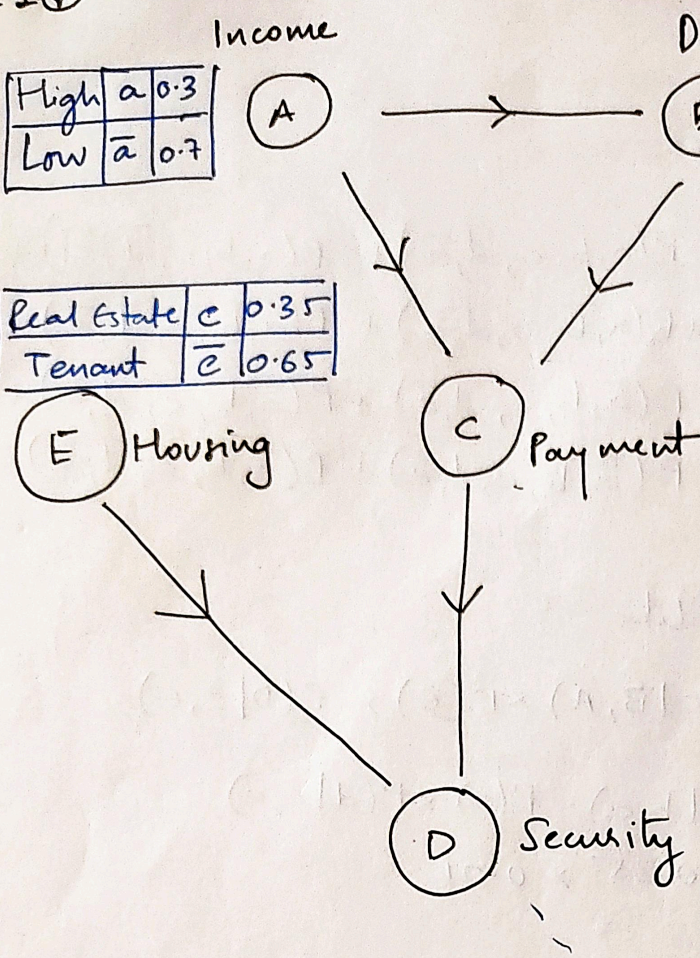
 Vinayak Nayak 




| | | |
|---|---|---|
| High | $a$ | 0.3 |
| Low | $\bar{a}$ | 0.7 |

| | | $a$ | $\bar{a}$ |
|---|---|---|---|
| Small | $\bar{b}$ | 0.9 | 0.4 |
| large | $b$ | 0.1 | 0.6 |

| | | |
|---|---|---|
| Real Estate | $c$ | 0.35 |
| Tenant | $\bar{c}$ | 0.65 |

| | | $a$ | | $\bar{a}$ | |
|---|---|---|---|---|---|
| | | $b$ | $\bar{b}$ | $b$ | $\bar{b}$ |
| Default | $c$ | 0.05 | 0.5 | 0.45 | 0.6 |
| Payback | $\bar{c}$ | 0.95 | 0.5 | 0.55 | 0.4 |

| | | $c$ | | $\bar{c}$ | |
|---|---|---|---|---|---|
| | | $e$ | $\bar{e}$ | $e$ | $\bar{e}$ |
| Security | $d$ | 0.01 | 0.5 | 0.75 | 0.31 |
| No Security | $\bar{d}$ | 0.99 | 0.5 | 0.25 | 0.69 |

Q.1 (i) False. There's an active information path $I \to P \to S$, hence telling something about security gives influences my belief in income and vice-versa.

(ii) True. Specifying payment breaks all active info paths between Income and security.

(iii) False. There's a direct connection between income & payment.

(iv) True. Specifying payment alone breaks all info flow between income and security.

(v) False. There's a direct connection between deposit & payment.

(vi) False. ~~Income~~ Income and payment have a direct edge. The influence of income on payment thru deposit might come to a stop by specifying deposit however there is still the direct connection influence alive.

Q.1 (ii) $P(A,B,C,D,E) = P(A) \times P(B|A) \times P(C|B,A) \times P(E|C,B,A) \times P(D|A,B,C,E)$

E is a root node i.e. no parents $\Rightarrow P(E|C,B,A) = P(E)$

Also D's parents are $C, E \Rightarrow P(D|A,B,C,E) = P(D|C,E)$

$$\boxed{\begin{aligned} \therefore P(A,B,C,D,E) &= P(A) \times P(B|A) \times P(C|B,A) \times P(E|C,B,A) \times P(D|A,B,C,E) \\ &= P(A) \times P(B|A) \times P(C|B,A) \times P(E) \times P(D|C,E) \end{aligned}}$$

