① solve the following recurrence relation

ⓐ $x(n) = x(n-1) + 5$ for $n > 1$ with $x(1) = 0$

sol ① write down the first two terms to identify the pattern

$$x(1) = 0$$
$$x(2) = x(1) + 5 = 5$$
$$x(3) = x(2) + 5 = 10$$
$$x(4) = x(3) + 5 = 15$$

② Identify the pattern (or) the general term

→ The first term $x(1) = 0$

The common difference $d = 5$

The general formula for the nth term of an AP is

$$x(n) = x(1) + (n-1)\, d$$

substituting the given values

$$x(n) = 0 + (n-1) \cdot 5 = 5(n-1)$$

The solution is

$$x(n) = 5(n-1)$$

ⓑ $x(n) = 3x(n-1)$. for $n > 1$ with $x(1) = 4$

① write down the first two terms to identify the pattern

$$x(1) = 4$$
$$x(2) = 3x(1) = 3 \cdot 4 = 12$$
$$x(3) = 3x(2) = 36$$
$$x(4) = 3x(3) = 10 \cdot 8$$

② Identify the general terms

The first term $x(1) = 4$

The common ratio $r = 3$

The general formula for the n^{th} term of a gp is

$$x(n) = x(1) \cdot r^{n-1}$$

substuting the given values

$$x(n) = 4 \cdot 3^{n-1}$$

The solution is $x(n) = 4 \cdot 3^{n-1}$

ⓒ $x(n) = x(n/2) + n$ for $n>1$ with $x(1)=1$ (solve for $n=2^n$)

For $n=2^k$, we can write recurrence in terms of k

① substitute $n=2k$ in the recurrence

$$x(2^k) = x(2^{k-1}) + 2^k$$

② write down the first few terms to identify the pattern

$$x(1) = 1$$

$$x(2) = x(2^1) = x(1) + 2 = 1+2=3$$

$$x(4) = x(2^2) = x(2) + 4 = 3+4=7$$

$$x(8) = x(2^3) = x(4) + 8 = 7+8=15$$

③ Identify the general term by finding the pattern we observe that :-

$$x(2^k) = x(2^{k-1}) + 2^k$$

we sum the series!

$$x(2^k) = 2^k + 2^{k-1} + 2^{k+2} + - -$$

since $a(1)=1$:

$$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + - -$$

The geometric series with the term $a=2$ and the last term 2^k except for the additional +1 term

The sum of geometric series is with ratio $r=2$ is given by

$$S = \frac{ar^n - 1}{r-1}$$

Hor $a=2, r=2$ and $n=k$

$$S = 2 \cdot \frac{2^k - 1}{2-1} = 2(2^k - 1) = 2^{k+1}$$

Adding The +1 term

$$x(2^k) = 2^{k+1} - 2 + 1 = 2^{k+1} - 1$$

solution is

$$x(2^k) = 2^{k+1} - 1$$

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(4) $x(n) = x(n/3)+1$ for $n>1$ with $(x(1)=1$ (solve for $n=3^k$)

for $n=3^k$ we can write the recurence in terms of k

(1) substitute $n=3^k$ in the recurrence

(2) write down the first few terms to identify the pattern

$x(1)=1$

$x(3)=x(3^1) = x(1)+1=1+1=2$

$x(9)=x(3^2) = x(3)+1 = 2+1=3$

$x(27) = x(3^3) = x(9)+1 = 3+1=4$

(3) Identify the general term!

we observe that:

$x(3^k) = x(3^{k-1})+1$

summing up the series

$x(3^k) = 1+1+1+\cdots+1$

$x(3^k) = k+1$

The solution is $x(3^k) = k+1$

(2) Evaluate the following recurrences complexity

(i) $T(n) = T(n/2)+1$ where $n=2^k$ for all $k\geq 0$

The recurrence relation can be solved using iteration method

(1) substitute $n=2^k$ in the recurrence

(2) iterate the recurrence

for $k=0$: $T(2^0) = T(1) = T(1)$

$k=1$: $T(2^1) = T(1)+1$

$k=2$ $= T(2^2) = T(4) = T(n)+1 = T(1)+2)+1 = T(1)+2$

$k=3$: $T(2^3) = T(8) = T(n)+1 = T(1)+2)+1 = T(1)+3$

(3) generalize the pattern

$T(2^k) = T(1)+k$

since $n=2^k$, $k=\log_2 n$

$T(n) = T(2^k) = T(1)+\log_2 n$

(i) Assume $T(1)$ is a constant c.

