School of Geography and Geology
McMaster University

Applied Spatial Statistics

Point Pattern Analysis I & II

This session:

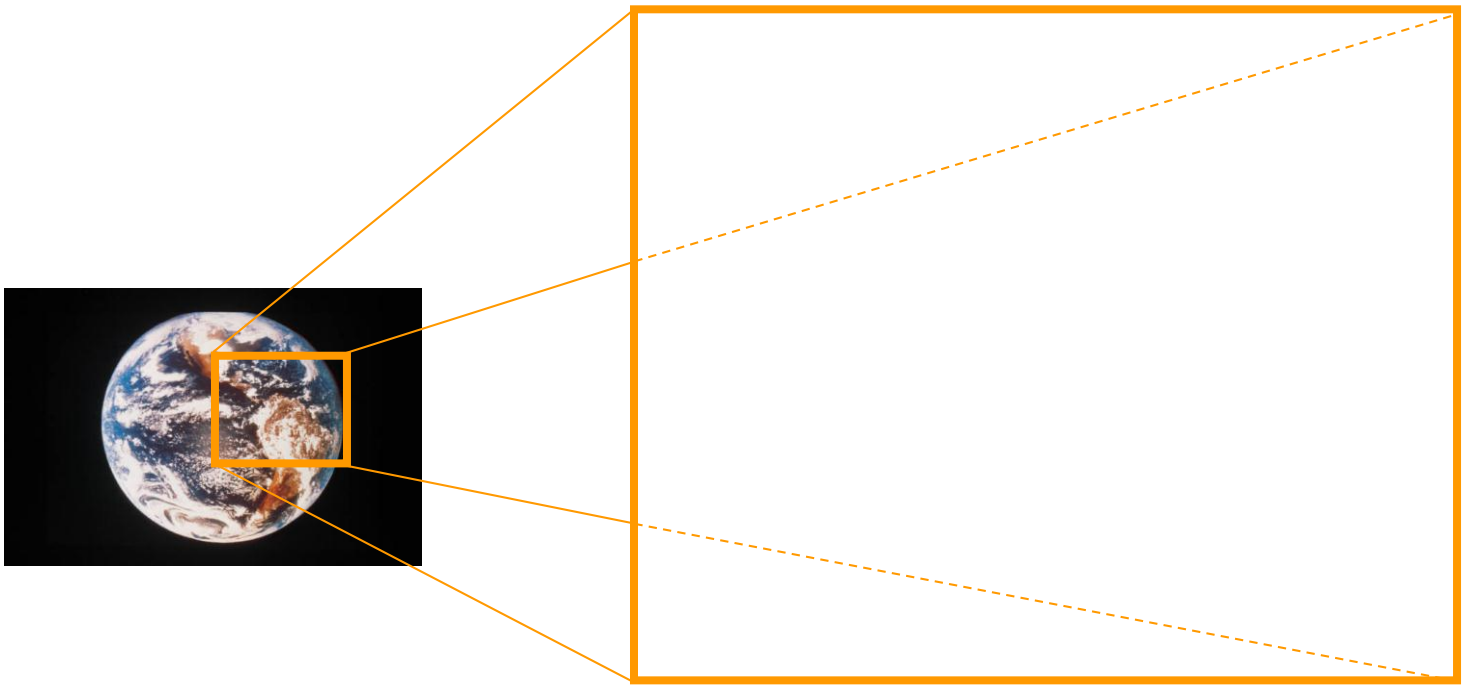
- Point Pattern Analysis I
  - Definitions
  - First and Second Order Properties
  - Visualizing Point Patterns
  - Exploring Point Patterns
    - Quadrat counts
    - Quadrat Analysis

Definitions

- Region
- Point
- Event
- Attributes
- Patterns
  - Random
  - Clustered
  - Dispersed/Regular

Definitions: Region

- **Region (*R*) – Specific area over the surface of the earth that is of interest**



