# **Topological Data Analysis**

Topological data analysis(TDA) is potential to find better representation of the data.

TDA can visualize the high dimensional data and characterize the intrinsic invariants of the data.

It is close to computational geometry, manifold learning and computational topology.

It is one kind of descriptive representation learning.

As The NIPS 2012 workshop on Algebraic Topology and Machine Learning puts:

Topological methods and machine learning have long enjoyed fruitful interactions as evidenced by popular algorithms

like ISOMAP, LLE and Laplacian Eigenmaps which have been borne out of studying point cloud data through the lens of geometry.

More recently several researchers have been attempting to also understand the algebraic topological properties of data.

Algebraic topology is a branch of mathematics which uses tools from abstract algebra to study and classify topological spaces.

The machine learning community thus far has focused almost exclusively on clustering as the main tool for unsupervised data analysis.

Clustering however only scratches the surface, and algebraic topological methods aim at extracting much richer topological information from data.

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- https://www.wikiwand.com/en/Topological\_data\_analysis
- TDA overview
- Studying the Shape of Data Using Topology
- Topological Data Analysis
- Why TDA works?
- Topology-Based Active Learning
- The NIPS 2012 workshop on Algebraic Topology and Machine Learning.
- Topological Data Analysis Part 4 Persistent Homology
- Topological Methods in Data Analysis and Visualization @springer
- https://jsseely.github.io/notes/TDA/
- Extracting insights from the shape of complex data using topology.
- Applied topology
- WORKSHOP ON TOPOLOGY AND NEUROSCIENCE
- https://www.h-its.org/event/workshop-grg-2018/
- Dragon Applied Topology Conference

- https://icerm.brown.edu/tripods/tri17-1-gtd/
- https://www.ipam.ucla.edu/programs/long-programs/geometry-and-learning-from-data-in-3d-and-beyond/
- http://kurlin.org/index.php#group
- http://chomp.rutgers.edu/
- http://chomp.rutgers.edu/Projects/Topological\_Data\_Analysis.html
- https://www.jstage.jst.go.jp/article/tjsai/32/3/32\_D-G72/\_pdf
- Computational & Algorithmic Topology, Sydney
- Oxford Topology
- https://cs.nyu.edu/~yap/classes/
- Wang Bei
- Computational Topology and Geometry (CompTaG)
- Topological Methods for Machine Learning: An ICML 2014 Workshop in Beijing, China
- · Towards topological machine learning

## **Topology**

Topology focuses on the invariants under continuous mapping.

It pays more attention to the geometrical or discrete properties of the objects such as the number of circles or holes. It is not distance-based.

**Definition**: Let X be a non-empty set. A set  $\tau$  of subsets of X is said to be a **topology** if

- X and the empty set  $\emptyset$  belong to  $\tau$ ;
- the union of any number of sets in  $\tau$  belongs to  $\tau$ ;
- the intersection of any two sets inn  $\tau$  belongs to  $\tau$ . The pair  $(X,\tau)$  is called a **topological space**.

As the definition shows the topology may be really not based on the definition of distance or measure. The set can be countable or discountable.e3

**Definition**: Let  $(X, \tau)$  be a topological space. Then the members of  $\tau$  (the subsets of X) is said to be **open set**. If X-S is open set, we call S as **closed set**.

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- https://www.wikiwand.com/en/Topology
- http://www.topologywithouttears.net/
- https://www.ics.uci.edu/~eppstein/junkyard/topo.html
- http://brickisland.net/DDGSpring2016/2016/01/20/reading-2-topology/
- https://www.ayasdi.com/blog/artificial-intelligence/relationships-geometry-artificial-intelligence/

#### **TDA**

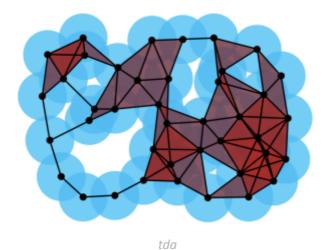
Topological data analysis as one data processing method is selected topic for some students on computer science and applied mathematics.