$T(n) = c + \log_2 n$

The solution is $T(n) = O(\log n)$

(ii) $T(n) = T(n/3) + T(\frac{2n}{3}) + n$ (where c is constant and n is input size.

The recurence can be solved using the master's theorem for divide and conquer recurrence of the form

$T(n) = aT(n/b) + f(n)$

where $a = 2, b = 3$ and $f(n) = cn$

let's determine the value of $\log_b a$

$\log_b a = \log_3 2$

using the properties of logarithms

$\log_3 2 = \frac{\log 2}{\log 3}$

Now we compare $f(n) = cn$ with $n^{\log_3 2}$:

$f(n) = O(n)$

$n = n^1$

since $\log_3 2$ we are in the third case of master's theorem

$f(n) = O(n^c)$ with $c > \log_b a$

The solution is

$T(n) = \Theta(f(n)) = O(cn) = O(n)$

(3) consider the following recurrence algorithm:

```
min [A(0 --- n-2)]
  if n=1 return A[0]
  else  temp= min1[A[0 --- n-2])
        if temp <= A [n-1] return temp
   else
        Return A(n-1]
```

(a) what is this algorithm compute.

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Sol: The given algorithm min [A[0 ... n-1]] computes the minimum value in the array 'A' from index 0 for n-1: it does this by recursively finding the minimum value in the sub array A[0 ... n-2] and then comparing it with the last element A[n-1] to determine the overall maximum value

(b) setup a recurrence relation for the algorithm basic operation count and solve it

The solution is $T(n) = n$

This means the algorithm performs n basic operations for an input array of size n

(4) Analyze the order of growth

(i) $f(n) = 2^n + 5$ and $g(n) = 7n$ use the $\Omega(g(n))$ notation

To analyze the order of growth and use the Ω notation, we need to compare the given function f(n) and g(n)

given functions:

$$f(n) = 2n^2 + 5$$
$$g(n) = 7n$$

order of growth using $\Omega(g(n))$ notation:

The notation $\Omega(g(n))$ describes a lower bound on the growth rate that for sufficiency large n, f(n) grows at least as for as g(n)

$$f(n) \geq c \cdot g(n)$$

let's analyze $f(n) = 2n^2 + 5$ with respect to $g(n) = 7n$

(1) Identify Dominant terms

The dominant terms in f(n) is $2n^2$ since it grows faster than the constant terms as n increases

The dominant term in g(n) is 7n

(2) establish the inequality

we want to find constants c and n_0 such that:

$$2n^2 + 5 \geq c \cdot 7n \quad \text{for all } n \geq n_0$$

③ simplify the inequality.

ignore the lower order term 5 for larger

$$2n^2 \geq 7cn$$

Divide both sides by n

$$2n \geq 7c$$

solve for n:

$$n \geq 7c/2$$

④ choose constants

let $c = 1$

$$n \geq \frac{7 \cdot 1}{2} = 3.5$$

∴ for $n \geq n$, The inequality holds:

$$2n^2+5 \geq 7n \text{ for all } n \geq n$$

we have shown that there exist constants $c=1$ and $n_0 = n$ such that for all $n \geq n_0$:-

$$2n^2+5 \geq 7n$$

Thus we can conclude that!

$$f(n) = 2n^2+5 = \Omega(7n)$$

in Ω notation the dominant term $2n^2$ in $f(n)$ clearly grows faster than $f(n)$. Hence

$$f(n) = \Omega(n^2)$$

However, for the specific comparision asked $f(n) = \Omega(7n)$ is also correct

showing that $f(n)$ grows at least as fast as $7n$