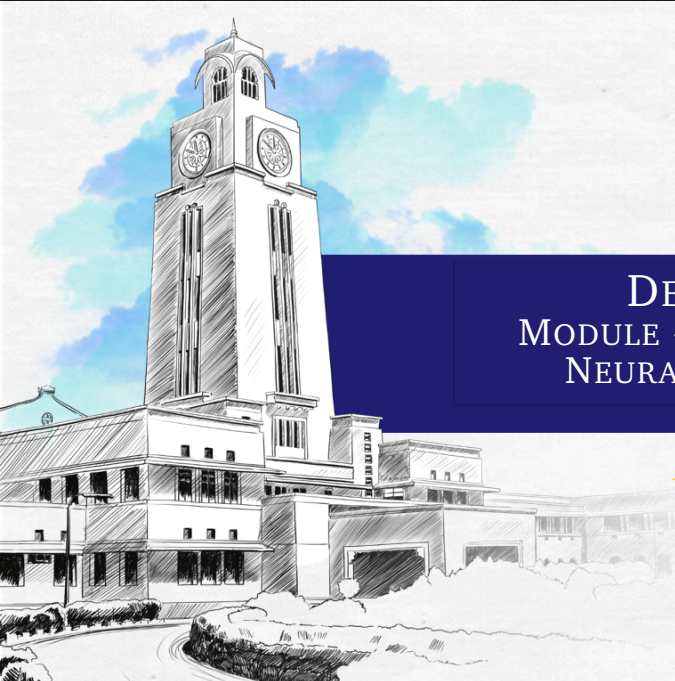

BITS Pilani
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Deep Learning
Module # 5 : Convolutional Neural Network [CNN]

Seetha Parameswaran
Asst Prof, BITS Pilani

The instructor is gratefully acknowledging the authors who made their course materials freely available online.

In This Segment

Image Classification

64x64 x3

Cat? (0/1)

Cat? (0/1)

Classify into 3 classes

Face Detection

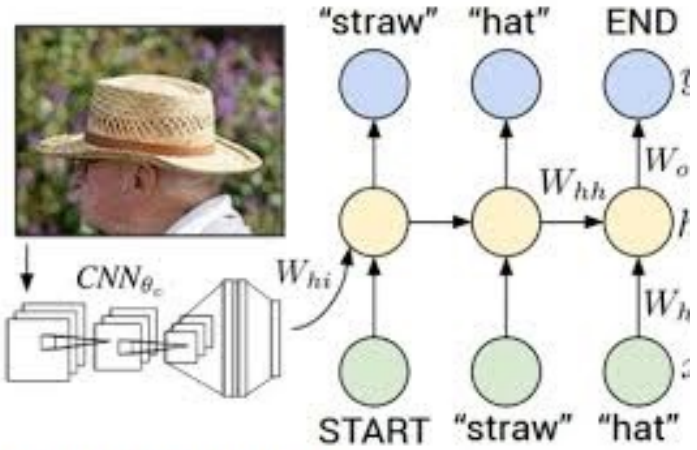
Object Recognition

"SIFT" & Object Recognition, David Lowe, 1999

Image is public domain

Image is CC BY-SA 2.0

Image Captioning

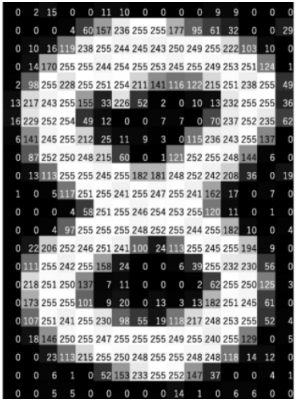


Andrej Karpathy, Fei-Fei Li

In This Segment

Digital Grayscale Image

Pixel intensities = $I(x, y)$



```
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  0   0   0   4  60 157 236 255 255 177  95  61  32   0   0  29
  0  10  16 119 238 255 244 245 243 250 249 255 222 103  10   0
  0  14 170 255 255 244 254 255 253 245 255 249 253 251 124   1
  2  98 255 228 255 251 254 211 141 116 122 215 251 238 255  49
 13 217 243 255 155  33 226  52   2   0  10  13 232 255 255  36
 16 229 252 254  49  12   0   0   7   7   0  70 237 252 235  62
  6 141 245 255 212  25  11   9   3   0 115 236 243 255 137   0
  0  87 252 250 248 215  60   0   1 121 252 255 248 144   6   0
  0  13 115 255 255 245 255 182 181 248 252 242 208  36   0  19
  1   0   5 117 251 255 241 255 247 255 241 162  17   0   7   0
  0   0   0   4  58 251 255 246 254 253 255 120  11   0   1   0
  0   0   4  97 255 255 255 248 252 255 244 255 182  10   0   4
  0  22 206 252 246 251 241 100  24 113 255 245 255 194   9   0
  0 111 255 242 255 158  24   0   0   6  39 255 232 230  56   0
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  0 173 255 255 101   9  20   0  13   3  13 182 251 245  61   0
  0 107 251 241 255 230  98  55  19 118 217 248 253 255  52   4
  0  18 146 250 255 247 255 255 255 249 255 240 255 129   0   5
  0   0  23 113 215 255 250 248 255 255 248 248 118  14  12   0
  0   0   6   1   0  52 153 233 255 252 147  37   0   0   4   1
  0   0   5   5   0   0   0   0   0  14   1   0   6   6   0   0
```

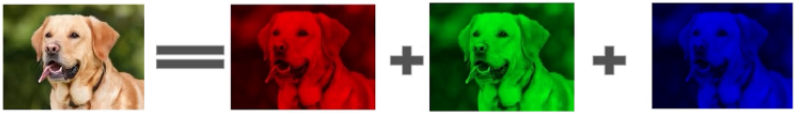

```
  0   2  15   0   0  11  10   0   0   0   0   9   9   0   0   0
  0   0   0   4  60 157 236 255 255 177  95  61  32   0   0  29
  0  10  16 119 238 255 244 245 243 250 249 255 222 103  10   0
  0  14 170 255 255 244 254 255 253 245 255 249 253 251 124   1
  2  98 255 228 255 251 254 211 141 116 122 215 251 238 255  49
 13 217 243 255 155  33 226  52   2   0  10  13 232 255 255  36
 16 229 252 254  49  12   0   0   7   7   0  70 237 252 235  62
  6 141 245 255 212  25  11   9   3   0 115 236 243 255 137   0
  0  87 252 250 248 215  60   0   1 121 252 255 248 144   6   0
  0  13 115 255 255 245 255 182 181 248 252 242 208  36   0  19
  1   0   5 117 251 255 241 255 247 255 241 162  17   0   7   0
  0   0   0   4  58 251 255 246 254 253 255 120  11   0   1   0
  0   0   4  97 255 255 255 248 252 255 244 255 182  10   0   4
  0  22 206 252 246 251 241 100  24 113 255 245 255 194   9   0
  0 111 255 242 255 158  24   0   0   6  39 255 232 230  56   0
  0 218 251 250 137   7  11   0   0   0   2  62 255 250 125   3
  0 173 255 255 101   9  20   0  13   3  13 182 251 245  61   0
  0 107 251 241 255 230  98  55  19 118 217 248 253 255  52   4
  0  18 146 250 255 247 255 255 255 249 255 240 255 129   0   5
  0   0  23 113 215 255 250 248 255 255 248 248 118  14  12   0
  0   0   6   1   0  52 153 233 255 252 147  37   0   0   4   1
  0   0   5   5   0   0   0   0   0  14   1   0   6   6   0   0
```


