School of Geography and Geology
McMaster University

Advanced Topics in Spatial Statistics

Spatially Continuous Data I & II

This session:

- **Spatially Continuous Data I & II**
  - Definitions
  - Visualization
  - Exploration:First Order Properties
    - Moving Averages
    - Kernel Estimation
    - Tesselation Methods
  - Exploration: Second Order Properties
    - Covariogram and Variogram

The difference between point patterns and spatially continuous data

- Location of fires in Kitchener, Ontario

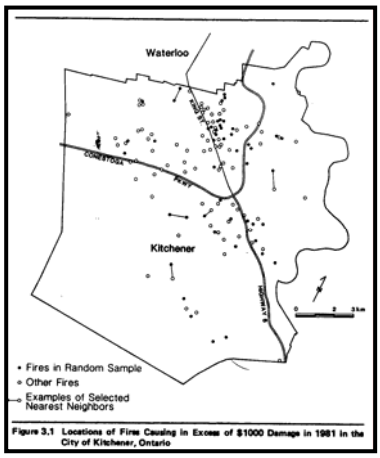
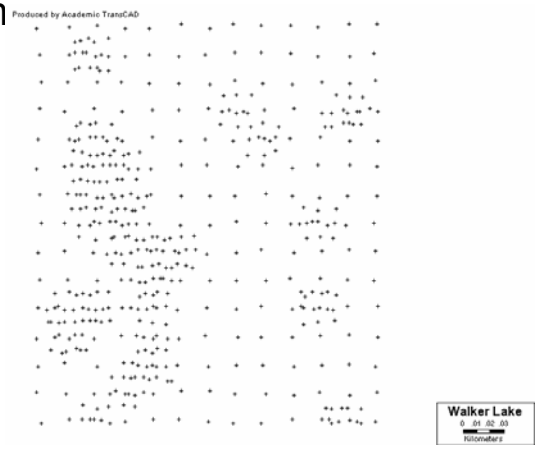


Figure 3.1 Locations of Fires Causing in Excess of $1000 Damage in 1981 in the City of Kitchener, Ontario

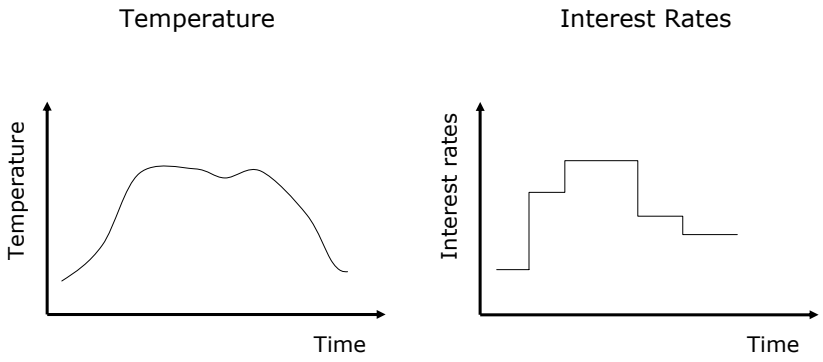
The difference between point patterns and spatially continuous data

- Concentration of a contaminant in ppm

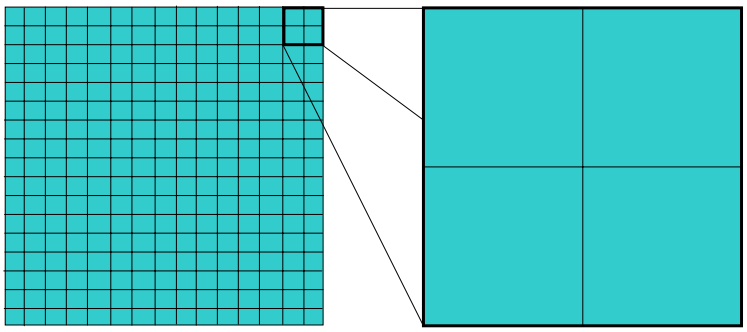


The difference between spatially continuous data and area data

- Time series



Scale of Analysis

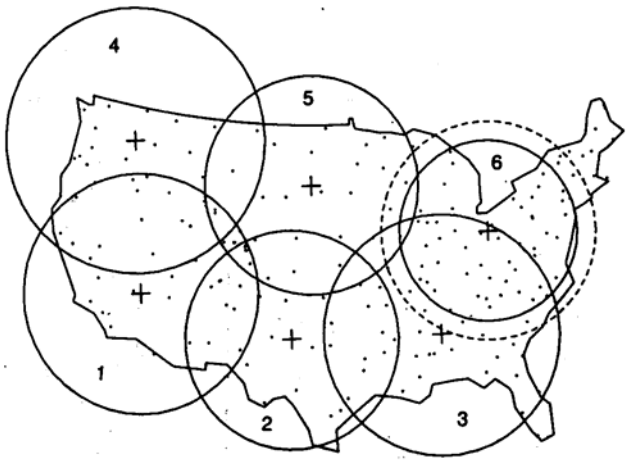


Examples of Applications

- In the environmental sciences

US NADP/NTN*: Stations in the US

*National Atmospheric Deposition Program/
National Trends Network



US NADP/NTN: Stations in the US

- Damage to environments from acid rain
- Forest effects: die-off of trees at high elevations
- Long term effects on the growth of commercial stands

Examples of Applications

- In the social sciences

Goals of Applications

- The description of important features
- Estimation of an average value over large areas
- The estimation of an average value over small areas
- Estimation of an unknown value at a particular location

(first order and second order effects)

Goals of Applications: PP

- The description of important features
- The identification of clustered, random or regular patterns

(first order and second order effects)

Spatially Continuous Data

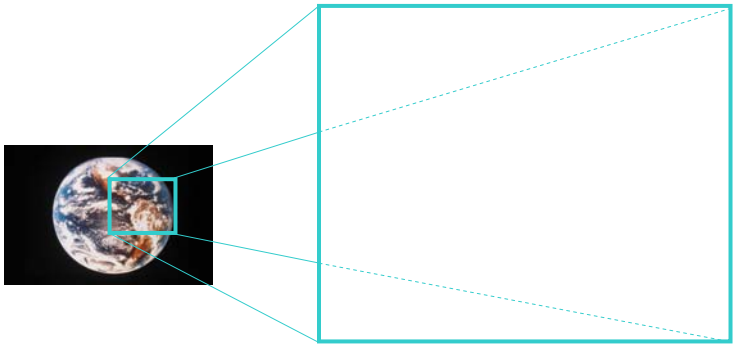
- Visualization
  - Proportional symbol maps
  - Triangulated irregular networks (TIN)
- Exploration

