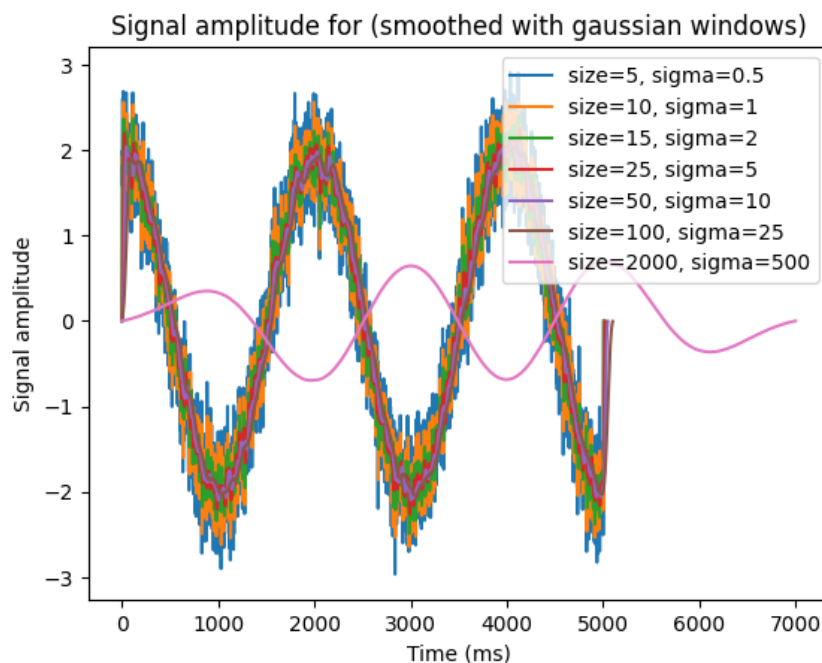


SDA H.W 2**Q1:**

a)

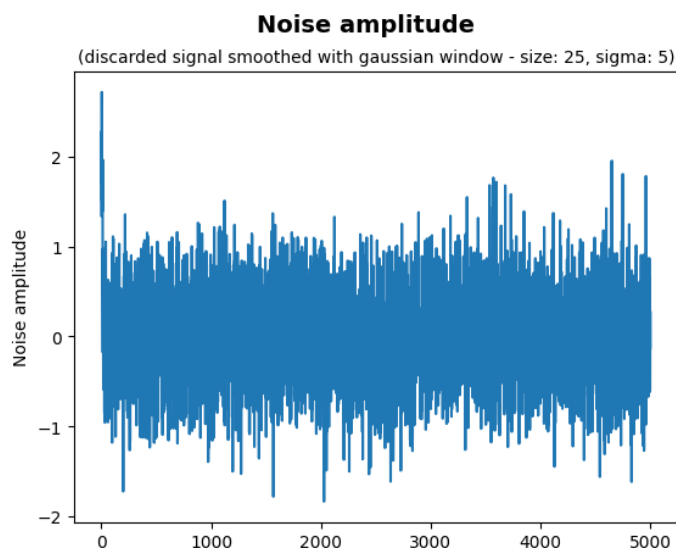


We can see that the smaller windows with smaller sigmas still kept some of the noise, and didn't do a very good job in smoothing out the noise (size,sigma - 5,0.5; 10, 1; 15, 2).

While the larger windows smoothed the signal better we can see slight distortions and shifts of the signal happening starting at window (50,10) due to a long time till the window overlaps fully with the signal such that the convolution result is longer and the peaks are shifted when the largest shift is evident using window (2000, 500) and the amplitude is being greatly attenuated due to a more flat gaussian (std=500) with small weights per time point.

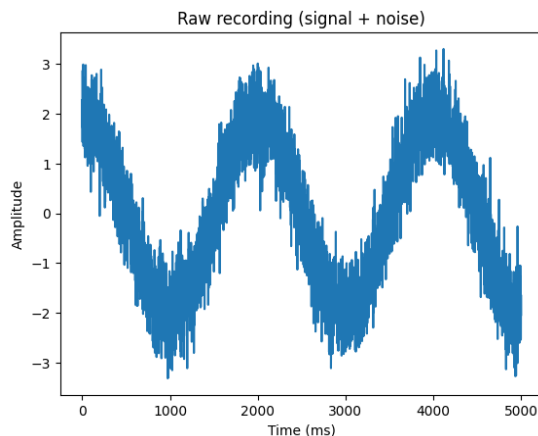
Due to these results, I would recommend using window (25, 5) for signal extraction cut up to index 5000 (signal recording length) due to the longer convolution result ($m+n-1$).

b)



It looks like the noise was extracted well, any oscillatory activity is not evident, otherwise it would indicate some signal residues. Plus we can see that the noise is a classic white noise taken from a random normal distribution (like most noises we encounter in neural recordings) where the mean is around 0 and the range is symmetric around 0: $\sim (-1, 1)$ with constant variance across time.