https://mozanunal.com/2019/11/img2sh/

Digital Color Image

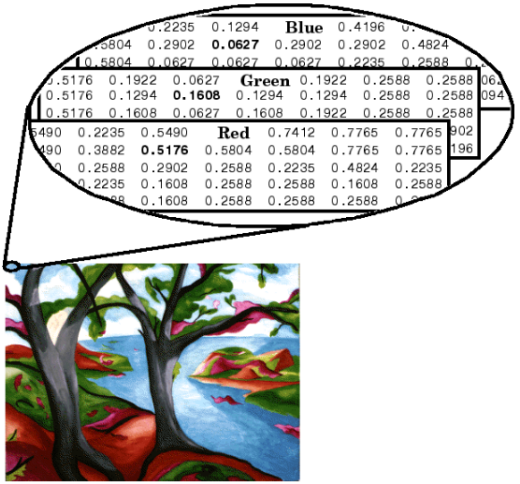
Colour Image

This image is composed of many colors and almost all colors can be generated from the three primary colors- **Red, Green, and Blue.** We can say that each colored image is composed of these three colors or 3 channels- Red, Green, and Blue-



https://www.analyticsvidhya.com/blog/2021/03/grayscale-and-rgb-format-for-storing-images/

Digital Color Image



https://www.mathworks.com/help/matlab/creating_plots/image-types.html

In This Segment

Image Classification - Challenges

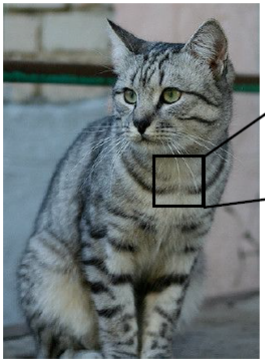
Image Classification: A core task in Computer Vision

