Control transitions in a classical spin chain

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Motivation:

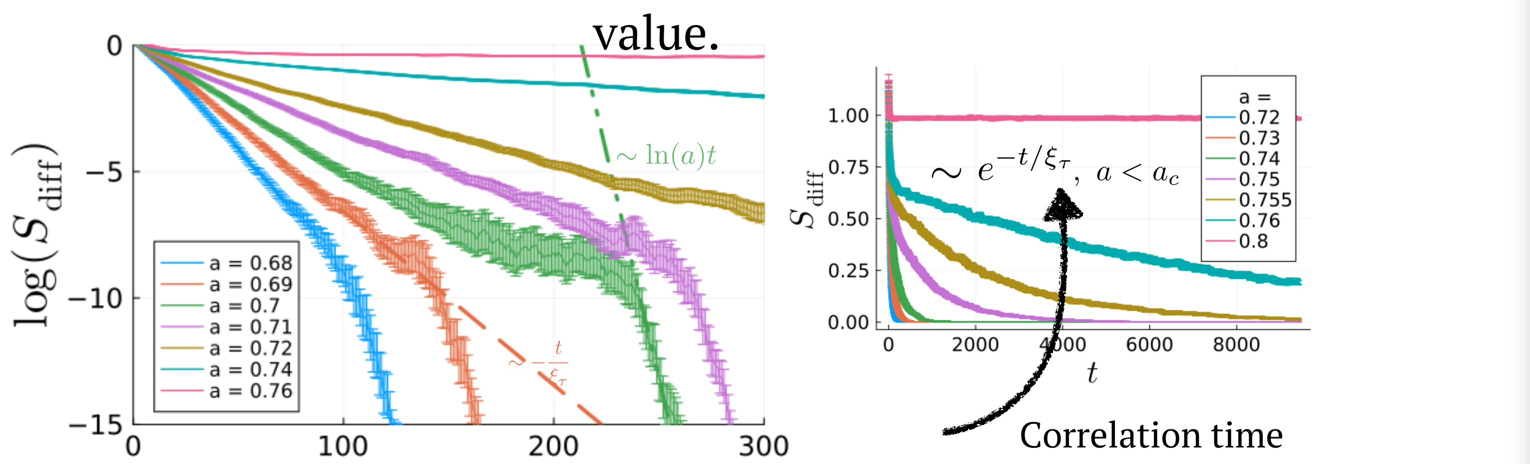
Controlling classically chaotic systems is a central problem in non-equilibrium dynamics due to its far reaching applications. We are interested in studying the behavior of non-equilibrium phase transitions that are in the presence of an "observer" that can make a control operation on the system. These principals can be applied to diverse fields such as robotics, computing, and statistical mechanics.

The full evolution involves hamiltonian dynamics, control map, and J_x, J_y randomization.

$$\tau : \xrightarrow{\boxed{H}} t \xrightarrow{\boxed{\hat{C}}} t \longrightarrow \boxed{\begin{cases} J_x \in \{-J, J\} \\ J_y \in \{-J, J\} \\ J_z = 1 \end{cases}} \xrightarrow{\boxed{H}} \cdots$$

$$S_{\text{diff}}(t) := \frac{1}{L} \sum_j |\delta \mathbf{S}_j(t)|$$

: In the control phase it decays exponentially, whereas in the chaotic phase it rapidly assumes and maintains a constant value.



By preforming FSS on the correlation time, we can approximate the values of critical exponents:

Critical point $\boxed{a_c}$

Spacial correlation length exponent $\boxed{\nu}$

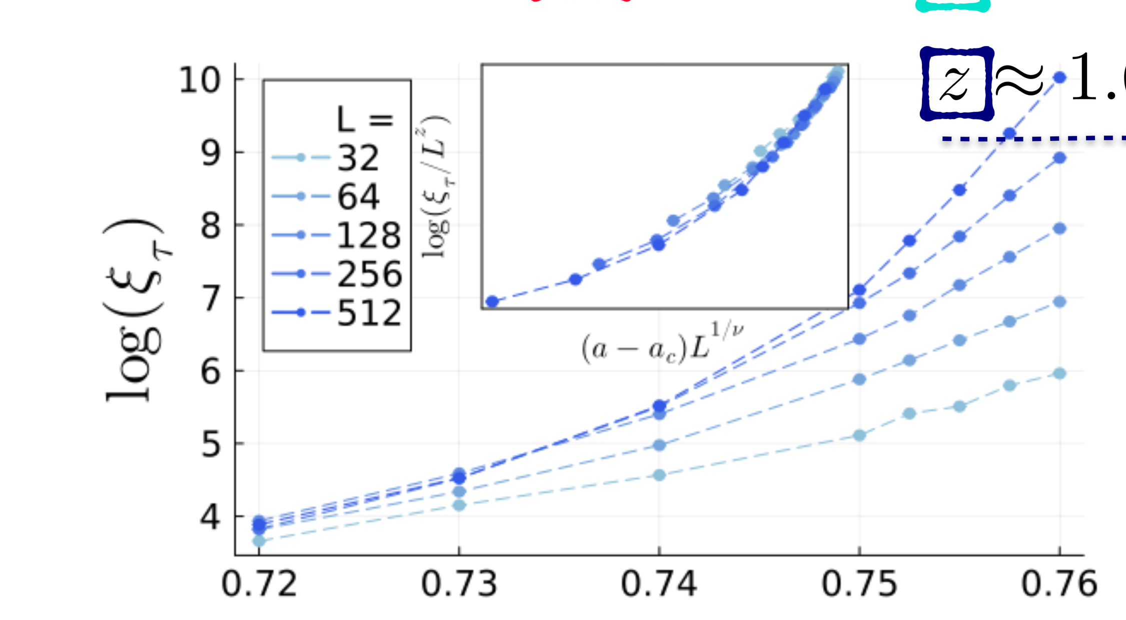
Dynamical scaling exponent $\boxed{z}$

$$\xi_\tau \sim L^{\boxed{z}} f[(a - \boxed{a_c}) L^{1/\boxed{\nu}}]$$

$\boxed{a_c} \approx 0.762 \pm 0.001$

$\boxed{\nu} \approx 1.9 \pm 0.3$

$\boxed{z} \approx 1.65 \pm 0.1$



The Classical Heisenberg spin chain:

The dynamics of an arbitrary spin configuration are in general chaotic.

$$\boxed{H} = \sum_{i=1}^{L} [J_x(t) S_i^x + J_y(t) S_i^y + J_z(t) S_i^z]$$

There do exist unstable fixed points such as $\mathbf{S}_j^0 := (0, \cos(\pi j/2), \sin(\pi j/2))$

$$\left.\begin{aligned} \mathbf{S}_j^0(t) &= \mathbf{S}_j^0(0) \\ \mathbf{S}_j(t) &= \mathbf{S}_j^0 + \delta \mathbf{S}_j(t) \end{aligned}\right\}$$

Control map: $\boxed{\hat{C}}$ We directly control control the evolution of a spin chain by pushing it towards this unstable fixed point.

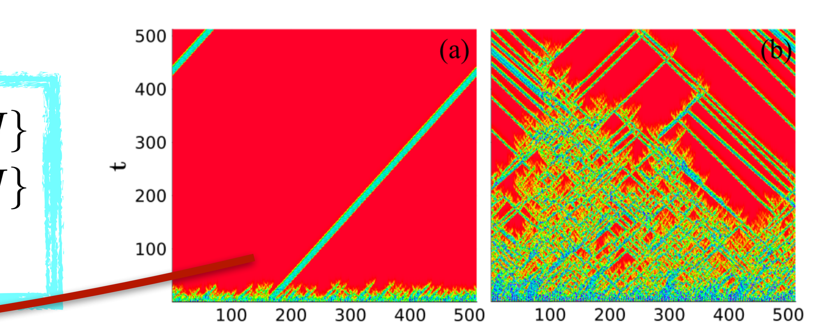
$$\hat{C}\mathbf{S}_j(t') = \frac{(1-a)\mathbf{S}_j^0 + a\mathbf{S}_j(t')}{|(1-a)\mathbf{S}_j^0 + a\mathbf{S}_j(t')|}$$

$0 < a < 1$ is our control parameter:

The unmodified hamiltonian + control dynamics leads to soliton like features that make it difficult to distinguish the two phases.