Definitions: Region

- **Examples of regions - Continent**

Definitions: Region

- **Examples of regions - Province**

Definitions: Region

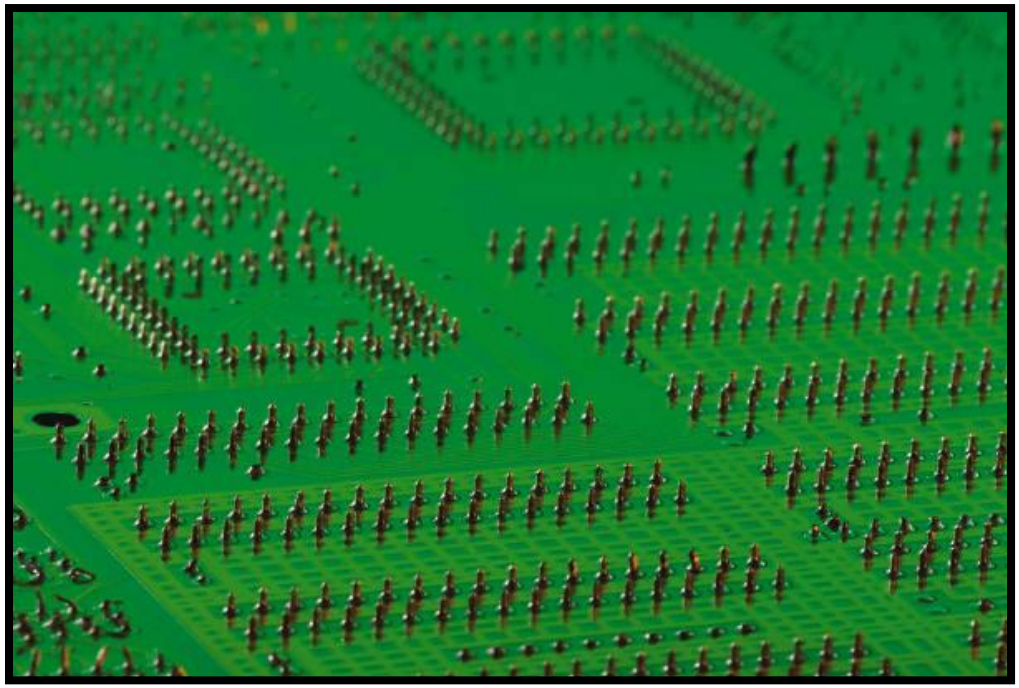
- **Examples of regions**
  - **City**

Definitions: Region

- **Examples of regions**
  - **Land plot**

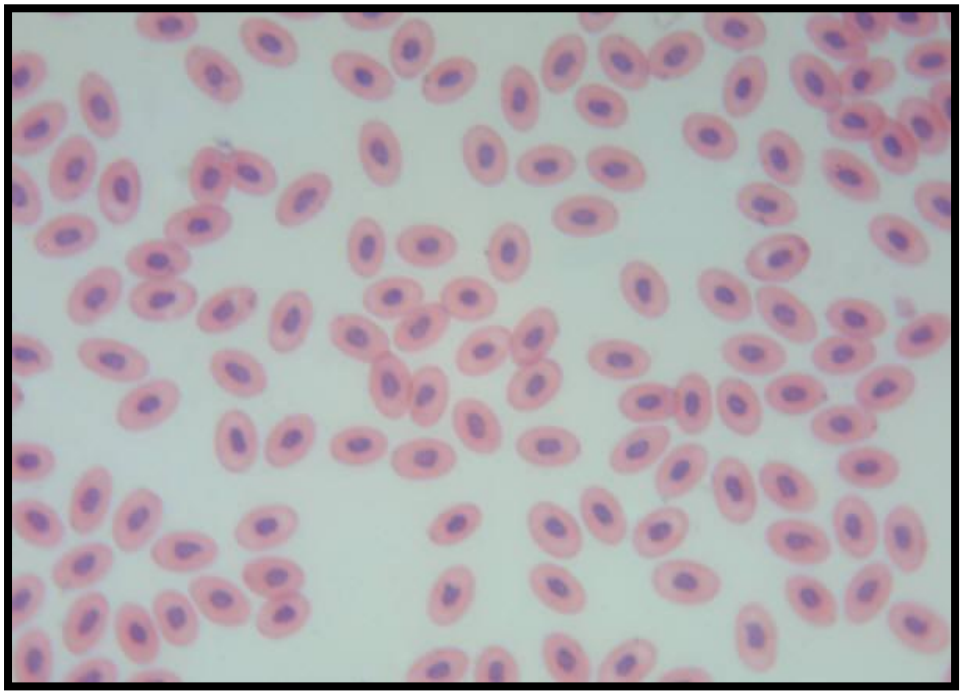
Definitions: Region

- **Examples of regions**
  - **Computer chip**



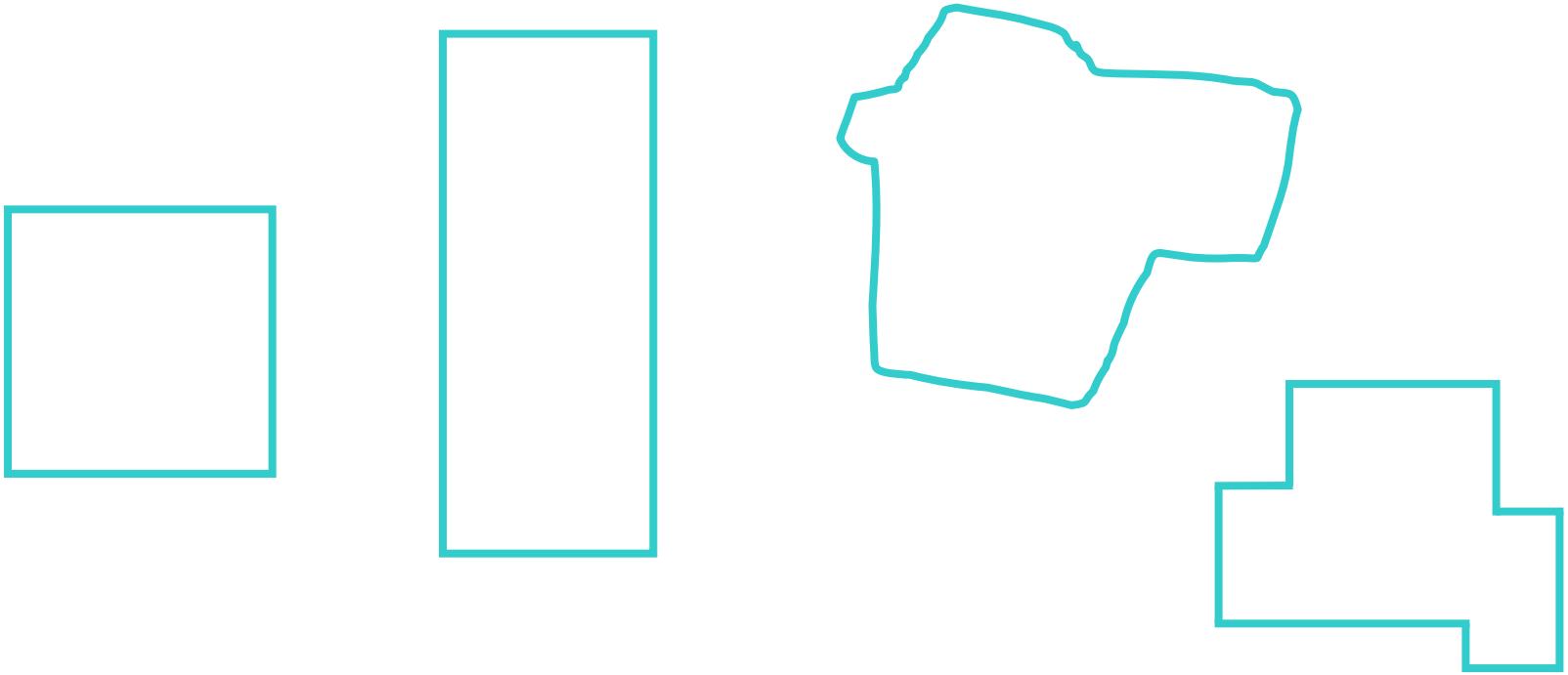
Definitions: Region

- Examples of regions
  - Section of tissue



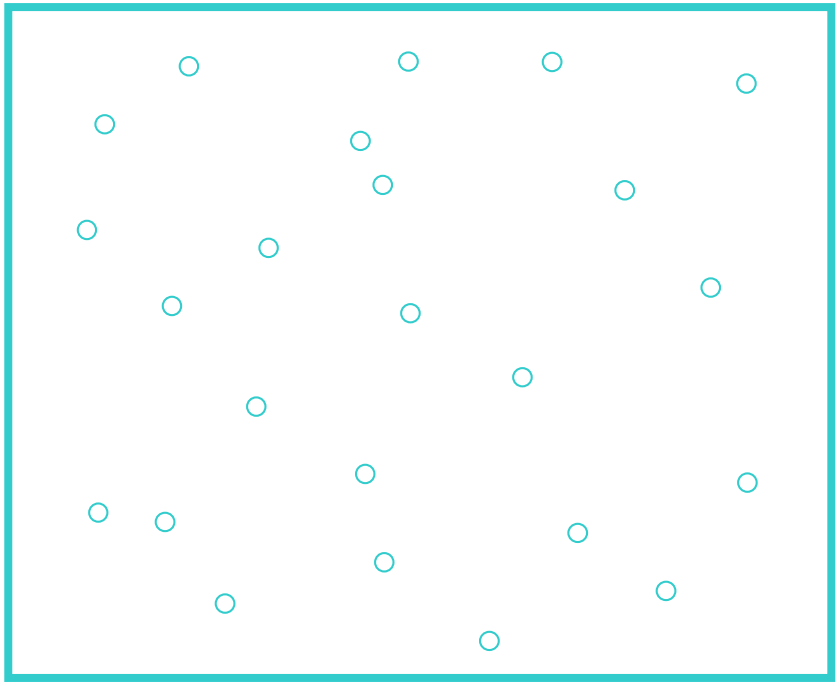
Definitions: Region

- *Regular* region
- *Irregular* region



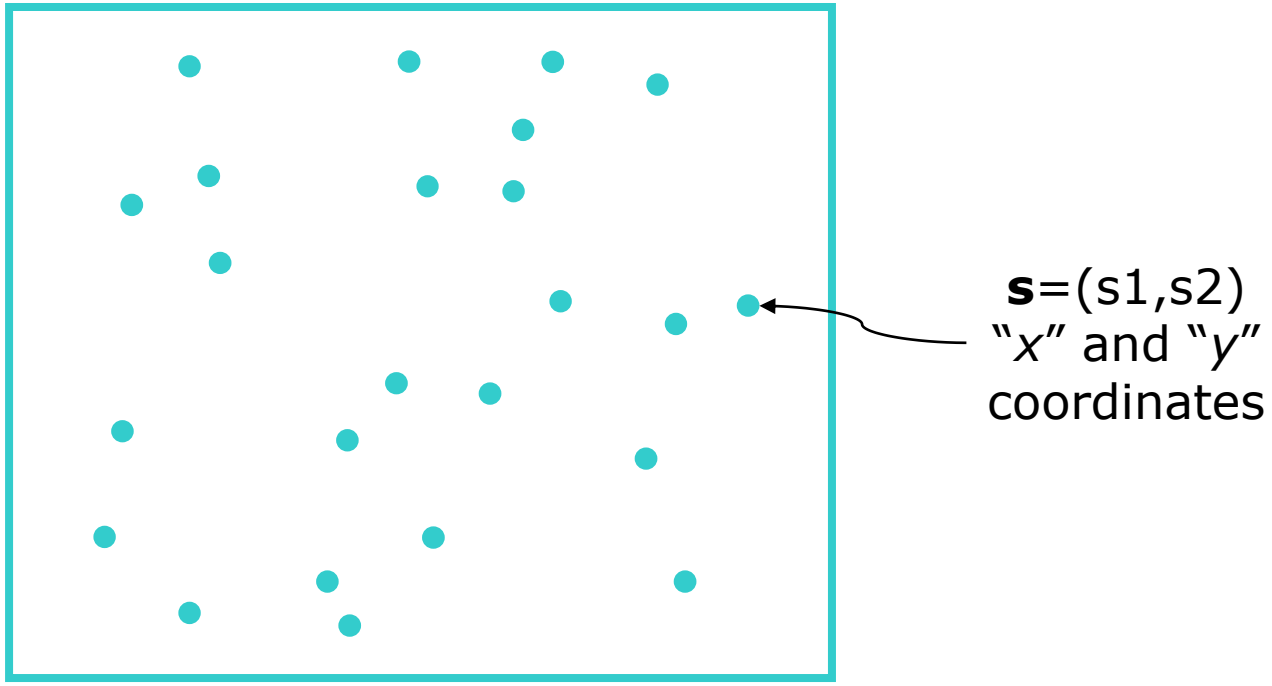
Definitions: Point

- Point – An arbitrary location in region *R*

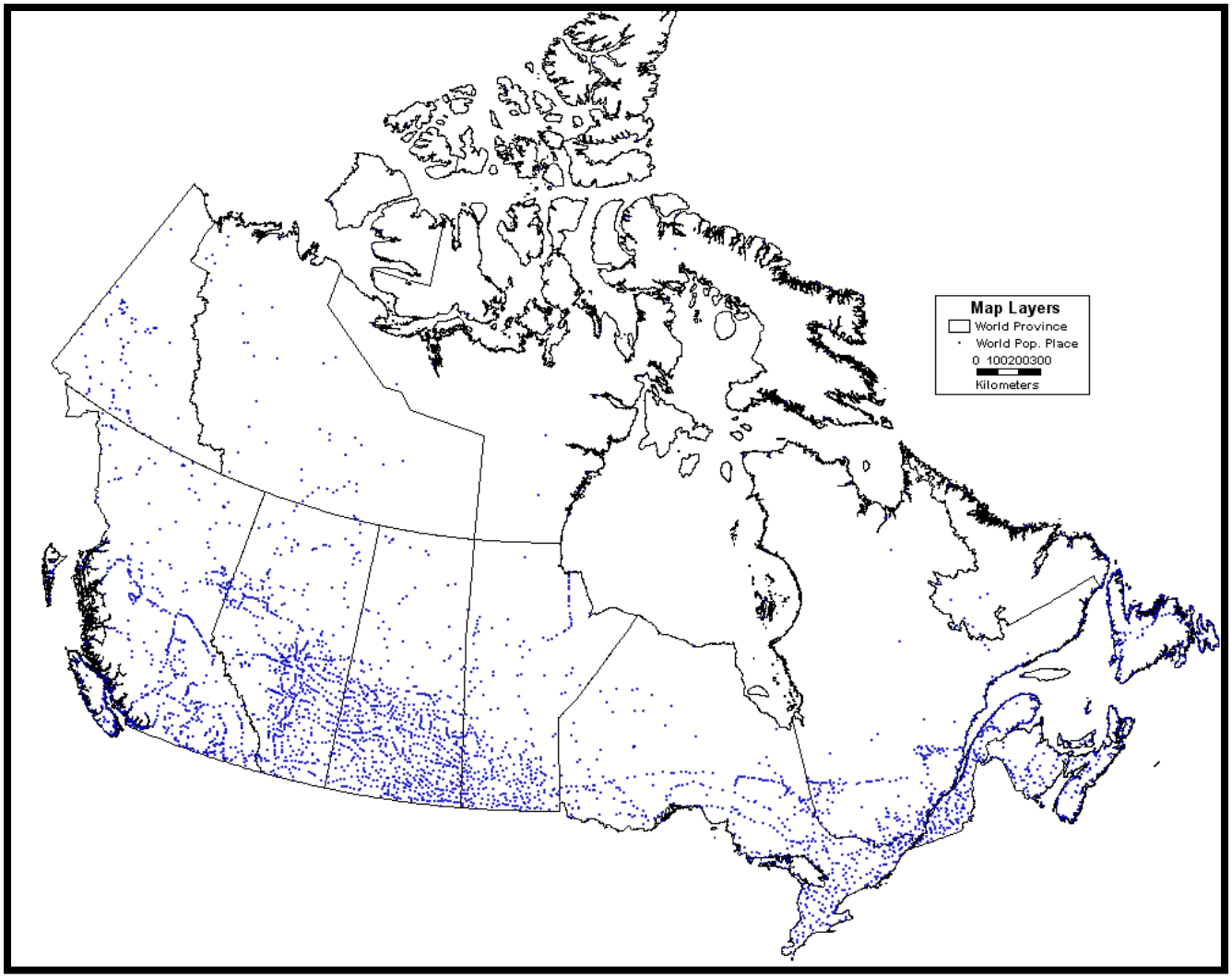