(assume given set of discrete labels)
{dog, cat, truck, plane, ...}

→ cat

Image Classification - Challenges

The Problem: Semantic Gap

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 [ 99  81  81  93 120 131 127 100  95  98 102  99  96  92 101  94]
 [105  91  61  64  65  91  88  85 101 107 105  98  75  84  96  95]
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 [122 121 102  80  82  86  94 117 145 148 153 102  58  76  92 107]
 [122 164 148 103  71  56  78  83  93 103 110 130 102  61  69  84]]
```

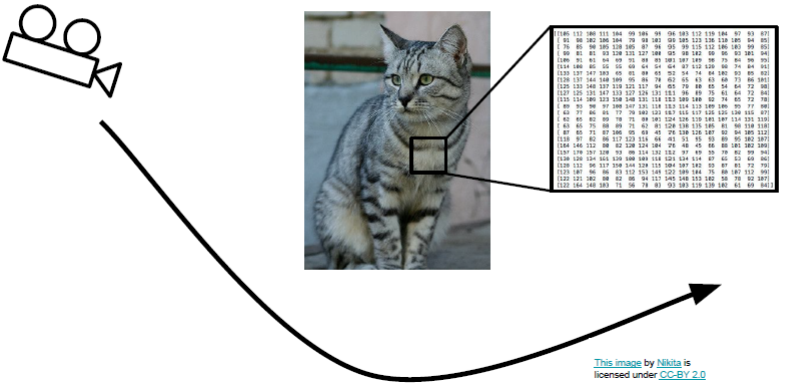
What the computer sees

An image is just a big grid of numbers between [0, 255]:

e.g. 800 x 600 x 3
(3 channels RGB)

Image Classification - Challenges

Challenges: Viewpoint variation

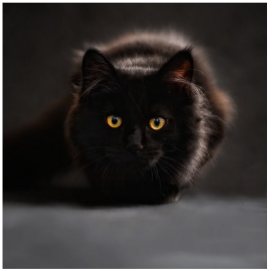


All pixels change when the camera moves!

Image Classification - Challenges

Challenges: Illumination

This image is CC0 1.0 public domain



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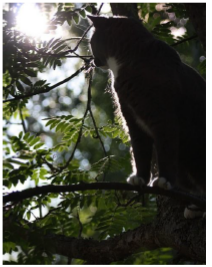
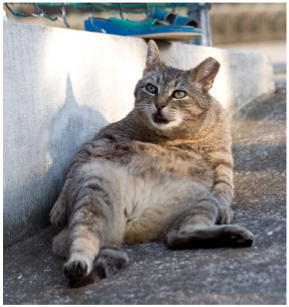
This image is CC0 1.0 public domain



This image is CC0 1.0 public domain

Image Classification - Challenges

Challenges: Deformation



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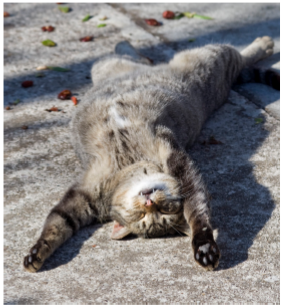

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Challenges: Occlusion

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Challenges: Background Clutter

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This image is CC0 1.0 public domain

Image Classification - Challenges

Challenges: Intraclass variation

Image Classification - Challenges

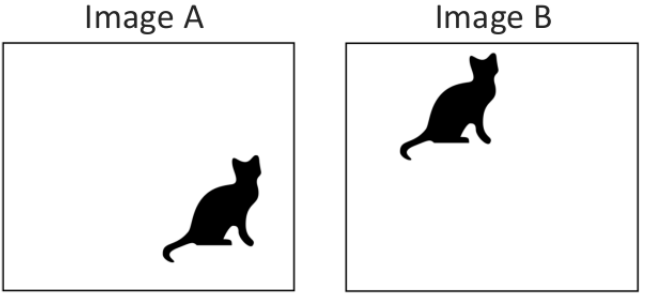
Pixelated Images

Larger Images

- A 50x50 Image requires 2500 input dimensions.
- A HD image 1920x1080 is about 2 million inputs. (2 Mega Pixels)

Image Invariance - Challenges

Object can appear anywhere in the image.

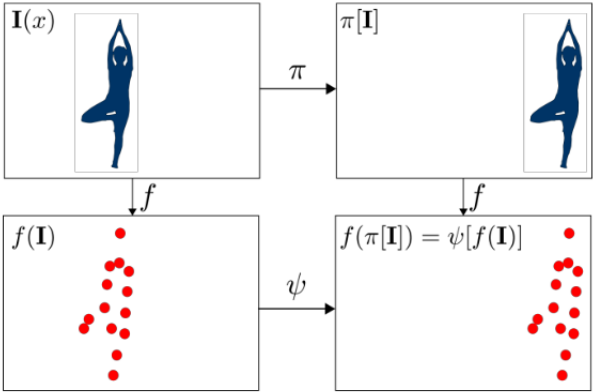


Model predicts “cat” for both images!

Source: https://penkovsky.com/neural-networks/day5/

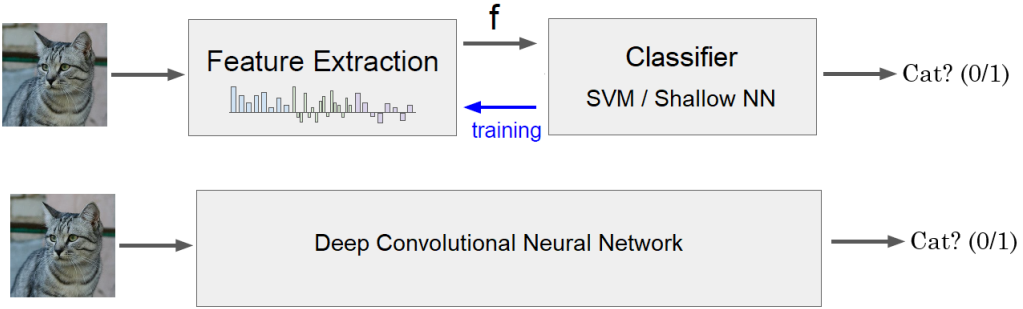
Image Equivariance - Challenges

Changes to the input predictably (equivalently) should not affect the output.



Source: https://towardsdatascience.com/translational-invariance-vs-translational-equivariance-f9fbc8fca63a

Image Classification - Solution



In This Segment

Convolutional Neural Networks (CNN)

- Convolutional networks are simply neural networks that use convolution in place of general matrix multiplication in at least one of their layers.
- Special kind of neural network for processing data that has a known, grid-like topology.
- Eg: Time-series data – 1D grid taking samples at regular time intervals.
- Eg: Image data – 2D grid of pixels.
- Network employs a mathematical operation called **convolution**.

Example of CNN – LeNet-5

- Handwritten Character recognition
- 10-categories classification problem

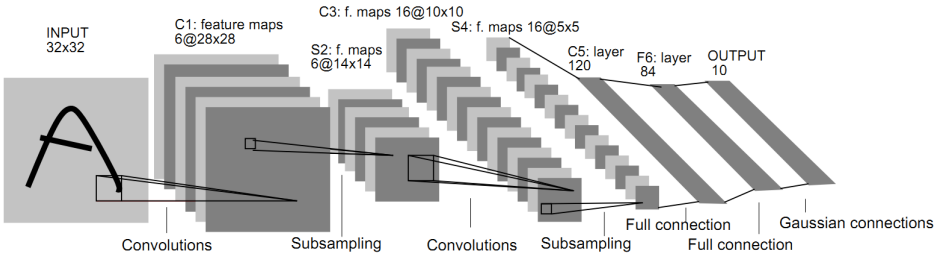


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

Courtesy: Yann LeCun

Why CNN for Images

- Some patterns are much smaller than the whole image.

Courtesy: Akka Zemmari

Why CNN for Images

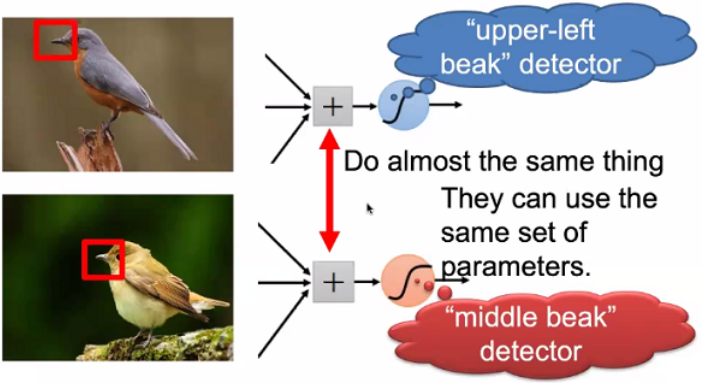
- Some patterns are much smaller than the whole image.

A neuron does not have to see the whole image to discover the pattern.

Courtesy: Akka Zemmari

Why CNN for Images

- Same pattern may appear in different regions.



Courtesy: Akka Zemmari

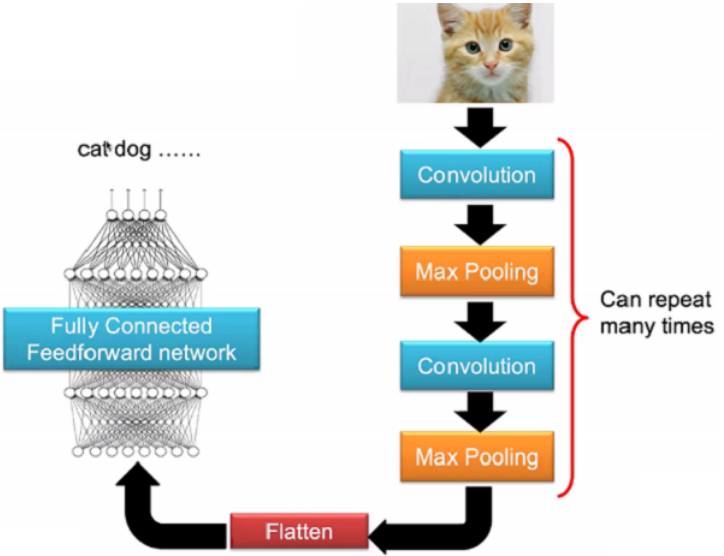
Why CNN for Images

- Sub-sampling the pixels will not change the object.

