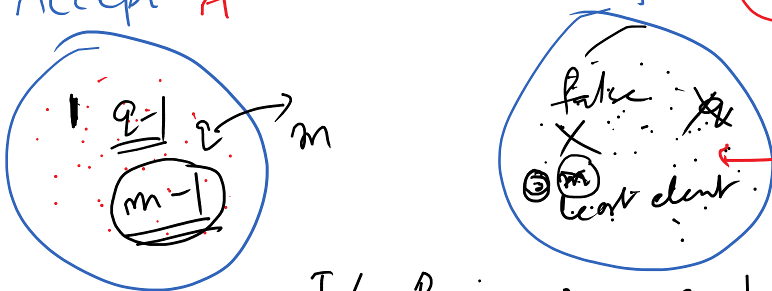
12 February 2022 11:35

Validated $P(1)$ $n=1$

$\boxed{\text{Prove : } P(k) \to P(k+1)}$ $\Bigg\}$ $\underline{P(n)}$ $\forall n$

Accept A Reject $R = \phi$



$P(j)$ is false

If R is non-empty, then R will have a least element (by WOP) m.

Can $m = 1$? $m \neq 1$. $m \geq 2$

$m-1 \in R$? No $m-1 \in A$

$1+2+3+4+ \dots + 5000$

$S_n = 1 + 2 + 3 + \dots + n$

$S_n = n + (n-1) + n-2 + \dots + 1$

$2S_n = n(n+1)$

$S_n = \dfrac{n(n+1)}{2}$

General $(n+1)^2 - \cancel{n^2} = n^2 + 2n + 1 - n^2 = 2n + 1$

General method

$$6^2 - 5^2 = 2.5 + 1$$

$$
\begin{aligned}
&(n+1)^2 - \cancel{n^2} = n^2 + 2n + 1 - n^2 = 2n + 1 \\
+\ &\cancel{n^2} - \cancel{(n-1)^2} = 2(n-1) + 1 \\
+\ &\vdots \\
+\ &\cancel{2^2} - 1^2 = 2.1 + 1 \\
\hline
&(n+1)^2 - 1 = 2(1+2+3+\dots n) + n
\end{aligned}
$$

$$n^2 + 2n + \cancel{1} - \cancel{1} = 2.S_n + n$$

$$n^2 + n = 2.S_n$$

$$S_n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$
\begin{aligned}
&(n+1)^3 - \cancel{n^3} = n^3 + 3n^2 + 3n + 1 - n^3 \\
&\qquad = 3n^2 + 3n + 1 \\
+\ &\cancel{n^3} - \cancel{(n-1)^3} = 3(n-1)^2 + 3(n-1) + 1 \\
+\ &\vdots \\
&\cancel{2^3} - 1^3 = 3.1^2 + 3.1 + 1
\end{aligned}
$$

$$(n+1)^3 - 1^3 = 3(1^2 + 2^2 + \dots + n^2) + 3(1+2+3+\dots n) + n$$

$A = \{1, 2, 3\}$

Subsets, $S_1\ \phi^{1}$, $S_2\ \{1\}^{2}$, $S_3\ \{2\}^{3}$, $S_4\ \{3\}^{4}$ 2^3

$S_5\ \{1,2\}^{5}$ $S_6\ \{1,3\}^{6}$ $S_7\ \{2,3\}^{7}$

$S_8\ \{1,2,3\}^{8}$ $2^0 = 1$

Suppose $A = \phi^{0}$, Subsets of $A = \underline{\underline{\phi}}$

Set $S = \{a_1, a_2, \ldots a_k, a_{k+1}\}$

Subsets S_1 S_2 . . . . $\underline{S_{2^k}}$.

$S_1 \cup \{a_{k+1}\}$ $S_2 \cup \{a_{k+1}\}$ . . . $S_{2^k} \cup \{a_{k+1}\}$

$2 \cdot 2^k = 2^{k+1}$ Subsets

$P(k) \rightarrow P(k+1)$

Any integer ≥ 2 can be written as product of primes.

$n=2 = 2^1$ prime.

$9 = 3^2 = 3 \cdot 3$

$26 = 2 \cdot 13$

$3 = 3$

$57 = 19 \cdot 3$

$\underbrace{P(2), P(3) \ldots P(k)}$ is true

$P(k+1)$ is true

$P(k+1)$ is true

$P(k) \to k$ can be expressed as product of primes

$P(k+1)$ is true

$k+1 \to$ prime (our result is true)

$k+1 \to$ Composite

$k+1 = a \cdot b$ $\quad a \le k,\ b \le k$

$p_1^{r_1} p_2^{r_2} \cdots p_\ell^{r_\ell} \cdot p_1^{a_1} p_2^{q_2} \cdots p_m^{q_m}$

$a, d \quad a \geq d$ all are +ve integers

$$S = \{ \overset{>0}{a - d\cdot 1},\ \overset{>0}{a - d\cdot 2},\ \overset{>0}{a-d\cdot 3},\ \ldots \ldots \underbrace{a - d\cdot q}_{\geq 0} \}\quad \underset{<0}{a-d(q+1)}$$

S has a least element, call it r

$a - dq = r \qquad a = dq + r$

$0 \le r < d$

q & r are unique, $q_1, r_1 \quad q_2, r_2$

$a = dq_1 + r_1 \qquad 0 \le r_1 < d$

$a = dq_2 + r_2 \qquad 0 \le r_2 < d$

$7 < 10$

$3 < 10$

$7 - 3 > 10$?

$0 = d(q_1 - q_2) + (r_1 - r_2)$

$\boxed{d}(q_2 - q_1) = (r_1 - r_2) \quad \text{— (1)}$

$r_1 - r_2$ is less than d.

(1) is possible only if $q_2 = q_1$.

& $r_1 = r_2$