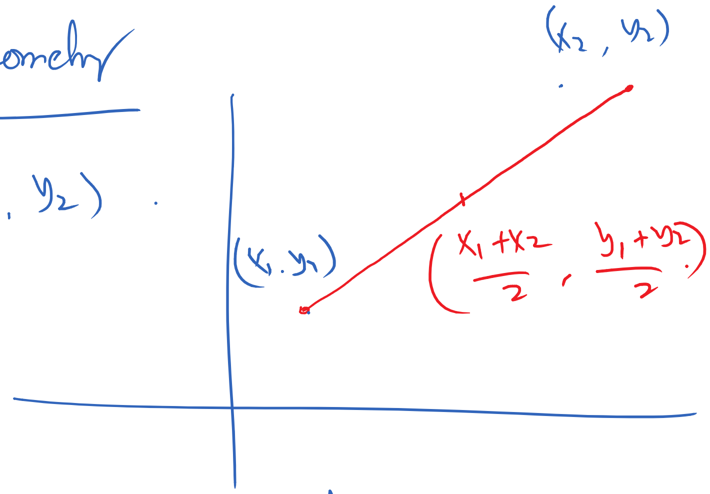
19 March 2022 13:55

Problem: Co-ordinate geometry

(x_1, y_1) (x_2, y_2).



Pigeonhole principle:

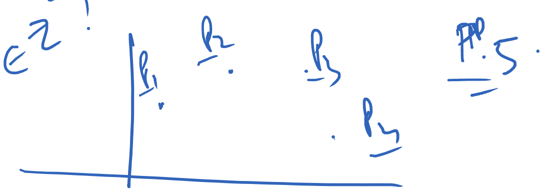
$(n+1)$ pigeons to be put in n pigeonholes, then atleast one of the pigeonholes will have 2 or more pigeons.

Problem: $\overset{P_1}{(a_1, b_1)}$, $\overset{P_2}{(a_2, b_2)}$, $\overset{P_3}{(a_3, b_3)}$, $\overset{P_4}{(a_4, b_4)}$ & $\overset{P_5}{(a_5, b_5)}$

All a_i, b_i $i = 1$ to 5 are integers.

$(5,-2)$ $(2,-1)$ $\in \mathbb{Z}^2$?

$(3.5, -1.5)$ ✗



$\exists$ at least one pair amongst $P_1, P_2, \ldots P_5$ s.t. the mid pt of the line segment has $\boxed{\text{integer value}}$.

$P_1 = (0, 0)$

$P_2 = (1, 0)$

$P_3 = (3, 3)$

$P_4 = (7, 8)$

$P_5 = (2, 2)$

$\overset{Q_1}{(\text{Even}, \text{even})}$, $\overset{Q_2}{(\text{even}, \text{odd})}$, $\overset{Q_3}{(\text{odd}, \text{odd})}$, $\overset{Q_4}{(\text{odd}, \text{even})}$

$\to$ pigeonholes

$\boxed{P_1, P_2, P_3, P_4 \text{ \& } P_5}$ $\to$ pigeons

... Q_i will have two pigeons.

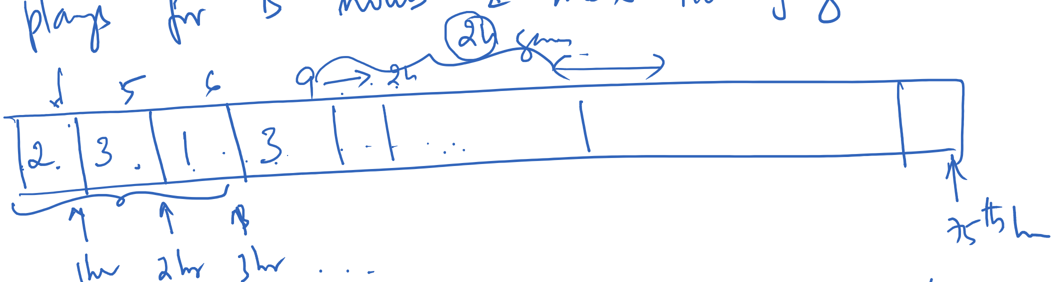
At least one of the θ_i's will have two pigeons.

θ_1 has two pigeons : (even, even) (even, even) $\to \in \mathbb{Z}\times\mathbb{Z}$

θ_2 (even, odd) (even, odd) $\in \mathbb{Z}\times\mathbb{Z}$

1 hr $\to$ 1 game at least & at the most 3 games.