Definitions: Event

- Event – An "interesting" occurrence in region *R*



Definitions: Event – Examples



Definitions: Event – Examples

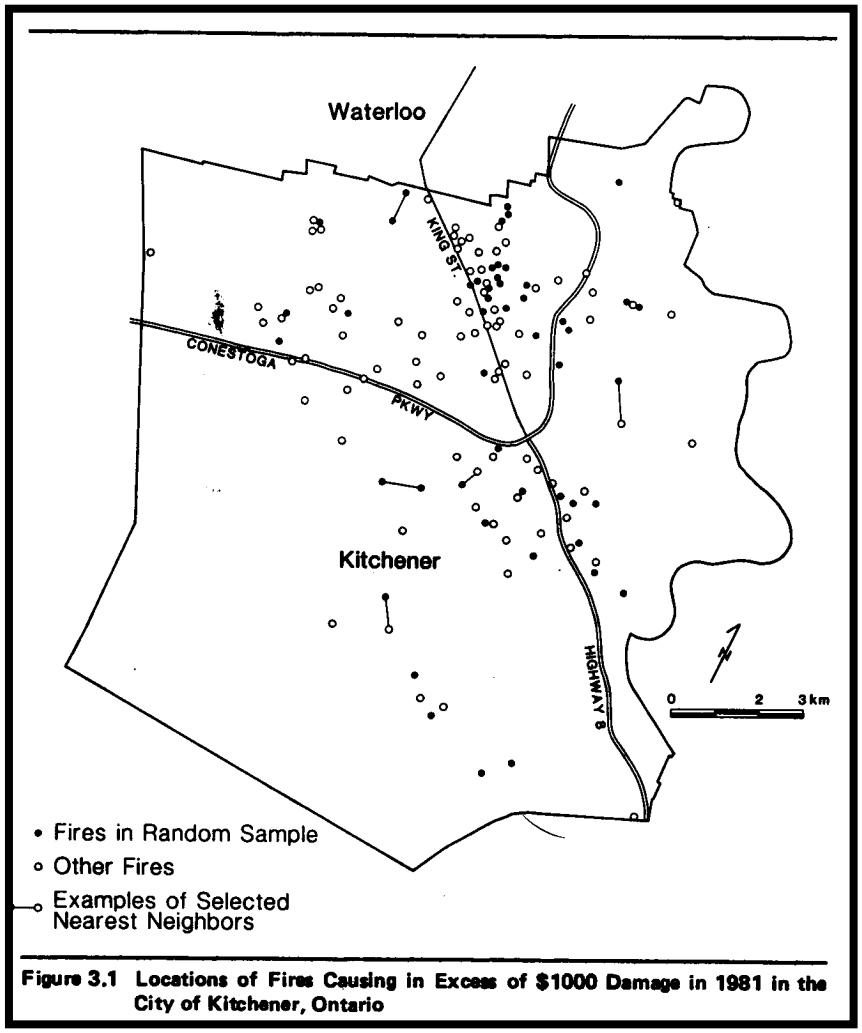


Figure 3.1 Locations of Fires Causing in Excess of $1000 Damage in 1981 in the City of Kitchener, Ontario

Note:
What about the underlying population?

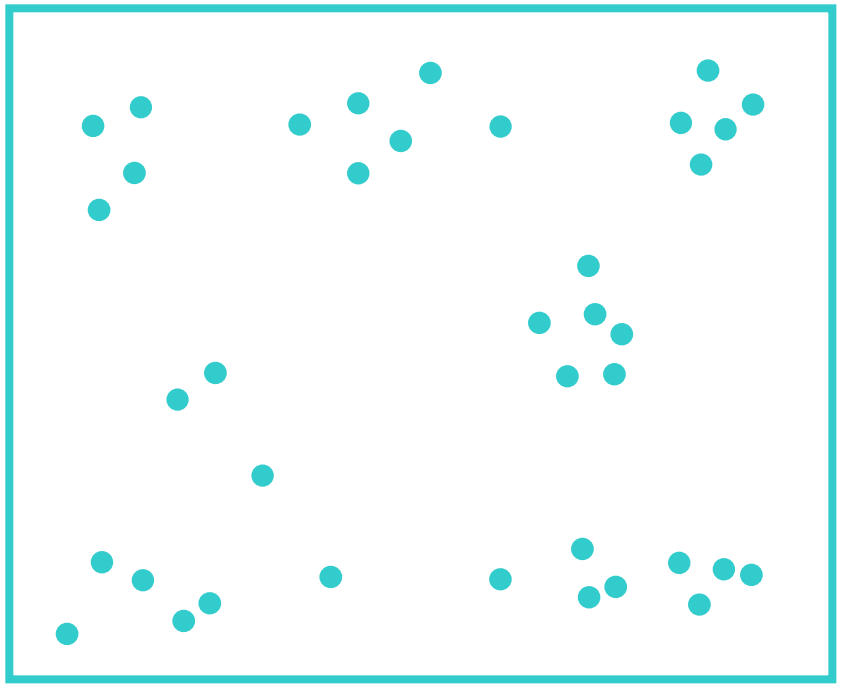
Definitions: Event – Examples

- Event – Further examples

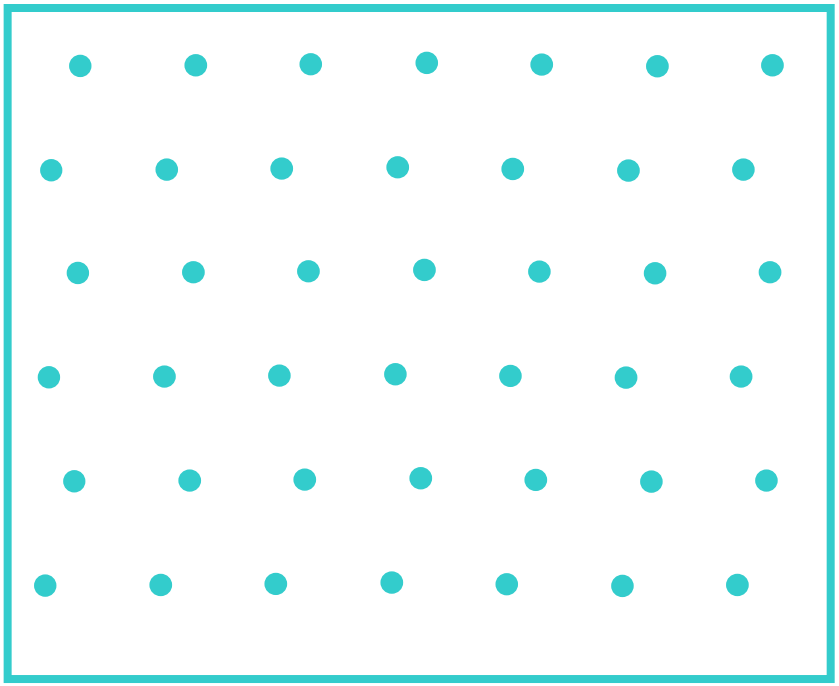
Definitions: Attributes

- s_1, s_2: the "x" and "y" coordinates of the event
- Other values attached to event
  - Type of occurrence
    - Type of building
    - Tree species
    - …
  - Time of occurrence
  - Other characteristics
    - Depth of borehole
    - Age
    - …