Definitions

- Region
- Location
- Attributes
- Observation
- First order effects
- Second order effects

Definitions: Region

- Region (R) – Specific area over the surface of the earth that is of interest



Definitions: Region

Definitions: Region

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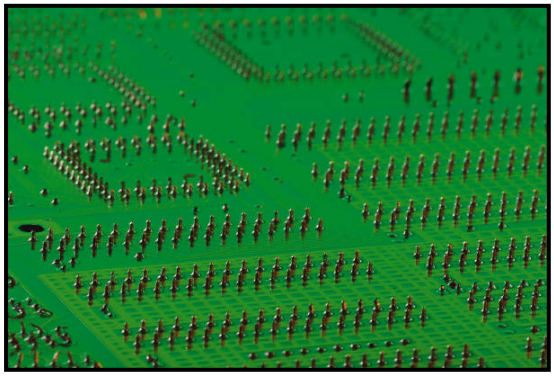
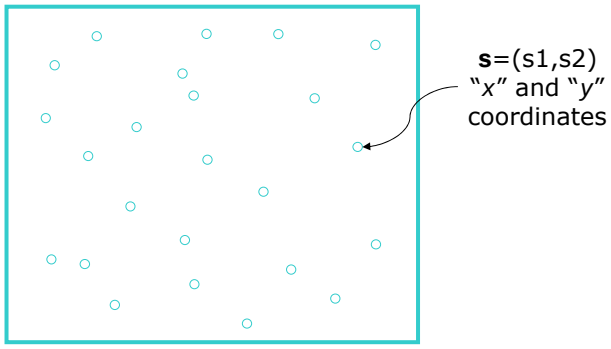


Definitions: Location

- An arbitrary point in region *R*



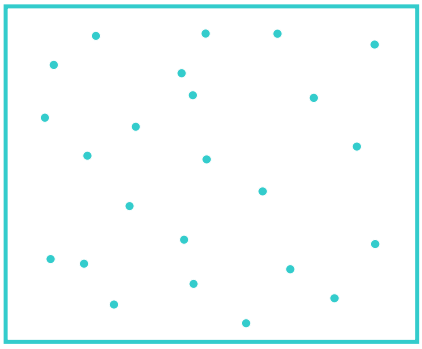
Definitions: Attribute

- A characteristic of interest about a location

Definitions: Observation

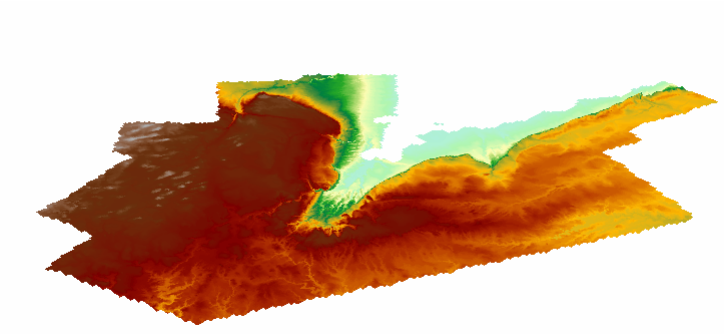
- A location for which one or more attributes have been measured

