Lecture 10

Wednesday, February 6, 2019 9:05 AM

all atomic formulas are WFF

WFF/statements

- formula phi where FV(phi) = 0
- atomic formulas are closed under PC operators
- if P is a WFF, replace a name in P with variable x. Call this P*.
    - there exists a P* that is WFF
    - for all P*, they are WFF

PL Statements

- definition: an interpretation M consists of
    - a universe U (sometimes called a "domain") where U is not the empty set
    - an assignment to each name of L a definite object in U
    - an assignment to each n-ary function symbol some n-ary function in U
    - an assignment to each n-ary predicate some n-ary predicate in U
        - f(x,y) --> [| f |]^v = +

Truth Conditions for WFF

- all truth conditions from PC are imported in
- there exists a Px that is true IFF there exists a u within U such that [|P|]^M holds of u.
- for all Px, Px are true IFF for all u within U, [|P|]^M holds of u.
- x = y is true IFF x and y refer to the same u within U (e.g. the morning star is the same as the evening star)

Supposing:

- predicate p
- name a
- let M be such that U = { 1, 2, 3, …}
- [|a|]^M = 1
- [|P|] = <
- Check: ExPx, AxPx, AxEy P(x,y)

PL Trees

- assuming operators and quantifiers: E, A, =
- definition: universal instantiation (UI) of phi with form Ax phi(x), which occupies an open path psi
    - if some name n appears in phi, write phi(n) at the end, unless phi(n) is already in psi.
    - if no name appears in our path, choose some name phi(n) and write phi(n)
    - don't tick
- example: "all the tables in this room are black" --> "this table is black"
- example: names are r, j; L is a 2-ary predicate
    - validity: Ax(Lxr-->Ljx) **apply I(1), apply UI(4)**
    - Lrr
    - ~Ljj
    - Lrr --> Ljr **apply I(2)**
    - branches into ~Lrr and Ljr
        - ~Lrr is **closed** and we **apply N1(3)**
        - Ljr branches into Ljr --> Ljj from applying UI(4), **apply I(5)**
            - ~Ljr
            - Ljj
    - desired conclusion: Ljj

Person(x) --> EyFather(y,x): be careful of this, a "database that explodes in size" - every person has a father, who has a father, etc.

Example for Satisfiability:

- predicates are U, F, P and Q
- S = {Ax(Ux-->Fx), Ax(Ux-->~Fx)}
- Ax(Ux-->Fx), **apply UI(1)**
- Ax(Ux-->~Fx), **apply UI(2)**
- Ua --> Fa, **apply I(3)**
- Ua --> ~Fa, **apply I(4)**
- branches into
    - ~Ua
        - ~Ua(open)
        - ~Fa(open)
    - Fa
        - ~Ua(open)
        - ~Fa(closed) **satisfiable, because there are open paths**

Note - quantifiers are intersomethingsomething (think discrete)

- ~Ex phi(x) ≡ Ax ~phi(x)
- ~Ax phi(x) ≡ Ex ~phi(x)
- ~(A v B) ≡ ~A ^ ~B

AxPx --> Pa ^ Pb ^ Pc
ExPx --> Pa v Pb v Pc
Consider: ~ExPx ≡ ~(Pa v Pb v Pc)

Negated Quantification (NQ)

- if ~Ax phi(x) or ~Ex phi(x) appears on an open path, tick it (discharge statement) and write "Ex~" in place of "~Ax" (or vice versa)

Existential Instantiation

- Given unticked Ex phi(x) on a path phi, check psi for phi(n)
- if phi(n) doesn't occur in psi, write phi(n) at the end
- example: Ex phi(x)
    - psi (path)
    - phi(n)
- sometimes dis boi is misapplied, check notes to see if there is an example

identity property

- two statements P, Q
- if psi (the path) has a statement of form x=y and P where x or y occurs at least once, you can write Q where Q is P w/ all x's replaced with y's or vice versa
- example:
    - x = y
    - P (original statement)
    - Q
- identity property allows us to get to transitivity, reflexivity, all that other good stuff