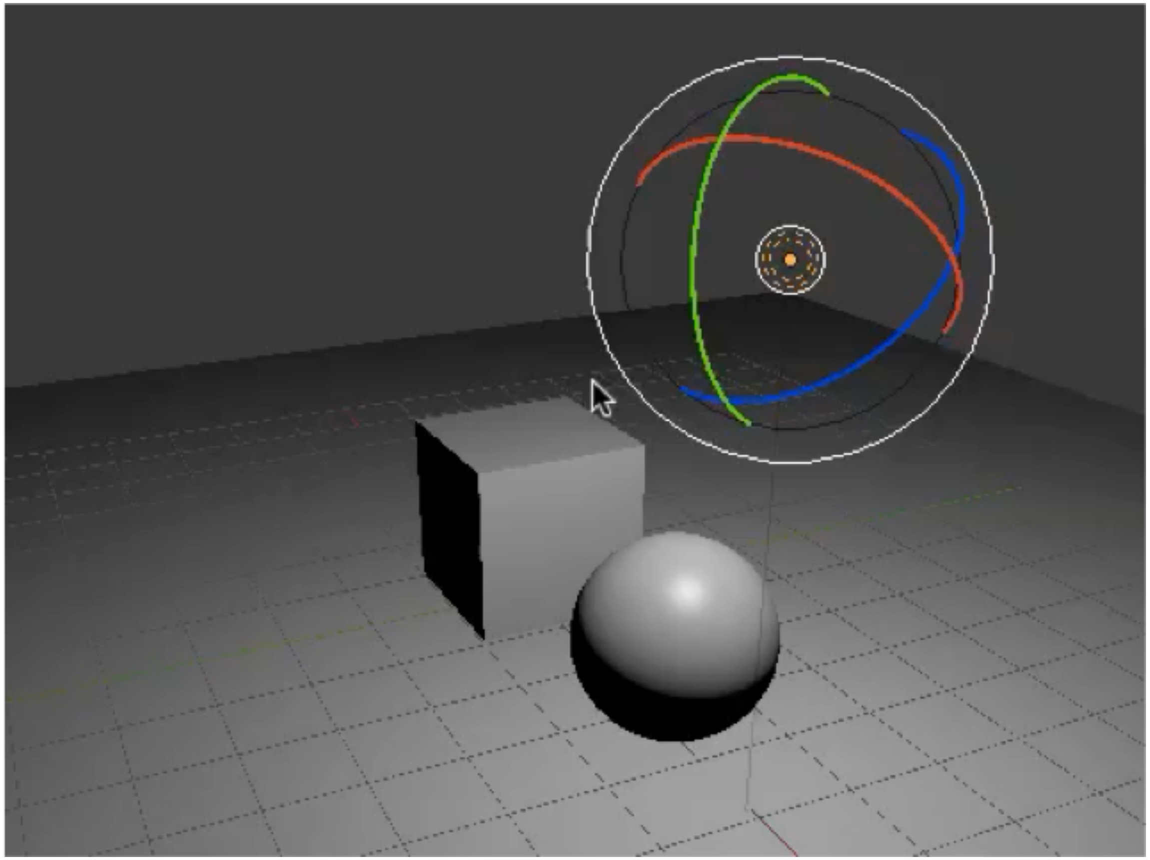
$$\boxed{L(l, \overrightarrow{lx})} = \boldsymbol{l_l} \qquad \boxed{\overrightarrow{lx}} = \boldsymbol{d_l}$$

In case of a single direct light, the rendering equation reduces to:

$$L(x, \omega_r) = \boldsymbol{l} * f_r(x, \boldsymbol{d}, \omega_r)$$

Point light models

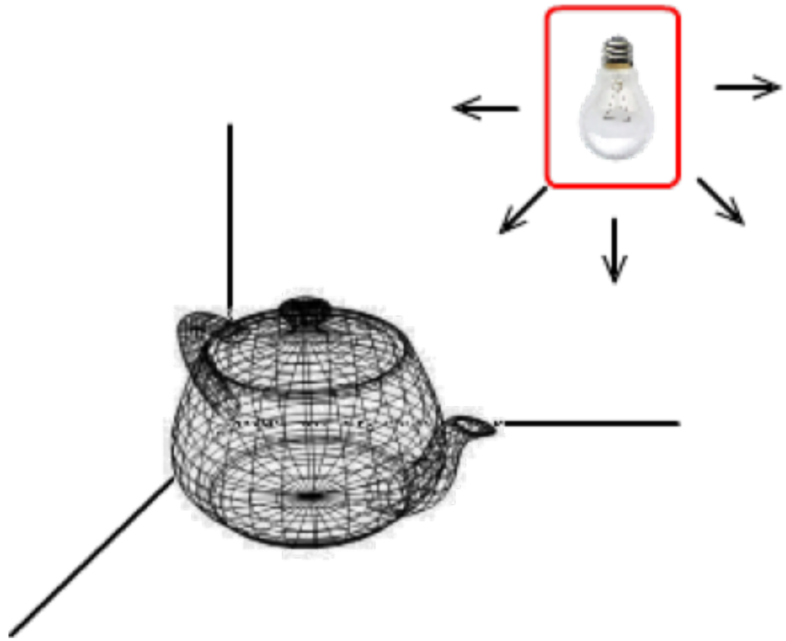
Point lights are sources that emit light from fixed points in the space, and do not have a direction.



