Homework 1

CS 4710

Due: 2/11 at 11:59PM EST

Problem 1

Prove that $\{\neg, \wedge\}$ is an **expressively complete** set of logical operators for PC. (Note: being expressively complete means being able to express *any possible* logical operator.)

Problem 2

We are already familiar with modus ponens (MP):

$$P, P \rightarrow Q \vdash Q$$

Define a new rule of inference called *modus tollens* ("denying the consequent") as follows:

$$P \rightarrow Q, \neg Q \vdash \neg P$$

Prove that modus tollens is a valid inference rule in propositional calculus.

Problem 3

Recall the statement of *soundness* for propositional calculus:

$$\text{If } S \vdash P, \text{ then } S \vDash P$$

Prove soundness for PC using mathematical induction.

Problem 4

Let A be a set of atomic statements, and $|A|$ denote the number of elements in A (its *cardinality*). Let V_{all} denote the set of all possible valuations for A.

Prove that $|V_{all}| = 2^{|A|}$.

Problem 5

Define a new logical operator, the "Peirce arrow", written "$\downarrow$", as follows:

P	Q	$P \downarrow Q$
$\top$	$\top$	$\bot$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\top$

Prove that the Peirce arrow is expressively complete.