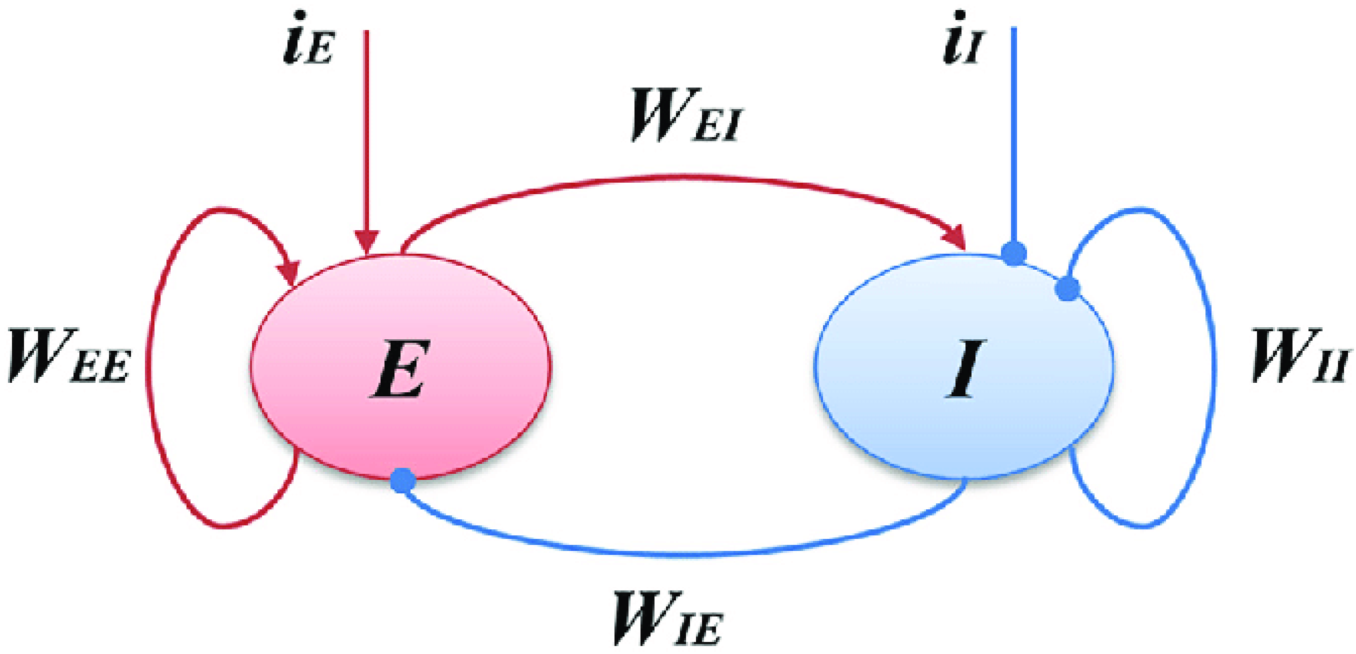
Wilson Cowan model

A complete overview



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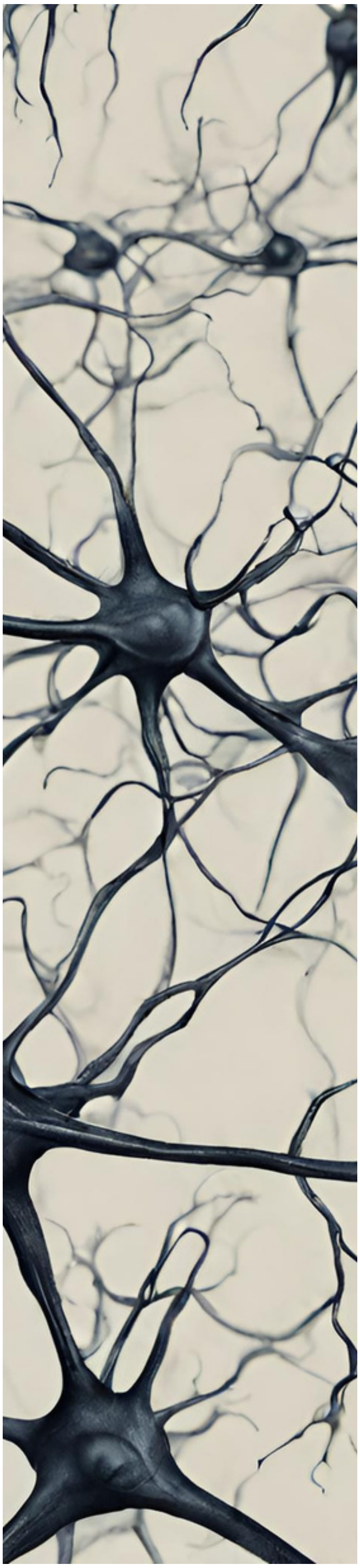
Introduction

