**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer :

B

The average time increase from 45min to 55min. because the work begins after 10 minutes.

for a normal distribution :-

z = (value-mean)/sd

= (60-55)/8

= 0.625

**1-stats.norm.cdf(60,loc=55,scale=8)**

=0.2659

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answers:

1. **FALSE**

z = (value-mean)/sd

=(44-38)/6

=1.33

 People above 44 age = 100 - 84.13 =  15.87%   63 out of 400

For 38 employees

=(38-38)/6

=0 =50%

Hence age between 38 & 44   = 84.13 - 50 = 34.13 %  137 out of 400

Hence More employees at the processing center are older than 44 than between 38 and 44 .so it is F**ALSE**

**B.**

**TRUE**

Z score for 30

= (30 - 38)/6

=  -1.33  =  9.15  %   36 out of 400

A training program for employees under the age of 30 at the center would be expected to attract about 36 employees, hence it is  **TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answers:

As we know that *X*~ *N*(μ1, σ1^2) and *Y*~ *N*(μ2, σ2^2) are two independent random variables

then X+Y~N(μ1+ μ2, σ1^2 + σ2^2) and X-Y~N(μ1- μ2, σ1^2 + σ2^2)

Similarly if Z = aX + bY , where X and Y are as defined above ,

i.e z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

Therefore from question 2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answers :

As we need to find values, *a* and *b*,

The probability of random variable taking a value between them is 0.99,

The Probability of getting value between a and b should be 0.99.

The Probability outside the a and b area is 0.01 (ie. 1-0.99). The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2). So since we have the probabilities of a and b, we need to calculate X.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Answers:

A:

Total profit of the company =( Profit 1 + Profit 2 ) ~N((5+7),(3 2 +4 )) =N(12,25)

Here we want to find Rupee range such that it contains 95% probability for the annual profit of the company.

Total area under the normal curve is 1.

Therefore excluded area is 1-0.95 = 0.05 excluded area is 0.025 in each of the left and right tails of the normal curve.

Thus, we want to find the 2.5 th and 97.5 th percentiles.

To find the 2.5 th percentile value of profit we have to find Z .025 =-1.96 To find the 97.5 th percentile value of profit

we have to find Z .975 =1.96 ,

Therefore lower rupee range = X= (-1.96\*5 )+12=2.2$ =2.2\*45=99 rs upper rupee range = X= (1.96\*5 )+12 =21.8 $ =21.8\*45 =981 rs

B:

To find the 5th percentile value of profit we have to find

Z alpha =Z 0.05 = - 1.644584 , Therefore 5th percentile value of

profit =( -1.644584 \* 5) +12 =3.777 $ =3.777\*45

=170 rs

C:

Division 2 Probability of making loss means that there is a 0 profit.

Z value for first division is

Z= = -5/3 = -1.667 Z value for Second division is Z= = -7/4 = -1.75

Since second division has the least value of Z, it has smaller probability of making loss