(1)

Based on the chain-rule, we can know

$$\frac{\partial E}{\partial a_k} = \frac{\partial E}{\partial y_k} \cdot \frac{\partial y_k}{\partial a_k}$$

Because
$$\begin{cases} E(w) = -\sum_{n=1}^{N} \{t_n \ln y_n + (1-t_n)\ln(1-y_n)\} & \cdots ① \\ y_k = \dfrac{1}{1+\exp(-a_k)} & \cdots ② \end{cases}$$

so
$$\frac{\partial E}{\partial y_k} = -\left[\frac{t_k}{y_k} + (1-t_k)\cdot\frac{1}{1-y_k}\cdot(-1)\right] = -\left(\frac{t_k-1}{1-y_k} + \frac{t_k}{y_k}\right) \cdots ③$$

$$\frac{\partial y_k}{\partial a_k} = -\left[1+\exp(-a_k)\right]^{-2} \cdot \exp(-a_k) \cdot (-1)$$

$$= \left[1+\exp(-a_k)\right]^{-2} \exp(-a_k) \cdots ④$$

from ① we can get

$$\exp(-a_k) = \frac{1}{y_k} - 1$$

so ④ could be written as:

$$\frac{\partial y_k}{\partial a_k} = y_k^2\left(\frac{1}{y_k} - 1\right) = y_k - y_k^2 = y_k(1-y_k)$$

Thus:
$$\frac{\partial E}{\partial a_k} = y_k(1-y_k)\cdot\left(\frac{t_k-1}{1-y_k} + \frac{t_k}{y_k}\right)(-1)$$

$$= \left[y_k(t_k-1) + (1-y_k)t_k\right](-1)$$

$$= y_k - t_k$$

(2)

$$\frac{\partial E}{\partial a_k} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial a_k}$$

because $\frac{\partial y_i}{\partial a_k} = \frac{\frac{\partial a_i}{\partial a_k} \exp(a_i)}{\Sigma_j \exp(a_j)} - \frac{\exp(a_i) \cdot \exp(a_j)}{(\Sigma_j \exp(a_j))^2}$

$$= \frac{\partial a_i}{\partial a_k} \cdot y_i - y_i y_k$$

$$= y_i \left(\frac{\partial a_i}{\partial a_k} - y_k \right) = \begin{cases} y_i(1-y_k) & i=k \\ -y_i y_k & i \neq k \end{cases}$$

Thus,

$$\frac{\partial E}{\partial a_k} = -\sum_{i=1}^{k} t_i \frac{1}{y_i} \frac{\partial y_i}{a_k}$$

$$= -\sum_{i=1}^{k} t_i \frac{1}{y_i} y_i \left(\frac{\partial a_i}{\partial a_k} - y_k \right)$$

$$= -\sum_{i=1}^{k} t_i \left(\frac{\partial a_i}{\partial a_k} - y_k \right)$$ when $i=k$ $t_i = 1$

otherwise $t_i = 0$

$$= y_k - t_k$$