Point light models

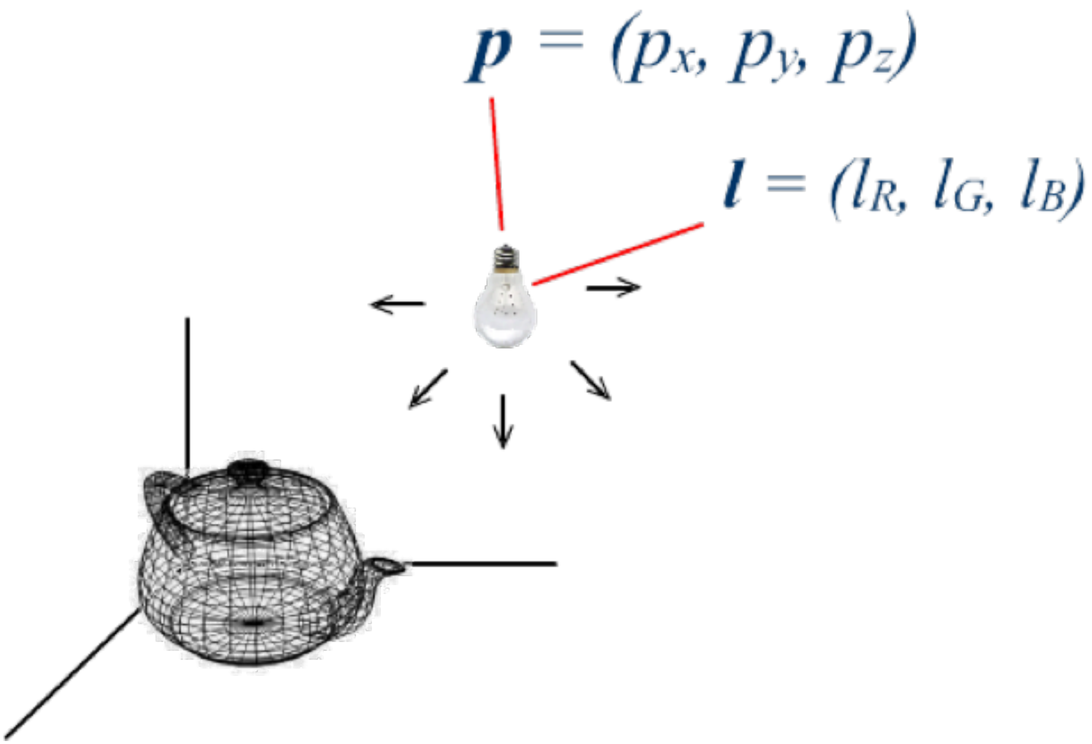
They are used to model sources that emit light in all directions, starting from a specific position in the scene.

For example, they can reproduce lamps, bulbs and other omnidirectional light sources.



Point light models

The position $\boldsymbol{p} = (p_x, p_y, p_z)$ and the color $\boldsymbol{l} = (l_R, l_G, l_B)$ characterize a point light.

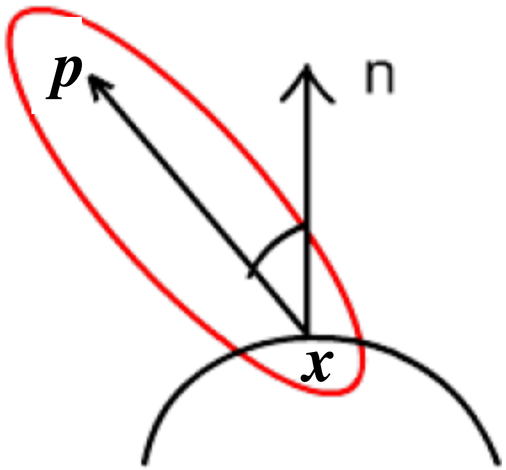


Point light models

The direction goes from point x to the center of the light, varying on the surface of the object it is illuminating.

Note that the light direction should be normalized to make it an unitary vector.

$$\overrightarrow{lx} = \frac{\boldsymbol{p} - \boldsymbol{x}}{|\boldsymbol{p} - \boldsymbol{x}|}$$

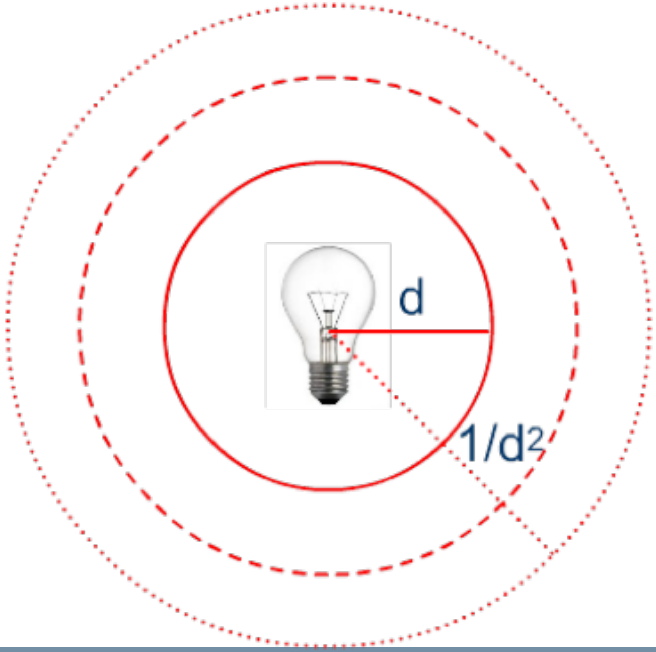


Also note that we write $p - x$ because the ray is oriented from the object to the light, as for the direct light case.