He plays for 75 hours & max no. of games is 125



$a_i \to$ no. of matches he has played till time i $\quad 1 \le i \le 75$

$a_3 \to ?\ 6 \qquad a_1 \ge 1 \qquad \cancel{a_i} \le \boxed{a_{i+1} - a_i \le 3}$

$$\boxed{1 \le a_1 < a_2 < a_3 \ldots < a_{75} \le 125} \to (1)$$

$b_i = a_i + 24$

$$25 \le a_1 + 24 < a_2 + 24 < \ldots < a_{75} + 24 \le 125 + 24 = 149$$

$$25 \le b_1 < b_2 \ldots < b_{75} \le 149$$

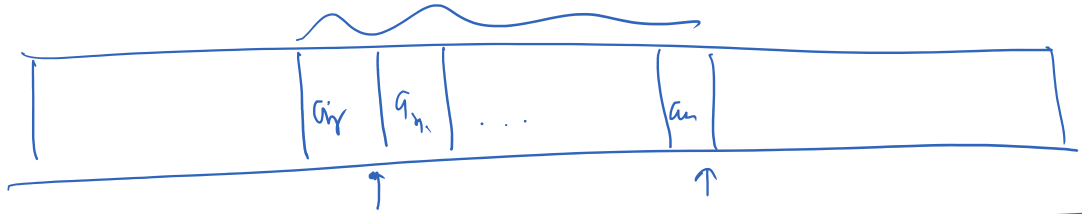
Call $\boxed{a_1, a_2, \ldots a_{75}, b_1, b_2, \ldots b_{75}}$ as pigeons

Nos: $\underbrace{1, 2, 3, \ldots, 145}$ as pigeonholes

$1 \leq j \leq 145$, hole j has two pigeons

Pigeons: a_k a_r. b_k, b_r

$\boxed{a_k \quad b_r}$ $a_k = b_r$. $\boxed{a_k = a_r + 24}$



Let x be an irrational number & j be an integer less than n

$$|jx - [jx]| < 1/n$$

Thm: $\boxed{\sqrt{2}, \sqrt{3}, \sqrt{5} \ldots, \; x \text{ is irrational} \Rightarrow}$

$x \neq \frac{p}{q}$ where p & q are coprimes.

$x = \sqrt{2} = 1.414\ldots$ $\quad |2.828 - 3| = .172$

$j = 2 \Rightarrow jx = 2.828$

$|jx - [2.828]| = \cancel{0.828}$ $0.172 < 1/n$

$jx - [jx] \in [0, 1)$

$\underbrace{\ldots, {}^{0}/_{n}}_{1} \; \underbrace{{}^{0}/_{n}}_{2} \ldots$

Intervals: $[0, \tfrac{1}{n})$, $[\tfrac{1}{n}, \tfrac{2}{n})$, ..., $[\tfrac{n-1}{n}, \tfrac{n}{n})$

Can $jx - [jx] = \frac{R}{n}$? It can't.

Suppose $jx - [jx] = k/n$

$jx = \frac{k}{n} + \text{integer} = \text{rational}$

$\Rightarrow x = \frac{\text{rational}}{j} = \text{ratio}$, a contradiction

---

$$\frac{(1-x^4)^3}{(1-x)^3} = \frac{1 + 3(-x^4) + 3(x^8) - x^{12}}{(1-x)^3}$$

$(a+b)^n = \sum_{r=0}^{n} {}^{n}C_r \, a^{n-r} b^r$

$a^r b^{n-r}$

$a=1$

$(1+b)^n = \sum_{r=0}^{n} \binom{n}{r} b^r$

$(1+x^4)^n = \sum_{r=0}^{n} \binom{n}{r} (x^4)^r$

$(1-x^4)^n = \sum_{r=0}^{n} \binom{n}{r} (-x^4)^r$

① $\times$ coeff of x^8 in $\frac{1}{(1-x)^3}$

$+$

$-3 \cdot \times$ coeff of x^4 in $\frac{1}{(1-x)^3}$

$+$

$3 \times$ coeff of const in $\frac{1}{(1-x)^3}$

$$\frac{1-9x}{(1-8x)(1-10x)} = \frac{A^{1/2}}{(1-8x)} + \frac{B^{1/2}}{(1-10x)}$$

$$\boxed{\frac{1}{(1-8x)(1-10x)}} \qquad \frac{1}{(1-8x)} \qquad (1-10x)$$

$$1-9x = A(1-10x) + B(1-8x)$$

$\boxed{\text{Put } x = 1/10}$

$$1-9/10 = B(1-8/10)$$

$$1/10 = B(2/10)$$

$$\Rightarrow 2B = 1 \quad \text{or} \quad B = 1/2$$

Put $x = 1/8$

$$1-9/8 = A(1-10/8)$$

$$-1/8 = A(-2/8) \Rightarrow 2A = 1$$

$$\text{or } A = 1/2$$

$$A = 1/2 \quad B = 1/2$$