Q.1 (iii) $P(C) = ?$

$$P(C) = \sum_{A,B,D,E} P(A,B,C,D,E)$$

$$= P(a,b,c,d,e) + P(a,b,c,\bar{d},e) + P(a,b,c,d,\bar{e}) + P(a,b,c,\bar{d},\bar{e}) +$$
$$P(a,\bar{b},c,d,e) + P(a,\bar{b},c,\bar{d},e) + P(a,\bar{b},c,d,\bar{e}) + P(a,\bar{b},c,\bar{d},\bar{e}) +$$
$$P(\bar{a},b,c,d,e) + P(\bar{a},b,c,\bar{d},e) + P(\bar{a},b,c,d,\bar{e}) + P(\bar{a},b,c,\bar{d},\bar{e}) +$$
$$P(\bar{a},\bar{b},c,d,e) + P(\bar{a},\bar{b},c,\bar{d},e) + P(\bar{a},\bar{b},c,d,\bar{e}) + P(\bar{a},\bar{b},c,\bar{d},\bar{e})$$

Solving for few terms,

We know the JPDF decomposes like

$$P(A,B,C,D,E) = P(A) \times P(B|A) \times P(C|B,A) \times P(E) \times P(D|C,E)$$

$$\therefore P(a,b,c,d,e) = P(a) \times P(b|a) \times P(c|b,a) \times P(e) \times P(d|c,e)$$
$$= 0.3 \times 0.1 \times 0.05 \times 0.35 \times 0.01$$
$$= 5.25 \times 10^{-6}$$

$$P(a,b,c,\bar{d},e) = P(a) \times P(b|a) \times P(c|b,a) \times P(e) \times P(\bar{d}|c,e)$$
$$= 0.3 \times 0.1 \times 0.05 \times 0.35 \times 0.99$$
$$= 5.1975 \times 10^{-4}$$

$$P(a,b,c,d,\bar{e}) = P(a) \times P(b|a) \times P(c|b,a) \times P(\bar{e}) \times P(d|c,\bar{e})$$
$$= 0.3 \times 0.1 \times 0.05 \times 0.65 \times 0.5$$
$$= 4.875 \times 10^{-4}$$

$$P(a,b,c,\bar{d},\bar{e}) = P(a) \times P(b|a) \times P(c|b,a) \times P(\bar{e}) \times P(\bar{d}|c,\bar{e})$$
$$= 0.3 \times 0.1 \times 0.05 \times 0.65 \times 0.5$$
$$= 4.875 \times 10^{-4}$$

~~Similarly, we can compute other probabilities and eventually, we would get~~

$$\cancel{P(c) = 5.25 \times 10^{-6} + 5.1975 \times 10^{-4} + 4.875 \times 10^{-4} + 4.875 \times 10^{-4} + }$$

Computing other probabilities,

$$P(a,\bar{b},c,d,e) = P(a) \times P(\bar{b}|a) \times P(c|\bar{b},a) \times P(e) \times P(d|c,e)$$
$$= 0.3 \times 0.9 \times 0.5 \times 0.35 \times 0.01$$
$$= 4.725 \times 10^{-4}$$

$$P(a,\bar{b},c,\bar{d},e) = P(a)\times P(\bar{b}|a)\times P(c|a,\bar{b})\times P(e)\times P(\bar{d}|c,e)$$
$$= 0.3\times 0.9\times 0.5\times 0.35\times 0.99$$
$$\cancel{= 2.3625\times 10^{-2}}$$
$$= 4.678\times 10^{-2}$$

$$P(a,\bar{b},c,d,\bar{e}) = P(a)\times P(\bar{b}|a)\times P(c|a,\bar{b})\times P(\bar{e})\times P(d|c,\bar{e})$$
$$= 0.3\times 0.9\times 0.5\times 0.65\times 0.5$$
$$= 4.3875\times 10^{-2}$$

$$P(a,\bar{b},c,\bar{d},\bar{e}) = P(a)\times P(\bar{b}|a)\times P(c|\bar{b},a)\times P(\bar{e})\times P(\bar{d}|c,\bar{e})$$
$$= 0.3\times 0.9\times 0.5\times 0.65\times 0.5$$
$$= 4.3875\times 10^{-2}$$

$$P(\bar{a},b,c,d,e) = P(\bar{a})\times P(b|\bar{a})\times P(c|b,\bar{a})\times P(e)\times P(d|c,e)$$
$$= 0.7\times 0.6\times 0.45\times 0.35\times 0.01$$
$$= 6.615\times 10^{-4}$$

$$P(\bar{a},b,c,\bar{d},e) = P(\bar{a})\times P(b|\bar{a})\times P(c|b,\bar{a})\times P(e)\times P(\bar{d}|c,e)$$
$$= 0.7\times 0.6\times 0.45\times 0.35\times 0.99$$
$$= 6.54885\times 10^{-2}$$

$$P(\bar{a},b,c,d,\bar{e}) = P(\bar{a})\times P(b|\bar{a})\times P(c|b,\bar{a})\times P(\bar{e})\times P(d|c,\bar{e})$$
$$= 0.7\times 0.6\times 0.45\times 0.65\times 0.5$$
$$= 6.1425\times 10^{-2}$$