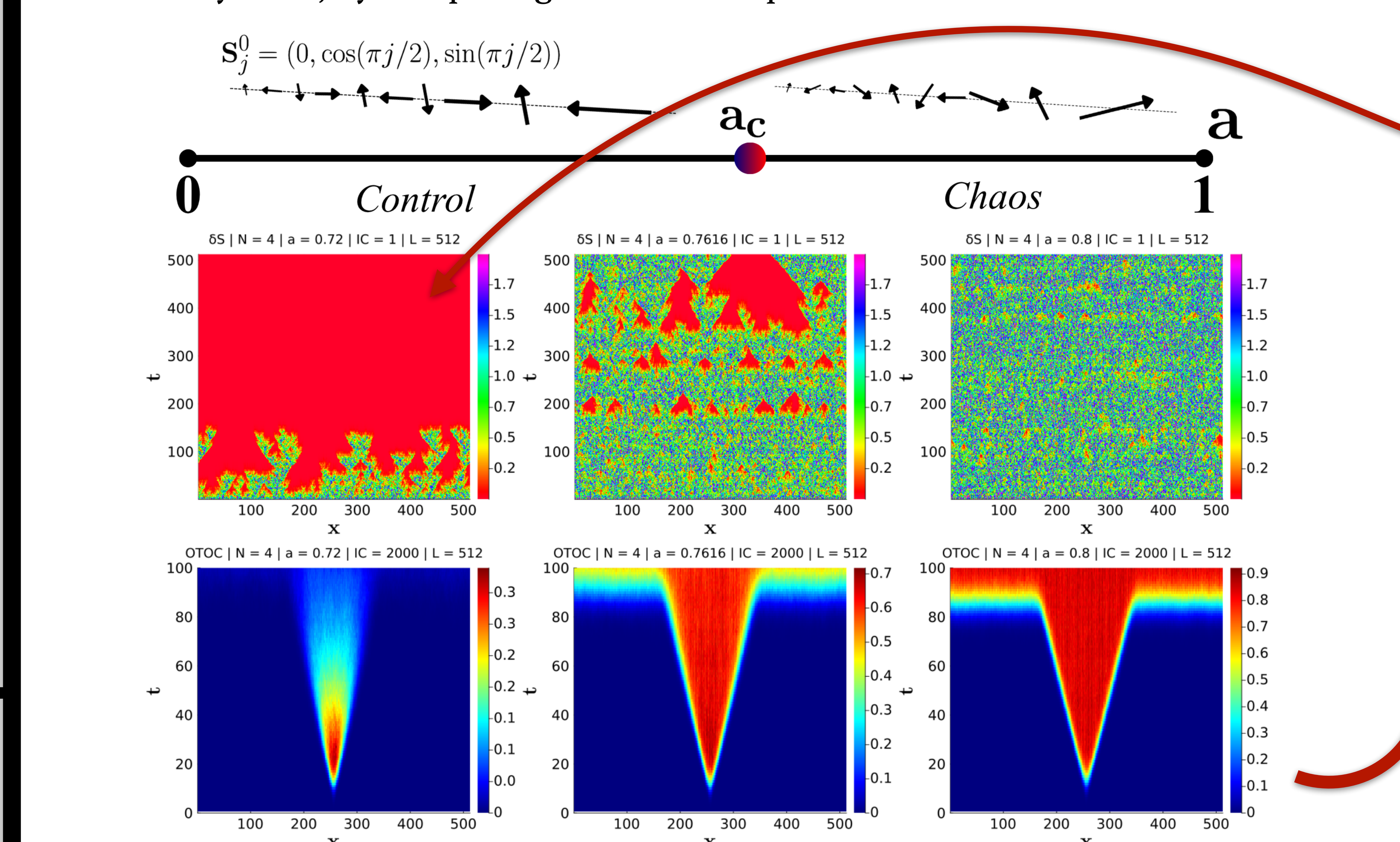
We randomize the sign of J_x, J_y to break up these wave packets:

$$\begin{cases} J_x \in \{-J, J\} \\ J_y \in \{-J, J\} \\ J_z = 1 \end{cases}$$



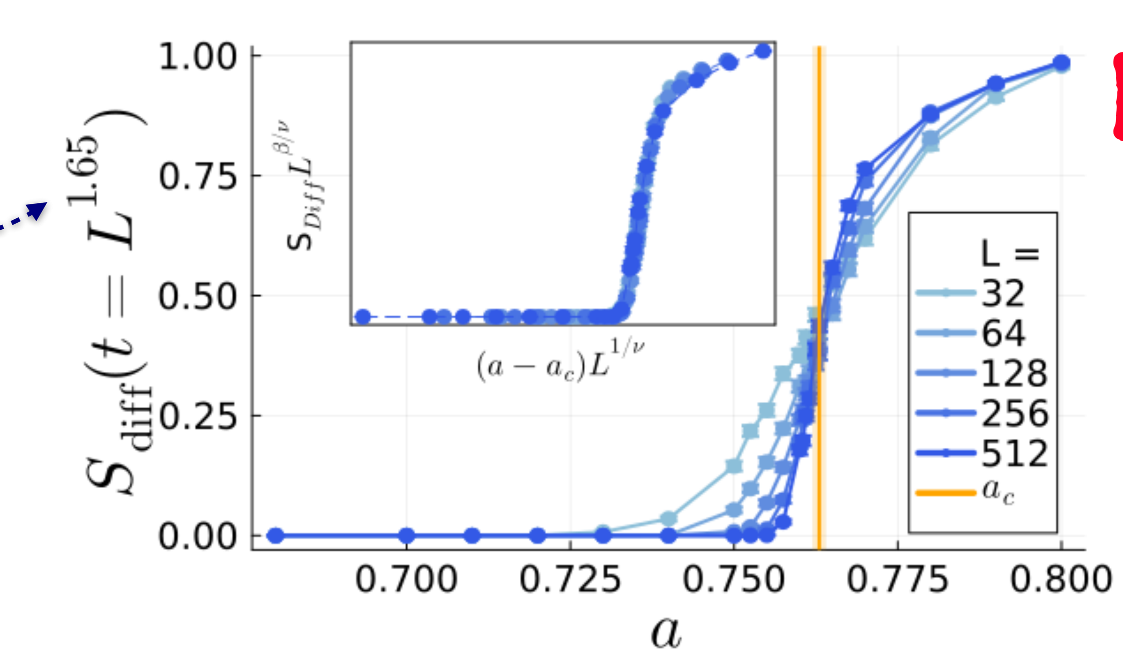
Phase Transition between chaos and control:

We demonstrate that the transition is **not** of the directed percolation (DP) universality class, by computing its critical exponents.



Preforming FSS on S_{diff} gives us another approximation of the critical exponents.

$$S_{\text{diff}}(t = L^{\boxed{z}}, a) := \langle |\delta \mathbf{S}_j| \rangle \sim L^{\boxed{\nu}/\boxed{\beta}} g[(a - \boxed{a_c}) L^{1/\boxed{\nu}}]$$