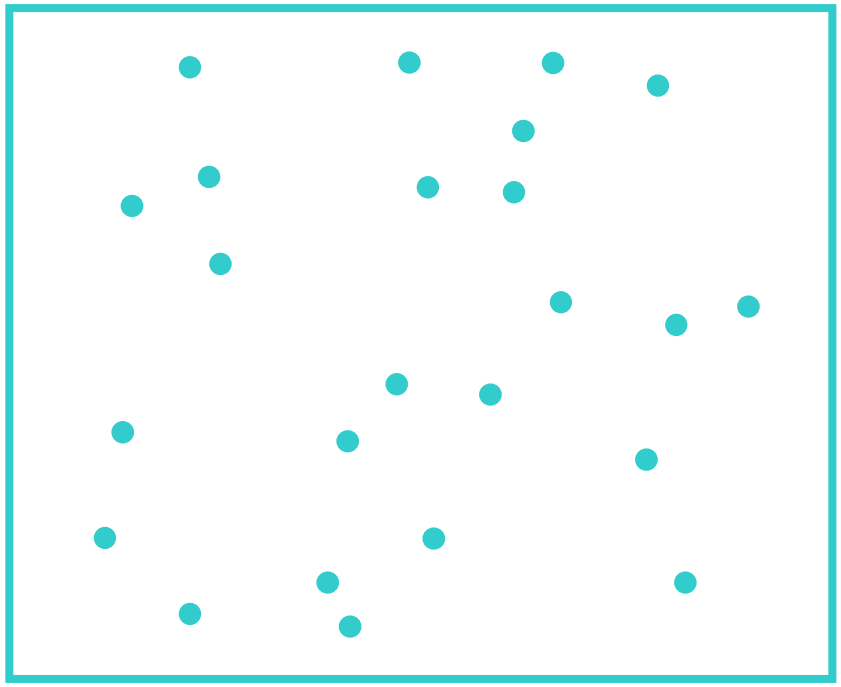
Definitions: Patterns



Definitions: Patterns

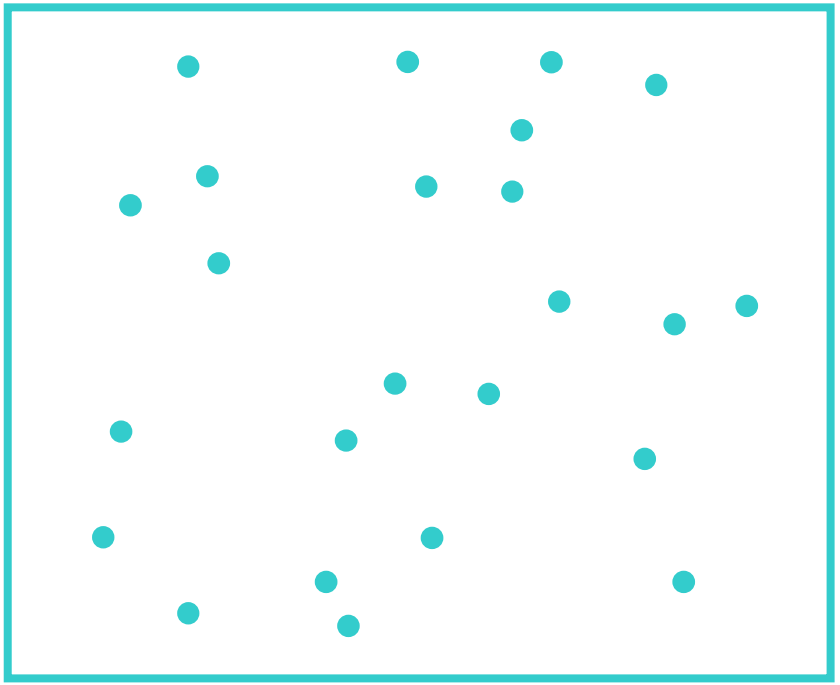


Definitions: Patterns



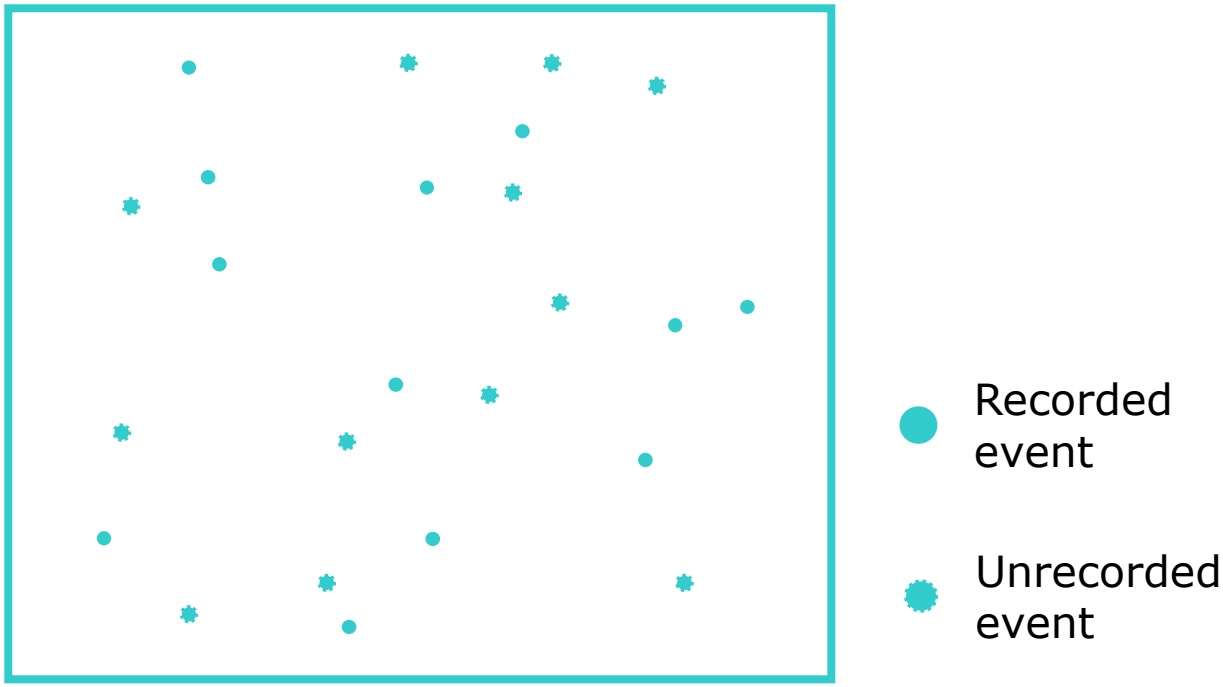
Definitions: Population

- *Mapped* point pattern – A complete map of events in *R*



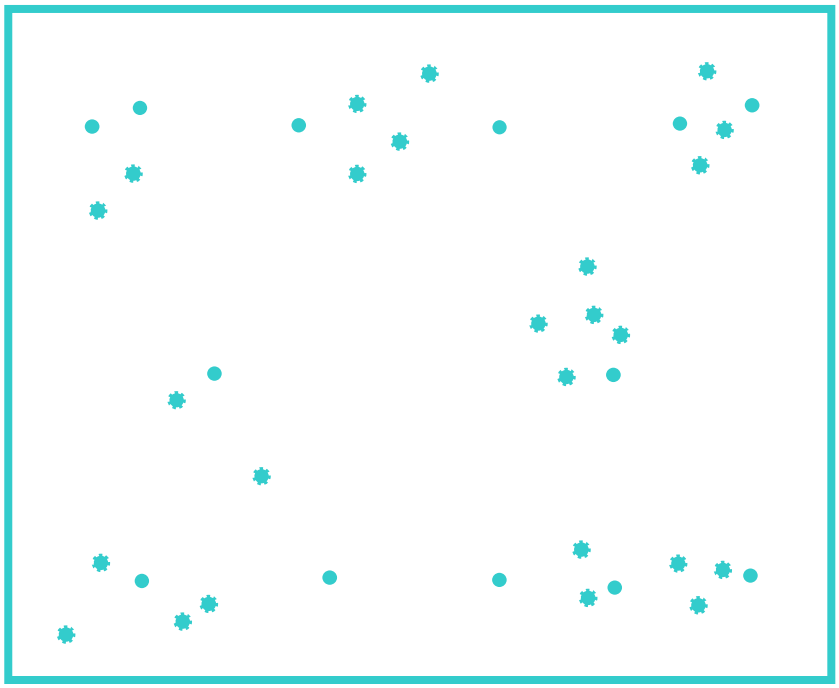
Definitions: Sample

- ***Sampled* point pattern – Some events in *R* have not been recorded**



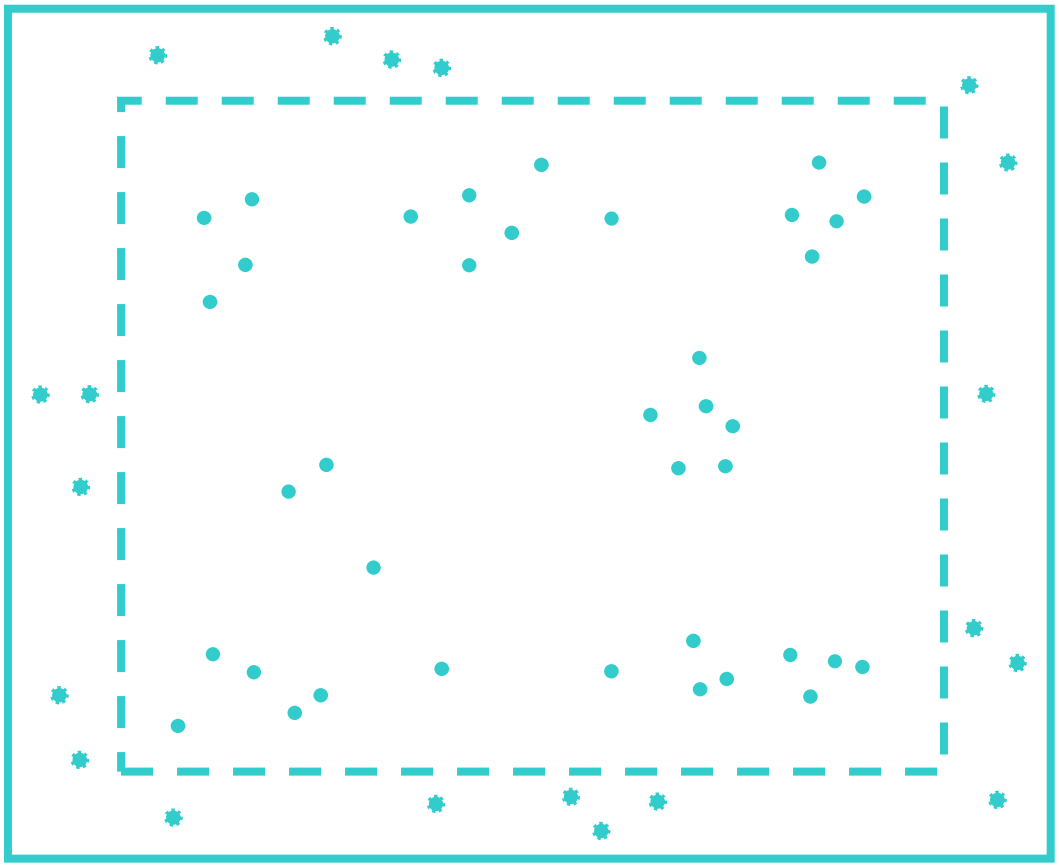
Definitions: Sample

- **Limitations - Example**



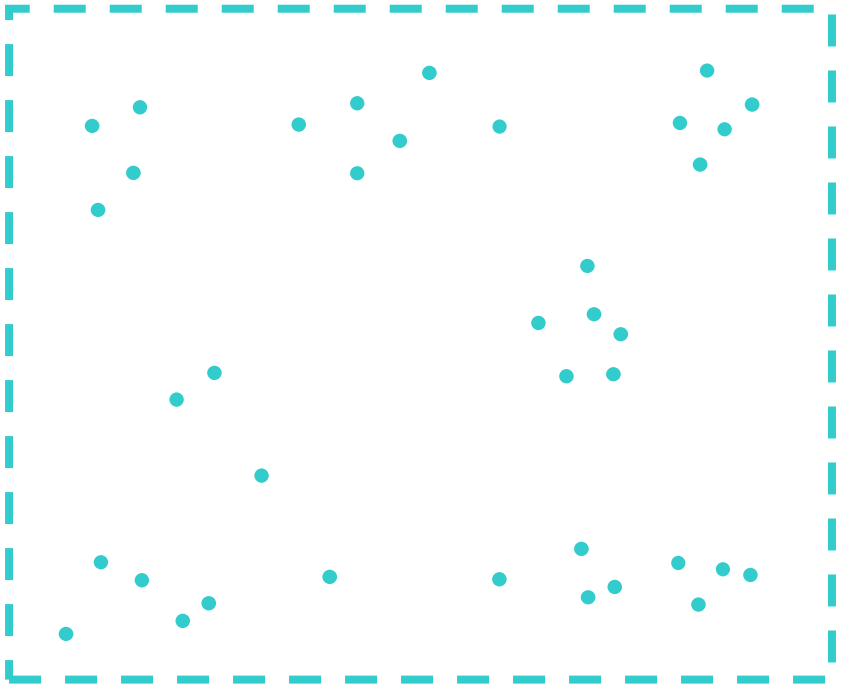
Definitions: Edge Effects

- **Edge Effects**



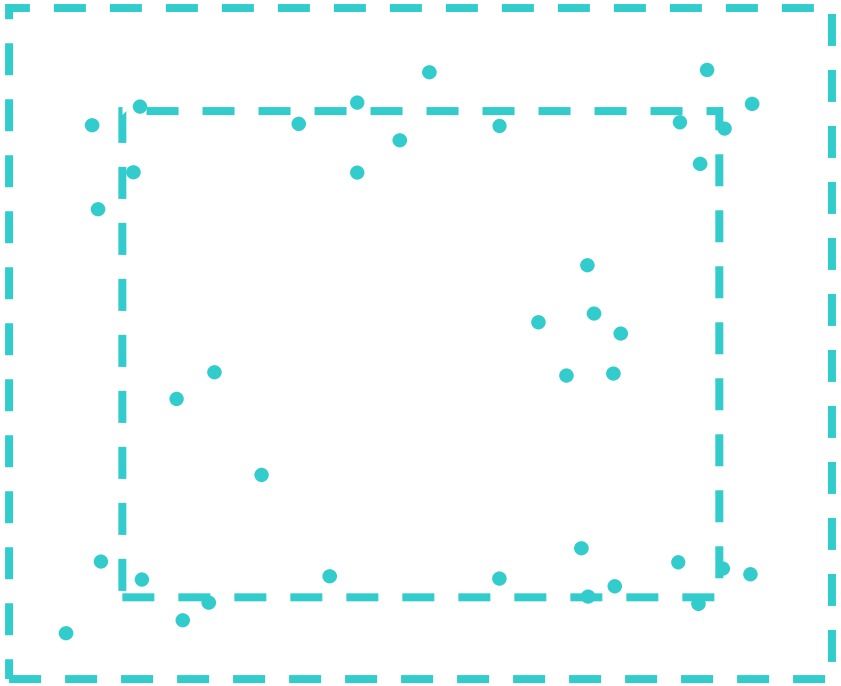
Definitions: Edge Effects

- **Possible Solution – Creating a torus**



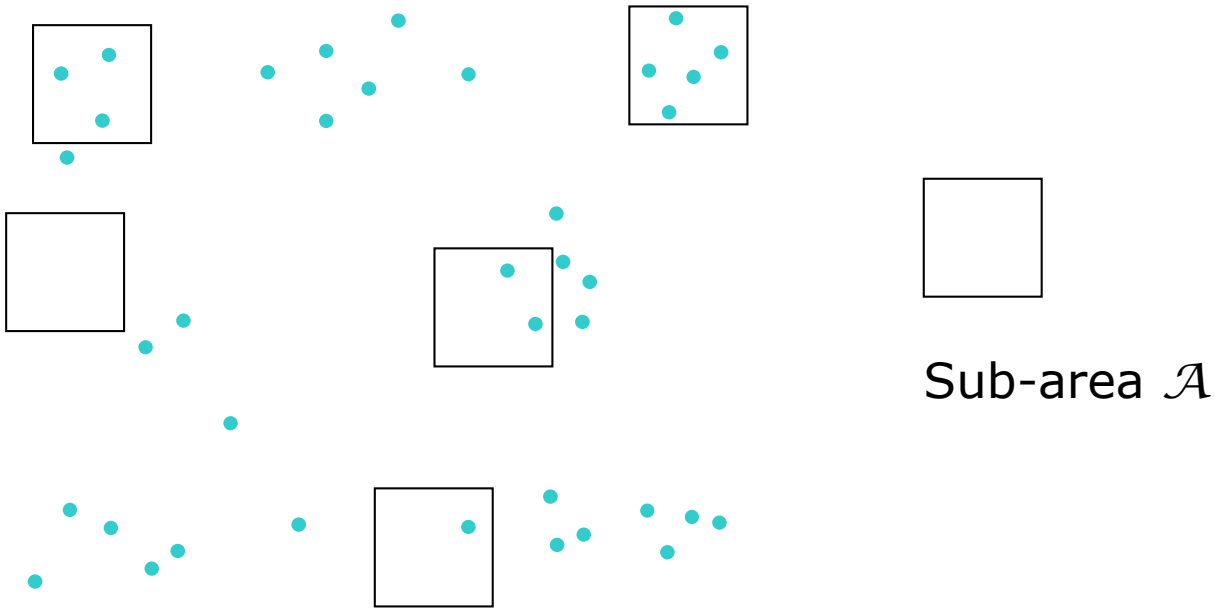
Definitions: Edge Effects

- **Possible Solution – Guard Area**



First & Second Order Properties

- **Number of Events per Unit Area**