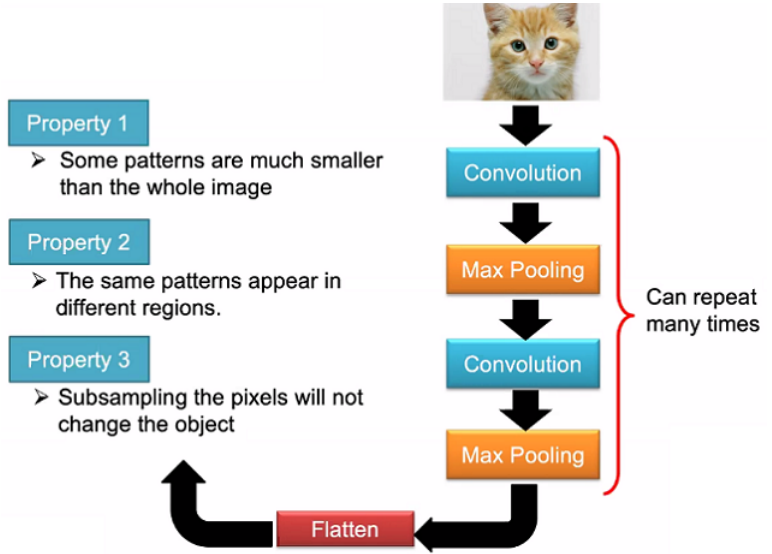
- We can sub-sample the image to make it smaller.
- Less parameters for the network to process the image.

Courtesy: Akka Zemmari

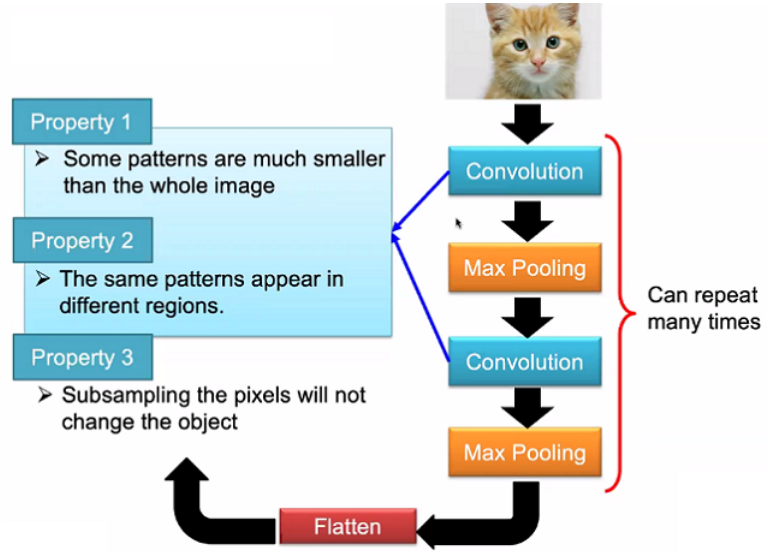
The Whole CNN



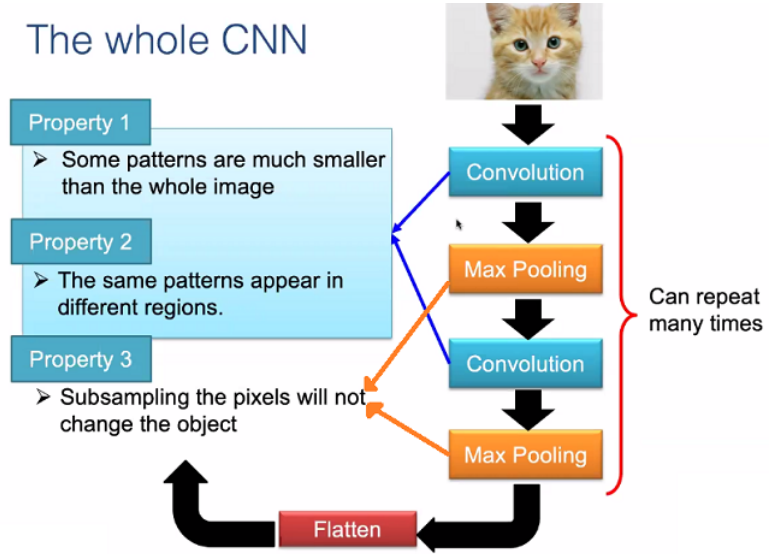
The whole CNN



The Whole CNN



The Whole CNN



In This Segment

Convolution Layer

- Perform Convolution operation.
- In mathematics, convolution is a mathematical operation on two functions that produces a third function that expresses how the shape of one is modified by the other. [Wikipedia]
- The term convolution refers to both the result function and to the process of computing it. [Wikipedia]
- We will assume that the kernel is square matrix.
- We will assume that the kernel is centered on the pixel of interest. (One reason why kernel is odd matrix.)
- Convolution operation

$$S_{ij} = (I * K)_{ij} = \sum_{a=\lfloor -\frac{m}{2} \rfloor}^{\lfloor \frac{m}{2} \rfloor} \sum_{b=\lfloor -\frac{n}{2} \rfloor}^{\lfloor \frac{n}{2} \rfloor} I_{i-a,j-b} K_{\frac{m}{2}+a,\frac{n}{2}+b}$$

1-D Convolution

- Suppose we track position of an object using a sensor or GPS device at discrete time intervals.
- Assume that the sensor is noisy.
- To obtain a less noisy estimate, we would like to take a weighted average.
- We would only sum over a small window or a weight array (filter) (w).

$$s_t = \sum_{a=0}^{k} x_{t-a} w - a$$

- Slide the filter over the input and convolve.
- The input and the kernel are one dimensional.

1-D Convolution

$s = x_5 w_0 + x_4 w_1 + x_3 w_2 + x_2 w_3 + x_1 w_4 + x_0 w_5$

X	1.00	1.10	1.20	1.40	1.70	1.80	1.90	2.10	2.20	2.40	2.50
W	0.01	0.02	0.03	0.04	0.4	0.5					
S						1.80					
W		0.01	0.02	0.03	0.04	0.4	0.5				
S						1.80	1.96				
W			0.01	0.02	0.03	0.04	0.4	0.5			
S						1.80	1.96	2.11			

2D Convolution Operation

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

Input Image 6×6

These values inside the matrix are to be learned by the network. We call these as parameters or weights.

1	-1	-1
-1	1	-1
-1	-1	1

Kernel / Filter 1

-1	1	-1
-1	1	-1
-1	1	-1

Kernel / Filter 2

Property 1: Each filter detects a small pattern (3×3)

Convolution Operation

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

Input Image 6×6

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

Start at top left corner.
Perform element wise product and add all the values.

Output

3

Convolution Operation

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

Input Image 6×6

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

Slide by one pixel towards right. Perform element wise product and add all the values.

Output

3 -1

Convolution Operation

stride $s = 1$

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

Input Image 6×6

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

Slide by one pixel towards right. Perform element wise product and add all the values.

Output

3 -1 -3

Convolution Operation

stride $s = 1$

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

Input Image 6×6

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

Slide by one pixel towards right. Perform element wise product and add all the values.

Output

3 -1 -3 -1

Convolution Operation

stride $s = 1$

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

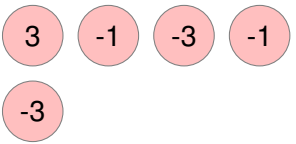
Input Image 6×6

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

Jump to next row, left most pixel.
The filter has to completely superimpose the pixels of input.
NOT PARTIALLY.

