

$$\begin{aligned}
 \alpha &= \frac{1+\sqrt{5}}{2} ; \quad \alpha^2 = \frac{1}{4} (1 + 2\sqrt{5} + 5) \\
 &= \frac{1}{4} + \frac{\sqrt{5}}{2} + \frac{5}{4} \\
 &= \frac{\sqrt{5}}{2} + \frac{6}{4} = \frac{\sqrt{5}}{2} + \frac{3}{2} \\
 &= \frac{\sqrt{5}}{2} + \frac{1}{2} + 1 \\
 &= \frac{\sqrt{5}+1}{2} + 1 = \alpha + 1
 \end{aligned}$$

$$\therefore \boxed{\begin{array}{l} \alpha \text{ satisfies} \\ \alpha^2 = \alpha + 1 \end{array}}$$

GCD \rightarrow positive integers

Let a, b be positive integers

$d = \gcd(a, b)$ when

Greater
Common
divisor

(i) d divides a & d divides b
 $\underline{d|a}$ & $\underline{d|b}$

(ii) $c|a$ & $c|b \Rightarrow c|d$

I $(3, 9)$

3 $(1, 3)$

$\textcircled{3}|3$ $\textcircled{3}|9$

9 $(1, 3, 9)$

$1|3$ $1|9$

$1|3$

II $(3, 18) = 3$

3 $(1, 3)$

18 $(1, 3, 6, 9, 18)$

$\textcircled{3}|18$ $\textcircled{3}|3$

$1|18$ $1|3$

$1|3$

(i) $d = \gcd(a, b)$, \exists unique integers

$$x \text{ e } y \text{ s.t. tal } d = ax + by$$

$$(ii) \quad d = \text{g.c.d}(a, b), \quad \text{g.c.d}\left(\frac{a}{d}, \frac{b}{d}\right) = 1$$

$$(iii) \quad d = \text{g.c.d}(a, b) \Rightarrow \text{g.c.d}(a^n, b^n) = d^n$$

$$(iv) \quad \underbrace{\text{l.c.m}(a, b)} \cdot \text{g.c.d}(a, b) = a \cdot b$$

$$\underline{\underline{45}} = r_1 \left[\begin{array}{l} 53 = r_0 \\ 45 \end{array} \right] \quad 1 = q_1$$

$$r_2 = \underline{\underline{8}} \left[\begin{array}{l} 45 = r_1 \\ 40 \end{array} \right] \quad 5 = q_2$$

$$r_3 = \underline{\underline{5}} \left[\begin{array}{l} 8 = r_2 \\ 5 \end{array} \right] \quad 1 = q_3$$

$$r_4 = \underline{\underline{3}} \left[\begin{array}{l} 5 = r_3 \\ 3 \end{array} \right] \quad 1$$

$$\underline{\underline{2}} \left[\begin{array}{l} 3 = r_4 \\ 2 \end{array} \right] \quad 1$$

$$\textcircled{1} \left[\begin{array}{l} 2 = r_5 \\ 2 \end{array} \right] \quad 2$$

$$\textcircled{0}$$