First & Second Order Properties

- **Number of Events per Unit Area**
  - Y($\mathcal{A}_i$): Number of events per unit area *i*
  - E[Y($\mathcal{A}$)]: Mean number of events per unit area
  - COV[Y($\mathcal{A}_1$), Y($\mathcal{A}_2$)]: Covariance between $\mathcal{A}_1$ and $\mathcal{A}_2$

First & Second Order Properties

- **Number of Events per Unit Area**

  Depends on the size of $\mathcal{A}$

First order properties

- **Intensity – Mean number of events per unit area at point *s***

$$\lambda(\mathbf{s}) = \lim_{ds \to 0} \left\{ \frac{E[Y(\mathbf{ds})]}{\mathrm{d}s} \right\}$$

- **Stationary process: $\lambda(s)$ is a constant → λ**

Second order properties

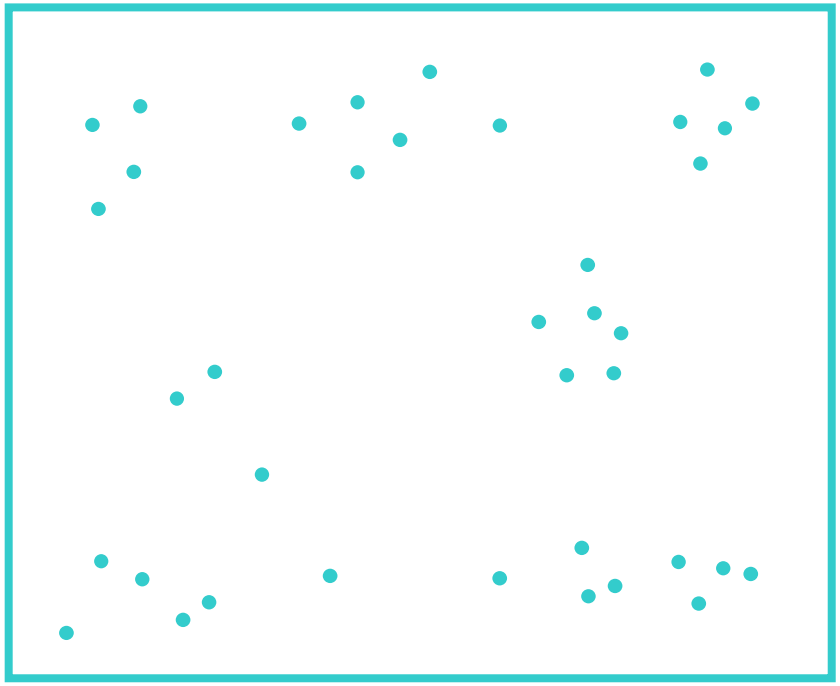
- **Second Order Intensity**

$$\gamma(\mathbf{s}_i, \mathbf{s}_j) = \lim_{ds_i, ds_j \to 0} \left\{ \frac{E[Y(\mathbf{ds}_i)Y(\mathbf{ds}_j)]}{\mathrm{d}s_i \mathrm{d}s_j} \right\}$$