$Y = (y_1, y_2, \ldots, y_n)$



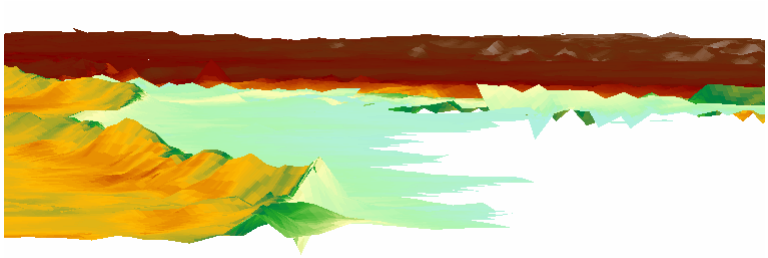
Definitions: First Order Effects

- Large scale variation

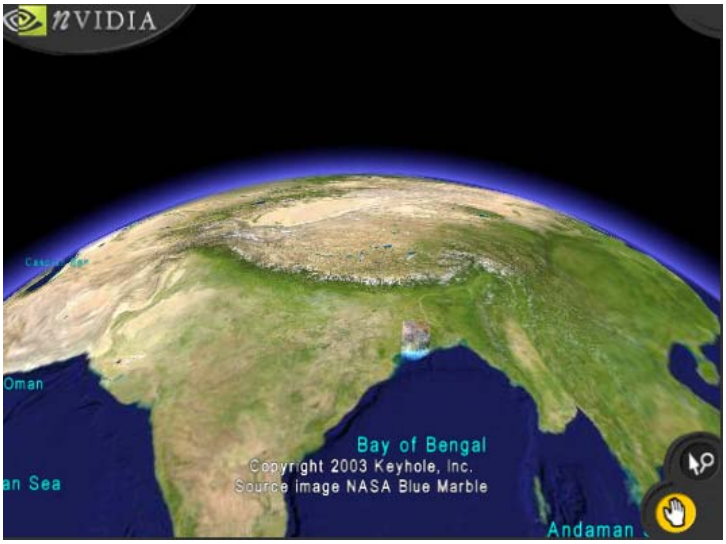


Definitions: Second Order Effects

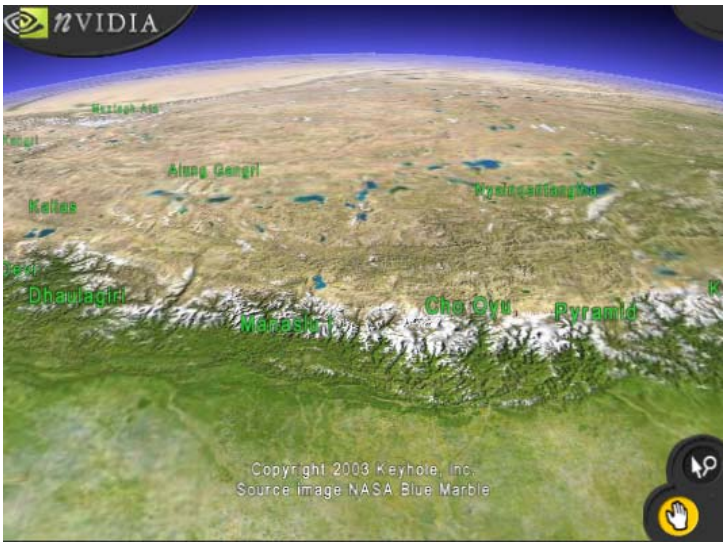
- Small scale variation



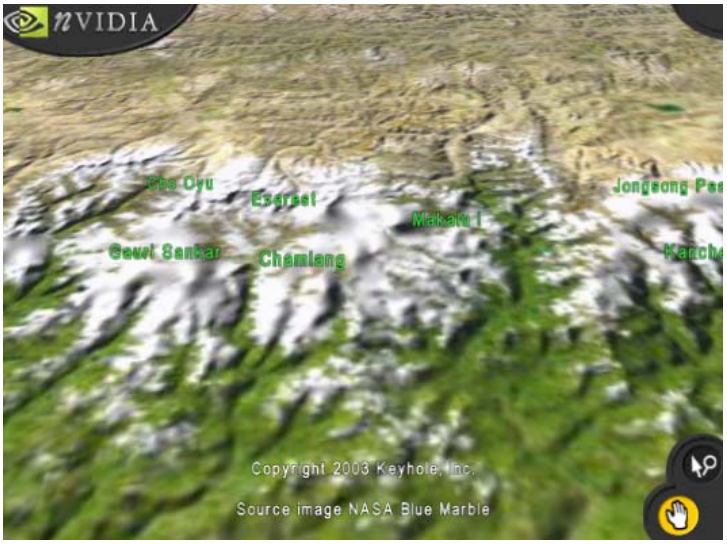
Definitions: Large Scale Variation



Definitions: Small Scale Variation



Definitions: Small Scale Variation



First Order Effects

- Expected value of Y at $\boldsymbol{s}$

$$\mu(\boldsymbol{s}) = E[Y(\boldsymbol{s})]$$

(systematic, deterministic)

Second Order Effects

- Covariance between of $Y(\boldsymbol{s}_i)$ and $Y(\boldsymbol{s}_j)$

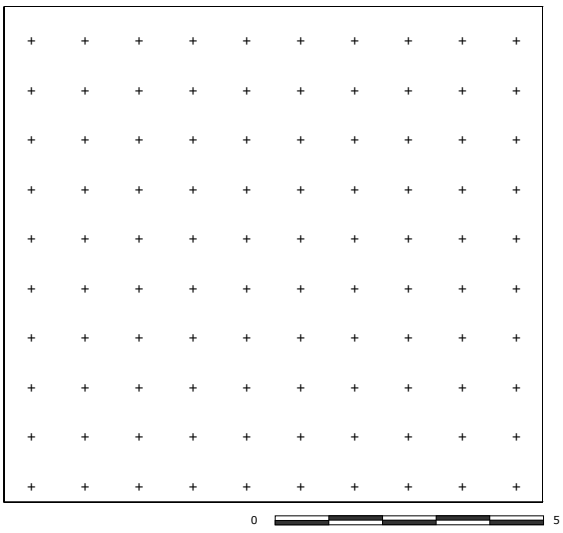
$$COV(Y(\boldsymbol{s}_i), Y(\boldsymbol{s}_j))$$

(independent, unpredictable? dependent, deterministic?)

Visualizing Spatially Continuous Data

- Symbol maps
- Proportional symbol maps
- Indicator maps

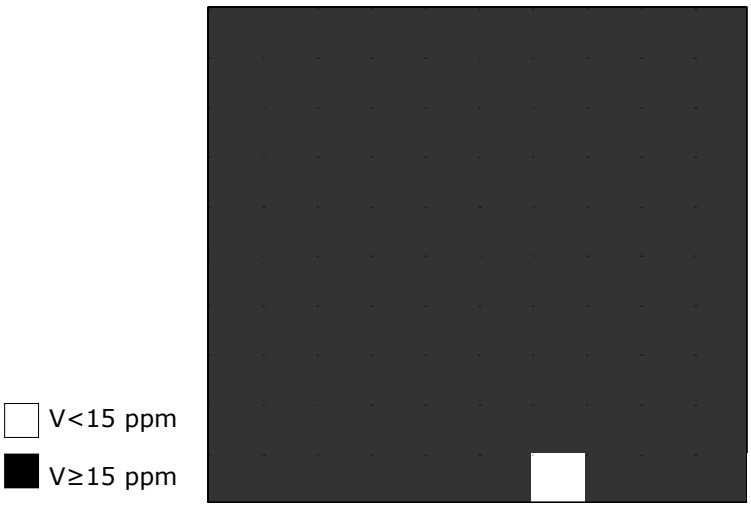
Visualization: Symbol Map (Walker Lake)



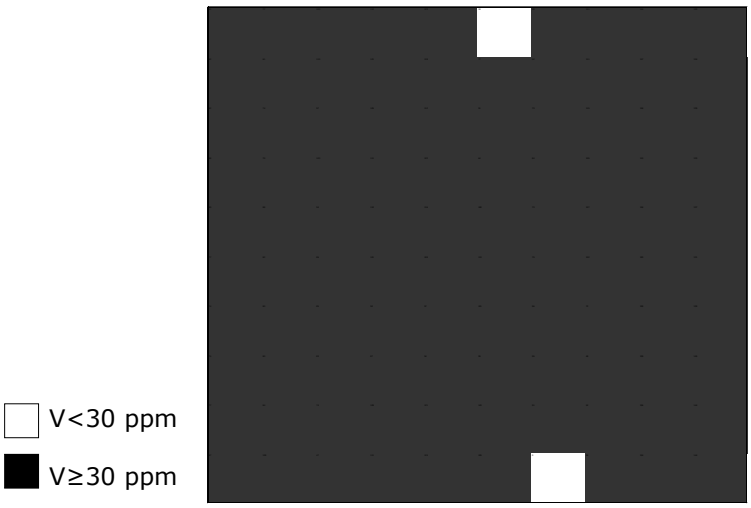
Visualization: Symbol map w/labels

81	77	103	112	123	19	40	111	114	120
82	61	110	121	119	77	52	111	117	124
82	74	97	105	112	91	73	115	118	129
88	70	103	111	122	64	84	105	113	123
89	88	94	110	116	108	73	107	118	127
77	82	86	101	109	113	79	102	120	121
74	80	85	90	97	101	96	72	128	130
75	80	83	87	94	99	95	48	139	145
77	84	74	108	121	143	91	52	136	144
87	100	47	111	124	109	0	98	134	144