$$P(\bar{a},b,c,\bar{d},\bar{e}) = P(\bar{a})\times P(b|\bar{a})\times P(c|b,\bar{a})\times P(\bar{e})\times P(\bar{d}|c,\bar{e})$$
$$= 0.7\times 0.6\times 0.45\times 0.65\times 0.5$$
$$= 6.1425\times 10^{-2}$$

$$P(\bar{a},\bar{b},c,d,e) = P(\bar{a})\times P(\bar{b}|\bar{a})\times P(c|\bar{b},\bar{a})\times P(e)\times P(d|c,e)$$
$$= 0.7\times 0.4\times 0.6\times 0.35\times 0.01$$
$$= 5.88\times 10^{-4}$$

$$P(\bar{a},\bar{b},c,\bar{d},e) = P(\bar{a})\times P(\bar{b}|\bar{a})\times P(c|\bar{b},\bar{a})\times P(e)\times P(\bar{d}|c,e)$$
$$= 0.7\times 0.4\times 0.6\times 0.35\times 0.99$$
$$= 5.8212\times 10^{-2}$$

Vinayak Nayak (4)

$$P(\bar{a}, \bar{b}, c, d, \bar{e}) = P(\bar{a}) \times P(\bar{b}|\bar{a}) \times P(c|\bar{b},\bar{a}) \times P(\bar{e}) \times P(d|c,\bar{e})$$
$$= 0.7 \times 0.4 \times 0.6 \times 0.65 \times 0.5$$
$$= 5.46 \times 10^{-2}$$

$$P(\bar{a}, \bar{b}, c, \bar{d}, \bar{e}) = P(\bar{a}) \times P(\bar{b}|\bar{a}) \times P(c|\bar{b},\bar{a}) \times P(\bar{e}) \times P(\bar{d}|c,\bar{e})$$
$$= 0.7 \times 0.4 \times 0.6 \times 0.65 \times 0.5$$
$$= 5.46 \times 10^{-2}$$

$$\therefore P(c) = \sum_{A,B,D,E} P(A, B, C, D, E)$$

$$= 5.25 \times 10^{-6} + 5.1975 \times 10^{-4} + 4.875 \times 10^{-4} + 4.875 \times 10^{-4} + 4.725 \times 10^{-4} + 4.678 \times 10^{-2} + 4.3875 \times 10^{-2} + 4.3875 \times 10^{-2} + 6.615 \times 10^{-4} + 6.5488 \times 10^{-2} + 6.1425 \times 10^{-2} + 6.1425 \times 10^{-2} + 5.88 \times 10^{-4} + 5.8212 \times 10^{-2} + 5.46 \times 10^{-2} + 5.46 \times 10^{-2}$$

$$\boxed{P(c) = 0.49346}$$

c is default ⇒ payment is false. Hence, we can say that in the absence of any prior information, default is likely to happen with slightly lower odds than toss of a fair coin.

---

Q.1. (iv) $P(\bar{c}|\bar{a}) = ?$

$$P(\bar{c}|\bar{a}) = \frac{P(\bar{c} \& \bar{a})}{P(\bar{a})}$$

We know $P(\bar{a}) = 0.7$

And $P(\bar{c} \& \bar{a}) = \sum_{i \in B,D,E} P(\bar{c}, \bar{a}, b_i, d_i, e_i)$

$$= P(\bar{a}, b, \bar{c}, d, e) + P(\bar{a}, \bar{b}, \bar{c}, d, e) + P(\bar{a}, b, \bar{c}, \bar{d}, e) + P(\bar{a}, \bar{b}, \bar{c}, \bar{d}, e) + P(\bar{a}, b, \bar{c}, d, \bar{e}) + P(\bar{a}, \bar{b}, \bar{c}, d, \bar{e}) + P(\bar{a}, b, \bar{c}, \bar{d}, \bar{e}) + P(\bar{a}, \bar{b}, \bar{c}, \bar{d}, \bar{e}) \quad \text{——— (1)}$$

Vinayak Nayak (5)

$$P(\bar{a}, b, \bar{c}, d, e) = P(\bar{a}) \times P(b|\bar{a}) \times P(\bar{c}|b,\bar{a}) \times P(e) \times P(d|\bar{c},e)$$
$$= 0.7 \times 0.6 \times 0.55 \times 0.35 \times 0.75$$
$$= 0.06064$$

$$P(\bar{a}, b, \bar{c}, \bar{d}, e) = P(\bar{a}) \times P(b|\bar{a}) \times P(\bar{c}|b,\bar{a}) \times P(e) \times P(\bar{d}|\bar{c},e)$$
$$= 0.7 \times 0.6 \times 0.55 \times 0.35 \times 0.25$$
$$= 0.02021$$

