Notes: Week of June 23

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June 24, 2025

1 Analytic derivatation of Lyop-Exp

1.1 Expansion of functions

1.1.1 $f(x)$

Let

$$f(x) = \sqrt{1 - x^2}$$

We expand $f(x_0 + \delta x)$ to leading order in δx using a Taylor expansion:

$$f(x_0 + \delta x) \approx f(x_0) + f'(x_0)\,\delta x$$

The derivative:

$$f'(x) = \frac{d}{dx}\left(\sqrt{1 - x^2}\right) = \frac{-x}{\sqrt{1 - x^2}}$$

Then:

$$f'(x_0) = \frac{-x_0}{\sqrt{1 - x_0^2}}$$

Thus:

$$f(x_0 + \delta x) \approx \sqrt{1 - x_0^2} - \frac{x_0}{\sqrt{1 - x_0^2}}\,\delta x$$