The Wilson-Cowan model is a conceptual framework used to describe the dynamics of neural populations within the brain. Developed by Hugh Wilson and Jack Cowan in the early 1970s, this model represents one of the foundational approaches in computational neuroscience for understanding how networks of excitatory and inhibitory neurons interact to produce complex brain activities. Unlike models that describe the behavior of individual neurons, the Wilson-Cowan model focuses on the aggregate activity of neuron populations.



Objectives

Our notebook focuses on the practical application of the Wilson-Cowan model, aiming to:

- Illustrate the use of numerical integration, particularly the Euler method, in simulating the time evolution of neural population activities.
- Simulate and analyze the interplay between excitatory (E) and inhibitory (I) neural populations, exploring how these interactions shape the network's behavior.
- Employ visualization techniques to interpret the model's outcomes, enhancing our understanding of neural dynamics and their implications.

Mathematical formulation

The model can be formulated as follows for the excitatory (E) and inhibitory (I) populations:

$$\frac{dE}{dt} = -E + (1 - rE)F(w_{EE}E - w_{EI}I + P_E + S_E)$$

$$\frac{dI}{dt} = -I + (1 - rI)F(w_{IE}E - w_{II}I + P_I + S_I)$$

Where:

- E and I are the activity levels of the excitatory and inhibitory populations, respectively.
- w_{XY} represents the connection strength from population Y to population X, with $X, Y \in \{E, I\}$.
- P_E and P_I represent external inputs to the excitatory and inhibitory populations, respectively.
- S_E and S_I are the self-feedback terms for the excitatory and inhibitory populations, respectively.
- $F(\cdot)$ is a sigmoid function that transforms the net input into an output activity level.
- rE and rI are refractory parameters that limit the maximum firing rate of the neurons in each population.

Tools

At the heart of our simulation lies a series of defined functions, each serving a specific purpose in the modeling process:

- **Visualization Functions**: Crafted to plot the firing rate functions and their inverses, these tools allow us to visually interpret the model's parameters and their impact on neural activity.
- **Simulation Functions**: Designed to implement the Wilson-Cowan equations, these functions facilitate the numerical simulation of excitatory and inhibitory population dynamics, offering a window into the temporal evolution of neural activities.

Building the model

A critical step in simulating the Wilson-Cowan model involves setting up the model parameters and initial conditions, which serve as the foundation for understanding neural dynamics. This section outlines the parameters essential for our simulation:

- **Excitatory and Inhibitory Parameters**: These include the rates at which excitatory and inhibitory neurons fire, their thresholds for activation, and the time constants governing their decay. Each parameter plays a pivotal role in shaping the dynamics of neural populations.
- **Connection Strengths**: The strengths of connections between excitatory and inhibitory populations determine the influence they exert on each other, directly impacting the network's overall behavior.
- **External Inputs**: External inputs to the excitatory and inhibitory populations simulate the effect of stimuli from outside the neural network, introducing variability and complexity into the model's dynamics.

Initial conditions for the excitatory and inhibitory populations are crucial for determining the trajectory of the system's dynamics. These conditions represent the initial states of neural activities, from which we explore how the system evolves over time under the influence of internal interactions and external stimuli.