Output



Convolution Operation

stride $s = 1$

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

Input Image 6×6

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

After sliding through the entire image.

3	-1	-3	-1
-3	-1	0	-3
-3	-2	0	1
3	-2	-2	-1

Convolution Operation

stride $s = 1$

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

Input Image 6×6

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

3	-1	-3	-1
-3	-1	0	-3
-3	-2	0	1
3	-2	-2	-1

Property 2 is satisfied.

Convolution Operation

stride $s = 1$

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

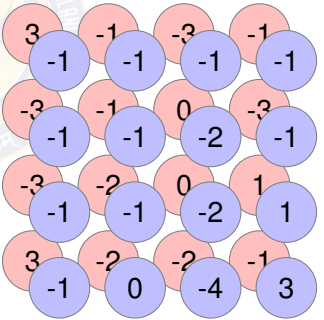
Input Image 6×6

Filter 1

1	-1	-1
-1	1	-1
-1	-1	1

Filter 2

-1	1	-1
-1	1	-1
-1	1	-1



Feature Maps

Convolution Operation

stride $s = 3$

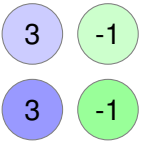
1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

Input Image 6×6

Filter1

1	-1	-1
-1	1	-1
-1	-1	1

Output



Convolution with Striding

- Stride means skip by how many pixels.

$$\begin{aligned} \text{Input Size} &: n \times n \\ \text{Stride} &: s \\ \text{Kernel Size} &: f \times f \\ \text{Output Size} &: \left\lfloor \frac{n-f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n-f}{s} + 1 \right\rfloor \end{aligned}$$

- The filter must lie completely inside the input.

Padding

- When filter is convolved with the input image
  - Image shrinks – 6×6 image became a 4×4 image.
  - Pixels in the corners are used only once, when compared the pixels in the middle. The data in the corners are thrown away.
- To resolve, apply padding to the input image before convolution.
- Padding means append zeros around the boundary of the input.

Convolution with Padding

Input Image with padding 8×8

0	0	0	0	0	0	0	0
0	1	0	0	0	0	1	0
0	0	1	0	0	1	0	0
0	0	0	1	1	0	0	0
0	1	0	0	0	1	0	0
0	0	1	0	0	1	0	0
0	0	0	1	0	1	0	0
0	0	0	0	0	0	0	0

*

Filter1

1	-1	-1
-1	1	-1
-1	-1	1

Padding

- Two kinds of padding
  - Valid Padding
    - Padding is not applied.

$$p = 0$$

    - Input size $\neq$ output size.
    - Output shrinks when compared to the input.
  - Same Padding
    - Padding is applied.

$$p = \frac{f-1}{2}$$

    - Input size $=$ output size.
    - We maintain the size of input and output as the same.

Padding

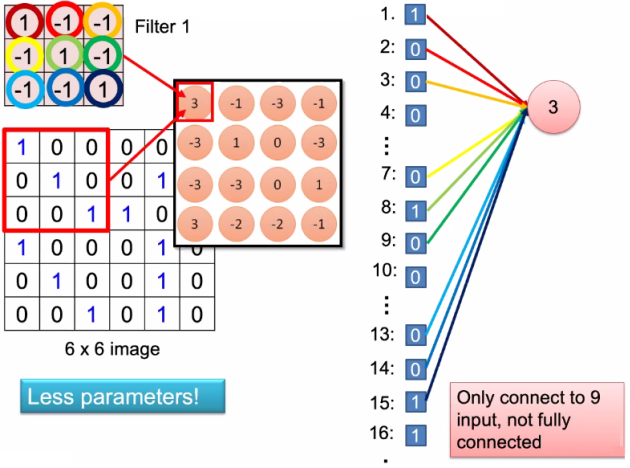
- How much to pad?

$$\text{Padding} : p = \frac{f-1}{2}$$
$$\text{Kernel } 3 \times 3 : p = 1$$
$$\text{Kernel } 5 \times 5 : p = 2$$
$$\text{Kernel } 7 \times 7 : p = 3$$

- Padding preserves the input size.

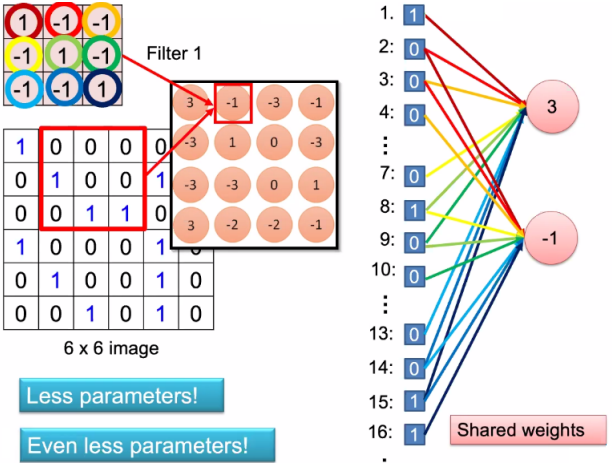
$$\text{Input Size} : n \times n$$
$$\text{Padding} : p$$
$$\text{Kernel Size} : f \times f$$
$$\text{Output Size} : (n + 2p - f + 1) \times (n + 2p - f + 1)$$

Convolutional Layer



Courtesy: Akka Zemmari

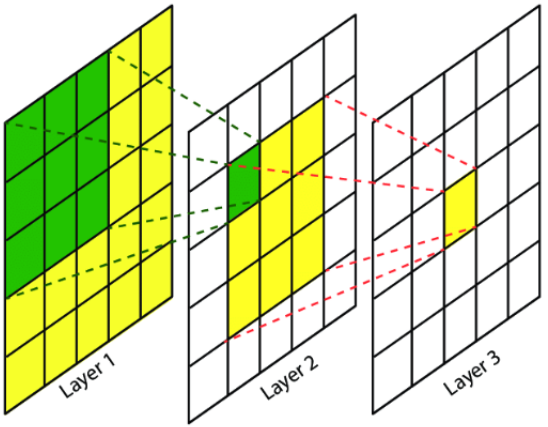
Convolutional Layer



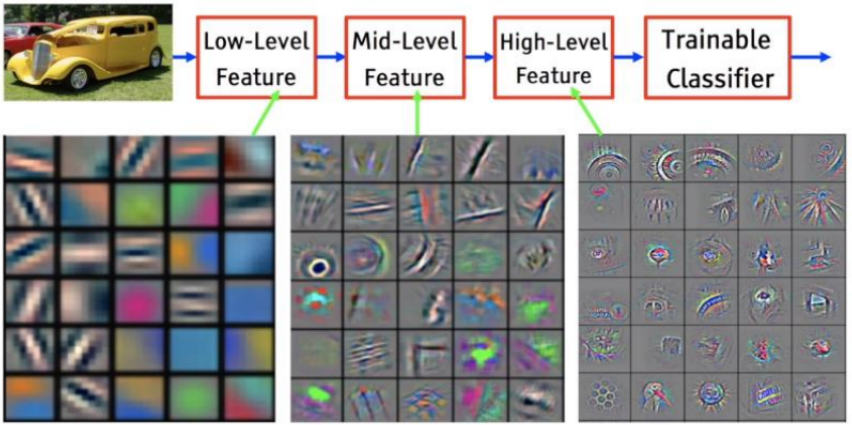
Courtesy: Akka Zemmari

Activations or Receptive Fields

- Effective area "seen" by a neuron
- What is the effective number of pixels seen at each layer?
  - Input image x
  - $h_1 = Conv2d(x, k = 3)$
  - $h_2 = Conv2d(h_1, k = 3)$
  - $h_3 = Conv2d(h_2, k = 3)$
  - ...
  - $h_n = Conv2d(h_{n-1}, k = 3)$
- Homework: What happens if the kernel size was 5?



Local to Global



Yann LeCun's presentations on ConvNets

Parameter Size in Convolution Layer

- Input image: $B \times W_{in} \times H_{in} \times C_{in}$
  - Input data has B samples
  - Input or Image size is $W_{in} \times H_{in} \times C_{in}$
- Convolution filter
  - Weight parameters: $C_{in} \times C_{out} \times K_W \times K_H$
