INTRODUCTION TO STATISTICAL METHODS
ASSIGNMENT-1

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Section 3

Q.1] For two observations 'a' and 'b' show that the standard deviation is half of the distance between them.

Observations - $[a, b]$
No. of Observations $= |[a, b]| = 2$

Arithmetic mean $\mu = \frac{1}{N} \sum_{i=1}^{N} x_i$

$$\therefore \mu = \frac{a+b}{2}$$

Variance $V = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2$

$$\therefore V = \frac{1}{2}\left[\left(a - \left(\frac{a+b}{2}\right)\right)^2 + \left(b - \left(\frac{a+b}{2}\right)\right)^2\right]$$

$$= \frac{1}{2}\left[\left(\frac{2a-a-b}{2}\right)^2 + \left(\frac{2b-a-b}{2}\right)^2\right]$$

$$= \frac{1}{2}\left[\left(\frac{a-b}{2}\right)^2 + \left(\frac{b-a}{2}\right)^2\right]$$

$$= \frac{1}{2}\left[\left(\frac{a-b}{2}\right)^2 + \left(\frac{a-b}{2}\right)^2\right] \quad - (\because y = x^2 \text{ is a symmetric function})$$

$$= \frac{1}{\cancel{2}} \cdot \cancel{2} \cdot \left(\frac{a-b}{2}\right)^2$$

$$\therefore \text{Variance} = \left(\frac{a-b}{2}\right)^2$$

Standard deviation $= +\sqrt{\text{Variance}} = +\sqrt{\left(\frac{a-b}{2}\right)^2} = \left|\frac{a-b}{2}\right|$

$\therefore$ For given observations a & b, the standard deviation of these observations is half of the distance between these observations.

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Q.2] An insurance company insured 1000 taxi drivers, 2000 car drivers, 3000 truck drivers respectively. Probability of their accident is 0.05, 0.15 and 0.1 respectively. One of the insured person meets with an accident. What's the probability that he is a truck driver?

Solution

Consider a universe of taxi, car and truck drivers respectively. i.e. Any given person must be only one of these three.

Let D_1 = Event that a person is a taxi driver
D_2 = Event that a person is a car driver
D_3 = Event that a person is a truck driver.

Given the universe that we're considering, we can say safely that D_1, D_2, D_3 are exhaustive and mutually exclusive.

$$P(D_1) = \frac{\text{No. of taxi drivers}}{\text{Total drivers}} = \frac{1000}{(1000+2000+3000)} = \frac{1}{6}$$

$$P(D_2) = \frac{\text{No. of car drivers}}{\text{Total drivers}} = \frac{2000}{(1000+2000+3000)} = \frac{2}{6} = \frac{1}{3}$$

$$P(D_3) = \frac{\text{No. of truck drivers}}{\text{Total drivers}} = \frac{3000}{(1000+2000+3000)} = \frac{3}{6} = \frac{1}{2}$$

Let A = Event that a person meets with an accident.

Then we can say from the given information

$$P\left(\frac{A}{D_1}\right) = 0.05 \; ; \quad P\left(\frac{A}{D_2}\right) = 0.15 \quad P\left(\frac{A}{D_3}\right) = 0.1$$

And we need to find the probability that a person was a truck driver given that she met with an accident

i.e. $P\left(\frac{D_3}{A}\right) = ?$

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By Baye's theorem, we can say

$$P\left(\frac{D_3}{A}\right) = \left[P\left(\frac{A}{D_3}\right) \cdot P(D_3)\right] \cdot \frac{1}{P(A)} \quad \text{———— ①}$$

We know the numerator terms but not the denominator.
We know that D_1, D_2, D_3 are mutually exclusive and exhaustive events.

$\therefore$ By total law of probability, we can say

$$P(A) = \sum_{i=1}^{3} P\left(\frac{A}{D_i}\right) \cdot P(D_i)$$

$$= P\left(\frac{A}{D_1}\right) \cdot P(D_1) + P\left(\frac{A}{D_2}\right) \cdot P(D_2) + P\left(\frac{A}{D_3}\right) \cdot P(D_3)$$

$$= 0.05 \times \frac{1}{6} + 0.15 \times \frac{1}{3} + 0.1 \times \frac{1}{2}$$

$$= \frac{0.05 \times 1 + 0.15 \times 2 + 0.1 \times 3}{6}$$

$$P(A) = \frac{0.05 + 0.30 + 0.30}{6} = \frac{0.65}{6} = \frac{65}{600} = \frac{13}{120}$$

Now from eqn ①, substituting all probability values we get,

$$P\left(\frac{D_3}{A}\right) = \left(0.1 \times \frac{1}{2}\right) \times \frac{1}{13/120} = \frac{12}{26} = \frac{6}{13}$$

Ans] Given that an accident occurred, the probability of the person being a truck driver is $6/13 = 0.4615$

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Q.3] Past surveys show that 40% of the officers at a certain industry own cars. Suppose six officers are selected at random from this industry with replacement.