$$P(\bar{a}, b, \bar{c}, d, \bar{e}) = P(\bar{a}) \times P(b|\bar{a}) \times P(\bar{c}|b,\bar{a}) \times P(\bar{e}) \times P(d|\bar{c},\bar{e})$$
$$= 0.7 \times 0.6 \times 0.55 \times 0.65 \times 0.31$$
$$= 0.04655$$

$$P(\bar{a}, b, \bar{c}, \bar{d}, \bar{e}) = P(\bar{a}) \times P(b|\bar{a}) \times P(\bar{c}|b,\bar{a}) \times P(\bar{e}) \times P(\bar{d}|\bar{c},\bar{e})$$
$$= 0.7 \times 0.6 \times 0.55 \times 0.65 \times 0.69$$
$$= 0.1036$$

$$P(\bar{a}, \bar{b}, \bar{c}, d, e) = P(\bar{a}) \times P(\bar{b}|\bar{a}) \times P(\bar{c}|\bar{b},\bar{a}) \times P(e) \times P(d|\bar{c},e)$$
$$= 0.7 \times 0.4 \times 0.4 \times 0.35 \times 0.75$$
$$= 0.0294$$

$$P(\bar{a}, \bar{b}, \bar{c}, \bar{d}, e) = P(\bar{a}) \times P(\bar{b}|\bar{a}) \times P(\bar{c}|\bar{b},\bar{a}) \times P(e) \times P(\bar{d}|\bar{c},e)$$
$$= 0.7 \times 0.4 \times 0.4 \times 0.35 \times 0.25$$
$$= 0.0098$$

$$P(\bar{a}, \bar{b}, \bar{c}, d, \bar{e}) = P(\bar{a}) \times P(\bar{b}|\bar{a}) \times P(\bar{c}|\bar{b},\bar{a}) \times P(\bar{e}) \times P(d|\bar{c},\bar{e})$$
$$= 0.7 \times 0.4 \times 0.4 \times 0.65 \times 0.31$$
$$= 0.02257$$

$$P(\bar{a}, \bar{b}, \bar{c}, \bar{d}, \bar{e}) = P(\bar{a}) \times P(\bar{b}|\bar{a}) \times P(\bar{c}|\bar{b},\bar{a}) \times P(\bar{e}) \times P(\bar{d}|\bar{c},\bar{e})$$
$$= 0.7 \times 0.4 \times 0.4 \times 0.65 \times 0.69$$
$$= 0.0502$$

From eqn (1) we can say,

$$P(\bar{c} \,\&\, \bar{a}) = 0.06064 + 0.02021 + 0.04655 + 0.1036 + 0.0294 + 0.0098 + 0.02257 + 0.0502$$

$$\boxed{P(\bar{c} \,\&\, \bar{a}) = 0.34297}$$

Also $P(\bar{a}) = 0.7$

$$\Rightarrow \boxed{P(\bar{c}|\bar{a}) = 0.34297 / 0.7 = 0.4899}$$

A] Probability of getting payment given that the income is low is 0.4899

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(V) Got payment $\rightarrow \bar{c}$
Low income $\rightarrow \bar{a}$
large deposits $\rightarrow b$
To find: $P(\bar{c} \mid \bar{a}, b)$

By conditional prob. definition,

$$P(\bar{c} \mid \bar{a}, b) = \frac{P(\bar{c} \,\&\, \bar{a} \,\&\, b)}{P(\bar{a} \,\&\, b)}$$

$$P(\bar{c} \,\&\, \bar{a} \,\&\, b) = \sum_{D,E} P(\bar{a}, b, \bar{c}, d_i, e_i)$$

$$= P(\bar{a}, b, \bar{c}, d, e) + P(\bar{a}, b, \bar{c}, d, \bar{e}) + P(\bar{a}, b, \bar{c}, \bar{d}, e) + P(\bar{a}, b, \bar{c}, \bar{d}, \bar{e})$$

We have already computed these 4 joint probas in previous q.
Hence using them we get

$$P(\bar{c} \,\&\, \bar{a} \,\&\, b) = 0.06064 + 0.04655 + 0.02021 + 0.1036$$

$$\boxed{P(\bar{c} \,\&\, \bar{a} \,\&\, b) = 0.231}$$

Also,

$$P(\bar{a} \,\&\, b) = \sum_{C,D,E} P(\bar{a}, b, c_i, d_i, e_i)$$

$$= P(\bar{a}, b, c, d, e) + P(\bar{a}, b, \bar{c}, d, e) + P(\bar{a}, b, c, \bar{d}, e) + P(\bar{a}, b, \bar{c}, \bar{d}, e) + P(\bar{a}, b, c, d, \bar{e}) + P(\bar{a}, b, \bar{c}, d, \bar{e}) + P(\bar{a}, b, c, \bar{d}, \bar{e}) + P(\bar{a}, b, \bar{c}, \bar{d}, \bar{e})$$

$P(\bar{c}, \bar{a}, b)$ computed above.

