⑥ Big Omega Notation prove that $g(n) = n^3+2n^2+4n$ is $\Omega(n^3)$

Sol: $g(n) \geq c \cdot n^3$

$g(n) = n^3+2n^2+4n$

for finding constants c and n_0

$n^3+2n^2+4n \geq c \cdot n^3$

Divide both sides with n^3

$1+\frac{2n^2}{n^3}+\frac{4n}{n^3} \geq c$

$1+\frac{2}{n}+\frac{4}{n^2} \geq c$

Here $\frac{2}{n}$ and $\frac{4}{n^2}$ approaches 0

$1+\frac{2}{n}+\frac{4}{n^2}$

Example. $c = 1/2$

$1+\frac{2}{n}+\frac{4}{n^2} \geq 1/2$

$1+\frac{2}{n}+\frac{4}{n^2} \geq 1 \quad (1 \geq 1/2,\ n \geq 1)$

$1+\frac{2}{n}+\frac{4}{n^2} \geq 1/2 \quad (n \geq 1, n_0 = 1)$

Thus, $g(n) = n^3+2n^2+4n$ is indeded $\Omega(n^3)$

⑦ Big Theta Notation: Determine whether $h(n) = 4n^2+3n$ is $\Theta(n^2)$ or not.

Sol: $c_1 \cdot n^2 \leq h(n) \leq c_2 n^2$

In upper bound $h(n)$ is $O(n^2)$

In lower bound $h(n)$ is $\Omega(n^2)$

upper bound $(O(n^2))$:

$h(n) = 4n^2+3n$

$h(n) \leq c_2 n^2$

$4n^2+3n \leq c_2 n^2$

$4n^2+3n \leq 5n^2$

let $c_2 = 5$

Divide both sides by n^2

$$4 + 3/n \leq 5$$

$h(n) = 4n^2 + 3n$ is $O(n^2)$ $(c_2 = 5, n_0 \geq 1)$

<u>lower bound</u>:

$$h(n) = 4n^2 + 3n$$

$$h(n) \geq c_1 n^2$$

$$4n^2 + 3n \geq c_1 n^2$$

let's $c_1 = 4 \Rightarrow 4n^2 + 3n \geq 4n^2$

Divide both sides by n^2

$$4 + \frac{3}{n} \geq 4$$

$h(n) = 4n^2 + 3n$ $(c_1 = 4, n_0 \geq 1)$

$h(n) = 4n^2 + 3n$ is $\Theta(n^2)$

8. let $f(n) = n^3 - 2n^2 + n$ and $g(n) - n^2$ show whether $f(n) = \Omega(g(n))$ is true or false and justify your answer.

<u>Sol:</u> $f(n) \geq c g(n)$

substituting $f(n)$ and $g(n)$ into this inequality we get

$$n^3 - 2n^2 + n \geq c(-n^2)$$

find c and n_0 holds $n \geq n_0$

$$n^3 - 2n^2 + n \geq -cn^2$$

$$n^3 - 2n^2 + n \geq c \cdot n^2 \geq 0$$

$$n^3 + (c-2)n^2 + n \geq 0$$

$$n^3 + (c-2)n^2 + n \geq 0 \quad (n^3 \geq 0)$$

$$n^3 + (1-2)n^2 + n = n^3 - n^2 + n \geq 0 \quad (c = 2)$$

$f(n) = n^3 - 2n^2 + n$ is $\Omega(g(n)) = \Omega(-n^2)$

There fore the statement $f(n) = \Omega(g(n))$ is true.

(9) Determine whether $h(n) = n\log n + n$ is $\theta(n\log n)$ prove a Rigorous proof for your conclusion

Sol: $c_1 \cdot n\log n \leq h(n) \leq c_2 n\log n$

upper bound:

$h(n) \leq c_2 n\log n$

$h(n) = n\log n + n$

$n\log n + n \leq c_2 n\log n$

Divide both sides by $n\log n$

$1 + \frac{n}{n\log n} \leq 2$

$1 + \frac{1}{\log n} \leq c_2$ (simplify)

$1 + \frac{1}{\log n} \leq 2$ $(c_2 = 2)$

Then $h(n)$ is $o(n\log n)$ $(c_2 = 2, n_0 = 2)$

lower bound:

$h(n) \geq c_1 \cdot n\log n$

$h(n) = n\log n + n$

$n\log n + n \geq c_1 n\log n$

Divide both sides by $n\log n$

$1 + \frac{n}{n\log n} \geq c_1$

$1 + \frac{1}{\log n} \leq c_1$ (simplify)

$1 + \frac{1}{\log n} \geq 1$ $(c_1 = 1)$

$\frac{1}{\log n} \geq n$ for all $n > 1$

$h(n)$ is $\Omega(n\log n)$ $(c_1 = 1, n_0 = 1)$

$h(n) = n\log n + n$ is $\theta(n\log n)$

(10) Solve the following Recurrence relations and find the order of growth for solutions $T(n) = 4T(n/2)+n^2$, $T(1)=1$

Sol:

$T(n) = 4T(n/2)+n^2$, $T(1)=1$

$T(n) = aT(n/2) + f(n)$

$a=4$, $b=2$, $f(n) = n^2$

applying master Theorem

$$T(n) = aT(n/b) + f(n)$$

$f(n) = O(n^{\log_b^a - \epsilon})$ $\left(\begin{matrix}\epsilon > 0 \\ T(n) = \theta(n^{\log_b^a})\end{matrix}\right)$

$f(n) = \theta(n^{\log_b^a})$, then $T(n) = \theta(n^{\log_b^a} \log n)$

$f(n) = \Omega(n^{\log_b^a + \epsilon})$, then $T(n) = f(n)$

calculating $\log_b^a$:

$\log_b^a = \log_2 4 = 2$

$f(n) = n^2 = \theta(n^2)$

$f(n) = \theta(n^2) = \theta(n^{\log_b^a})$, (case 2)

$t(n) = 4T(n/2) + n^2$

$T(n) = \theta(n^{\log_b^a} \log n) = \theta(n^2 \log n)$

order of growth

$T(n) = 4T(n/2)+n^2$ with $T(1) = 1$ is $\theta(n^2 \log n)$