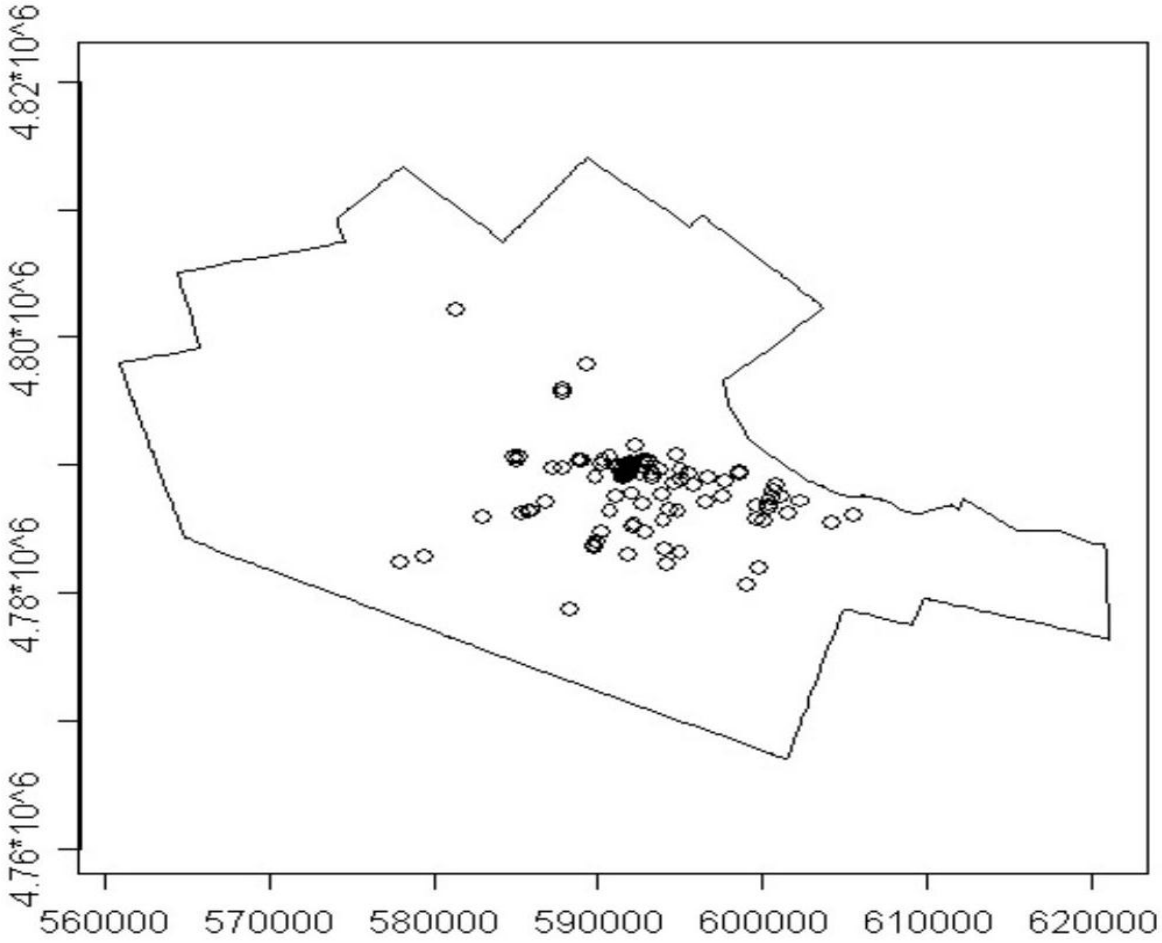
- **Stationary, isotropic process: $\gamma(s)$ depends on distance only**

Visualizing Point Patterns

- Dot Map



Example: Shops in Hamilton, ON



Visualizing Point Patterns

- Dot Map with Attributes

Example: Shops in Hamilton, ON

Example: Cities in Eastern Asia

- Dot Map with Attributes

Example: Cities in Eastern Asia

- Dot Map with Attributes

Visualizing Point Patterns

- Dot Map with Attributes
  - Represent underlying population

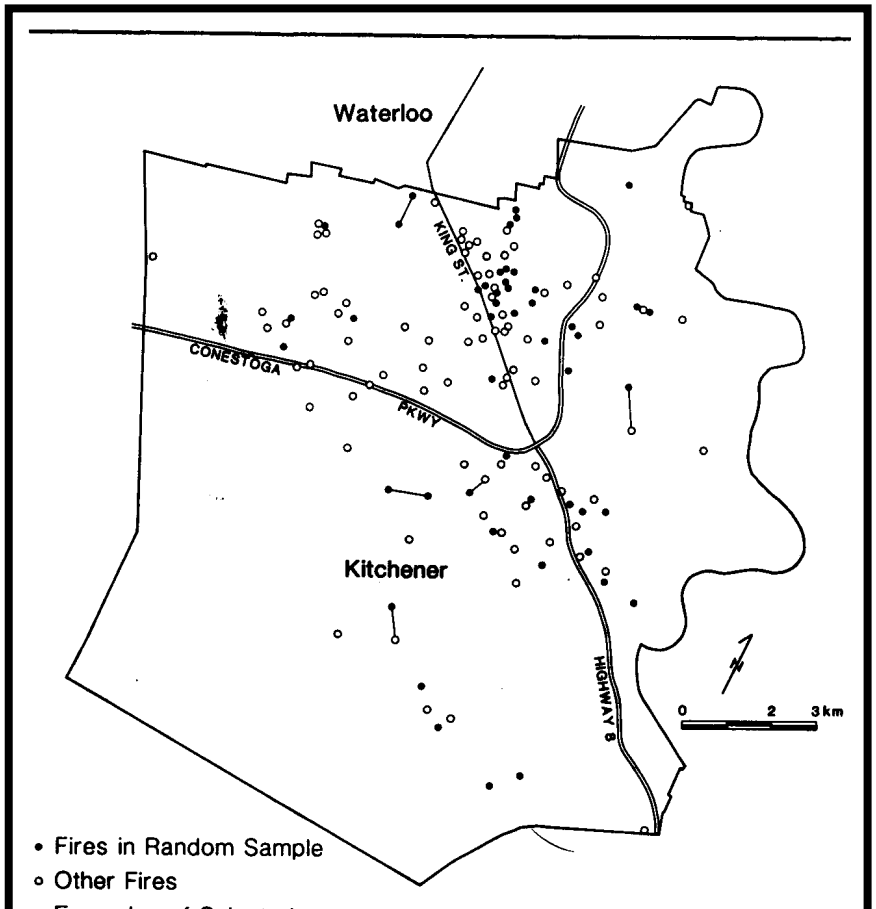


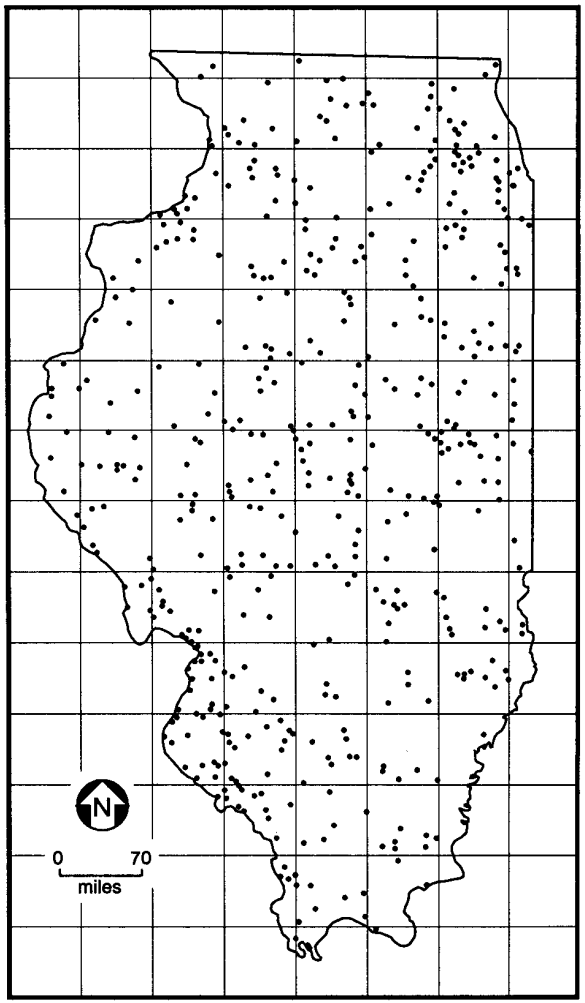
Figure 3.1 Locations of Fires Causing in Excess of $1000 Damage in 1981 in the City of Kitchener, Ontario

Exploring Point Patterns

- Methods
  - Quadrat Counts/Analysis
  - Kernel Estimation

Quadrat Analysis

Figure 5.5 Illinois Tornado Pattern with Quadrats Superimposed



Quadrat Analysis

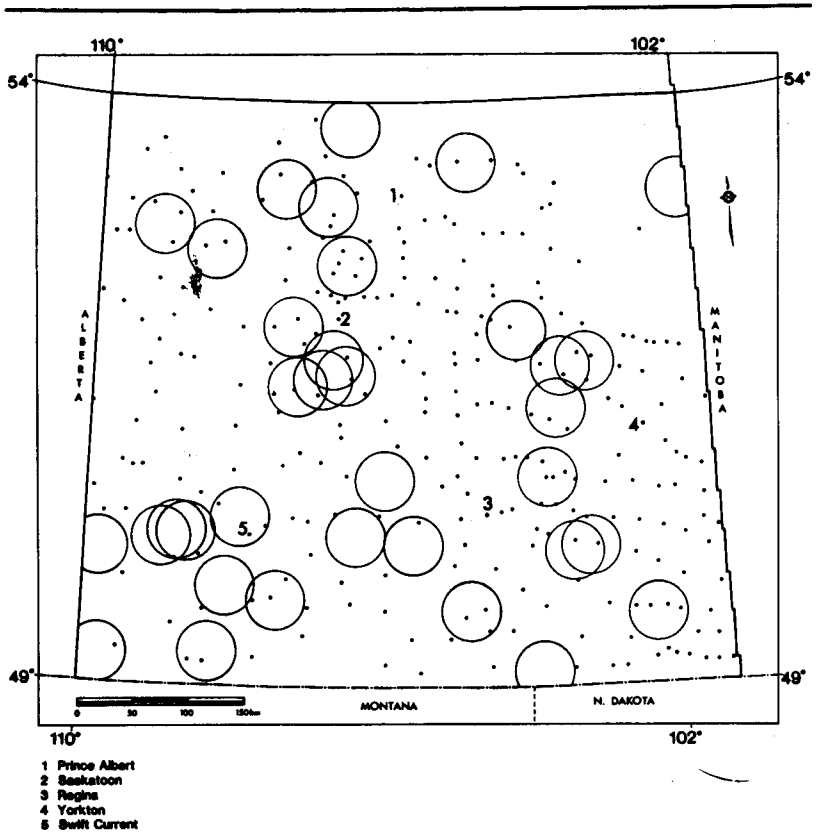


Figure 2.1 Location of Settlements of 300 or Greater Population in 1971 in Southern Saskatchewan