Probabilities are computed in 1.(iv) when marginalizing over A, B, D, E respectively.

$$\therefore P(\bar{a} \,\&\, b) = 6.615 \times 10^{-4} + 0.06549 + 0.061425 + 0.061425 + 0.231$$

$$P(\bar{a} \,\&\, b) = 0.4200$$

$$\boxed{\therefore P(\bar{c} \mid \bar{a}, b) = \frac{P(\bar{c}, \bar{a}, b)}{P(\bar{a}, b)} = \frac{0.231}{0.4200} = 0.55}$$

A] Probability of getting payment given income is low and you have high deposits is 0.55

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(vi) Did not default $\rightarrow \bar{c}$

High income $\rightarrow a$

No security $\rightarrow \bar{d}$

To find: $P(\bar{c} \mid a, \bar{d})$

By def. of conditional probability,

$$P(\bar{c} \mid a, \bar{d}) = \frac{P(\bar{c} \,\&\, a \,\&\, \bar{d})}{P(a \,\&\, \bar{d})}$$

$$P(\bar{c}, a, \bar{d}) = \sum_{B,E} P(a, b_i, \bar{c}, \bar{d}, e_i)$$

$$= P(a, b, \bar{c}, \bar{d}, e) + P(a, \bar{b}, \bar{c}, \bar{d}, e)$$
$$P(a, b, \bar{c}, \bar{d}, \bar{e}) + P(a, \bar{b}, \bar{c}, \bar{d}, \bar{e})$$

$$P(a, b, \bar{c}, \bar{d}, e) = P(a) \times P(b \mid a) \times P(\bar{c} \mid b, a) \times P(e) \times P(\bar{d} \mid \bar{c}, e)$$
$$= 0.3 \times 0.1 \times 0.95 \times 0.35 \times 0.25$$
$$= \cancel{0.001312}$$
$$= 0.002494$$

$$P(a, \bar{b}, \bar{c}, \bar{d}, e) = P(a) \times P(\bar{b} \mid a) \times P(\bar{c} \mid \bar{b}, a) \times P(e) \times P(\bar{d} \mid \bar{c}, e)$$
$$= 0.3 \times 0.9 \times 0.5 \times 0.35 \times 0.25$$
$$= 0.01181$$

$$P(a, b, \bar{c}, \bar{d}, \bar{e}) = \cancel{P(a) \times P(\bar{c})}$$
$$P(a) \times P(b \mid a) \times P(\bar{c} \mid b, a) \times P(\bar{e}) \times P(\bar{d} \mid \bar{c}, \bar{e})$$
$$= 0.3 \times 0.1 \times 0.95 \times 0.65 \times 0.69$$
$$= 0.01278$$

$$P(a, \bar{b}, \bar{c}, \bar{d}, \bar{e}) = P(a) \times P(\bar{b} \mid a) \times P(\bar{c} \mid \bar{b}, a) \times P(\bar{e}) \times P(\bar{d} \mid \bar{c}, \bar{e})$$
$$= 0.3 \times 0.9 \times 0.5 \times 0.65 \times 0.69$$
$$= 0.06055$$

$$P(\bar{c}, a, \bar{d}) = 0.002494 + 0.01181 + 0.01278 + 0.06055$$
$$P(\bar{c}, a, \bar{d}) = 0.087634$$

$$P(a, \bar{d}) = \sum_{B,C,E} P(a, b_i, c_i, \bar{d}, e_i)$$

$$= P(a, b, c, \bar{d}, e) + P(a, \bar{b}, c, \bar{d}, e)$$
$$+ P(a, b, \bar{c}, \bar{d}, e) + P(a, \bar{b}, \bar{c}, \bar{d}, e)$$
$$+ P(a, b, c, \bar{d}, \bar{e}) + P(a, \bar{b}, c, \bar{d}, \bar{e})$$
$$+ P(a, b, \bar{c}, \bar{d}, \bar{e}) + P(a, \bar{b}, \bar{c}, \bar{d}, \bar{e})$$

All the above probabilities are computed above and also in question (iii), hence substituting, we get