Point light models

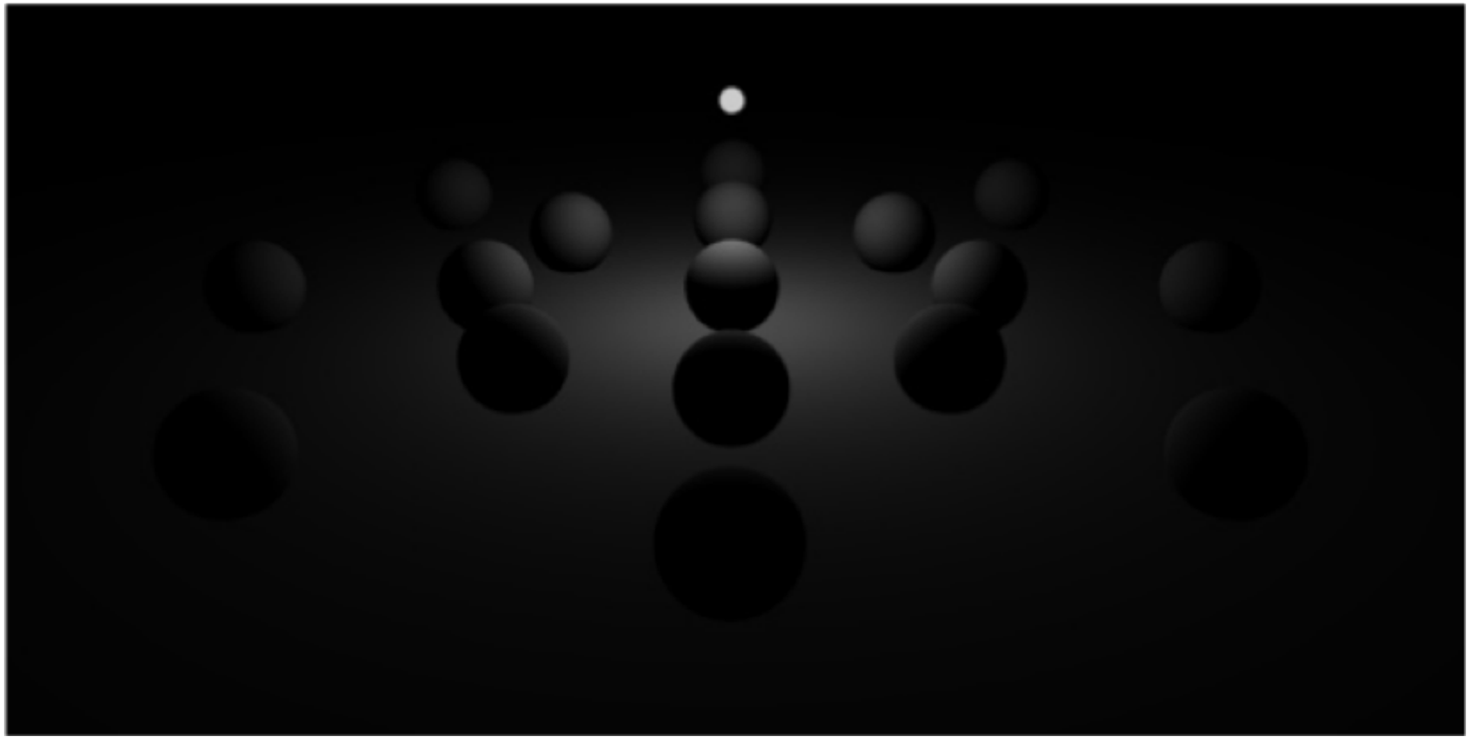
To reproduce the physical properties of light sources, point lights are characterized by a *decay factor*.

Physically, the intensity of a point light reduces at a rate that is proportional to the inverse of the square of the distance.



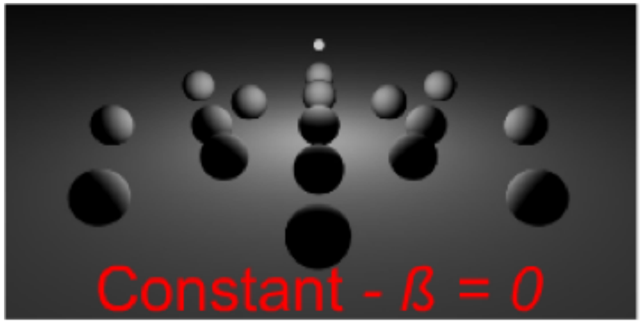
Point light models

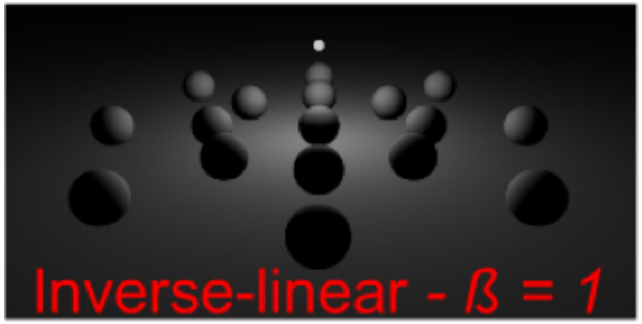
However this might lead to images that are too dark.