- c) The SNR is 8.588 dB, $SNR > 0$ which means that the signal power is larger than the noise (due to the log function: $SNR = 10 * \log_{10}(|signal|^2/|noise|^2)$, only $signal > noise$ would give us an $SNR > 0$). This is already evident in the raw recording where we can see that there is noise but still it does not overshadow the signal itself.



- d) Considering the two extremes that the time series contains no noise at all (0 noise at all values) or contains at all values that are non zero noise (1 noise at all non zero values) we can find the SNR boundaries of the time series.

First case (no noise):

The upper bound assuming no noise at all is infinity.

$$SNR = 10 / \log_{10} \frac{|signal|^2}{|noise|^2} \rightarrow \infty$$

Second case (noise at all non zero values):

This case decides the lower bound assuming full noise (besides zero which is known to have no noise, because noise is +1 and not -1, and assuming there is no negative count).

Original recording (signal + noise) = [0, 0, 1, 4, 5, 5, 0, 0, 1, 5, 5, 4, 0, 0, 1, 5, 4, 4].

$\text{Signal} = [0, 0, 0, 3, 4, 4, 0, 0, 0, 4, 4, 3, 0, 0, 0]$
 (without noise)

$$|(\text{signal})^2| = \frac{9 \cdot 0^2 + 4 \cdot 3^2 + 5 \cdot 4^2}{18} = \frac{116}{18} \approx 6.44$$

$\text{noise} = [0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1]$

$$|(\text{noise})^2| = \frac{12 \cdot 1^2}{18} \approx 0.666$$

$$\text{SNR} = 10 \log_{10} \left(\frac{6.44}{0.666} \right) \approx 9.85$$

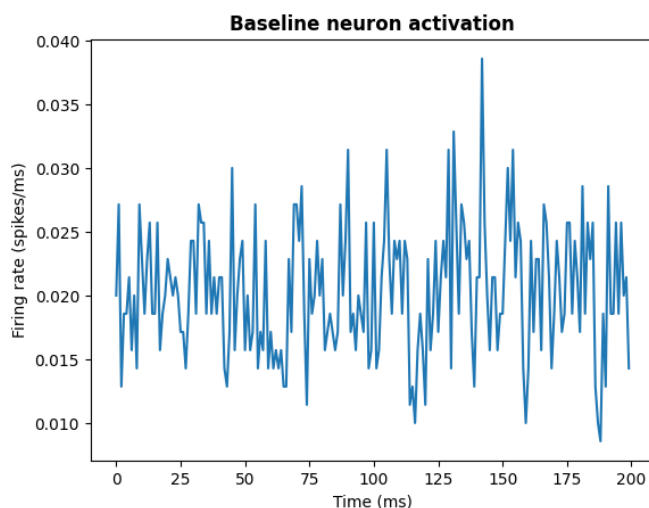
lower bound

Result: SNR > 9.85dB

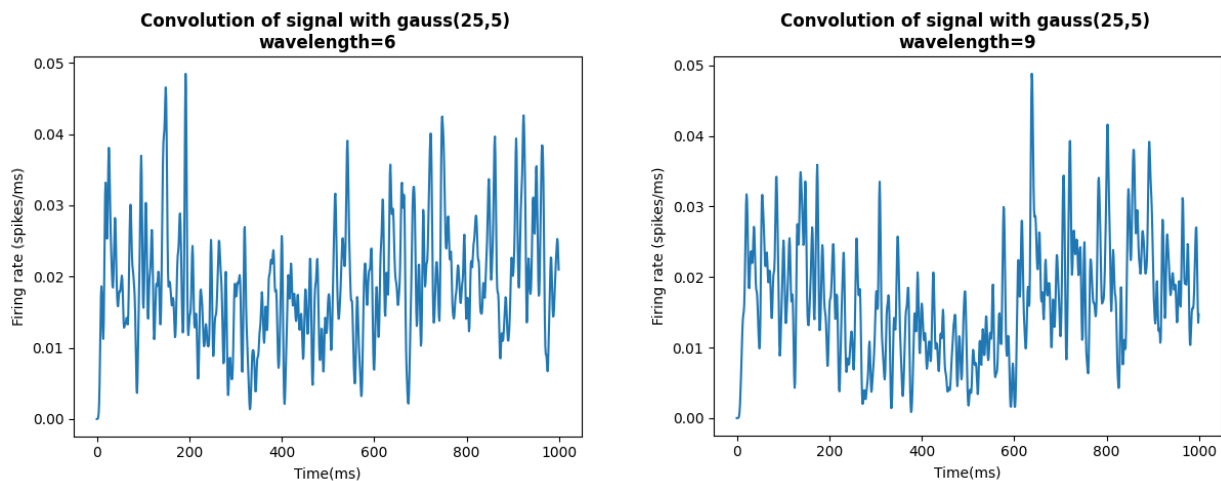
This is a good SNR that indicates that the signal can be averaged out of the recording because even in the assumption of full noise the SNR is larger than 1. Meaning the noise doesn't overshadow the signal.