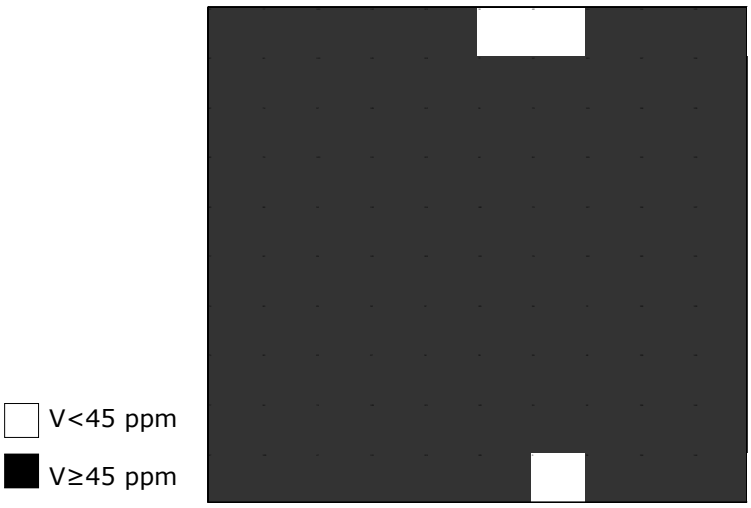
Visualization: Indicator Maps



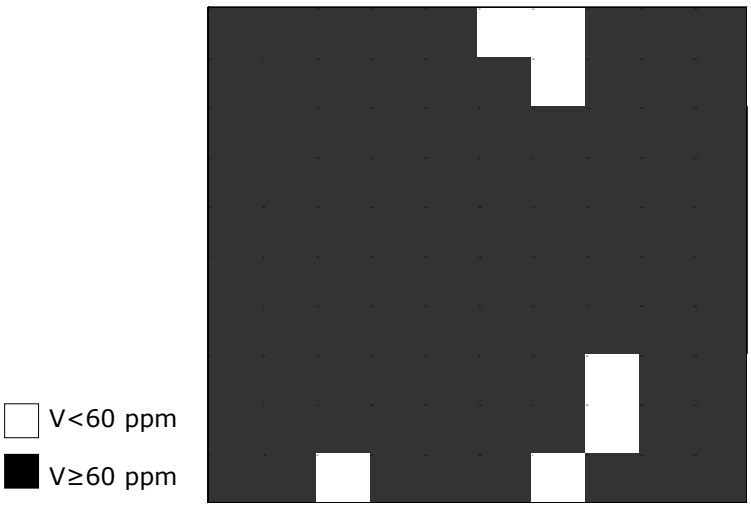
Visualization: Indicator Maps



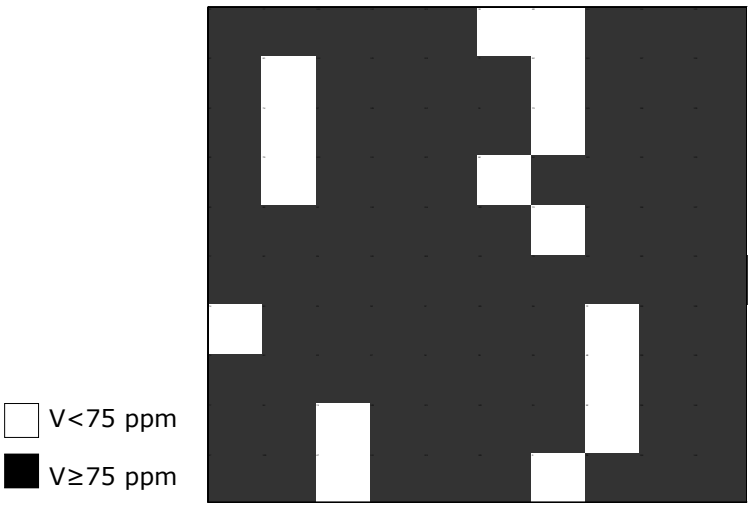
Visualization: Indicator Maps



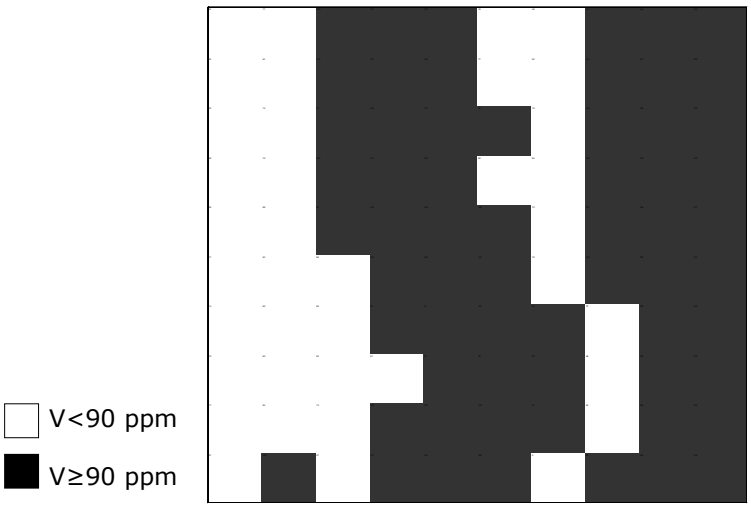
Visualization: Indicator Maps



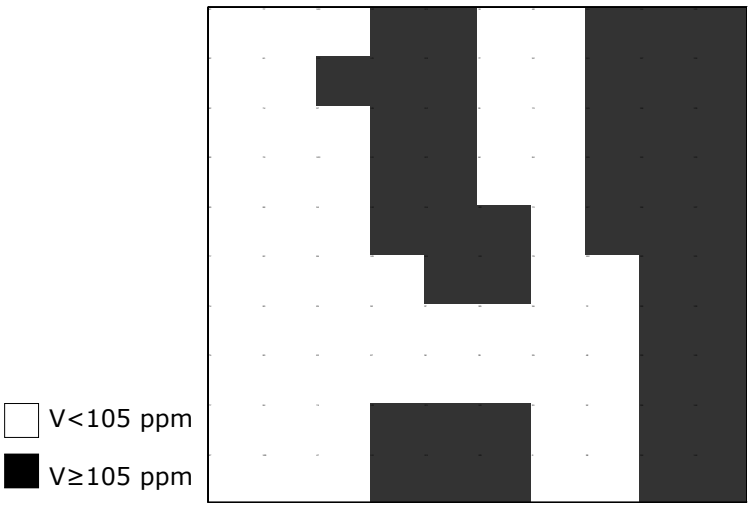
Visualization: Indicator Maps



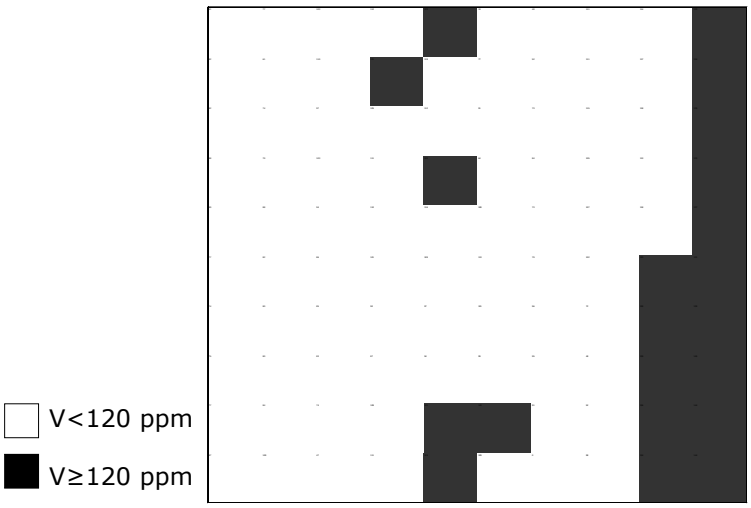