  - Bias parameters: C_{out}
- Output activation: $B \times W_{out} \times H_{out} \times C_{out}$
  - C_{out} filters of shape $C_{in} \times K_W \times K_H$ operate on each sample

Parameter Size in Convolution Layer

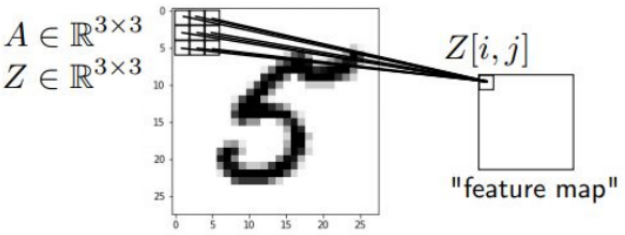
- Each filter gives one 2D output.
- K filter will give K such 2D outputs.
- The number of parameters depend on the number of kernels and the kernel size.

$$\text{Number of kernels in layer } (l-1) : K_{l-1}$$
$$\text{Kernel size} : f \times f$$
$$\text{Number of kernels in layer } (l) : K_l$$
$$\text{Number of Parameters in layer } (l) : K_{l-1} * f * f * K_l$$

Cross-Correlation vs Convolution

- Convolution in Deep Learning is actually cross-correlation.
- Cross-correlation is sliding dot product over the images.
- Cross-correlation

$$Z[i,j] = \sum_{u=-k}^{k} \sum_{-k}^{k} K[u,v] A[i+u, j+v]$$



Cross-Correlation vs Convolution

Cross-Correlation

$$Z[i,j] = \sum_{u=-k}^{k} \sum_{-k}^{k} K[u,v]A[i+u, j+v]$$

(Note the looping direction given in red.)

1) -1,-1	2) -1,0	3) -1,1
4) 0,-1	5) 0,0	6) 0,1
7) 1,-1	8) 1,0	9) 1,1

Convolution

$$Z[i,j] = \sum_{u=-k}^{k} \sum_{-k}^{k} K[u,v]A[i-u, j-v]$$

(Flipping the kernel.)

9) -1,-1	8) -1,0	7) -1,1
6) 0,-1	5) 0,0	4) 0,1
3) 1,-1	2) 1,0	1) 1,1

Cross-Correlation vs Convolution

- Convolution in Deep Learning is actually cross-correlation.
- Convolution (as in Signal Processing) has nice association property.

$$(A * B) * C = A * (B * C)$$

- In DL, we usually don't care about that (as opposed to traditional computer vision and signal processing applications.)
- Cross-correlation is easier to implement.
- Maybe the term "convolution" for cross-correlation because popular, because " Cross-correlation Neural Network" sounds weird.

Transposed Convolution

Regular Convolution:

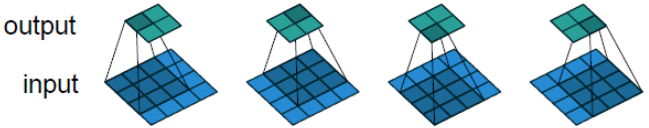
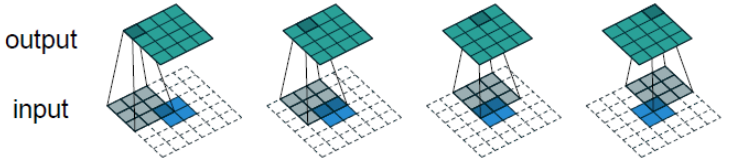


Figure 2.1: (No padding, unit strides) Convolving a $3 * 3$ kernel over a $4 * 4$ input using unit strides (i.e., $i = 4$, $k = 3$, $s = 1$ and $p = 0$).

Transposed Convolution (emulated with direct convolution):



Dumoulin, Vincent, and Francesco Visin. "A guide to convolution arithmetic for deep learning." *arXiv preprint arXiv:1603.07285* (2016).

Dilated Convolution

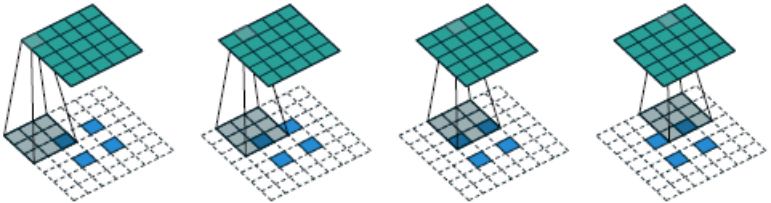
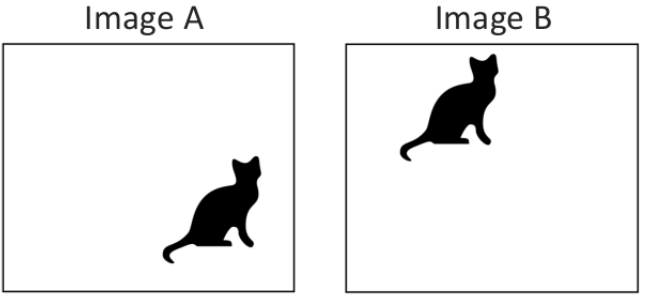


Figure 4.5: The transpose of convolving a $3 * 3$ kernel over a $5 * 5$ input using $2 * 2$ strides (i.e., $i = 5$, $k = 3$, $s = 2$ and $p = 0$). It is equivalent to convolving a $3 * 3$ kernel over a $2 * 2$ input (with 1 zero inserted between inputs) padded with a $2 * 2$ border of zeros using unit strides

Dumoulin, Vincent, and Francesco Visin. "A guide to convolution arithmetic for deep learning." *arXiv preprint arXiv:1603.07285* (2016).

Remember Image Invariance Challenge

Object can appear anywhere in the image.



Model predicts “cat” for both images!

Source: https://penkovsky.com/neural-networks/day5/

Pooling (Pool) Layer

- Helps to achieve image invariance. Objects can appear anywhere in the Image and still get detected.
- Used for sub-sampling
- Reduce the size of representation
- Speed up computation

Pooling

- Apply a filter of size f and stride s. Padding is not applied.
- The output dimensions will be

$$\left\lfloor \frac{n-f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n-f}{s} + 1 \right\rfloor$$

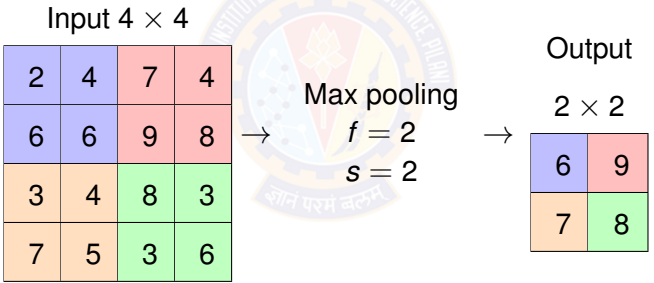
- Fixed computation
- Gradient descent is not applied as there are not parameters to be learned.