It is not popular for the statisticians, where there is no estimation and test.

Topological data analysis (TDA) refers to statistical methods that find structure in data. As the name suggests, these methods make use of topological ideas. Often, the term TDA is used narrowly to describe a particular method called **persistent homology**.

TDA, which originates from mathematical topology, is a discipline that studies shape. It's concerned with measuring the shape, by means applying math functions to data, and with representing it in forms of topological networks or combinatorial graphs.

There is another field that deals with the topological and geometric structure of data: computational geometry. The main difference is that in TDA we treat the data as random points, whereas in computational geometry the data are usually seen as fixed.



TDA can be applied to manifold estimation, nonlinear dimension reduction, mode estimation, ridge estimation and persistent homology.

- IDAC TDA Workshop: Topological Data Analysis for Discovery in Multi-scalar Biomedical Data Applications in Musculoskeletal Imaging
- International Workshop on Topological Data Analysis in Biomedicine (TDA-Bio) Seattle, WA, October 2, 2016
- Topological Data Analysis of fMRI data: 11 Apr 2018 by Manish Saggar
- Topological Data Analysis for Genomics and Applications to Cancer
- Topological Data Analysis and Machine Learning for Classifying Atmospheric River Patterns in Large Climate Datasets
- DBI: ABI Innovation: A Scalable Framework for Visual Exploration and Hypotheses Extraction of Phenomics Data using Topological Analytics
- Algebraic topology and neuroscience: a bibliography
- Deep Learning with Topological Signatures Persistent Homology and Machine Learning
- Topological data analysis for imaging and machine learning
- A series of blogs on TDA
- Topological Data Analysis @ Annual Review of Statistics and Its Application

- https://github.com/prokopevaleksey/TDAforCNN
- https://github.com/ognis1205/spark-tda
- https://github.com/stephenhky/PyTDA
- Topological Data Analysis by peterbubenik
- Applied Algebraic Topology Research Network
- Henry Adams interests in computational topology and geometry, combinatorial topology, and applied topology
- Robert Ghrist's research is in applied topology that is, applications of topology to engineering systems, data, dynamics,
   & more
- CSE 5559: Computational Topology and Data Analysis by Tamal K Dey
- CMU TopStat
- Data Visualization with TDA Mapper
- Topological & Functional Data Analysis @ CMU
- Topological Data Analysis: an Overview of the World's Most Promising Data Mining Methodology
- Index of /~beiwang/teaching/cs6170-spring-2017
- Topological Data Analysis: One Applied Mathematician's Heartwarming Story of Struggle, Triumph, and (Ultimately)
   More Struggle By Chad Topaz
- Scalable topological data analysis
- Topology, Computation and Data Analysis
- https://www-apr.lip6.fr/~tierny/topologicalDataAnalysisClass.html
- Topological Data Analysis and Persistent Homology

## **Computational Topology**

Computational topology is the mathematical theoretic foundation of topological data analysis. It is different from the deep neural network that origins from the engineering or the simulation to biological neural network.

Topological data analysis is principle-driven and application-inspired in some sense.

CS 598: Computational Topology Spring 2013 put that

Potential mathematical topics include the topology of <u>cell complexes</u>, topological graph theory, homotopy, covering spaces, simplicial homology, persistent homology, discrete Morse theory, discrete differential geometry, and normal surface theory. Potential computing topics include algorithms for computing topological invariants, graphics and geometry processing, mesh generation, curve and surface reconstruction, VLSI routing, motion planning, manifold learning, clustering, image processing, and combinatorial optimization.