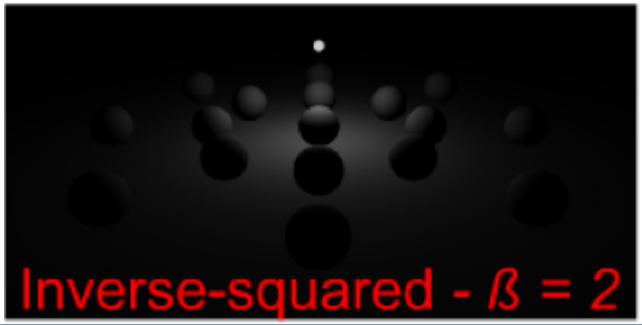


Point light models

For this reason, light models usually allow the user to specify a decay factor ß that is either constant, inverse-linear or inverse-squared.



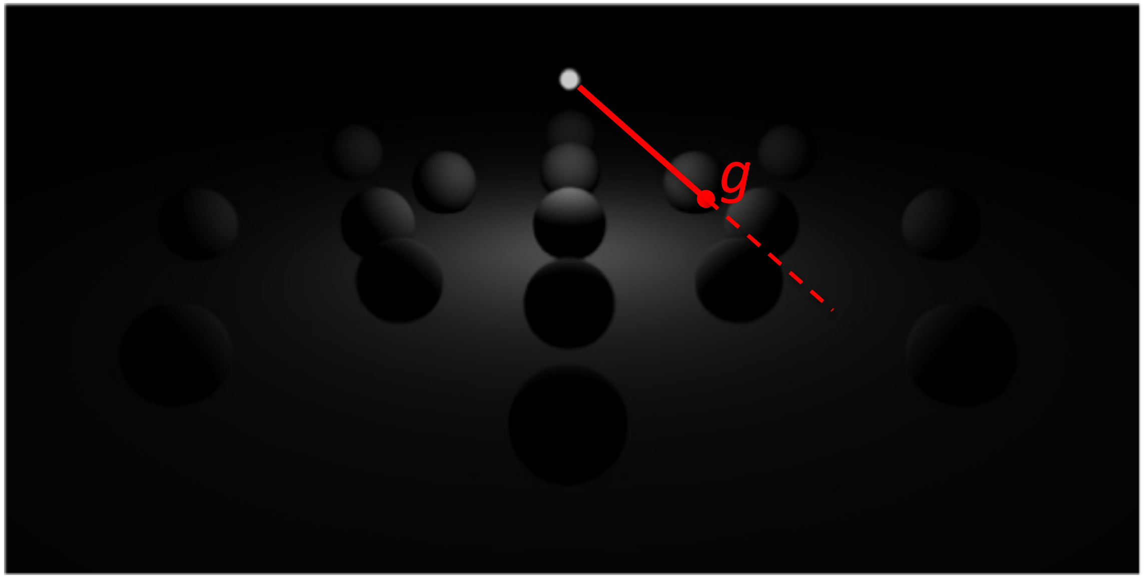




$$L(l,\overrightarrow{lx}) = \left(\frac{g}{|\boldsymbol{p}-\boldsymbol{x}|}\right)^{\beta} \boldsymbol{l}$$

Point light models

The model requires also another value *g that* represents the distance at which the light reduction is exactly 1 : intensity will be higher than $\boldsymbol{l}$ for distances shorter than *g*, and it will dim for longer distances.



$$L(l,\overrightarrow{lx}) = \left(\frac{g}{|\boldsymbol{p}-\boldsymbol{x}|}\right)^{\beta} \boldsymbol{l}$$

Point light models

To summarize, the direction of the light and the color used in the rendering equations become:

$$L(x, \omega_r) = \sum_l L(l, \overrightarrow{lx}) f_r(x, \overrightarrow{lx}, \omega_r)$$

$$\boxed{L(l, \overrightarrow{lx})} = \boldsymbol{l}\left(\frac{g}{|\boldsymbol{p} - \boldsymbol{x}|}\right)^{\beta} \qquad \boxed{\overrightarrow{lx}} = \frac{\boldsymbol{p} - \boldsymbol{x}}{|\boldsymbol{p} - \boldsymbol{x}|}$$

In case of a single point light, the rendering equation for one pixel is:

When no decay is considered

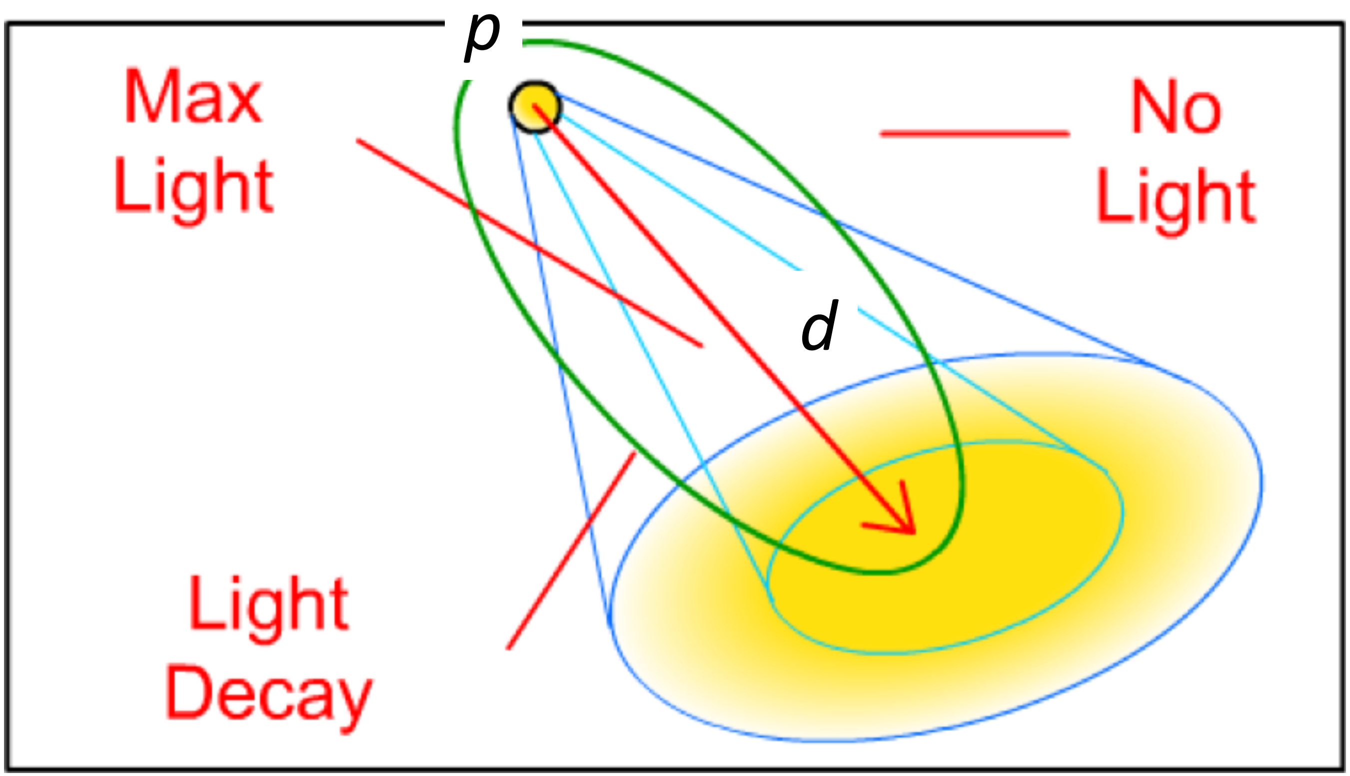
$$L(x, \omega_r) = \boldsymbol{l}\left(\frac{g}{|\boldsymbol{p} - \boldsymbol{x}|}\right)^{\beta} * f_r\left(x, \frac{\boldsymbol{p} - \boldsymbol{x}}{|\boldsymbol{p} - \boldsymbol{x}|}, \omega_r\right) = \boxed{\boldsymbol{l} * f_r\left(x, \frac{\boldsymbol{p} - \boldsymbol{x}}{|\boldsymbol{p} - \boldsymbol{x}|}, \omega_r\right)}$$

Spot light models

Spot lights are special projectors that are used to illuminate specific objects or locations.

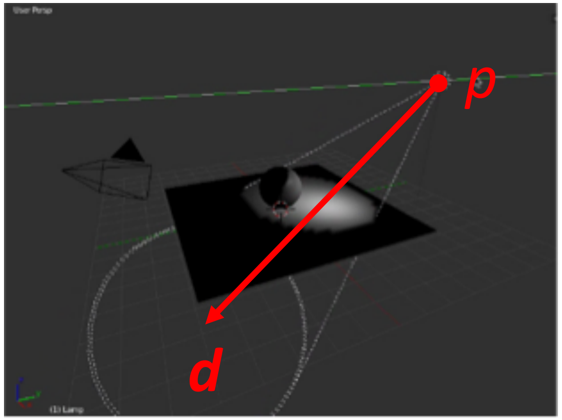
Spot light models

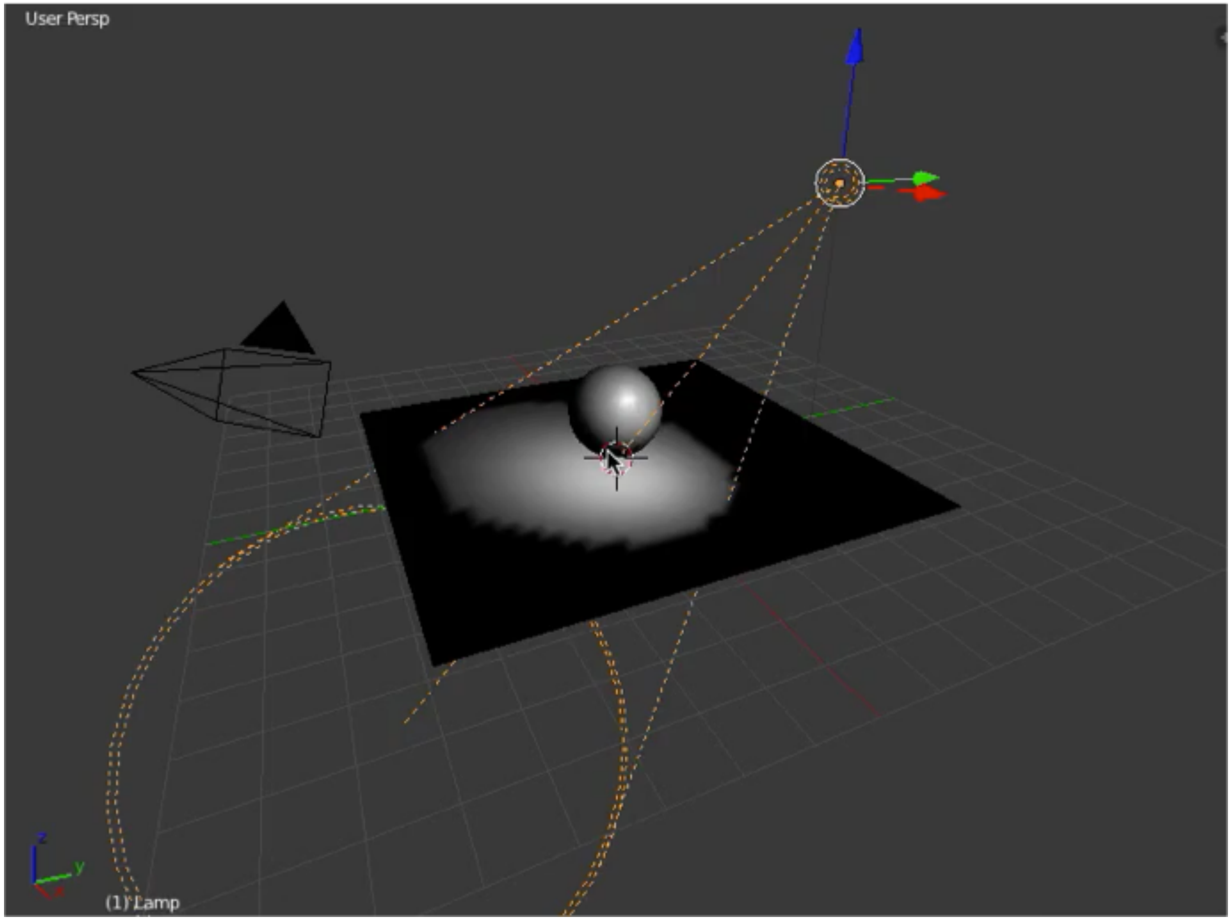
They are conic sources characterized by a direction *d* and a position *p*.