Numerical integration

At the core of our simulation lies the Euler method, a straightforward technique for numerical integration. This method enables us to approximate the solution to the differential equations governing the Wilson-Cowan model, updating the state of the neural populations at discrete time steps. By iteratively applying this method, we simulate the temporal evolution of excitatory and inhibitory activities, capturing the dynamics of the neural network.

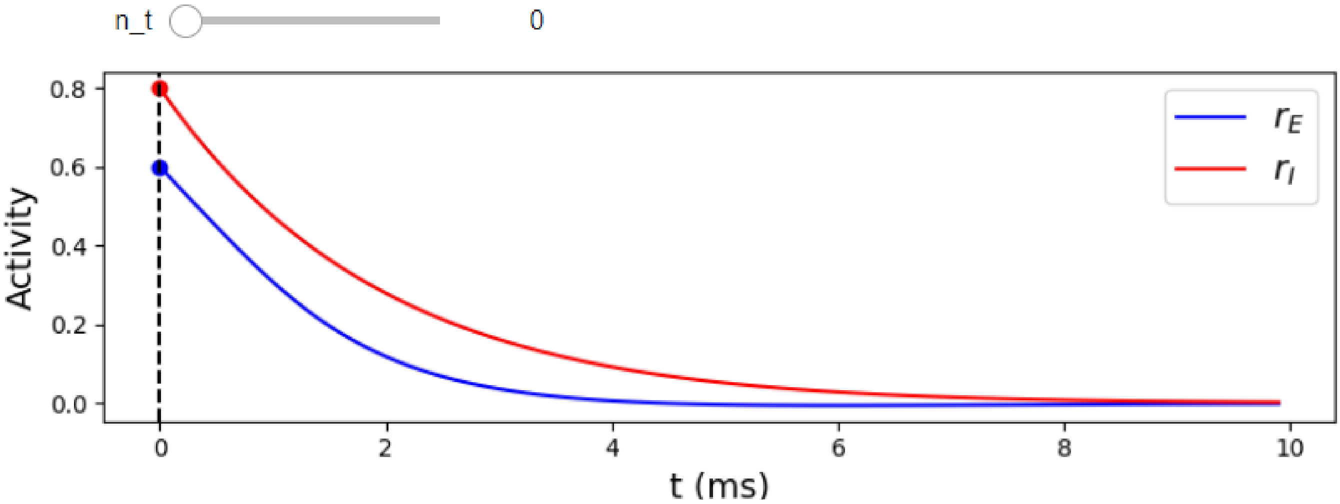
The simulation process unfolds through a series of meticulously defined steps:

- **Initialization**: We begin by establishing initial activity levels for the excitatory and inhibitory populations, setting the stage for their dynamic evolution.
- **Temporal Evolution**: At each time step, we calculate the derivatives of the E and I populations based on the current state, model parameters, and external inputs.
- **Updating the State**: Using the Euler method, we update the activities of the E and I populations, stepping forward in time to observe the unfolding dynamics.

This iterative process allows us to trace the trajectories of neural populations.