- Pooling is applied on each of the channels.

Pooling Layer

- Two types
  1. Max pooling
  2. Average pooling

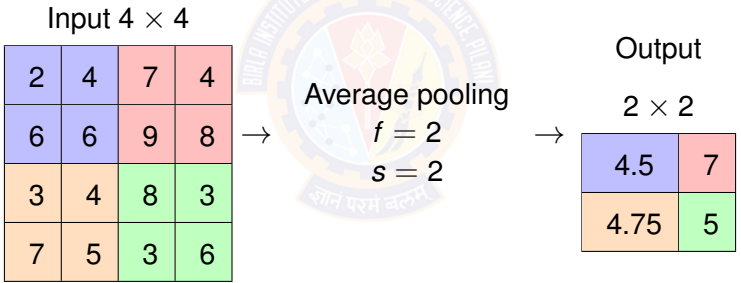
Max Pooling

- Take the maximum of the sub region that is considered.



Average Pooling

- Compute the average of the sub region that is considered.



Convolution + Max Pooling

Input Image 6×6

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

Filter 1

1	-1	-1
-1	1	-1
-1	-1	1

Filter 2

-1	1	-1
-1	1	-1
-1	1	-1

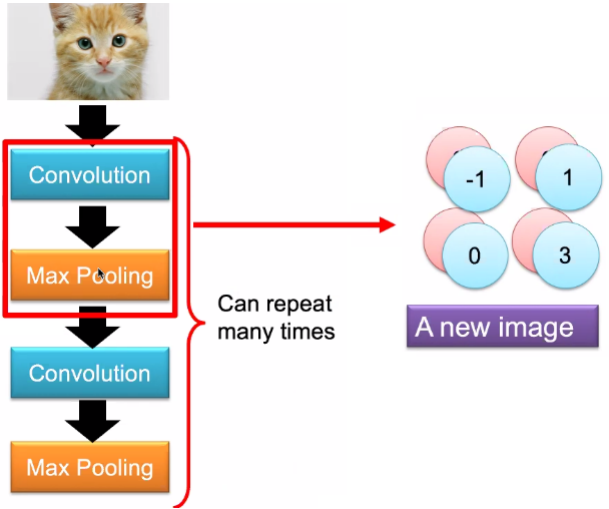
Convolve s=1

3	-1	-3	-1
-3	-1	0	-3
-3	-2	0	1
3	-2	-2	-1
-1	-1	-1	-1
-1	-1	-2	-1
-1	-1	-2	1
-1	0	-4	3

Maxpool f=2, s=2

3	0
3	1
-1	-1
0	3

Convolution + Max Pooling



Flatten

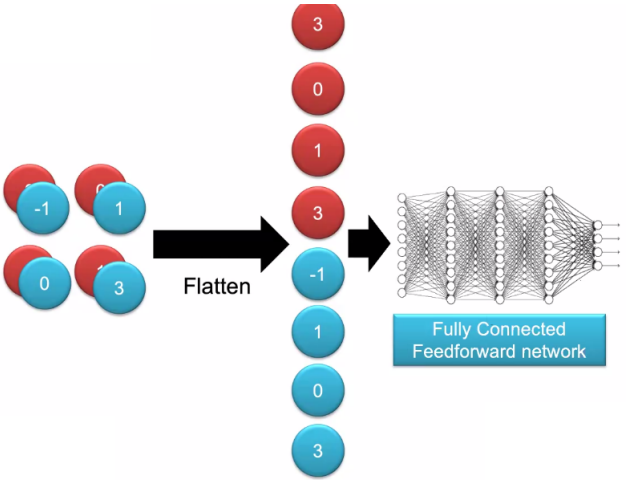
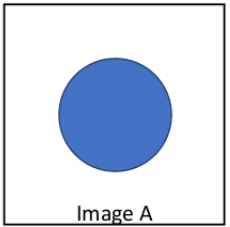
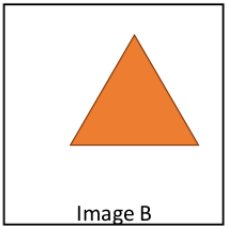
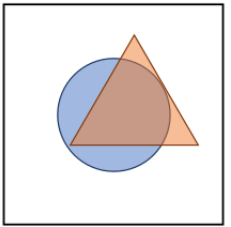
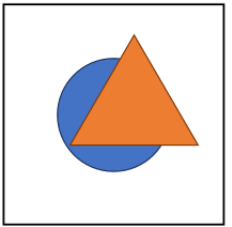

-1
1
0
3
Flatten
3
0
1
3
-1
1
0
3
Fully Connected
Feedforward network

Image is Highly Non-Linear



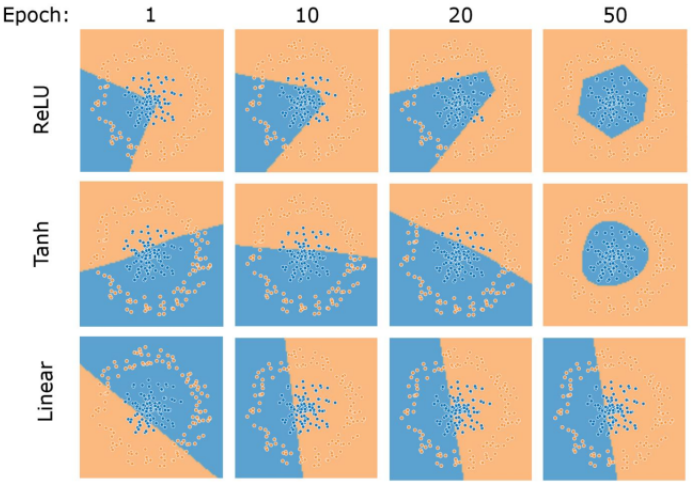






Rectified Linear Unit (ReLU)

- Very fast
- Gradients
- Piece-wise linear approximation



ReLU in Images

Jon Barron's talk at the MIT Vision and Graphics Seminar 2021 on Neural Radiance Fields

Loss for Training

- Mean Squared Error
- Binary Cross Entropy
- Multi class Cross Entropy
- Class-weighted cross-entropy losses (as a way to counter imbalance)
- L1 (MAE), L2 (MSE) losses
- Focal, Dice, Huber loss
- Margin ranking losses
- Generalized Intersection-over-Union (GIoU) loss
- Connectionist Temporal Classification (CTC) loss

Regularisation and Optimization

- Drop out
- Batch Normalisation
- RMSProp, Adam or any other optimizer can be used.

Classification

- $p \in [0, 1]$ - use Sigmoid activation function
- k classes - use softmax activation function

In This Segment

Putting Everything Together

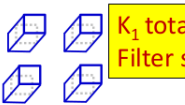
1. Identify the task as classification or regression. This helps in identify the output size.
2. Collect the N images. This helps in identifying input size as $W \times H \times C$.
3. Feature extraction
   - Decide on the number of Convolution layers. This can be derived by looking at how large or small pattern we are interested in.
   - Decide on the kernel size and number of kernels in each layer. Smaller patterns in images require smaller kernels. Number of kernels can be derived from the non-linearity.
   - Use ReLU activation function for better approximation.
4. Classification
   - Use Fully connected layers for classification.
   - The number of (dense) layers depends on the non-linearity.
   - The last (output) layer activation is decided based on Step 1.
5. Train and Test on the available dataset.

Convolutional Neural Network

- Input: 1 or 3 images
  - Grey scale or color
  - Will assume color to be generic

Convolutional Neural Network

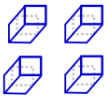
- Input is convolved with a set of K_1 filters or kernels.
  - Typically, K_1 is a power of 2 e.g 2,4,8,16,32,...
  - Filters are typically 5x5x3, 3x3x3 or 1x1x3 (better notation).
  - Typical stride is 1 or 2.



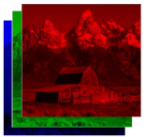
$I \times I$ *image*

Convolutional Neural Network

- Input is convolved with a set of K_1 filters or kernels.
  - Typically, K_1 is a power of 2 e.g 2,4,8,16,32,...
  - Filters are typically 5x5, 3x3 or 1x1.
  - Filters should be small enough to capture fine features, particularly important for scaled down images.



K_1 total filters
Filter size: $L \times L \times 3$