Spot light models

In particular, Spot lights emit in direction *d*, starting from point *p*.

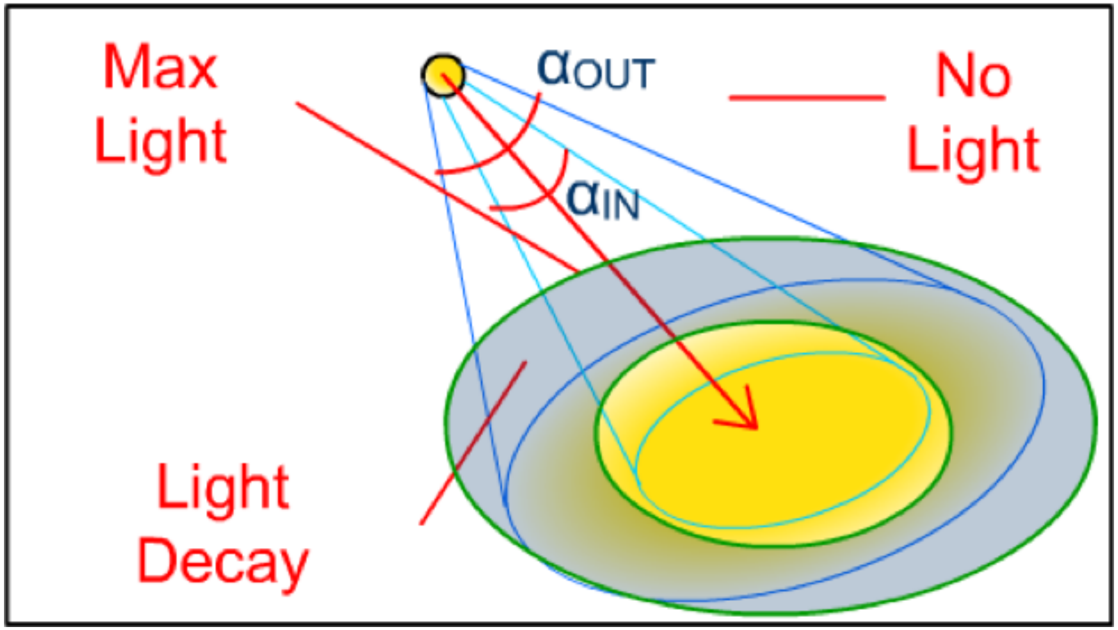




Spot light models

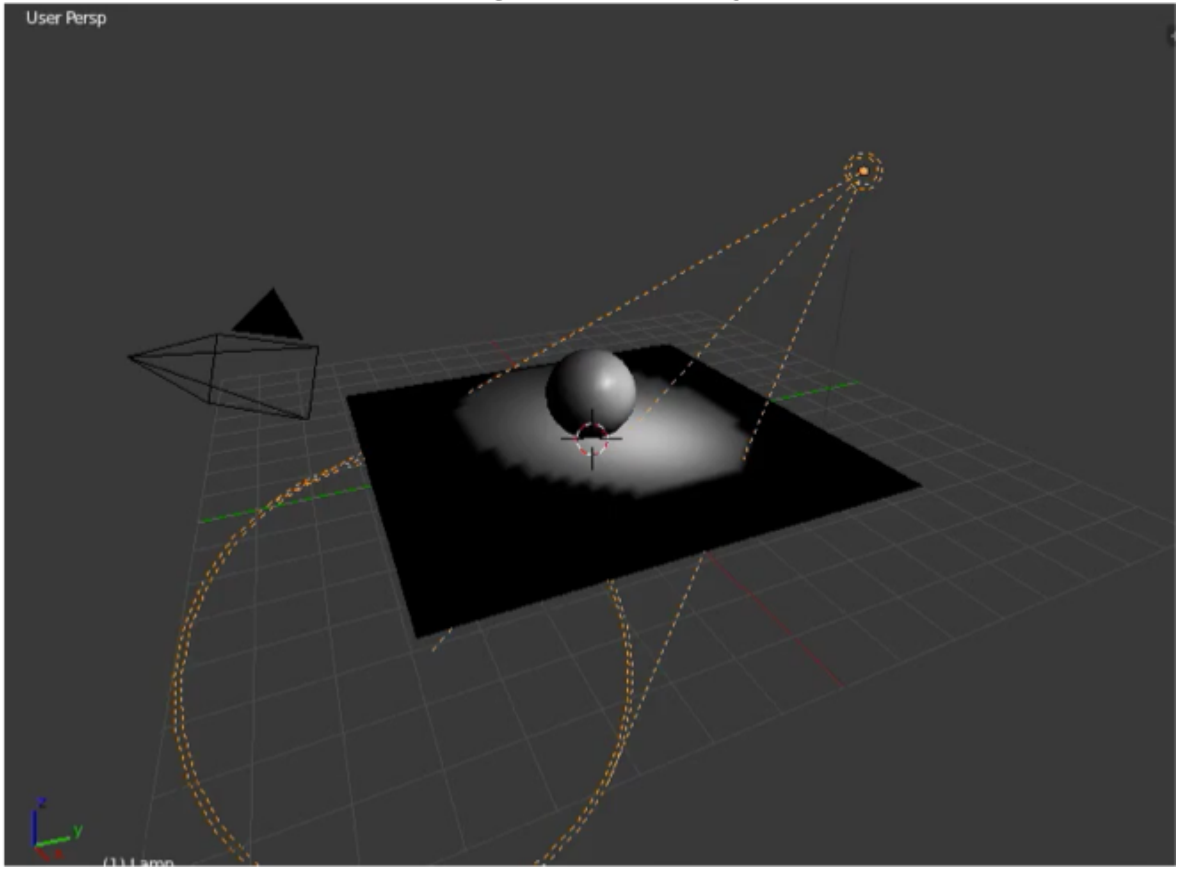
Spot lights are also characterized by two angles α_{IN} and α_{OUT} that divide the illuminated area into three zones: constant (inside α_{IN}), decay and absent (outside α_{OUT}).