$\boxed{a_c} \approx 0.763 \pm 0.001$

$\boxed{\nu} \approx 2.0 \pm 0.2$

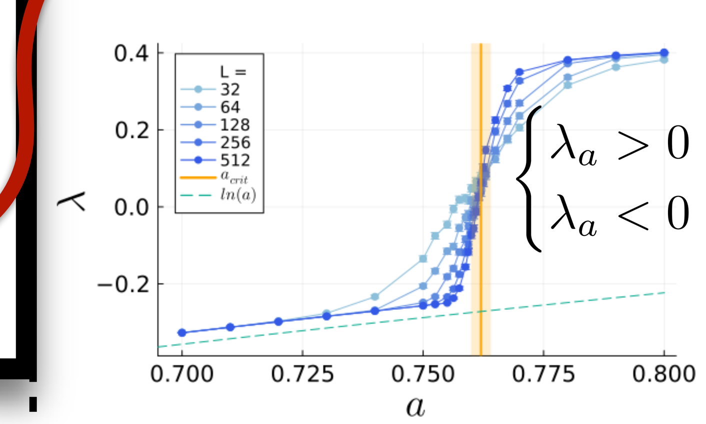
$\boxed{\beta} \approx 0.1 \pm 0.05$

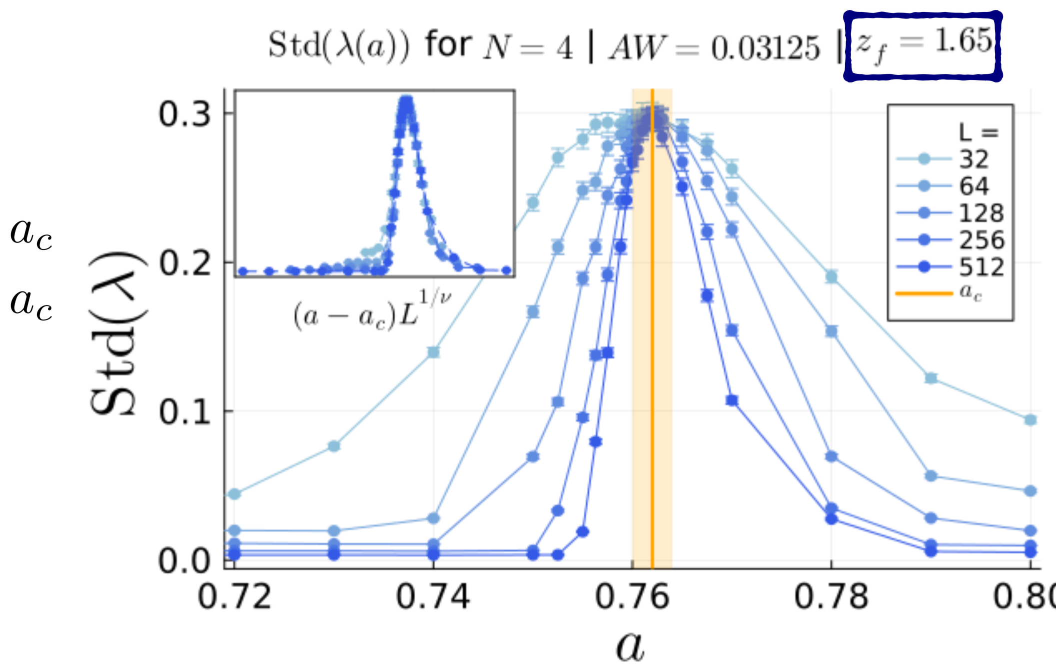
Order parameter exponent

Jump in Lyapunov Exponent:

$$D(x,t) := 1 - \langle \mathbf{S}_x^A \cdot \mathbf{S}_x^B \rangle \sim e^{\lambda_a t}$$

The dynamics cause two states which initially differ by some $\epsilon << 1$ to either come to together ($a < a_c$) or fly apart ($a > a_c$). This is characterized by a jump in the Lyapunov exponent:





$\boxed{a_c} \approx 0.762 \pm 0.002$

$\boxed{\nu} \approx 2.0 \pm 0.3$

$$\text{Std}(\lambda(t = L^{\boxed{z}}, a)) \sim h[(a - \boxed{a_c}) L^{1/\boxed{\nu}}]$$

Conclusion and Next Steps:

$$\begin{cases} J_x \in \{-J, J\} \\ J_y \in \{-J, J\} \\ J_z = 1 \end{cases}$$

The critical exponents have shown that our transition is not of the DP universality class, despite reasons to think that it could be.

We think that the transition might be of a yet unexplored **time random** universality class. This is possibility is the subject of current research.

We also plan to explore the effects of local control on the system.

1. Giancarlo Benettin. Kolmogorov entropy and numerical experiments. Physical Review A, 14(6):2338, 1976.