MIDSEM MAKEUP

Q1 a) $\begin{bmatrix} c_1 & 2 & 7 \\ c_1 & c_1 & 4 \\ c_1 & c_1 & c_1 \end{bmatrix}$

$\rightarrow \begin{bmatrix} c_1 & 2 & 7 \\ 0 & c_1-2 & -3 \\ 0 & c_1-2 & c_1-7 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} c_1 & 2 & 7 \\ 0 & c_1-2 & -3 \\ 0 & 0 & c_1-4 \end{bmatrix}$ (1/2 Mark)

$c_1=0$, $c_1=2$, $c_1=4$ are 3 values for which it will fail to have pivots (1 1/2 Mark)

1 b) Student U is correct

$S = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

$T = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ (2 Marks)

Student W is correct

$S = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right\}$

$T = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ (2 Marks)

From S $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ was Removed.

Q2 a) $T \circ T$ is identity transformation because

$(A^T)^T = A$ (½ Marks)

b) Kernel (T) is the matrix $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ because, its the only matrix whose transpose is $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (½ Marks)

c) Range (T) is set of all 2×2 matrices as any matrix B can be obtained from T(M) by giving $M = B^T$ Range (T) $= M(2 \times 2)$ (½ Marks)

d) $T(M) = -M$ is possible when M is a skew symmetric matrix (½ Marks)

2 b) $a_{i1} + a_{i2} + \cdots + a_{in} = 0 \quad \forall \; i$

implies that $x = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$ is in N(A) (1 Mark)

Nullity (A) ≥ 1

$\Rightarrow$ By Rank Nullity Theorem rank(A) $< n$ (1 Mark)

2c) Since some right hand sides b do not have a solution it means rank (A) $< m$ (1 Mark) ①

By Rank Nullity Theorem

rank (A^T) + Nullity $(A^T) = m$ [since rank(A) = rank(A^T)]

$\Rightarrow$ rank(A) + Nullity(A^T) $= m$

From ① Nullity $(A^T) \geq 1$

$A^T y = 0$ has nontrivial solutions (1 Mark)

Q3a) Eigen Values are $\lambda = 0, \pm\sqrt{2\cos(2x)}$ ↓ Calculation & final answer (1 Mark)

$x = \frac{\pi}{4}, \frac{3\pi}{4}$ ↓ (1 Mark)

Case 1 :- When $x = \frac{\pi}{4}, \frac{3\pi}{4}$ A is not diagonalizable (1 Mark)

Case 2 :- When $x \neq \frac{\pi}{4}, \frac{3\pi}{4}$ A is diagonalizable (1 Mark)

Q3b) $A = PQ$

SVD of $Q = U \Sigma V^T$

$A = \underbrace{PU} \Sigma V^T$

$A = P' \Sigma V^T$

Same Singular Values of A & P (1 Mark)

One Multiplication. (1 Mark)

Q4a) $l_{11} = 1$, $l_{21} = -1$, $l_{31} = 3$
$l_{22} = 2$, $l_{32} = -1$, $l_{33} = 3$ → 3 Marks

$$Ly = b \quad \begin{bmatrix} -1 & 0 & 0 \\ -1 & 2 & 0 \\ 3 & -1 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 15 \\ -35 \\ 94 \end{bmatrix} \quad y = \begin{bmatrix} 15 \\ -10 \\ 13 \end{bmatrix}$$

↓ (½ Marks)

$$Ux = L^T x = y \Rightarrow \quad x = \begin{bmatrix} -0.83333 \\ -2.83333 \\ 4.33333 \end{bmatrix}$$

↓ (½ Mark)

Q4b) Total Number of Operations

$U_{n \times n}$ is upper triangular matrix

$$= 2 \sum_{i=1}^{n} (n-i) + n$$

(½ Marks)

$$= \cancel{2} \frac{n(n-1)}{\cancel{2}} + n = n^2$$

Given that arithmetic operation takes 1 μ second on computing resource, hence time taken to solve system $Ux = b$ is $n^2 \mu$ seconds (½ Mark)

Time taken by lower triangular system is same (1 Mark)

$= n^2 \mu$ seconds

Q 5a) x_1 = Number of units of single bedroom homes

x_2 = Number of units of two bedroom homes

x_3 = Number of units of three bedroom homes

x_4 = Open Space in ft^2

Maximize $Z = 45000x_1 + 56250x_2 + 90000x_3 + 7500x_4$ (½ Mark)

subject to

$x_1 \leq 300$

$x_2 \leq 500$

$x_3 \leq 650$ → (3 Marks)

$x_3 \geq 0.6(x_1 + x_2)$

$x_4 \leq 10000$

$x_4 \geq 10x_3 + 15x_2 + 18x_1$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0$ (½ Mark)

Q 5b) Standard Form

Max $Z = 3x_1 - x_2 - x_3 + 0s_1 + 0s_2$ → (½ Mark)

subject to

$x_1 - 2x_2 + x_3 + s_1 = 11$

$-4x_1 + x_2 + 2x_3 - s_2 = 3$ → (1 Mark)

$-2x_1 + x_3 = +1$ (½ Mark)

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ s_1, s_2 \geq 0.$

s_1 is slack variable

s_2 is surplus variable