In the light decay zone between α_{IN} and α_{OUT}, the light intensity decreases linearly from the inner to the outer angle.



Spot light models

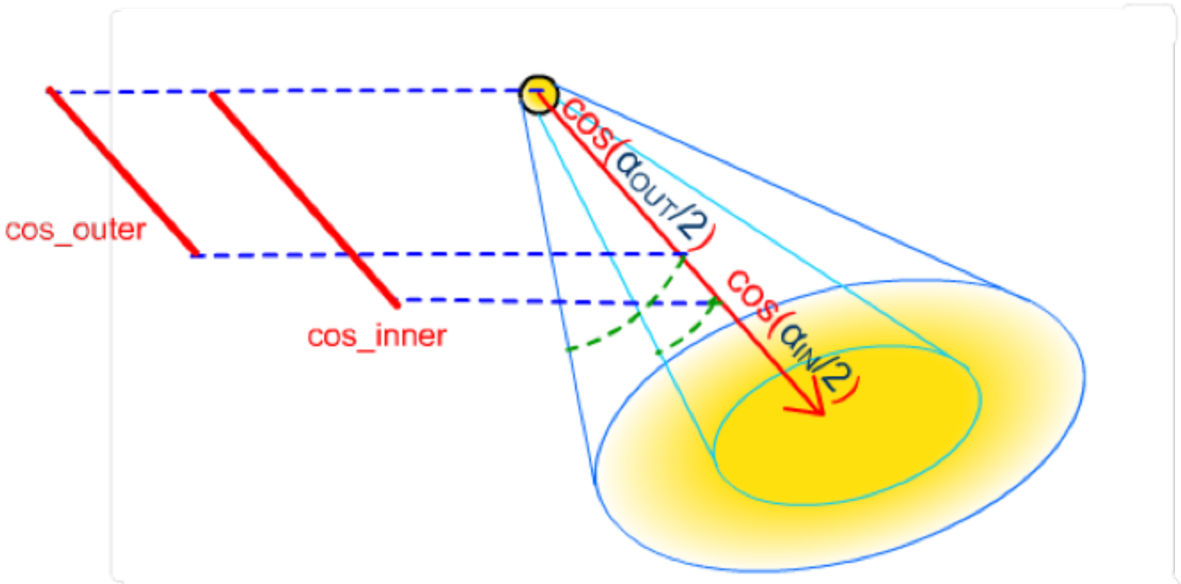
Using these two parameters, the light can be sized to concentrate its effect on a given subject.



Spot light models

For the implementation of the spot lights, usually the *cosine of the half-angles* of the *inner* and *outer cones* c_{in} and c_{out} are used.