<img src = "http://jeffe.cs.illinois.edu/teaching/comptop/Fig/codex-bugs.png" width = 40% />

- https://datawarrior.wordpress.com/
- http://graphics.stanford.edu/courses/cs468-09-fall/
- https://graphics.stanford.edu/courses/cs468-02-fall/schedule.html

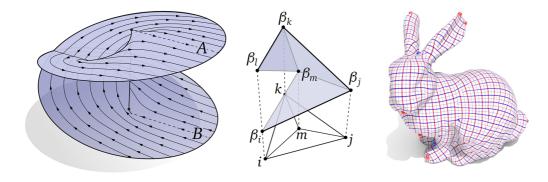
- http://people.maths.ox.ac.uk/nanda/source/RSVWeb.pdf
- https://jeremykun.com/tag/computational-topology/
- http://jeffe.cs.illinois.edu/teaching/comptop/
- http://www.enseignement.polytechnique.fr/informatique/INF556/
- https://topology-tool-kit.github.io/
- https://www.kth.se/student/kurser/kurs/SF2956?l=en
- https://cs.nyu.edu/~yap/classes/modeling/06f/
- https://courses.maths.ox.ac.uk/node/161
- https://www2.cs.duke.edu/courses/fall06/cps296.1/
- Math 574--Introduction to Computational Topology (Spring 2016)
- NSF-CBMS Conference and Software Day on Topological Methods in Machine Learning and Artificial Intelligence: May 13–17 and May 18, 2019. Department of Mathematics, College of Charleston, South Carolina
- Data science and applied topology

<img src = "http://www.math.wsu.edu/faculty/bkrishna/pics/MultipleTunnels.png" width= "20%" />

## **Computational Geometry**

https://shapeofdata.wordpress.com/

- Probabilistic Approach to Geometry
- · Applied Geometry Lab webpage!
- Titane: Geometric Modeling of 3D Environments
- http://www.multires.caltech.edu/research/research.htm
- Geometry in Graphics Group in Computer Science and Engineering@Michigan State University
- Computational Geometry Week (CG Week 2019
- https://drona.csa.iisc.ac.in/~gsat/Course/CGT/
- http://www.computational-geometry.org/
- Handbook of Discrete and Computational Geometry —Third Edition— edited by Jacob E. Goodman, Joseph O'Rourke, and Csaba D. Tóth
- CS171: Discrete Differential Geometry
- http://brickisland.net/DDGSpring2016/2016/01/22/reading-3-topological-data-analysis/
- http://graphics.stanford.edu/courses/cs468-14-winter/



discrete differential geomety

#### Geometric Data Analysis

- GEOMETRIC DATA ANALYSIS, U CHICAGO, MAY 20-23 2019
- Geometric Data Analysis Reading Group
- Foundations of Geometric Methods in Data Analysis
- CS233 Class Schedule for Spring Quarter '17-'18
- MA500 Geometric Foundations of Data Analysis
- Special Session on Geometric Data Analysis
- Workshop Statistics for geometric data and applications to anthropology
- CSIC 5011: Topological and Geometric Data Reduction and Visualization
- 4th conference on Geometric Science of Information

#### **Optimal Transport**



- The geometry of optimal transportation
- Transformations of PDEs: Optimal Transport and Conservation Laws by Woo-Hyun Cook
- Optimal transport, old and new
- Math 3015 (Topics in Optimal Transport). Spring 2010
- https://optimaltransport.github.io/
- http://www.math.ucla.edu/~wgangbo/Cedric-Villani.pdf
- https://pot.readthedocs.io/en/stable/
- Optimal Transport @ESI
- Optimal Transport Methods in Density Functional Theory (19w5035)
- Discrete OT
- https://www.mindcodec.com/an-intuitive-guide-to-optimal-transport-for-machine-learning/
- http://faculty.virginia.edu/rohde/transport/
- http://otml17.marcocuturi.net/
- Optimal Transport & Machine Learning
- Optimal Transport and Machine learning course at DS3 2018
- Hot Topics: Optimal transport and applications to machine learning and statistics
- https://anr.fr/Project-ANR-17-CE23-0012
- http://otnm.lakecomoschool.org/program/
- https://sites.uclouvain.be/socn/drupal/socn/node/113
- Topics on Optimal Transport in Machine Learning and Shape Analysis (OT.ML.SA)
- Optimal Transport in Biomedical Imaging
- Optimal transport for documents classification: Classifying news with Word Mover Distance

Monge-Kantorovich Optimal Transport – Theory and Applications