Visualization: Indicator Maps



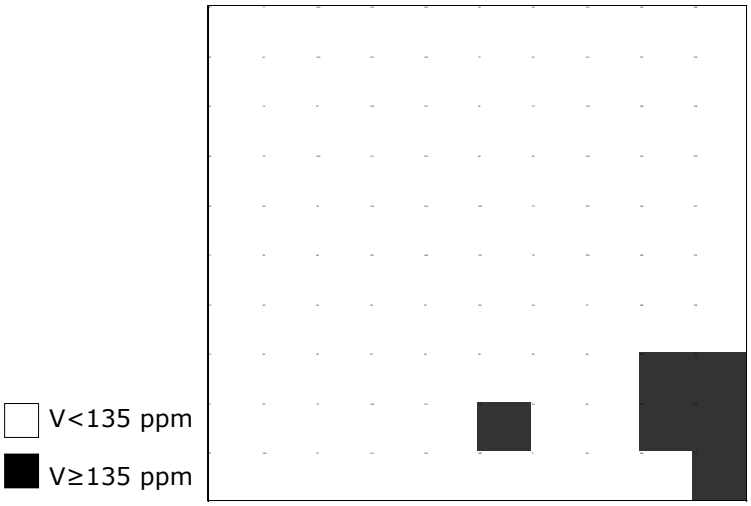
Visualization: Indicator Maps



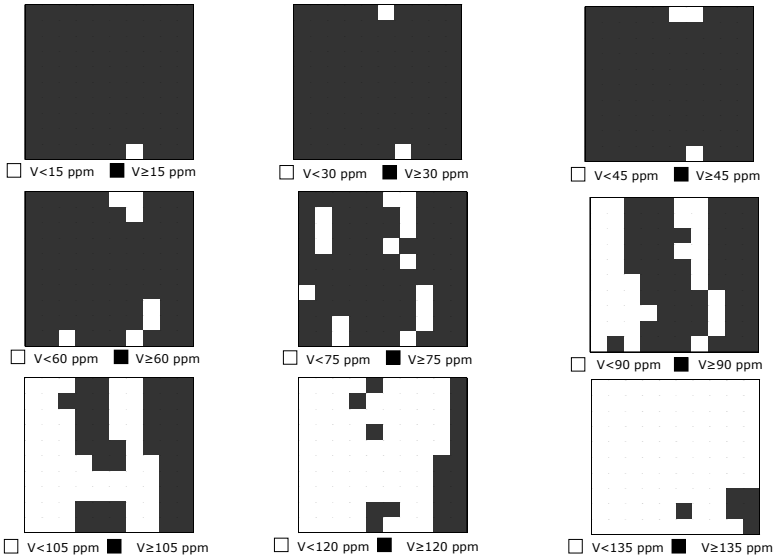
Visualization: Indicator Maps



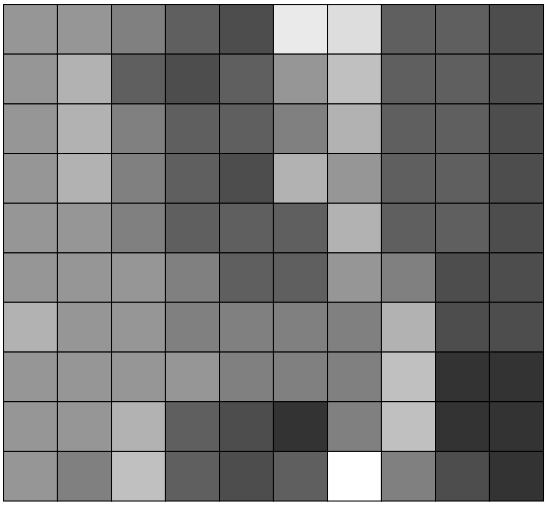
Visualization: Indicator Maps



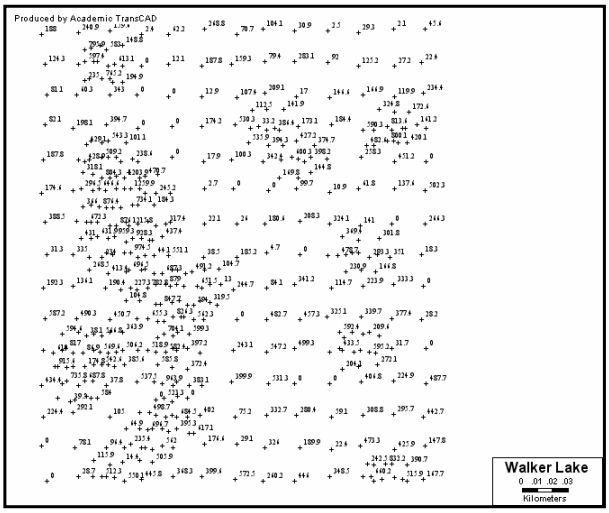
Visualization: Indicator Maps



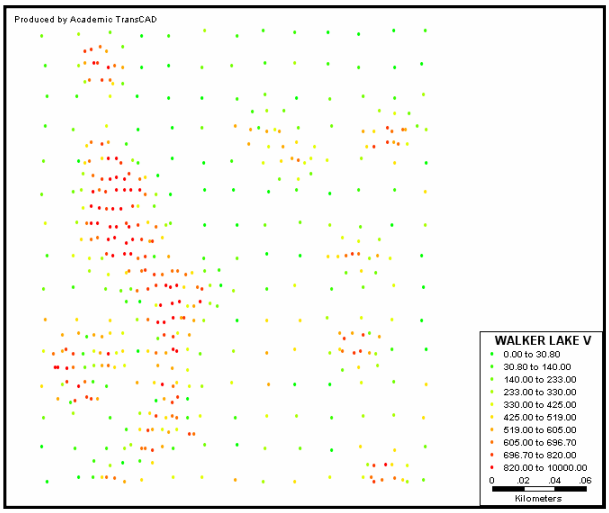
Visualization: Grayscale map