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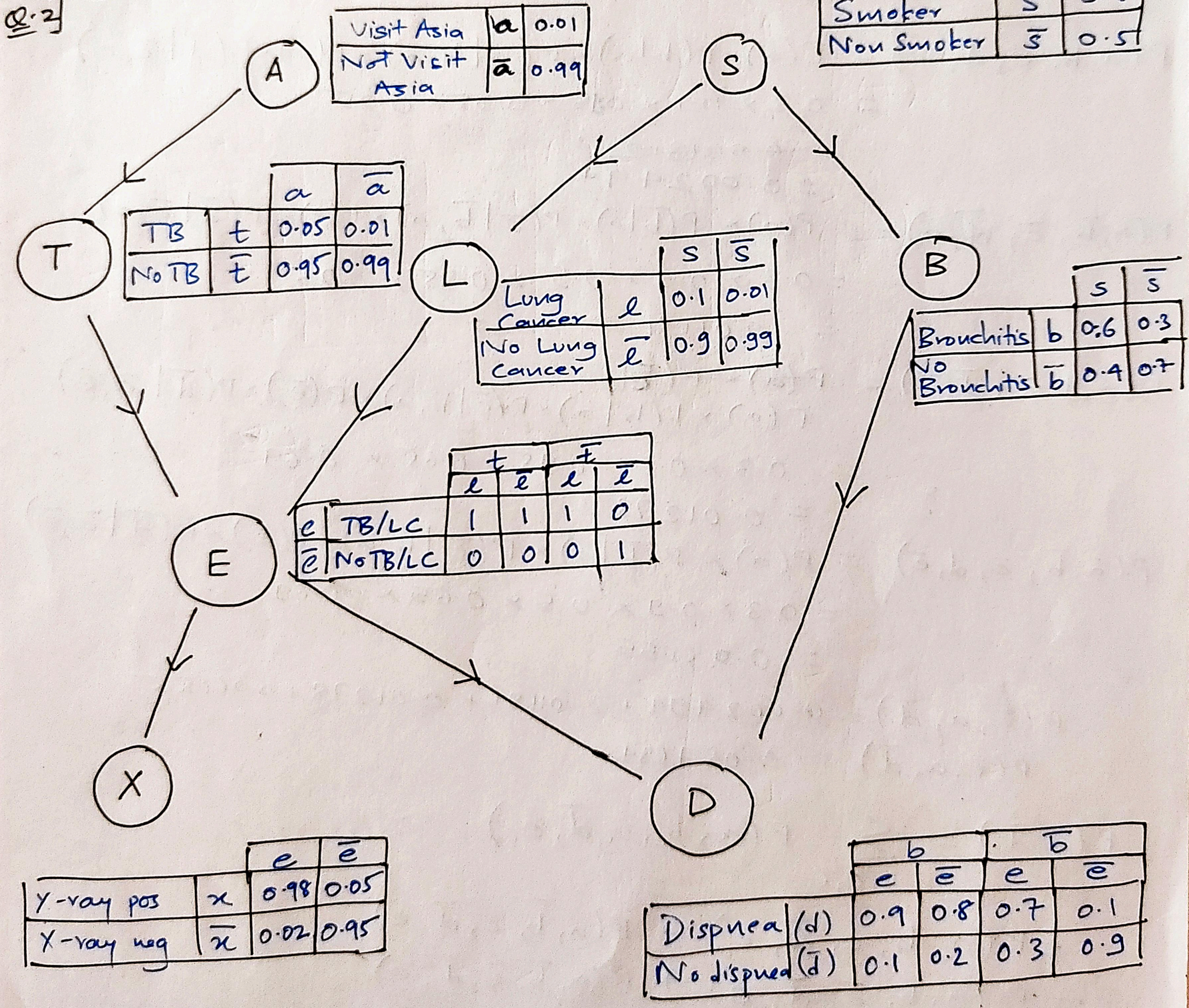
$$P(a,\bar{d}) = 5.1975\times10^{-4} + 4.678\times10^{-2} + 2.494\times10^{-3} + 1.181\times10^{-2} + 4.875\times10^{-4} + 4.3875\times10^{-2} + 1.278\times10^{-2} + 6.055\times10^{-2}$$

$$\therefore P(a,\bar{d}) = 0.1793$$

$$\boxed{\therefore P(\bar{e}\mid a,\bar{d}) = \frac{0.087634}{0.1793} = 0.4887}$$

A) Probability of not defaulting in payment (i.e. payback) given high income and no security is 0.4887

Q.2




| | | |
|---|---|---|
| Visit Asia | a | 0.01 |
| Not Visit Asia | $\bar{a}$ | 0.99 |

| | | |
|---|---|---|
| Smoker | s | 0.5 |
| Non Smoker | $\bar{s}$ | 0.5 |

| | | a | $\bar{a}$ |
|---|---|---|---|
| TB | t | 0.05 | 0.01 |
| No TB | $\bar{t}$ | 0.95 | 0.99 |

| | | s | $\bar{s}$ |
|---|---|---|---|
| Lung Cancer | l | 0.1 | 0.01 |
| No Lung Cancer | $\bar{l}$ | 0.9 | 0.99 |

| | | s | $\bar{s}$ |
|---|---|---|---|
| Bronchitis | b | 0.6 | 0.3 |
| No Bronchitis | $\bar{b}$ | 0.4 | 0.7 |

| | | t | | $\bar{t}$ | |
|---|---|---|---|---|---|
| | | l | $\bar{l}$ | l | $\bar{l}$ |
| e | TB/LC | 1 | 1 | 1 | 0 |
| $\bar{e}$ | NoTB/LC | 0 | 0 | 0 | 1 |

| | | e | $\bar{e}$ |
|---|---|---|---|
| Y-ray pos | x | 0.98 | 0.05 |
| X-ray neg | $\bar{x}$ | 0.02 | 0.95 |

| | b | | $\bar{b}$ | |
|---|---|---|---|---|
| | e | $\bar{e}$ | e | $\bar{e}$ |
| Dispnea (d) | 0.9 | 0.8 | 0.7 | 0.1 |
| No dispnea ($\bar{d}$) | 0.1 | 0.2 | 0.3 | 0.9 |

