05 March 2022 11:15

$A = \{Sh, Kannan, Jay, Nayak\} := \{S, K, J, N\}$

Select 2. SK, SJ, SN, KJ, KN, JN.

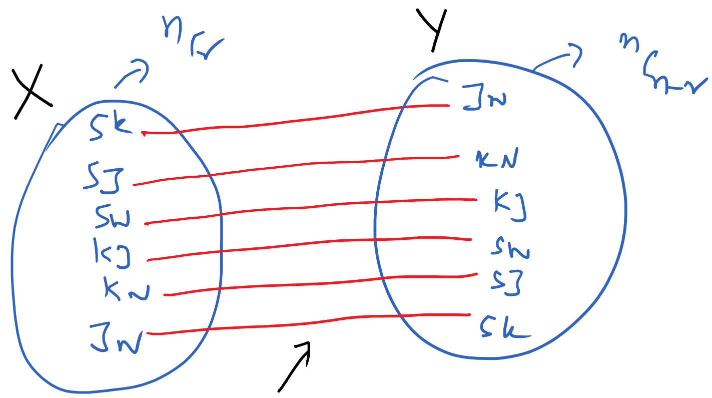
$\overline{JN}$ $\overline{KN}$ $\overline{KJ}$ $\overline{SN}$ $\overline{SJ}$ $\overline{SK}$

$^4C_2 = \frac{4!}{2!.2!} = 6$

$^nC_r = {}^nC_{n-r}$

Combinatorial proof

Bijective proof



Mapping $f : X \longrightarrow Y$

$f(A) = \bar{A}$, $\bar{A}$ is the complement of A

Let A & B be two subsets

$f(A) = f(B) \Rightarrow$

$\bar{A} = \bar{B}$

$(\bar{\bar{A}}) = (\bar{\bar{B}}) \Rightarrow A = B$

$\Rightarrow f$ is 1-1

$f(a) = f(b)$
$\Rightarrow a = b$ | 1-1

Given any $c \in Y$, take $\bar{c} \in X$

$f(\bar{c}) = (\bar{\bar{c}}) = c.$

$\therefore \bar{c}$ is the preimage of c under f

$\therefore$ C is the pre-image of ... under f

$\therefore$ f is onto.

1. CORMEN $\rightarrow$ Algorithms (Buy) $\hookrightarrow$ pdf

2. Sadgewick $\rightarrow$ Algorithms in C, C++, Java.

As long as X & Y are finite sets

$n(X) = n(Y)$

$\therefore$ ${}^nC_r = {}^nC_{n-r}$

$n=1 \quad (x+y)^1 = x + y \quad 2$

$n=2 \quad (x+y)^2 = x^2 + 2xy + y^2 \quad 3$

$n=3 \quad (x+y)^3 = x^3y^0 + 3x^2y^1 + 3x^1y^2 + y^3 \quad 4$

$n=11? \quad (x+y)^{11} = \underbrace{x^{11} \_ \_ \_ \_ \_ \_ \_ \_ y^{11}}_{12 \text{ terms}}$

2 observations

1) All terms in $(x+y)^n$ has degree n.

2) Altogether there are $(n+1)$ terms in $(x+y)^n$

$$(x+y)^n = \sum_{r=0}^{n} {}^nC_r \, x^{n-r} y^r.$$

$$\boxed{(x+y)^{11}} = \sum_{r=0}^{11} \underbrace{({}^{11}C_r) \, x^{11-r}}_{1} y^r = \sum_{r=0}^{11} {}^{11}C_r \, x^r y^{11-r} \quad 2$$

$$= \sum_{r} {}^{11}C_{11-r} \, x^r y^{11-r}.$$

$$= \sum_{r=0} {}^{11}C_{11-r} \, x \, y$$

$$ {}^{11}C \qquad \boxed{{}^{11}C_5} \, x^6 y^5 \qquad \boxed{{}^{11}C_6}$$

$\cdot 5$

Multinomial theorem

$$(x_1 + x_2 + x_3 + \dots + x_n)^m = \_\_\_\_^m \quad \_\_\_\_$$

$$x_1^{r_1} x_2^{r_2} x_3^{r_3} \dots x_n^{r_n} \rightarrow \sum_{i=1}^{n} r_i = m.$$

