Birla Institute of Technology and Science, Pilani

Work Integrated Learning Programmes Division

Cluster Programme - M.Tech. in Data Science and Engg.

II Semester 2019-20

Course Number	DSECL ZC416
Course Name	Mathematical Foundation for Data Science
Nature of Exam	Open Book
Weightage for grading	30%
Duration	90 minutes
Date of Exam	21/06/2020 (10:00 a.m - 11:30 a.m)

# Pages	2
# Questions	5

Instructions

1. All questions are compulsory
2. Answers without proper justification would not be awarded marks.

Q1a) Let $M \in \mathbb{R}^{n\times n}$ be a square matrix and $I_n \in \mathbb{R}^{n\times n}$ be the identity matrix. Consider a matrix E defined as $E = I_n - M$ and for which nullity$(E) = 0$. If F and G are constructed such that $F = ME^{-1}$ and $G = E^{-1}M$. Is it true in general that (1+1)

a) F and G are similar?
b) F and G are the same?

Give justifications for your answers.

b) Prof. X has two brilliant students Y and Z in his class. He introduces the concept of vector spaces, bases and dimension in his 3rd session. As an exercise, he gives a vector space V of dimension n and asks Y and Z to find the basis based on the following. Y should start with a set S which has a single non-zero element of V and build the basis by adding vectors to S such that S is linearly independent. The process should be stopped once S has n elements. Z is asked to start with a set T which has more than n elements, say $n+k$, with $k > 0$, and remove elements such that T becomes linearly independent. The next day Y and Z bring in their working. Prof. X asks the class if S and T, in the working of Y and Z, the same sets? While student U says yes, student W says need not and Prof. X says that both U and W could be correct. Justify the statement of Prof. X with suitable examples of V, S and T. (4)

Q2a) Assume A is a $n \times m$ matrix. Verify whether $T : M_{ln} \to M_{lm}$ with $T(B) = BA$ is linear transformation or not, where M_{rq} denotes the set of all $r \times q$ matrices. (2)

b) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be linear and suppose that $T((1, 0, -1)) = (-1, -1, 3)$ and $T((-2, 1, 0)) = (0, -2, -1)$. Determine $T((1, 3, -7))$ from the given data. Is this value unique? Justify. (1+1)

c) Verify that Rank-Nullity theorem for the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ defined by $T(x_1, x_2, x_3, x_4) = (x_1 + x_2, 2x_2 - x_3, x_1 + x_2 + 3x_3)$. (2)

Q3a) Applying Gaussian elimination with and without partial pivoting and 4 digit floating point arithmetic, solve the following system and compare the results. Show all the intermediate computations. (2+2)

$$0.1036x_1 + 0.2122x_2 = 0.7381$$
$$0.2081x_1 + 0.4247x_2 = 0.9327$$

b) Let $A_{n\times n}\mathbf{x} = \mathbf{b}$ be a system with unique solution. Compare the number of divisions, multiplications and additions used in solving the system using Gauss elimination and Gauss Jordan methods. Give the exact numbers and not the orders. (2)

Q4a) Find the QR factorization of the matrix (4)

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 1 & 3 & 7 \\ 1 & -1 & -4 \\ 1 & -1 & 2 \end{pmatrix}$$

b) Test using the spectral method or suitable matrix norms, the guaranteed convergence of Gauss Jacobi method for the following system (2)

$$\begin{aligned} 1x + 4z &= 8 \\ 4y + 2z &= 9 \\ 4x + 2y - 2z &= 10 \end{aligned}$$

Q5) ABC company in Pune has about 120 employees and produces two types of belts, B1 and B2. Product B1 is superior to B2 and the profits on selling them in the market are Rs. 30 and Rs. 22.5 per piece respectively.
Product B1 requires twice the processing time compared to B2. If the company were to produce B2 alone, they could do about 1200 pieces a day. But because of some restrictions in the internal processes, they are able to produce only a maximum of 1000 pieces of B1 and B2 together, in a day. Also, the market trend shows that at least 250 pieces of B2 be produced.

a) Assuming all of the products can be sold, formulate the LPP. (2)
b) Using proper scaling, solve the problem graphically. (2)
c) What are the changes permitted in the profits of B1 and B2 such that the present solution remains optimal? (1)
d) Would the solution change if we introduce a minimum quantity of B1 to be produced? List the various scenarios. (1)

→ All the best ←