Q2:

- a) Average firing rate over the whole baseline is 0.0203 spikes/ms. Averaged over all wavelengths and trials for the whole baseline period: 0-200 ms, because we assume that the baseline is stationary and ergodic. Every point in time in the baseline should be taken from the same random distribution (same mean and variance), before a stimulus was shown and affected the process, as seen in the figure below - therefore we can average across time. Additionally assuming that the baseline (prestimulus) activity is the same across all trials/wavelength labels - we'd say the baseline is ergodic and thus we can average across trials/wavelength.



- b) Again assuming that the activity of the signal is ergodic at specific time windows. We can average the trials to get an average estimation over time. In some wave-lengths a decrease in firing rate was evident in 200-600ms (when stimulus was presented), then there is an increase in activity in 600-1000ms (see figures below for example).

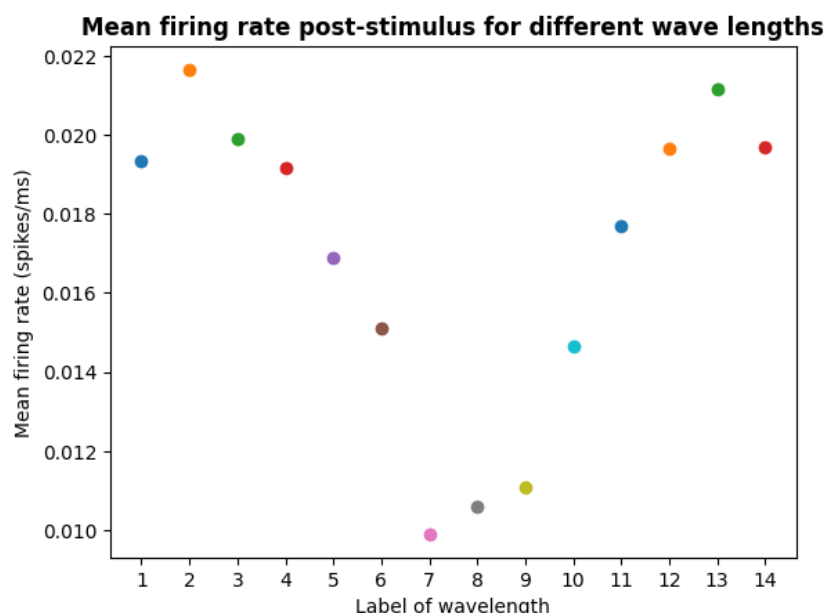


Thus we can't assume stationarity for all post-stimulus activity and we need to cut the post stimulus window to WSS time windows to average across time.

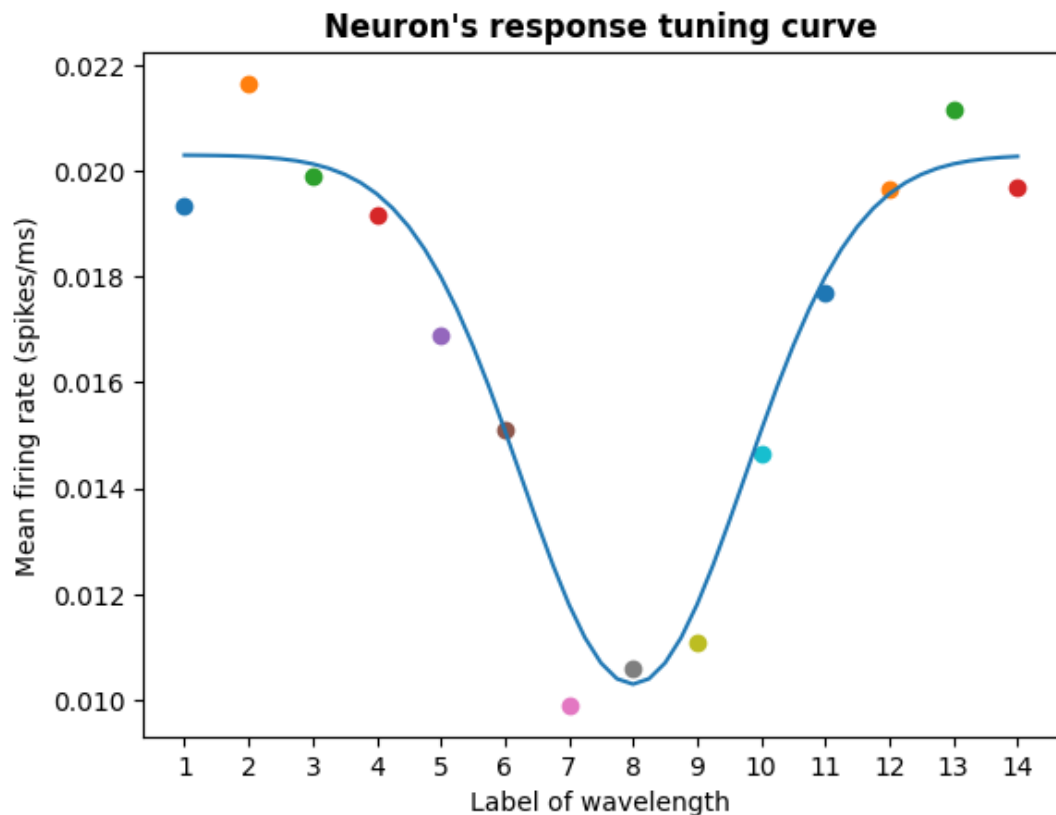
I chose to average the activity across 200-600ms because of prior knowledge that firing rate should decrease as a response to the stimulus and in that time window we see this decrease.