$$(x_1 + (x_2 + (x_3 + \dots + x_n)))^m$$

$$(x+y)^n = \sum_{r=0}^{n} {}^nC_r \, x^{n-r} y^r.$$

a) $x = 1, \ y = 1 \implies$

$$2^n = \sum_{r=0}^{n} {}^nC_r \, 1^{n-r} \cdot 1^r = \sum_{r=0}^{n} {}^nC_r.$$

$${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n.$$

b) $x = 1, \ y = -1$

$$0 = \sum_{r=0}^{n} {}^nC_r \, 1^{n-r} (-1)^r.$$

$$= {}^nC_0 - {}^nC_1 + {}^nC_2 + \dots$$

$$\underbrace{{}^nC_1 + {}^nC_3 + {}^nC_5 - \cdots -}_{= \frac{2^n}{2} = 2^{n-1}} = \underbrace{{}^nC_0 + {}^nC_2 + {}^nC_4 + \cdots}_{\frac{2^n}{2} = 2^{n-1}}$$

$$ {}^nC_{k-1} + {}^nC_k = \frac{n!}{(n-k+1)!\,(k-1)!} + \frac{n!}{(n-k)!\,.\,k!}$$

$5! = 5\times4\times3\times2\times1$
$= 5\times4!$

$$= \frac{n!}{(n-k+1)\,(n-k)!\,(k-1)!} + \frac{n!}{(n-k)!\,.\,k(k-1)!}$$

$$= \frac{n!}{(n-k)!\,(k-1)!}\left[\frac{1}{n-k+1} + \frac{1}{k}\right]$$

Algebraic proof

$$= \frac{n!}{(n-k)!\,(k-1)!}\left[\frac{\cancel{k} + n - \cancel{k} + 1}{(n-k+1)\,.\,k}\right]$$