(a) What is the probability that exactly 4 will own cars?

Here we can say that owning a car is success and there's only 2 outcomes → own/not own a car

⇒ Each trial is a Bernoulli trial.

We're sampling 6 officers ⇒ $n=6$

Owning car $= 0.4 = p =$ Success probability

Our distribution $= B(N=6, p=0.4)$

$$\therefore P(X=4) = {}^{n}C_{r}\, p^{r} (1-p)^{n-r}$$

$$= {}^{6}C_{4} \cdot (0.4)^{4} \cdot (1-0.4)^{6-4}$$

$$= \frac{6!}{2!\cdot 4!} \times 0.4^{4} \times 0.6^{2}$$

$$= 0.13824$$

A] Probability of exactly four people having a car $= 0.13824$

(b) What is the probability that at least one will own a car?

$P(X \geq 1) = P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$

But we know $\sum_{n=0}^{6} P(X=n) = 1 \Rightarrow P(0) + \sum_{n=1}^{6} P(X=n) = 1 \Rightarrow \sum_{n=1}^{6} P(X=n) = 1 - P(0)$

$$\therefore P(X \geq 1) = 1 - P(0)$$

$$= 1 - {}^{6}C_{0} \cdot p^{0} \cdot (1-p)^{6-0}$$

$$= 1 - 1 \cdot 1 \cdot (0.6)^{6}$$

$$= 0.953344$$

A] Probability that at least one will own a car $= 0.953344$

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Q.4] Let A and B be two possible outcomes of an experiment. Let $P(A) = 0.4$ $P(B) = 'p'$ & $P(A \cup B) = 0.7$

(i) For what choice of 'p' are A and B mutually exclusive?

We know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \text{ ——— (a)}$$

For two events to be mutually exclusive, $A \cap B = \phi$ & $P(A \cap B) = 0$

$\therefore P(A \cup B) = P(A) + P(B)$

$\Rightarrow 0.7 = 0.4 + p$

$\Rightarrow p = 0.7 - 0.4 = 0.3$

A] If $P(B) = p = 0.3$, events A & B will be mutually exclusive.

(ii) For what choice of 'p' are A and B independent?

If events are independent, then

$$P(X_1, X_2, X_3 \cdots X_i) = P(X_1) \cdot P(X_2) \cdot P(X_3) \cdots P(X_i)$$

Then $P(A \cap B) = P(A) \cdot P(B)$ ——— (b)

Using (a) & (b), we get

$P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

$\therefore 0.7 = 0.4 + p - 0.4 \times p$

$\therefore 0.7 - 0.4 = (1 - 0.4)p$

$\therefore 0.3 = 0.6p$

or $p = \frac{0.3}{0.6} = 0.5$

A] If $P(B) = p = 0.5$, events A & B will be independent.

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Q.5] A variable is normally distributed with mean of 120 and a standard deviation of 5. What is the probability that it is above 127?

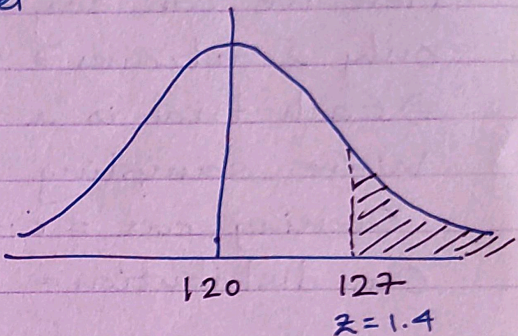
Solution

Given $X \sim N(120, 5)$

~~Non~~ Standardizing the given obs. we get

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{127-120}{5} = \frac{7}{5} = 1.4$$



To find $P(X > 127)$

i.e. $P(z > 1.4)$

From the normal distribution table, we find

$$P(z \leq 1.4) = 0.9192$$

$$\therefore P(z > 1.4) = 1 - P(z \leq 1.4)$$

$$= 1 - 0.9192$$

$$= 0.0808$$

$$= 8.08\%$$

A] For the given distribution $P(X > 127) = P(z > 1.4) = 8.08\% \ (0.0808)$