Quadrat Counts – Moving Window

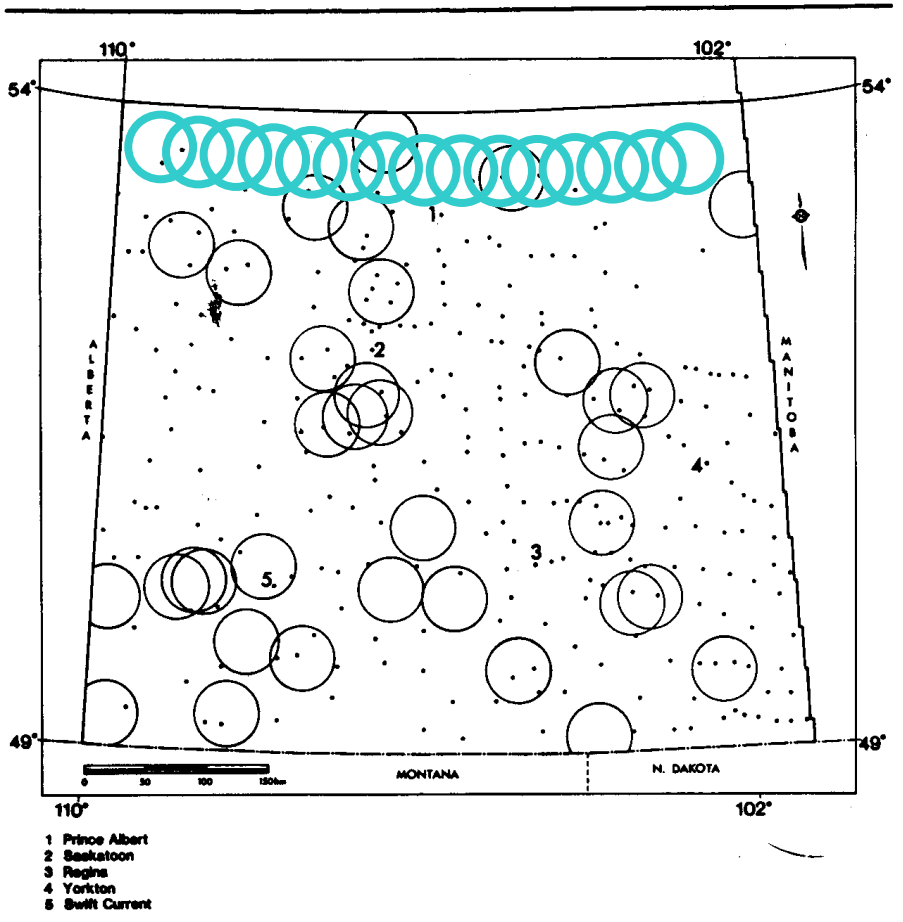
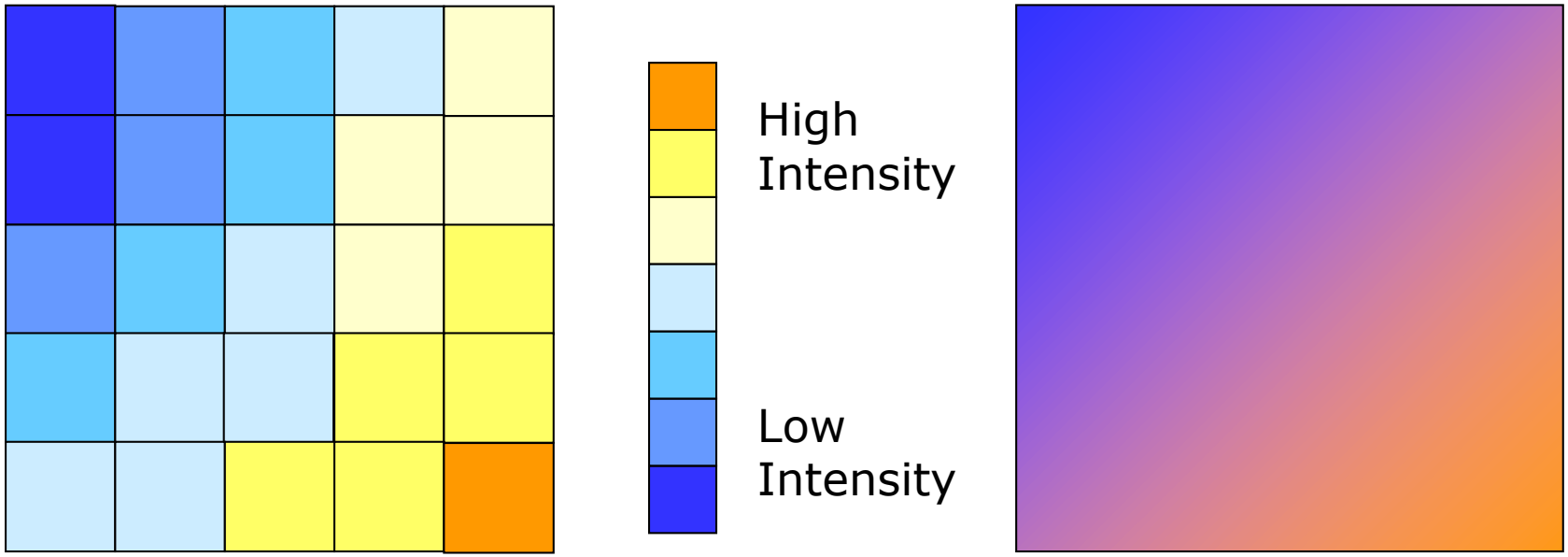


Figure 2.1 Location of Settlements of 300 or Greater Population in 1971 in Southern Saskatchewan

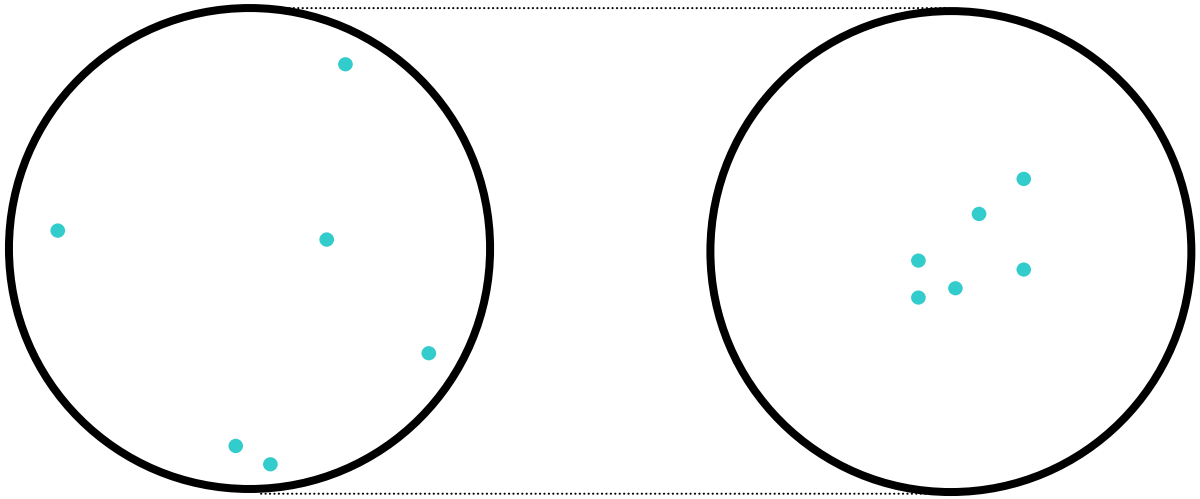
Quadrat Counts – Moving Window

- Smooth variation of intensity $\lambda(\mathbf{s})$ (Good)



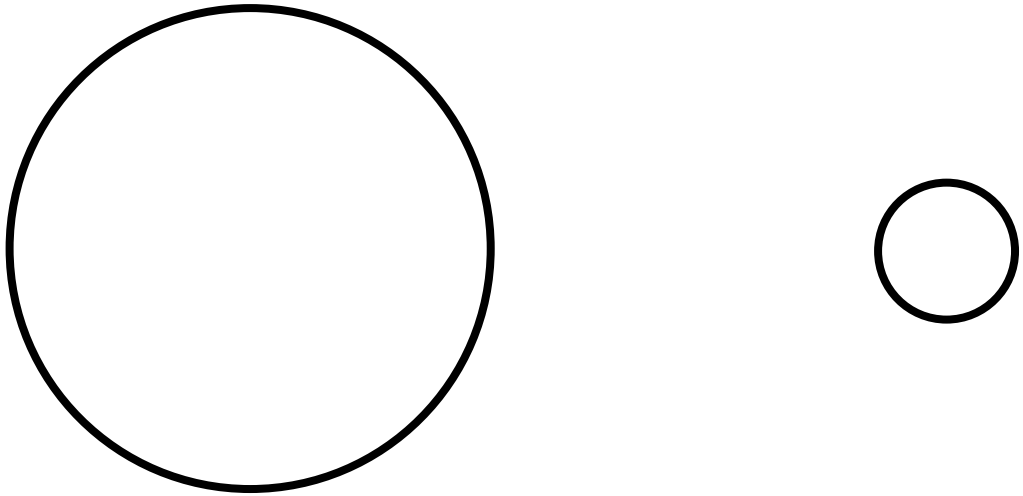
Quadrat Counts – Moving Window

- Relative location of points within window?