$$= \frac{(n+1)!}{(n-k+1)!\;k!} = {}^{n+1}C_k$$

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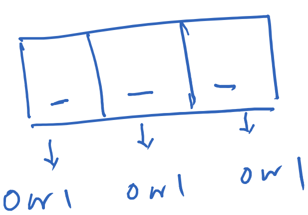
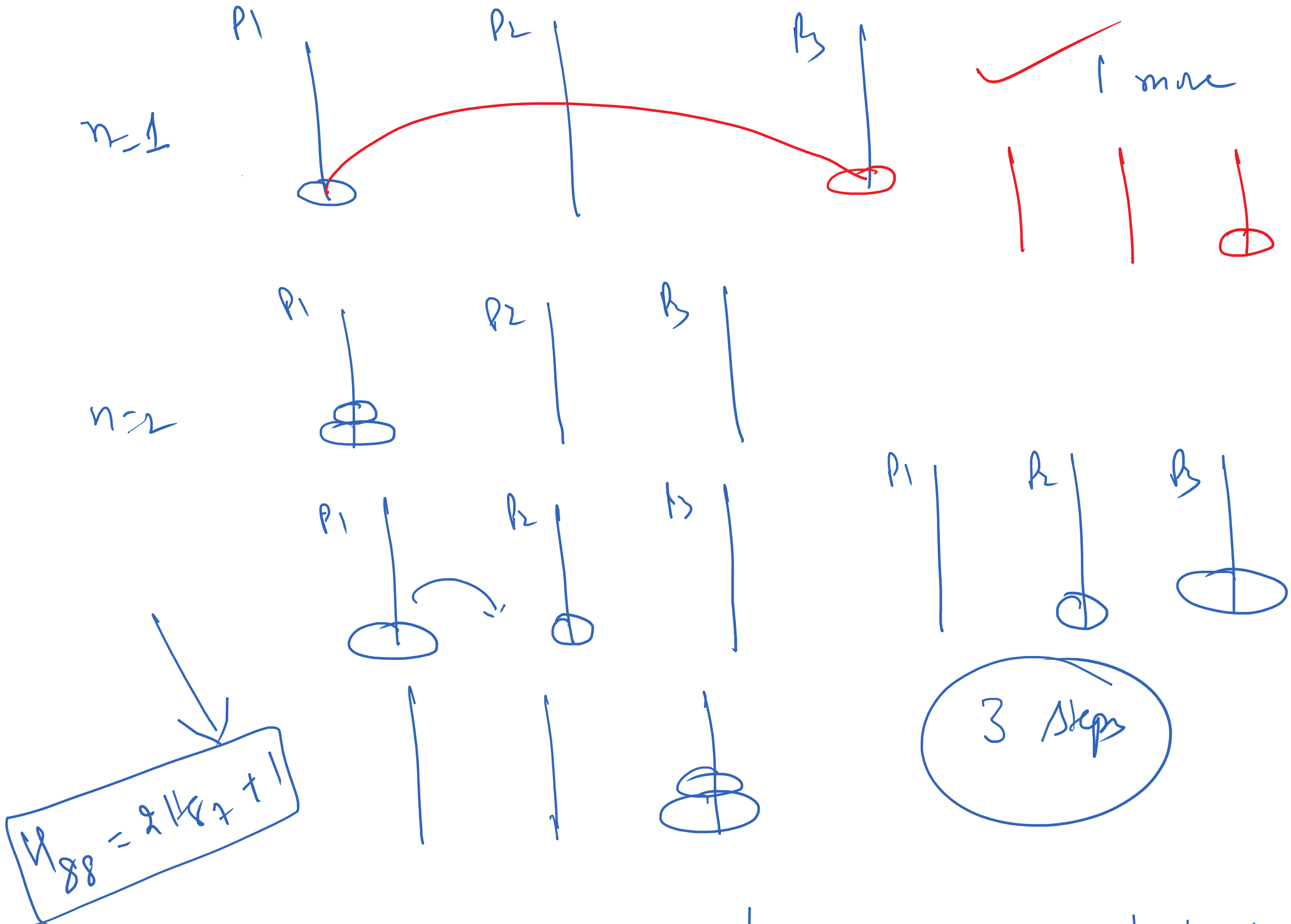
0 0 0
1 0 0
0 1 0
0 0 1
$\boxed{1\ 1\ 0}$
$\boxed{1\ 0\ 1}$
$\boxed{0\ 1\ 1}$
1 1 1