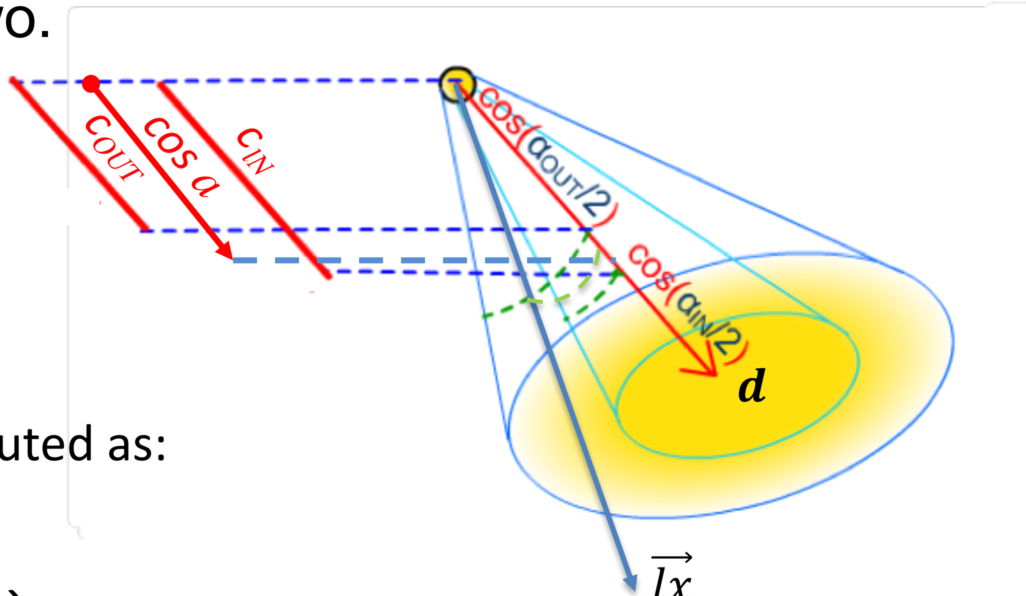
Note that the cosine of the inner angle is greater than the one of the outer angle.



Spot light models

The cosine of between the light direction vector $\overrightarrow{lx}$ and the direction of the spot ***d*** can be computed by performing the dot product between the two.

$$\cos\alpha = \overrightarrow{lx} \cdot \boldsymbol{d}$$



The cone dimming effect is computed as:

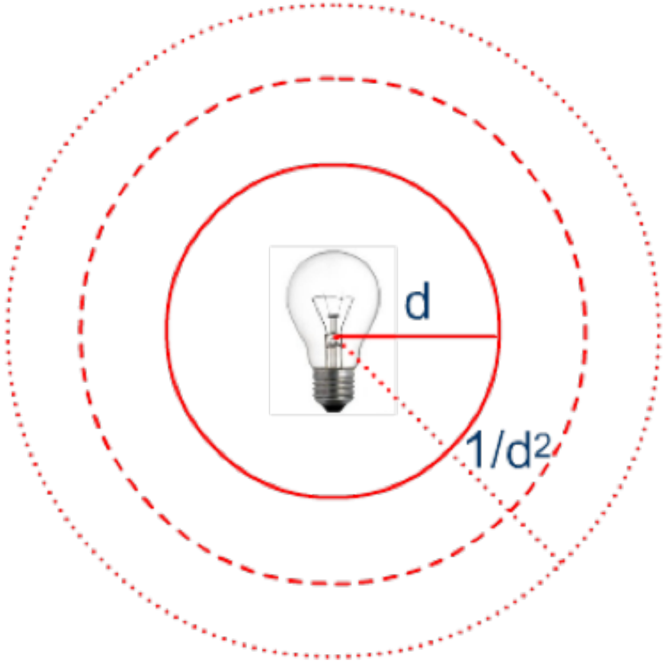
$$clamp\left(\frac{\cos\alpha - c_{OUT}}{c_{IN} - c_{OUT}}\right)$$

With $clamp(y) = \begin{cases} 0 & y < 0 \\ y & y \in [0,1] \\ 1 & y > 1 \end{cases}$

Spot light models

Since a spot light is an extension of a point light, it is also characterized by a decay factor β, a target distance g and a color vector $\boldsymbol{l}$.

Light direction is also computed as for the point light.



$$\boldsymbol{l}\left(\frac{g}{|\boldsymbol{p}-\boldsymbol{x}|}\right)^{\beta}$$

$$\overrightarrow{lx} = \frac{\boldsymbol{p}-\boldsymbol{x}}{|\boldsymbol{p}-\boldsymbol{x}|}$$

Spot light models

To summarize, the direction of the light and the color used in the rendering equations are the following:

$$L(x,\omega_r) = \sum_l L(l,\overrightarrow{lx}) f_r(x,\overrightarrow{lx},\omega_r)$$

$$\boxed{L(l,\overrightarrow{lx})} = \boldsymbol{l}\left(\frac{g}{|\boldsymbol{p}-\boldsymbol{x}|}\right)^{\beta} \cdot clamp\left(\frac{\frac{\boldsymbol{p}-\boldsymbol{x}}{|\boldsymbol{p}-\boldsymbol{x}|}\cdot \boldsymbol{d} - c_{OUT}}{c_{IN}-c_{OUT}}\right) \qquad \boxed{\overrightarrow{lx}} = \frac{\boldsymbol{p}-\boldsymbol{x}}{|\boldsymbol{p}-\boldsymbol{x}|}$$

In case of a spot light (without decay), the rendering equation for one pixel is:

$$L(x,\omega_r) = \boldsymbol{l}\cdot clamp\left(\frac{\frac{\boldsymbol{p}-\boldsymbol{x}}{|\boldsymbol{p}-\boldsymbol{x}|}\cdot \boldsymbol{d} - c_{OUT}}{c_{IN}-c_{OUT}}\right) * f_r\left(x,\frac{\boldsymbol{p}-\boldsymbol{x}}{|\boldsymbol{p}-\boldsymbol{x}|},\omega_r\right)$$