Lecture 12 Utility function

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<u>Decisions</u>

- Certainty
- Uncertainty

Data --> model --> DOM

<u>Utility functions:</u>

- **Def**: a utility function U is a <u>numeric</u> representation of a preference relation
- **Def:** an <u>ordinal utility function</u> U is a utility function such that the number denotes <u>**rank**</u>
    - E.g., if A $<=_P$ B and U is ordinal, then U(A) = 1, U(B) = 100 is a valid representation.

<u>Claim:</u> let s be a set of finite prospects, any weak order of S can be represented by an ordinal utility function.

(U(A) <= U(B)) <==> (A $<=_P$ B)

<u>Any order preserving transformation results in another ordinal U</u>

<u>Theorem: ordinal representation</u>

- Let S be a finite set of prospects and $<=_P$ be a weak preference relation on S
- Then there is an ordinal utility function U that represents $<=_P$ iff represents inducesa weak order on S

Lottery is a special type of prospects

Def: a lottery L is a prospect with a probability P and the possible "prizes."

$L = \{_p B, _{(1-p)} A\}$

Suppose we have $A <_p B$, $B <_p C$.

---A----------B-----------C----

This is an underline structure used in AB test

$L = \{_p C, _{(1-p)} A\}$
Suppose we have a free choice for p
Then there are some P such that $B \sim_p L$. this p provides a a measurement of U(B) (utility of B) that uses U(A) <= U (C) as an arbitrary scale

Example:
$U(B) = _p(U(C) - U(A)) + U(A)$
This could tell us the position of B, in another word, how far away are B from A and C

Def: the <u>expected utility</u> of a lottery can be calculated as following:

- Let S be a set of lotteries
- And Let L_i in S and payoff $O_{i\,k}$ be the prize of L_i in state K with probability $P_{i\,k}$
- Expected value of $U(L_i)$ = sum(through k) of $U(O_{i\,k})\ P_{i\,k}$
- If for any L_i, L_j belongs to S, $L_i \le_p L_j \iff E\,U(L_i) \le E\,U(L_j)$
- There exists an expected utility function that represents the agent's preferenc

-> Maximizing expected utility

Set of prospects / lotteries L is closed under probability mixture, for example, if L_i, L_j belongs to L, then {p(Li), (1-p) Lj} is also a lottery in L for all P belongs to [0,1]

Construct cardinal utility

- If we have the above set of thing provided, we can recover our utility function
- Continuity: suppose A <=$_p$ B , B <=$_p$ C, then there exists some O <= P <= 1 such that:

  {pC, (1-p)A} ~ pB

Independence:

Suppose A<=p B, for all C and all 0 <=p <= 1, {pA, (1-p)C} <=$_p$ {pB, (1-p)C}. In another word, you will prefer the right side.

Representation theorem

- Let O be a finite set of payoffs,
  - From here, we can know that there are some probability and payoff (structure) in our problem
- Let L be a set of lotteries closed under probability mixture, then <= pp satisfies iff there exists a function U: between 0 and Real numbers, Which is unique up to a positive linear transformation , which <=p represents maximizing EU over.