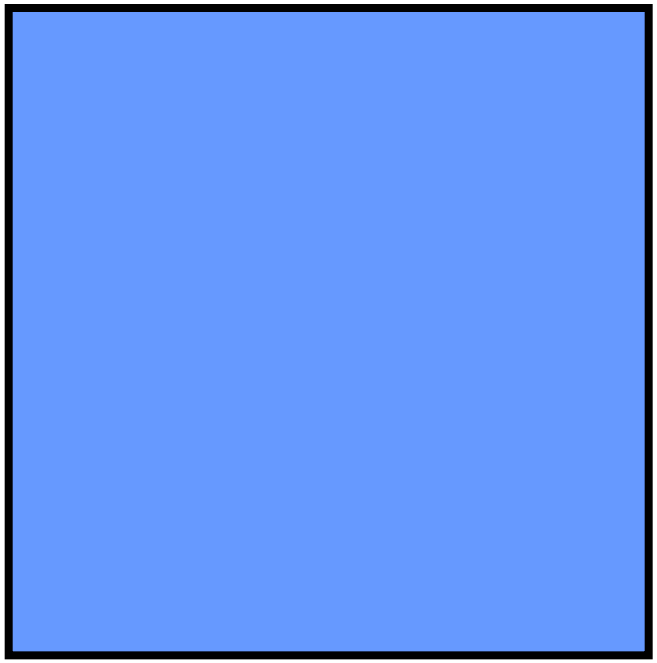
Quadrat Counts – Moving Window

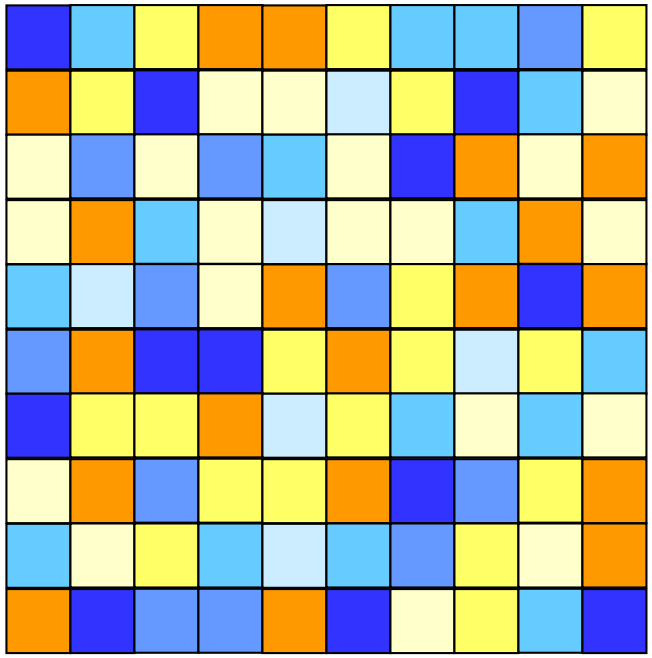
- Size of window?



Quadrat Counts – Moving Window

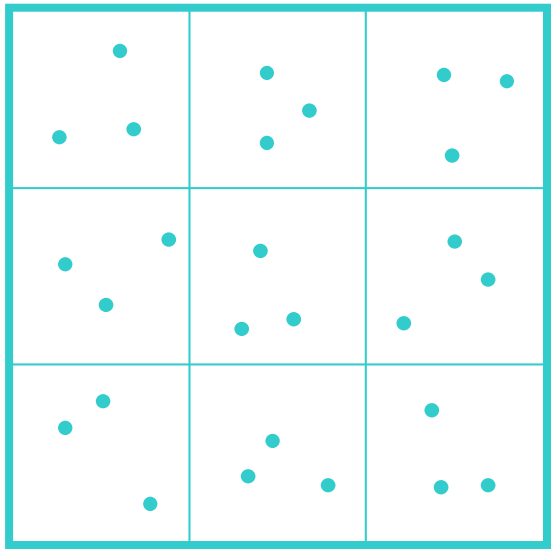
- Large window
- Small window

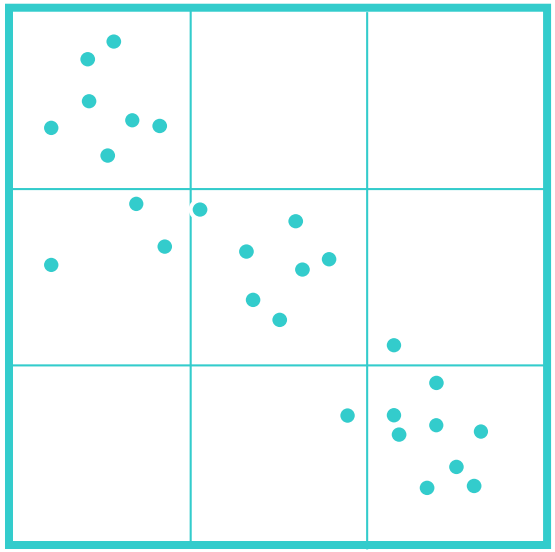


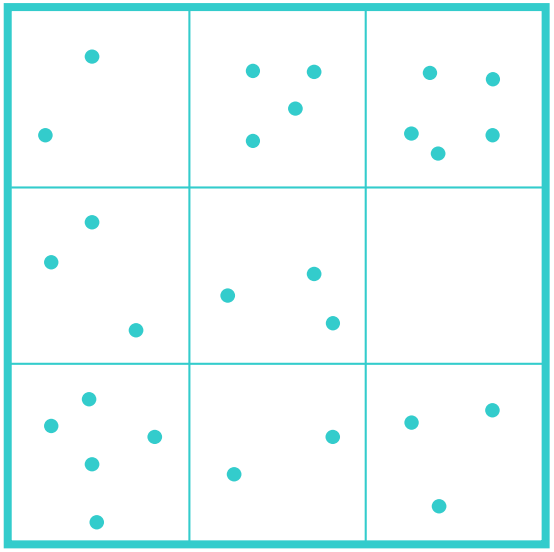


Quadrat Analysis

- Based on the frequency of events occurring in various parts of the region







Quadrat Analysis

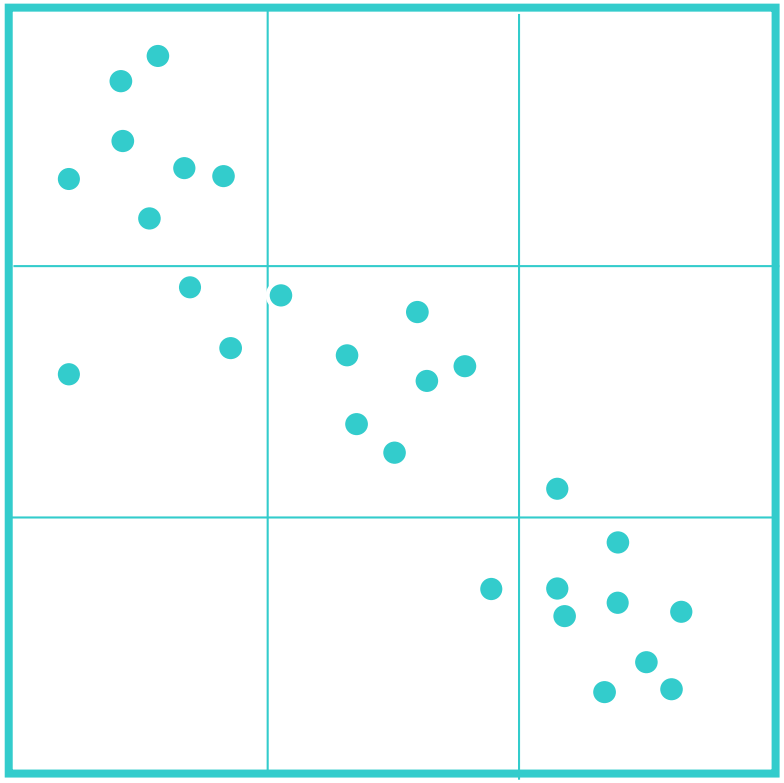
- Mean cell frequency
  - $MEAN = \#$ of events $(n)/\#$ of cells (m)
- Variance of the cell frequencies
  - $$VAR = \frac{\sum f_i X_i^2 - \left(\sum f_i X_i\right)^2 / m}{m-1}$$
- Variance–mean ratio
  - $VMR = VAR / MEAN$

Case 1

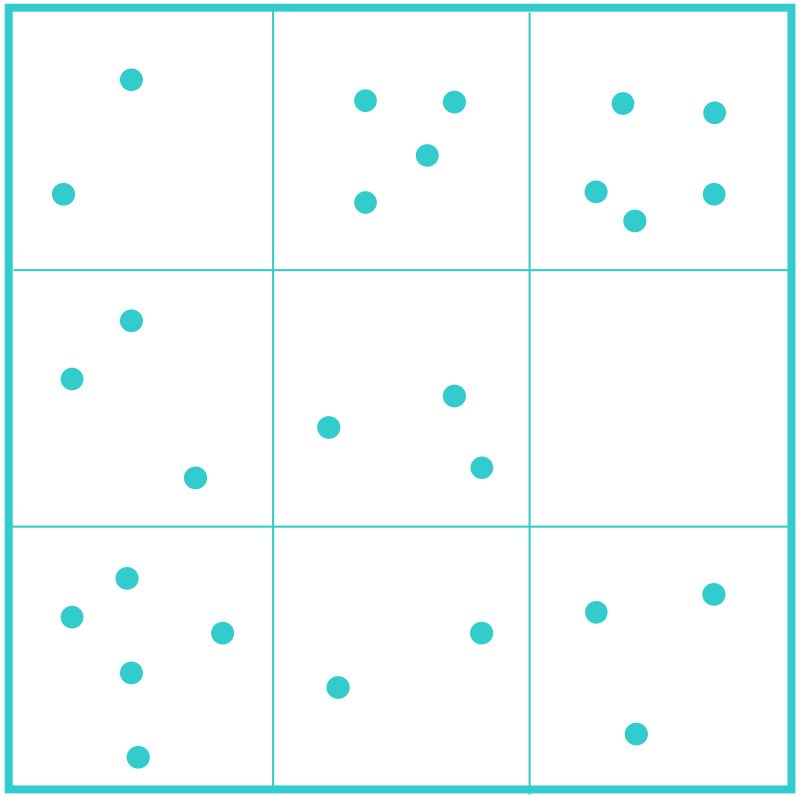
# of Events X	X^2	# of Cells with X Events (f)	fX	fX^2
X=0	0	0	0	0
X=1	1	0	0	0
X=2	4	0	0	0
X=3	9	9	27	81
X=4	16	0	0	0
X=5	25	0	0	0
		Sum=	27	81

$$MEAN = 27/9 = 3 \qquad VAR = \frac{81 - 27^2/9}{9-1} = 0 \qquad VMR = \frac{0}{3} = 0$$

Case 2



Case 3



Quadrat Analysis

- Variance-mean ratio
  - $VMR < 1$: The pattern is more dispersed than random
  - $VMR > 1$: The pattern is more clustered than random
  - $VMR = 1$: The pattern is random

Next…

- Kernel estimation
- Second order properties
- Nearest neighbor analysis
- K Functions