Q.2. (i) Lung cancer & bronchitis have a converging path at dispnea through E (Either LC or TB) 

So, my knowledge of lung cancer can influence my belif on bronchitis if I am fed additional info about dispnea.

As per the graph hence,

---

(ii) Having tuberculosis → t

Visiting Asia → a

Having lung cancer → l

⇒ Either lung cancer/TB → e

Positive X-ray → x

⇒ To find: $P(t \mid a, l, e, x)$

$$P(t \mid a, l, e, x) = \frac{P(t, a, l, e, x)}{P(a, l, e, x)}$$

$$P(t, a, l, e, x) = \sum_{S,B,D} P(t, a, l, e, x, s_i, b_i, d_i)$$

$$= P(t, a, l, e, x, s, b, d) + P(t, a, l, e, x, \bar{s}, b, d)$$
$$P(t, a, l, e, x, s, \bar{b}, d) + P(t, a, l, e, x, \bar{s}, \bar{b}, d)$$
$$P(t, a, l, e, x, s, b, \bar{d}) + P(t, a, l, e, x, \bar{s}, b, \bar{d})$$
$$P(t, a, l, e, x, s, \bar{b}, \bar{d}) + P(t, a, l, e, x, \bar{s}, \bar{b}, \bar{d})$$

Factorization of Distribution

$P(T, A, L, E, X, S, B, D)$ or

$$P(A, S, B, T, L, E, X, D)$$
$$= P(A) \cdot P(S \mid A) \cdot P(B \mid S, A) \cdot P(L \mid A, B, S) \cdot P(T \mid A, B, L, S) \cdot P(E \mid A, B, L, S, T)$$
$$\cdot P(X \mid A, B, E, L, S, T) \cdot P(D \mid A, B, E, L, S, T, X)$$
$$= P(A) \cdot P(S) \cdot P(B \mid S) \cdot P(L \mid S) \cdot P(T \mid A) \cdot P(E \mid T, L) \cdot P(X \mid E) \cdot P(D \mid E, B)$$

—(From the network, factorizing based on JPDF $= \prod P(X \mid Pa(X))$)

$$\therefore P(t, a, l, e, x, s, b, d) = 0.01 \times 0.5 \times 0.6 \times 0.1 \times 0.05 \times 1 \times 0.98 \times 0.9$$
$$= 1.323 \times 10^{-5}$$

$$P(t, a, l, e, x, \bar{s}, b, d) = 0.01 \times 0.5 \times 0.3 \times 0.01 \times 0.05 \times 1 \times 0.98 \times 0.9$$
$$= 6.615 \times 10^{-7}$$

$$P(t,a,l,e,x,s,\bar{b},d) = P(a)\,P(s)\,P(\bar{b}|s)\,P(l|s)\,P(t|a)\,P(e|t,l)\,P(x|e)\,P(d|e,\bar{b})$$
$$= 0.01 \times 0.5 \times 0.4 \times 0.1 \times 0.05 \times 1 \times 0.98 \times 0.7$$
$$= 6.86 \times 10^{-6}$$

similarly, we can find remaining terms as

$P(t,a,l,e,x,\bar{s},\bar{b},d) = 1.2\times10^{-6}$ $\quad P(t,a,l,e,x,\bar{s},b,\bar{d}) = 7.35\times10^{-8}$

$P(t,a,l,e,x,s,b,\bar{d}) = 1.47\times10^{-6}$ $\quad P(t,a,l,e,x,s,\bar{b},\bar{d}) = 2.94\times10^{-6}$