Plotting model

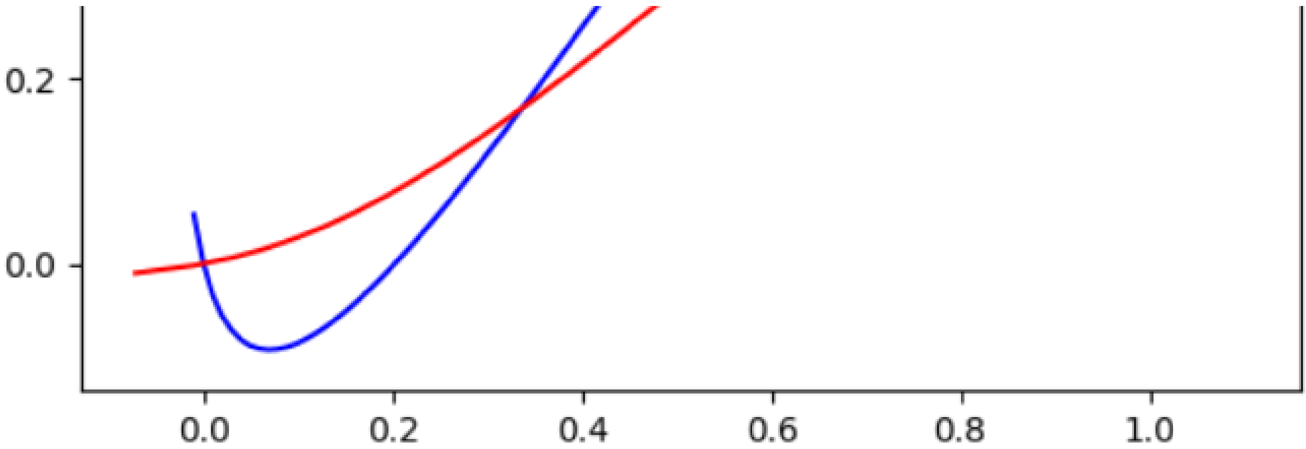
Visualization plays a crucial role in our exploration of the Wilson-Cowan model, offering intuitive insights into the dynamics of neural populations. By plotting the trajectories of excitatory and inhibitory activities, we observe how these populations evolve over time, under different initial conditions and parameter settings. Interactive plots further enrich our understanding, allowing us to manipulate conditions and instantly observe their effects on the system's behavior.



Exploring the Phase Plane

The phase plane analysis emerges as a powerful tool in our investigation, providing a graphical representation of the system's state space. Here, we plot the activities of the E and I populations against each other, revealing:

- **Nullclines**: Curves where the rate of change for either population is zero, highlighting equilibrium points.
- **Vector Fields**: Arrows indicating the direction and magnitude of the system's flow, offering clues about stability and potential oscillatory behavior.