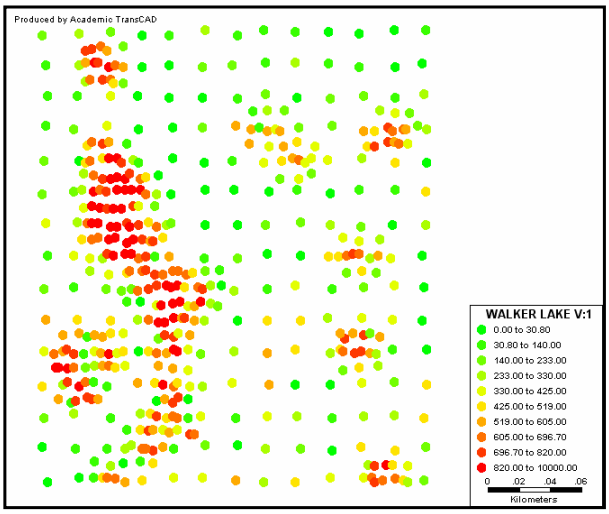
Visualization: Symbol Map w/labels



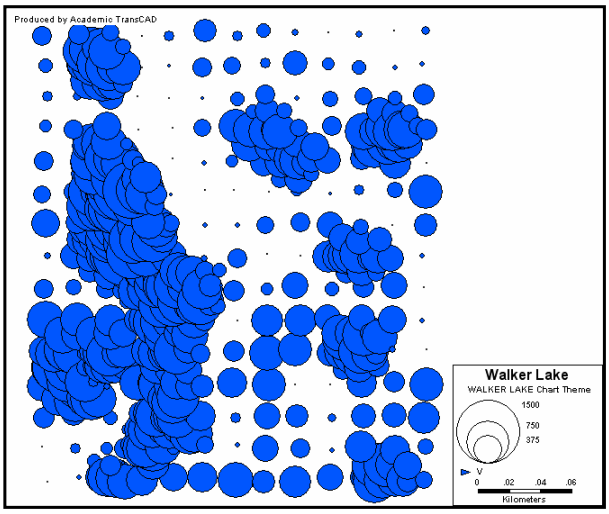
Visualization: Symbol Map w/colors



Visualization: Symbol Map w/colors



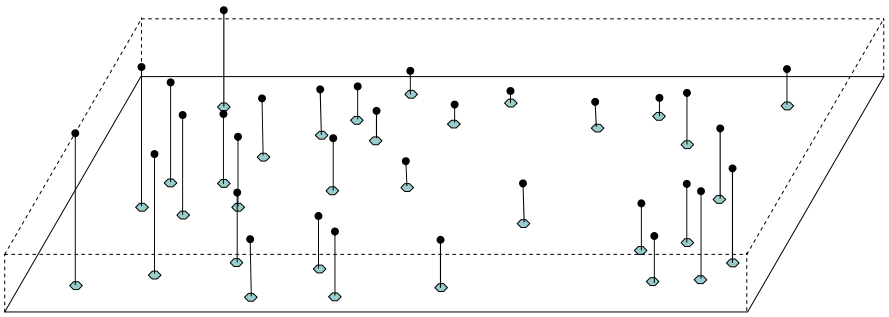
Visualization: Proportional Symbol Map



Visualization

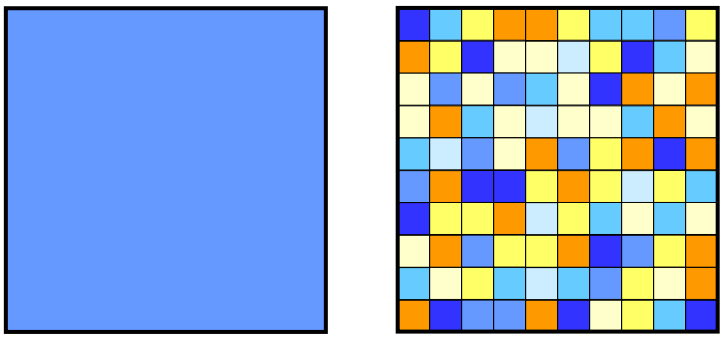
- Irregularly spaced observations
  - Indicator Maps?
  - Grayscale maps?