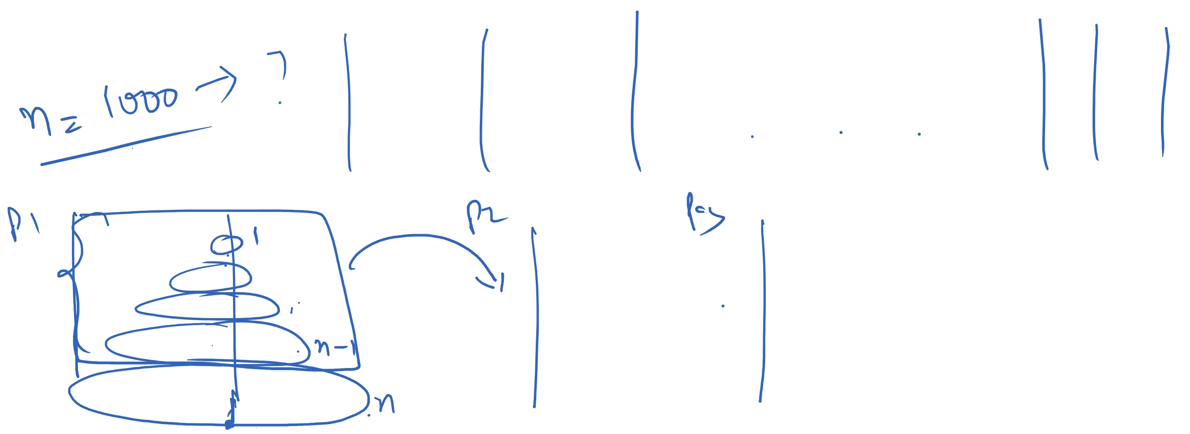


2 1s

$${}^3C_2 = \frac{3!}{2!\,.\,1!} = 3$$

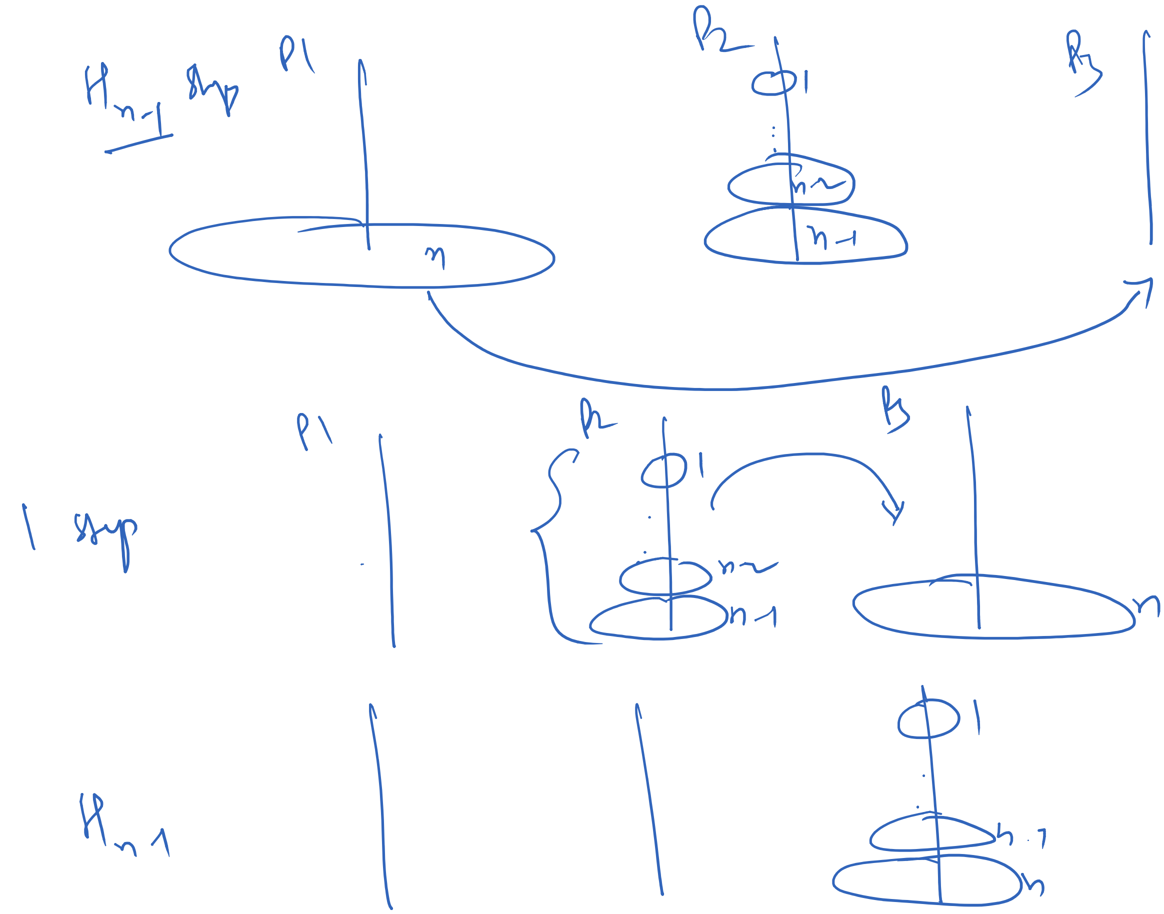
$(r+1)$ 1s. $\binom{n+1}{r+1}$.





[H_n steps to move <u>n discs</u> from peg 1 to peg 3 using peg 2

P1 P2 P3



$$H_n = H_{n-1} + 1 + H_{n-1}$$
$$= 2H_{n-1} + 1$$