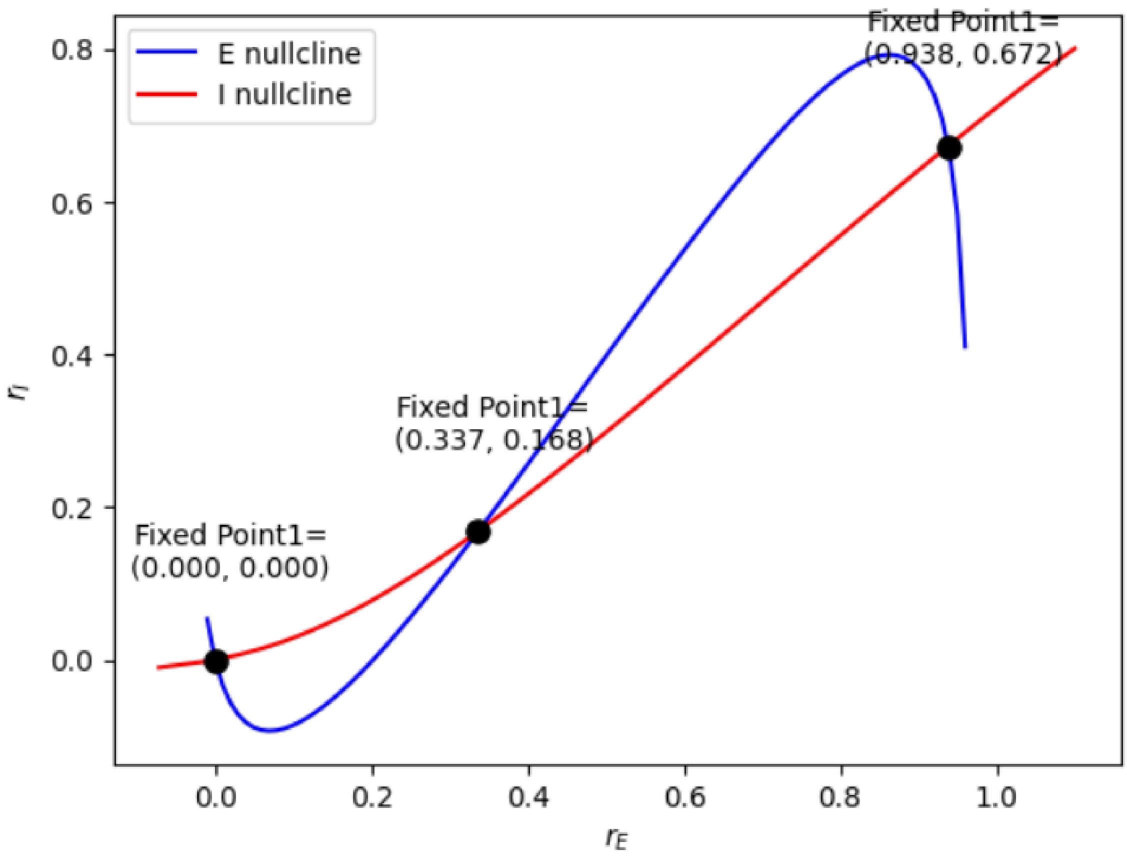


Fixed points

A key aspect of exploring this model involves identifying its stationary points, critical for understanding the stability and behavior of neural networks.
Stationary points, or fixed points, are states where the system's dynamics are in equilibrium. For the Wilson-Cowan model, these points occur when the rates of change of both excitatory and inhibitory populations are zero, indicating no further evolution in their activities.

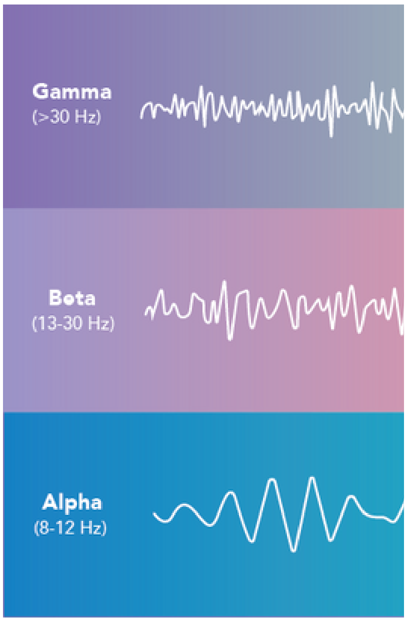
To locate these points, we employ the following steps:

- **Equation Setup:** Begin by setting the Wilson-Cowan differential equations to zero, creating a set of algebric equations that represent the conditions for stationary points.
- **Numerical Solutions**: Use numerical methods, such as the Newton-Raphson technique or optimization algorithms available in libraries like SciPy, to solve these equations. These methods help find solutions that satisfy the equilibrium conditions for E and I populations.
- **Analysis**: Evaluate the found solutions to determine their nature (stable, unstable, saddle points) by analyzing the Jacobian matrix at each stationary point. The eigenvalues of the Jacobian indicate the stability, guiding our understanding of how the system behaves near these points.



Simulating brain activity

In our exploration of the Wilson-Cowan model, an important moment come when we transitioned from theoretical understanding to the practical simulation of brain activity. This step not only exemplifies the model's utility in representing neural dynamics but also showcases its potential to mimic patterns of brain activity that reflect various cognitive and neurological states.



We decided to simulate the brain activity through the reproduction of:

- the alpha, beta and gamma waves
- the epilepsy related neural oscillations

To simulate brain activity, we begun by defining the parameters of the Wilson-Cowan model, including the excitatory and inhibitory populations, their interaction strengths, and external inputs. These parameters are crucial, as they dictate the dynamics and potential states the model can exhibit. Selection of initial conditions for the excitatory (E) and inhibitory (I) populations further shapes the trajectory of the simulation, influencing the resulting patterns of neural activity.

example: to simulate alpha waves, the parameters can be adjusted to reflect a stable, rhythmic oscillation in the lower frequency range. This typically involves tuning the interaction strengths between excitatory and inhibitory populations to facilitate such oscillatory behavior without pushing the system into hyperactivity.

fter simulating normal brain activity patterns, the transition to simulating epilepsy involves altering the model to reflect the abnormal, hyper-synchronous neuronal firing characteristic of seizures. Increasing excitatory connectivity or decreasing inhibitory control to simulate the conditions under which normal neural oscillations can escalate into seizure-like activity.