Exploration: Average (Large Area)



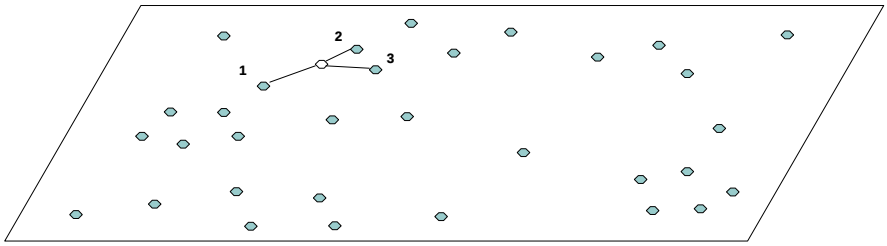
Exploration: Averages at Different Scales

- *n* observations
- 1 observation



Exploration: Moving Averages

- Three-point spatial moving average



$$\hat{\mu}(\mathbf{s}_1) = \frac{1}{3}\left(y_1 + y_2 + y_3\right)$$

Exploration: Moving Averages

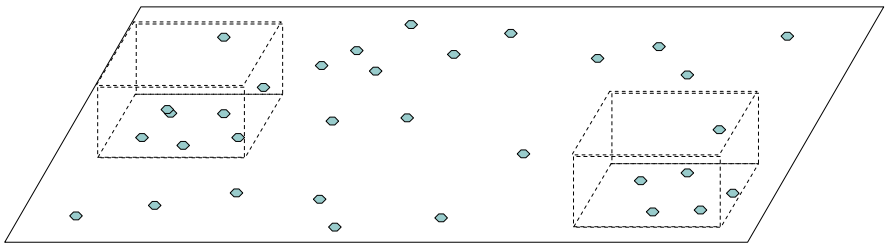
$$\hat{\mu}(\mathbf{s}) = \sum_{i=1}^{n} w_i(\boldsymbol{s}) y_i$$

$$\sum_{i=1}^{n} w_i(\boldsymbol{s}) = 1$$

Exploration: Moving Averages

$$\hat{\mu}(\mathbf{s}_1) = 0.333 y_1 + 0.333 y_2 + 0.333 y_3$$

Exploration: Moving Averages



Exploration: Moving Averages

$$\sum_{i=1}^{n} w_i(\boldsymbol{s}) = 1$$

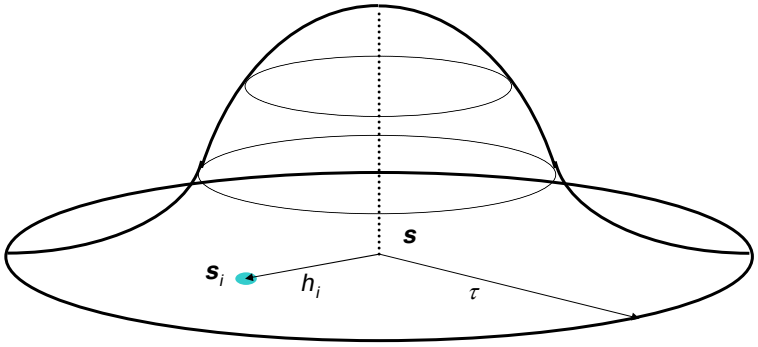
$$w_i(\boldsymbol{s}) \propto h_i^{-\alpha}$$

$$w_i(\boldsymbol{s}) \propto \mathrm{e}^{-\alpha h_i}$$

$\mathcal{M}$

Exploration: Kernel Estimation

- Kernel function



Exploration: Kernel Estimation

$$\hat{\mu}(\mathbf{s}) = \sum_{i=1}^{n} w_i(\mathbf{s}) y_i$$

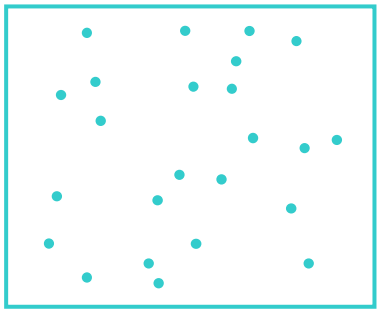
$$w_i(\mathbf{s}) = \frac{k\left(\frac{(\mathbf{s}-\mathbf{s}_i)}{\tau}\right)}{\sum_{i=1}^{n} k\left(\frac{(\mathbf{s}-\mathbf{s}_i)}{\tau}\right)}$$

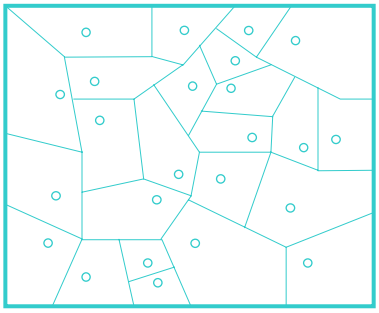
Exploration: Kernel Estimation

$$\hat{\mu}(\mathbf{s}) = \frac{\sum_{i=1}^{n} k\left(\frac{(\mathbf{s}-\mathbf{s}_i)}{\tau}\right) y_i}{\sum_{i=1}^{n} k\left(\frac{(\mathbf{s}-\mathbf{s}_i)}{\tau}\right)}$$

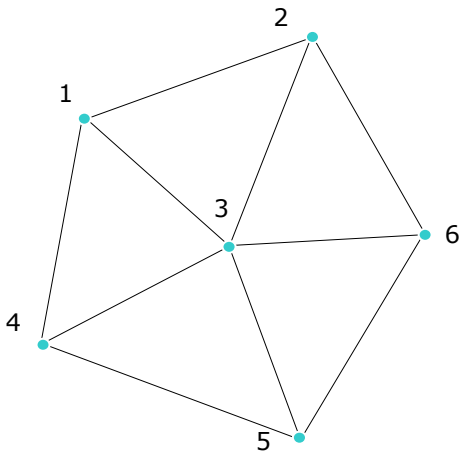
Exploration: Tessellations

- The objective is to produce "tiles"



