

Mathematics
for Computer Science Students (Math 403)
Worksheet No. (4)

Problem 1:

Let X be a random variable, such that the probability mass function (PMF) is given by:

X	-1	a	2	4
$f(x)$	0.1	0.25	0.3	b

If $E(X) = 2.15$. **Determine:**

- (i) The values of a and b .
- (ii) $E(2X + 3)$
- (iii) $Var(X)$
- (iv) $Var(2X + 3)$

Problem 2:

In tossing a coin three times. Let X is a random variable represents the number of heads. **Find:**

- (i) The probability mass function (PMF).
- (ii) The cumulative distribution function (CDF).
- (iii) $P(X \geq 1)$
- (iv) $P(X > 0.5)$
- (v) $P(0 \leq X \leq 0.5)$

Problem 3:

Two balls are drawn in succession without replacement from a box containing 4 red balls and 3 black balls. Let X is a random variable represents the number of red balls. **Find:**

- (i) $f(x)$
- (ii) $F(x)$
- (iii) $E(X)$
- (iv) $Var(X)$

Problem 4:

There are frequent volcanic eruptions at a particular site. The times of occurrences are unpredictable. From past observations, the probability mass function of occurrences $f(x)$ over 10 years is given by:

X	0	1	2	3
$f(x)$	0.1	0.3	0.4	0.2

- (i) **Find** the cumulative distribution function $F(x)$.
 - (ii) **Sketch** $f(x)$ and $F(x)$.
 - (iii) **Calculate** $E(X)$ and $Var(X)$.
 - (iv) **Calculate** $E(3X - 4)$ and $Var(3X - 4)$.
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Problem 5:

A contractor is submitting to do three jobs, A , B and C . The probabilities that he will do each of the three jobs $P(A) = 0.5$, $P(B) = 0.8$ and $P(C) = 0.2$, respectively. Assume that events A , B and C are statistically independent. Let X be the total number of jobs the contractor will do.

- (i) **Compute** and plot the probability mass function (PMF) of the random variable X .
 - (ii) **Compute** and plot the probability distribution function (CDF) of the random variable X .
 - (iii) **Determine** $P(X \leq 2)$.
 - (iv) **Determine** $P(0 < X \leq 2)$.
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Problem 6:

A random variable X has the following probability mass function:

X	0	1	2	3	4	5
$f(x)$	$\frac{-k+1}{10}$	$-\left(\frac{3k+2}{10}\right)$	$-\left(\frac{3k+2k^2}{10}\right)$	$\frac{k^2}{10}$	$\frac{3k^2-k}{10}$	$-\left(\frac{k}{10}\right)$

Find:

- (i) The value of k .
- (ii) $P(X > 2)$, $P(X \leq 2)$ and $P(1 < X \leq 5)$.
- (iii) $E(X)$ and $Var(X)$.

Problem 7:

If the discrete random variable X has the following CDF:

$$F(x) = \begin{cases} 0, & x < -1 \\ 1/4, & -1 \leq x < 1 \\ 1/2, & 1 \leq x < 3 \\ 3/4, & 3 \leq x < 5 \\ 1, & x \geq 5 \end{cases}$$

Find:

- (i) $f(x)$
 - (ii) $P(-0.4 < X < 3.6)$
 - (iii) $P(X = 5)$
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Problem 8:

If the discrete random variable X has the following CDF:

$$F(x) = \begin{cases} 0, & x < -3 \\ 1/25, & -3 \leq x < -2 \\ 3/25, & -2 \leq x < -1 \\ 7/25, & -1 \leq x < 0 \\ 18/25, & 0 \leq x < 1 \\ 22/25, & 1 \leq x < 2 \\ 24/25, & 2 \leq x < 3 \\ 1, & x \geq 3 \end{cases}$$

Find:

- (i) $f(x)$
- (ii) $P(-1.5 \leq X \leq 2.2)$
- (iii) $E(X + 4)$ and $Var(-2X + 1)$.