Problem: Let m & n be +ve integers, Consider a sequence of $mn+1$ real numbers. Then there exists a subsequence of $(m+1)$ numbers that is increasing or a subsequence of $(n+1)$ numbers that is decreasing.

(ex) 101 : random 101 elements (real nos)

$S: a_1 \quad a_2 \quad \ldots \quad a_{100} \quad a_{101}$

: $101 = 10 \times 10 + 1$

$m = 10, \; n = 10$ +ve integers

11 nos that is increasing

or 11 nos that is decreasing.

Proof: Consider $S = a_1, a_2, \ldots, a_{mn}, a_{mn+1}$ sequence of arbitrary real nos. Assume that the result is false. Let $x \in S$ be any member

$\Rightarrow \exists$ an index p s. that $a_p = x$.

$f: x \to (i, j)$

$i \to$ length longest subsequence in S beginning with x & increasing

$j \to$ length longest subsequence in S & ending with x & decreasing

$S = 2 \quad 6 \quad 7 \quad 4 \quad 1$

$5 \to 2 \times 2 + 1$

$m = n = 2$

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1 3 2 4 9

$S = 17 \quad 4 \quad 19 \quad 11 \quad 12 \quad 8$

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$f(2) = (3, 1)$

$S =$ 17 4 14 11 12 8

17 19

$a_1 \quad a_2, \ldots, a_{mn}, a_{mn+1}$

$f(a_1) = (\alpha, \beta)$ $\alpha = m+1$ ✓

$f(a_2) =$ $\beta = n+1$ ✓

$\vdots$

$f(a_{mn+1}) =$

$$\boxed{\begin{array}{c} 1 \leq \alpha \leq m \\ 1 \leq \beta \leq n \end{array}}$$ Assumptions

$mn+1$ numbers for which the right hand side has at the most ~~mn~~ entries
mn only

At least there are two numbers $\boxed{a_p \ \& \ a_q}$ for which the function value is the same.

$S = a_1 \ a_2, \ldots a_p \ldots a_q, \ldots a_{mn} \ a_{mn+1}$

$f(a_p) = f(a_q)$

$a_p < a_q \Rightarrow \#$

$a_q < a_p \Rightarrow \#$

$mn+1 = nm+1$ (Sy. in m & n)

$c = [1 \quad 2 \quad 3 \quad 4]$; $1x^3 + 2x^2 + 3x + 4 = 0$

roots (c);

$c = [1 \quad 5 \quad 6]$; $1x^2 + 5x + 6 = 0$

roots(c); $-3 \quad -2$

$c = [a_n \quad a_{n-1} \quad \ldots \quad a_0]$;

$$\boxed{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0 = 0} \quad (1)$$

$a_i \in \mathbb{R} \quad 0 \le i \le n$

$x+iy$ as a root of (1), then $x-iy$ is also a root.