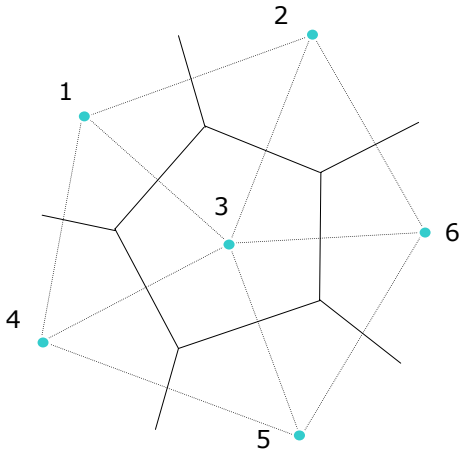


Tessellations: Delaunay Triangulation



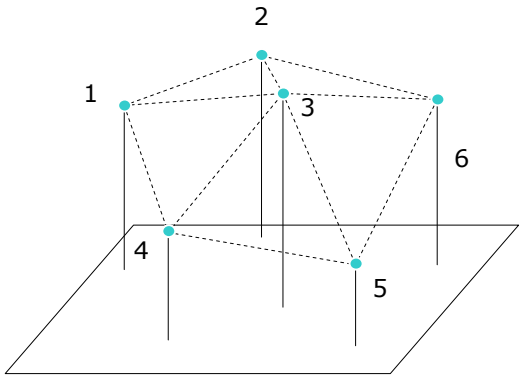
- Triangulated Irregular Network (TIN)
- Triangles are as close to equilateral as possible

Tessellations: Voronoi Polygons

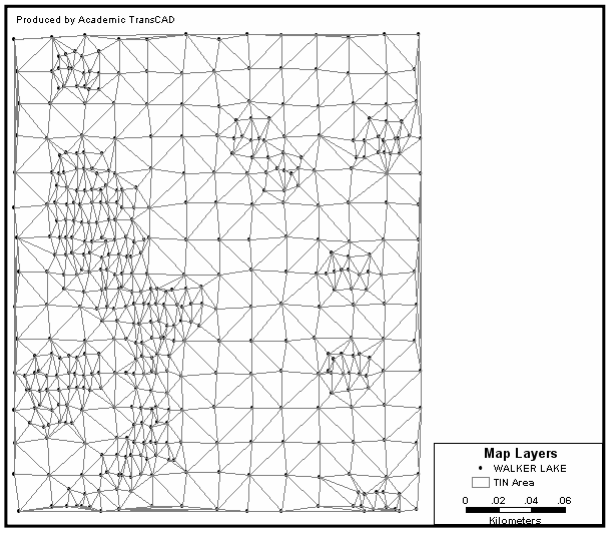


- Areas of Influence

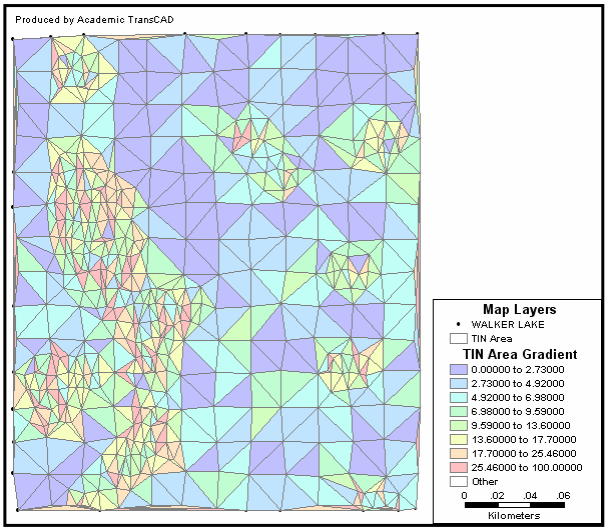
Tessellations: Delaunay Triangulation



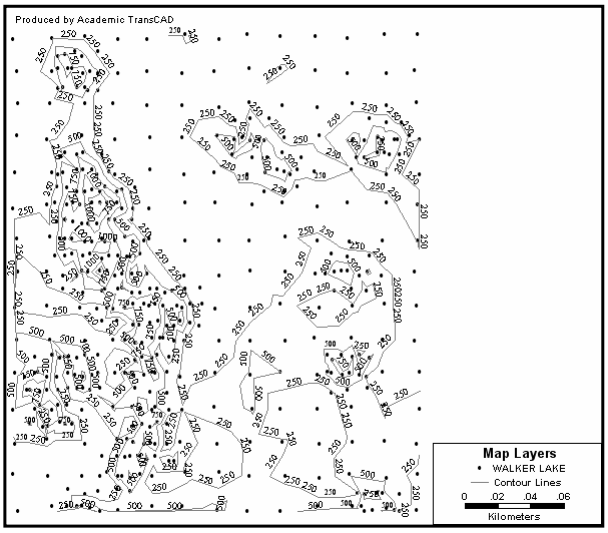
Exploration: Delaunay Triangulation



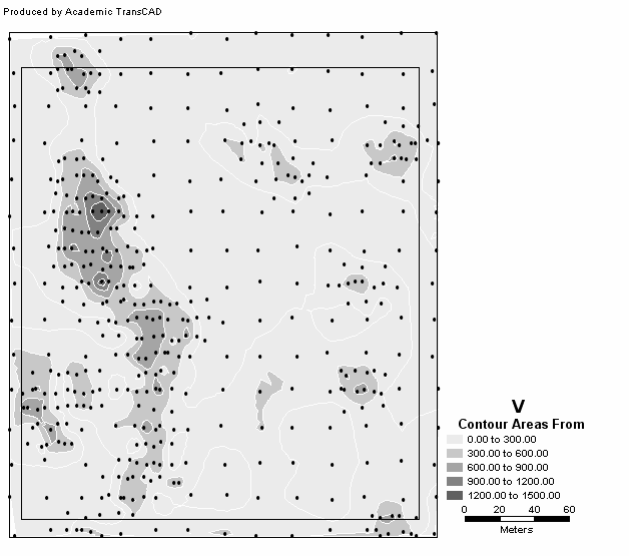
Exploration: Delaunay Triangulation



Exploration: Contours



Exploration: Contour Areas



Visualization/Exploration

- First order effects, large scale variation
- What about small scale variation?

Next…

- Exploring second order effects
  - Covariogram and variogram
- Modeling spatially continuous data