$P(t,a,l,e,x,\bar{s},\bar{b},\bar{d}) = 5.145\times10^{-7}$

$$\therefore P(t,a,l,e,x) = 1.323\times10^{-5} + 6.615\times10^{-7} + 6.86\times10^{-6} + 1.2\times10^{-6} + 7.35\times10^{-8} + 1.47\times10^{-6} + 2.94\times10^{-6} + 5.145\times10^{-7}$$
$$= 10^{-6}\times[13.23 + 0.6615 + 6.86 + 1.2 + 0.0735 + 1.47 + 2.94 + 0.5145]$$
$$= 10^{-6} \times 26.945$$

$$\boxed{\therefore P(t,a,l,e,x) = 2.694\times10^{-6}}$$

$$P(a,l,e,x) = \sum_{T,S,B,D} P(a,l,e,x,t_i,s_i,b_i,d_i)$$

$= P(a,l,e,x,t,s,b,d) + P(a,l,e,x,\bar{t},s,b,d)$
$P(a,l,e,x,t,s,b,\bar{d}) + P(a,l,e,x,\bar{t},s,b,\bar{d})$
$P(a,l,e,x,t,s,\bar{b},d) + P(a,l,e,x,\bar{t},s,\bar{b},d)$
$P(a,l,e,x,t,s,\bar{b},\bar{d}) + P(a,l,e,x,\bar{t},s,\bar{b},\bar{d})$
$P(a,l,e,x,t,\bar{s},b,d) + P(a,l,e,x,\bar{t},\bar{s},b,d)$
$P(a,l,e,x,t,\bar{s},b,\bar{d}) + P(a,l,e,x,\bar{t},\bar{s},b,\bar{d})$
$P(a,l,e,x,t,\bar{s},\bar{b},d) + P(a,l,e,x,\bar{t},\bar{s},\bar{b},d)$
$P(a,l,e,x,t,\bar{s},\bar{b},\bar{d}) + P(a,l,e,x,\bar{t},\bar{s},\bar{b},\bar{d})$

Terms on LHS have already been calculated in the previous computation. We need to compute RHS terms now.

$$P(a,l,e,x,\bar{t},s,b,d) = P(a)\times P(s)\times P(b|s)\times P(l|s)\times P(\bar{t}|a) \times P(e|\bar{t},l)\times P(x|e)\times P(d|e,b)$$
$$= 0.01\times0.5\times0.6\times0.1\times0.95\times1\times0.98\times0.9$$
$$= 2.51\times10^{-4}$$

Similarly we could compute all the joint probabilities and eventually, we get

$P(t,a,l,e,x) = 2.694\times10^{-6}$ $\quad P(a,l,e,x) = 5.39\times10^{-4}$

$$\boxed{\therefore P\left(\frac{t}{a,l,e,x}\right) = \frac{2.694\times10^{-6}}{5.39\times10^{-4}} = 0.05}$$

Vinayak Nayak 

(14) What is the probability that you have dispnea given that you have positive X-ray?

Having Dispnea - d; Positive xray - x

∴ To find : $P\left(\frac{d}{x}\right)$ or $P(d|x)$

$$P(d|x) = P(d,x)/P(x)$$

$$P(d,x) = \sum_{A,S,T,L,B,E} P(a_i, t_i, s_i, l_i, b_i, e_i, d, x)$$

This summation contains close to 64 terms since we're marginalizing over 6 binary variables ($2^6 = 64$). Accounting for all those and summing we finally obtain

$$P(d,x) = 0.0707$$

Similarly

$$P(x) = \sum_{A,S,T,L,B,E,D} P(a_i, t_i, s_i, l_i, b_i, e_i, d_i, x)$$

Same as above this expansion contains $2^7 = 128$ terms.

Summing over all these, we eventually obtain

$$P(x) = 0.1103$$

$$\boxed{\therefore P(d|x) = \frac{P(d,x)}{P(x)} = \frac{0.0707}{0.1103} = 0.6407}$$

A) ∴ Probability of having dispnea given positive X-ray is 0.6407

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(15) Find out all the independencies in the given graph.

There are a lot of independencies in the given graph. Some of the striking ones are as follows.

Asia ⊥ Bronchitis, Smoking, Lung Cancer

Asia ⊥ X-ray | Tb/Lc

Asia ⊥ Dispnea | Tb, Lung Cancer, Tb/Lc

Dispnea ⊥ Xray, Asia | Tb/Lc

Tb ⊥ Bronchitis, Smoking, Lung Cancer

~~Tb ⊥ Smoking |~~

Tb ⊥ X-ray | Tb/LC, LC

X-ray ⊥ Smoker | LC, Tb/LC

tb $\perp$ smoking | lung cancer, bronchitis

tb $\perp$ asia | tb or lc

tb $\perp$ x-ray | tb or lc

lung cancer $\perp$ asia | dispnea, smoking, x ray, tb

lung cancer $\perp$ tb | asia, bronchitis, ~~smoking~~

lung cancer $\perp$ tb | asia, tb

lung cancer $\perp$ xray | tb or lc

dispnea $\perp$ asia | bronchitis, x-ray, tb

dispnea $\perp$ smoker | lung cancer, bronchitis, xray

dispnea $\perp$ xray | tb or lc

smoking $\perp$ xray | dispnea, asia, tb or lc

smoking $\perp$ asia | tb

tb or lc $\perp$ asia | tb

tb or lc $\perp$ bronchitis | smoking

bronchitis $\perp$ asia | lung cancer, dispnea, tb

bronchitis $\perp$ tb | asia