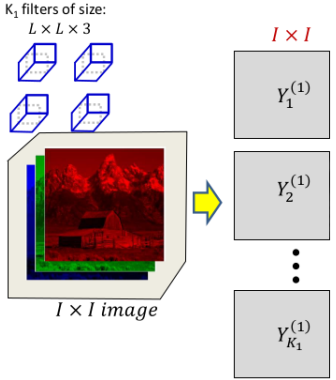
$I \times I$ *image*

Parameters to choose: K_1, L and S
1. Number of filters K_1
2. Size of filters $L \times L \times 3 + bias$
3. *Stride* of convolution S

Total number of parameters: $K_1(3L^2 + 1)$

Convolutional Neural Network

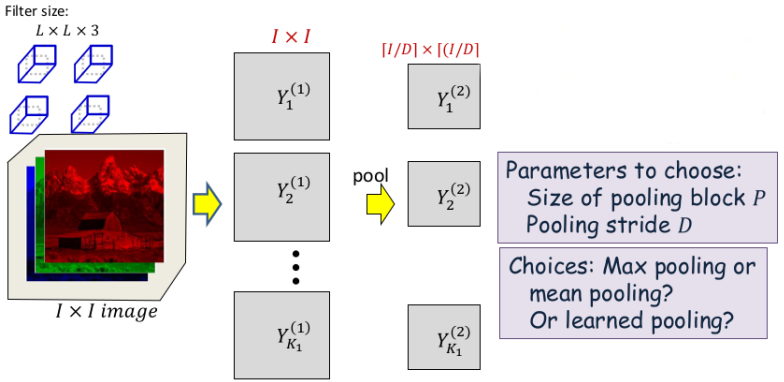
- First Convolutional layer:
  - Several convolutional filers, which are 3D.
  - Convolution is followed typically by a ReLU activation.
  - Each filter creates a singly 2D feature map.



The layer includes a convolution operation followed by an activation (typically RELU)

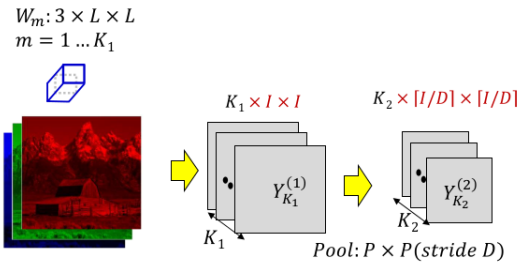
Convolutional Neural Network

- First Pooling layer:
  - For each $P \times P$ block of each map, pool down to a single value.



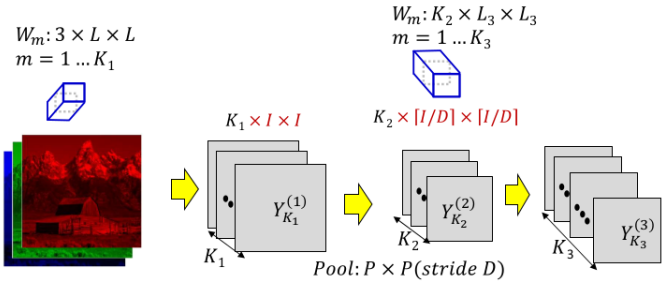
Convolutional Neural Network

- Better drawing



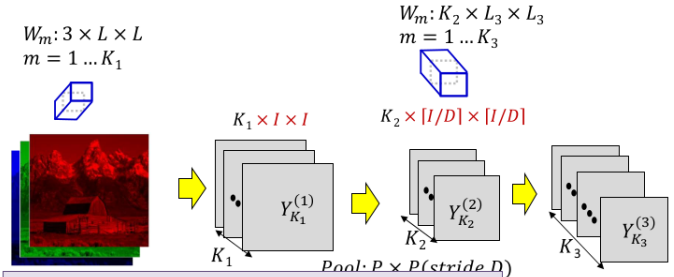
Convolutional Neural Network

- Second Convolutional layer" K_2 3D filters resulting in K_2 feature maps.



Convolutional Neural Network

- Second Convolutional layer



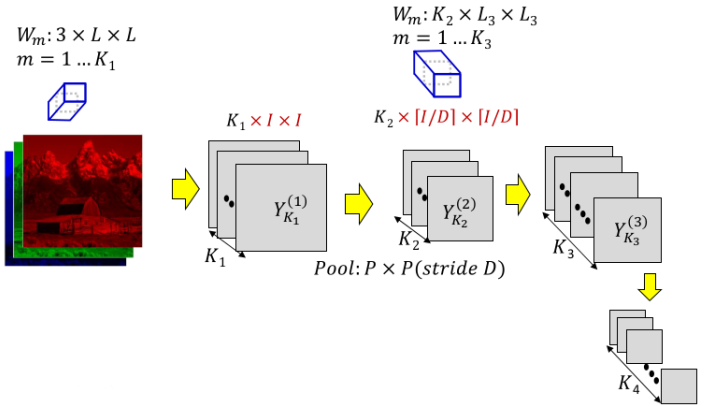
Parameters to choose: K_3, L_3 and S_3

1. Number of filters K_3
2. Size of filters $L_3 \times L_3 \times K_2 + bias$
3. *Stride* of convolution S_3

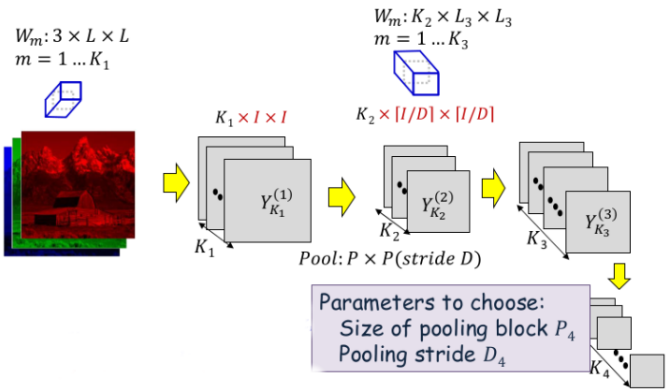
Total number of parameters: $K_3(K_2 L_3^2 + 1)$
All these parameters must be learned

Convolutional Neural Network

- Second Pooling layer: K_2 pooling operations results in K_2 reduced 2D maps.

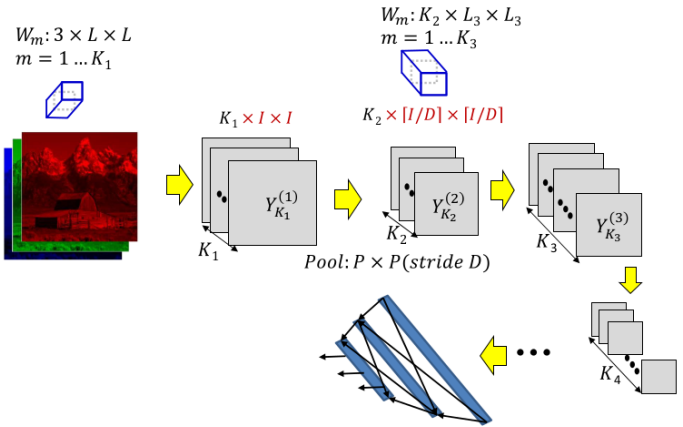


- Second Pooling layer



Convolutional Neural Network

- Fully connected layer: This continues for several layers until the final convolved feature maps are fed to MLP.



Convolutional Neural Network

Size of Layers

- Each convolution layer with stride 1 typically maintains the size of the image.
  - With appropriate zero padding.
  - If performed without zero padding it will decrease the size of the input.
- Each convolution layer will generally increase the number of maps from the previous layer.
  - Increasing layers reduces the amount of information lost by subsequent downsampling.
- Each pooling layer with stride D decreases the size of the maps by a factor of D.
- Filters within a layer must all be the same size, but sizes may vary with layer. Similarly for pooling D may vary with layer.
- In general the number of convolutional filters increases with layers.

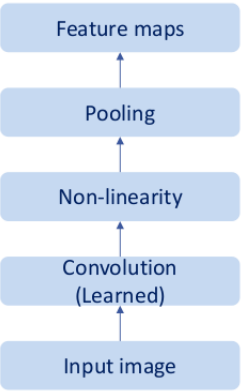
Convolutional Neural Network

Parameters to choose (design choices)

- Number of convolutional and downsampling layers
  - And arrangement (order in which they follow one another)
- For each convolution layer:
  - Number of filters K_i
  - Spatial extent of filter $L_i \times L_i$
    - ⋆ The "depth" of the filter is fixed by the number of filters in the previous layer K_{i-1}.
  - The stride S_i
- For each downsampling/pooling layer:
  - Spatial extent of filter $P_i \times P_i$
  - The stride D_i
- For the final MLP:
  - Number of layers
  - Number of neurons in each layer

CNN-Recap

- Neural network with specialized connectivity structure
- Feed-forward:
  - Convolve input
  - Non-linearity (rectified linear)
  - Pooling (local max)
- Supervised training
- Train convolutional filters by back-propagating error
- Convolution over time



References

1. Deep Learning with Python by Francois Chollet.
   `https://livebook.manning.com/book/deep-learning-with-python/`
2. Deep Learning by Ian Goodfellow, YoshuaBengio, Aaron Courville
   `https://www.deeplearningbook.org/`

Thank You!