Firing rate results (spikes/ms): [0.0194, 0.0216, 0.0199, 0.0192, 0.0169, 0.0151, 0.0099, 0.0106, 0.0111, 0.0147, 0.0177, 0.0196, 0.0212, 0.0197].

- c) As seen in the figure below the activity is generally around the mean baseline firing rate 0.0203 spikes/ms in the first 3 and last 3 waves (1, 2, 3, 12, 13, 14). Then after the first 3 the activity starts decreasing in the range of wavelengths 4-7 (7 has minimum firing rate amongst all). After 7 the activity increases and reaches baseline in the latter wavelengths. We can assume that the neuron is more sensitive/ its preferable stimuli is wavelength 7 and its neighbors due to the prior knowledge of decreased firing of the neuron to stimuli it is sensitive to and due to larger change in firing rate relative to baseline.



- d) The data is behaving according to an inverted bell curve / gaussian distribution. Such that its mean and minimum is around 8 (practically when putting the mean around 7.5 the inverted gaussian less fits to the data), its std=7, "amplitude" = -0.01 (close to the maximal change from baseline and negative for the inversion) and offset=0.0203 which is the baseline activity (if the baseline would start at 0 so there would be no offset needed, assuming that the data is distributed in a gaussian way and there are some wavelengths the neuron is not sensitive to).



Q3:

- a) I am using the 1s window only for calculation of FF of course because Cv is calculated across the whole time window. Reason counting spikes on the whole time window (for FF) won't yield a distribution while counting ISIs over the whole time window (for Cv) will yield a distribution.

$$FanoFactor(spikes) = \frac{var(spikes)}{mean(spikes)}$$

$$CoefficientofVariation(ISI) = \frac{std(ISI)}{mean(ISI)}$$

Results for 1 sec window:

FF1 = 0.058076152304609205, Cv1 = 0.127423395914091

FF2 = 608.6663574351977, Cv2 = 2.2718555754985745

We can see by the results that spike train 1 is very ordered due to $FF \rightarrow 0$ and $Cv \rightarrow 0$. But spike train 2 is very unordered (more unordered than a Poisson neuron) $FF \gg 1$, $Cv > 1$.

- b) Because the Cv is not calculated using a specific time window, its values stay the same as in a.

Using a 50 ms window for FF :

Spikes train 1: $FF = 0.5009999999999999$

Spikes train 2: $FF = 32.82731514324693$

We can see that the FF of spike train 1 increased by ~ 10 times, could be because it fires several spikes close by then has a pretty long ISI before firing again, while the 1s window could capture this regularity in number of spikes the 50ms may be smaller than the larger ISI and does not capture this regularity as well as the 1s window (e.g. some windows have same number of spikes and some don't have spikes at all) . Still the received value indicates a more regular neuron than Poisson.

FF of spike train 2 decreased by ~ 20 times, could be because the 1 second window captured larger differences and using smaller windows even in places where there was high firing rate the number of spikes fitting this window was pretty small and the firing was more even across windows. (for example [5, 0, 3] vs. [2, 3, 0, 0, 2, 1] hypothetically). Still the FF is pretty large indicating a less ordered neuron than Poisson.

- c) Because a Poisson's process measures (FF and Cv) are always $\rightarrow 1$, and looking at the Cv due to its stability I would say that none of the spike trains are of a poisson neuron.

I assume that spike train 1 is of a bursting neuron such that it fires several spikes in a regular manner, and that's why it's FF is higher for smaller time windows - more variability in firing rate between smaller windows - but still has a fairly small Cv that indicates some form of regularity. Spike train 2 is of very unordered, noisy neuron.