

- ① Given an array of  $\{4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9\}$  integers find the maximum and minimum product that can be obtained by multiplying two integers from the array

Sol: array is  $\{4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9\}$   
we need to consider the largest and smallest products that can be formed by selecting two numbers from the array

① Sort the array:

sorted array

$[-9, -8, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]$

② Identify possible candidates for maximum product

③ Identify possible candidates for minimum product

calculating maximum product:

• The two largest positive numbers are 10 and 11

$$-9 \times -8 = 72 \quad 10 \times 11 = 110$$

• The two smallest negative numbers are -9 and -8

$$-9 \times -8 = 72$$

The maximum product is 110

calculating minimum product:

The largest positive and negative number is 11 and -9

$$11 \times -9 = -99$$

The smallest negative numbers are

$$-9 \times -8 = 72$$

-99 is smaller than 72 so.

maximum product = 110 and minimum product = -99

② Demonstrate the Binary Search method to search for the key = 23 from the array = {2, 5, 8, 12, 16, 23, 38, 56, 72, 91}

Sol: Given key = 23 and array = {2, 5, 8, 12, 16, 23, 38, 56, 72, 91}

1. Initialize pointers

low = 0 and high = 9

$$\text{calculate mid} = \left\lfloor \frac{\text{low} + \text{high}}{2} \right\rfloor = \left\lfloor \frac{0 + 9}{2} \right\rfloor = 4$$

compare arr[mid] with key:

$$\text{arr}[4] = 16$$

since  $16 < 23$  update low = mid + 1 = 5

$$\text{calculate mid} = \left\lfloor \frac{\text{low} + \text{high}}{2} \right\rfloor = \left\lfloor \frac{5 + 9}{2} \right\rfloor = 7$$

compare arr[mid] with key

$$\text{arr}[7] = 56$$

since  $56 > 23$  update high = mid - 1 = 6

$$\text{mid} = \left\lfloor \frac{5 + 6}{2} \right\rfloor = 5$$

$$\text{arr}[\text{mid}] = \text{arr}[5] = 23$$

$$23 == 23$$

The key is found at index 5

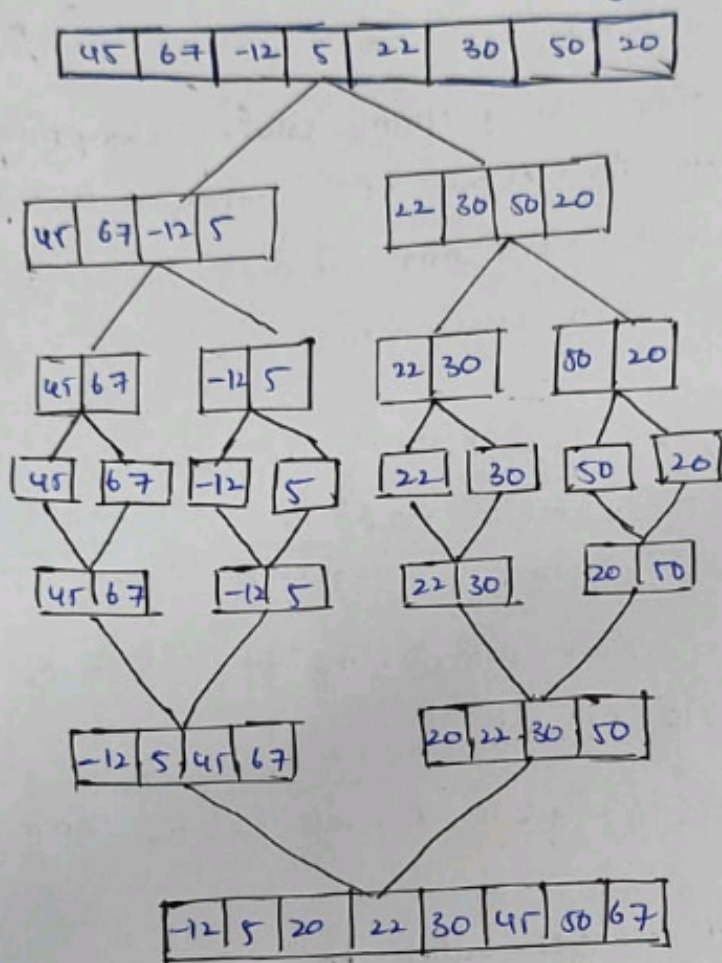
∴ The key = 23 is found at index 5.

③ Apply merge sort and other list of 8 elements, data d = [45, 16, 7, 12, 5, 22, 30, 50, 70]. set up a recurrence relation for the number of key comparisons made by merge sort



sol: Merge Sort.

given  $d = [45, 67, -12, 5, 22, 30, 50, 20]$



$\therefore$  The sorted list =  $[-12, 5, 20, 22, 30, 45, 50, 67]$

- ④ Find the no. of times to perform swapping for selection sort-  
also estimate the time.  
Recursive relation for comparison:

$$T(n) = 2 + (n/2) + O(n) \text{ if } n \geq 1, T(1) = 0$$

-At each level of recursion we make at most  $n-1$  comparisons  
to merge two levels of size  $n$  so it becomes

$$T(n) = 2 + (n/2) + (n-1)$$

solving recurrence relation we get

$$T(n) = n \log_2(n) - n + 1$$

The recurrence relation is  $T(n) = 2T(n/2) + O(n)$  or more precisely

$$T(n) = n \log_2(n) + n + 1$$

Find the no. of times to perform solving swapping for selection sort also estimate the time of notation set  $S = \{12, 7, 5, -2, 18, 11, 13, 14\}$

Sol: The selection sort algorithm always moves exactly  $n-1$  swaps in the worst case, where  $n$  is the no. of elements in the list

given  $S = \{12, 7, 5, -2, 18, 11, 13, 14\}$

No. of elements  $n = 8$

$$\text{No. of swaps} = n - 1 = 8 - 1 = 7$$

Time complexity: The time complexity of selection sort in Big-O notation is  $O(n^2)$

So, the number of swaps is 7 and the time complexity is  $O(n^2)$

⑤ Find the index of target value to using binary search from following list of elements  $[2, 4, 6, 8, 10, 12, 14, 16, 18, 20]$

Sol: given list  $= [2, 4, 6, 8, 10, 12, 14, 16, 18, 20]$  and value  $= 10$

low  $= 0$  and high  $= 9$

$$\text{mid} = \frac{\text{low} + \text{high}}{2} = \frac{0 + 9}{2} = 4$$

mid  $= 10$       mid  $= \text{value}$

since  $10 = 10$  the target is found at index 4

$\therefore$  The target value 10 is found at index 4