1 Transcription bursting model

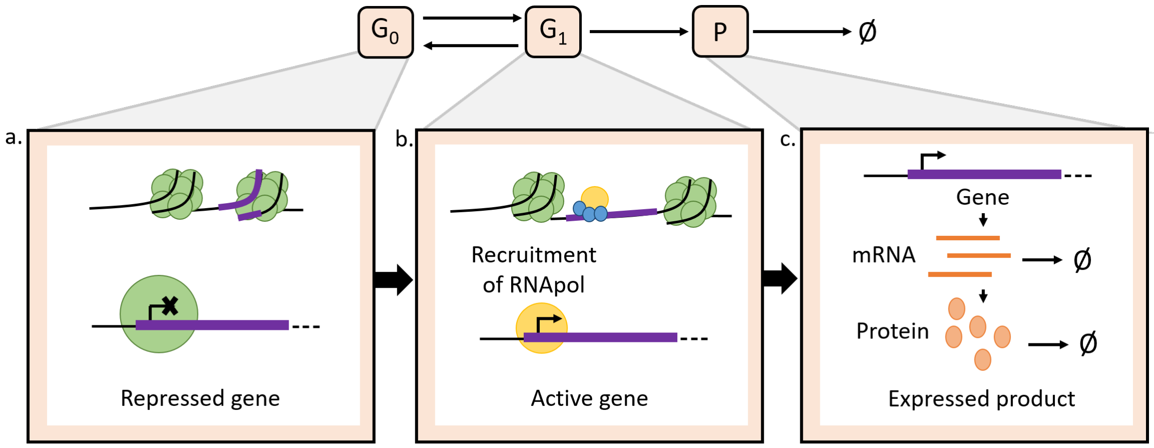

Figure 1: A stochastic model of transcriptional bursting. **a)** G_0 refers to the inactive gene (purple) which could be due to reduced access to the promoter due to chromatin (green) organisation or inhibitory regulatory factors. **b)** G_1 refers to the active gene (purple) due to possible re-modelling of the chromatin (green), production of the pre-initiation complex (blue) and recruitment of RNA polymerase (yellow). **c)** P refers to the expressed product (orange) which can be defined as mRNA or protein, at different stages of post-transcriptional and translational modifications.

2 Standard model

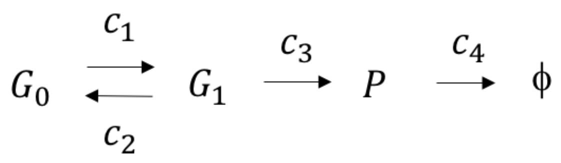

Figure 2

Where G_0 and G_1 represent inactive and active genes respectively, P represents the product and ϕ represents null space. Rate constants c_1, c_2, c_3 and c_4 represent gene activation and deactivation, and product production and degradation rates respectively.

Differential equations

$$\frac{dG_0}{dt} = G_1 c_2 - G_0 c_1$$

$$\frac{dG_1}{dt} = G_0 c_1 - G_1 c_2$$

$$\frac{dP}{dt} = G_1 c_3 - P c_4 \tag{1}$$

Stationary states

$$G_0 = G - G_1$$

$$G_1 = G \frac{c_1}{c_1 + c_2}$$

$$P = G \frac{c_3}{c_4} \frac{c_1}{c_1 + c_2} \tag{2}$$

where G = The total number of genes in the system and remaining variables are defined as above.

3 Feedback loops

3.1 Negative feedback: Activation

Negative feedback loop, where c_1 is the maximum activation rate, and is reduced depending on product abundance.

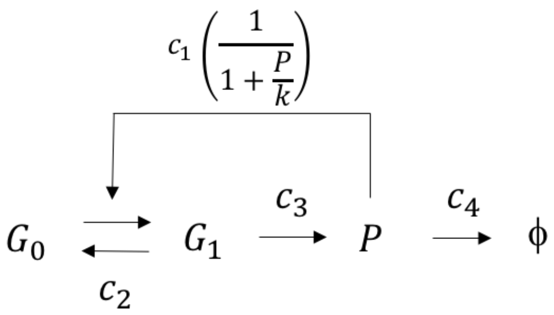

Figure 3

Differential equations

$$\frac{dG_0}{dt} = G_1c_2 - G_0c_1(\frac{1}{1+\frac{P}{k}})$$

$$\frac{dG_1}{dt} = G_0c_1(\frac{1}{1+\frac{P}{k}}) - G_1c_2$$

$$\frac{dP}{dt} = G_1c_3 - Pc_4 \tag{3}$$

Stationary states

$$G_0 = G - G_1$$

$$G_1 = \frac{c_1Gk}{c_1k + c_2(k+P)}$$

$$P = \frac{\sqrt{4c_1c_2c_3c_4Gk + (c_1c_4k + c_2c_4k)^2} - c_1c_4k - c_2c_4k}{2c_2c_4} \tag{4}$$

3.2 Positive feedback: De-activation

Positive feedback loop, where c_2 is the basal de-activation rate, and is increased depending on product abundance.

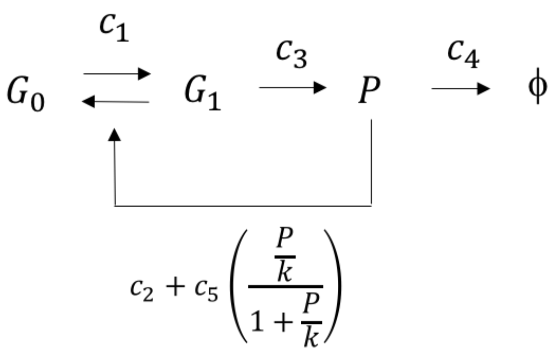

Figure 4

Differential equations

$$\frac{dG_0}{dt} = G_1c_2 + G_1c_5(\frac{\frac{P}{k}}{1+\frac{P}{k}}) - G_0c_1$$

$$\frac{dG_1}{dt} = G_0c_1 - G_1c_2 + G_1c_5(\frac{\frac{P}{k}}{1+\frac{P}{k}})$$

$$\frac{dP}{dt} = G_1c_3 - Pc_4 \tag{5}$$

Stationary states

$$G_0 = G - G_1$$

$$G_1 = \frac{c_1G(k+P)}{c_1(P+k)+c_2(P+k)+c_5P}$$

$$P = \frac{\sqrt{4c_1c_3c_4Gk(c_1+c_2+c_5)+(-c_1c_3G-c_1c_4k+c_2c_4k)^2}+c_1c_3G-c_1c_4k-c_2c_4k}{2c_4(c_1+c_2+c_5)} \tag{6}$$