School of Geography and Earth Sciences McMaster University

Applied Spatial Statistics

Point Pattern Analysis III & IV

Last session:

- **Point Pattern Analysis I & II**
  - Definitions
  - First and Second Order Properties
  - Visualizing Point Patterns
  - Exploring Point Patterns
    - Quadrat analysis
    - Kernel Estimation

This session:

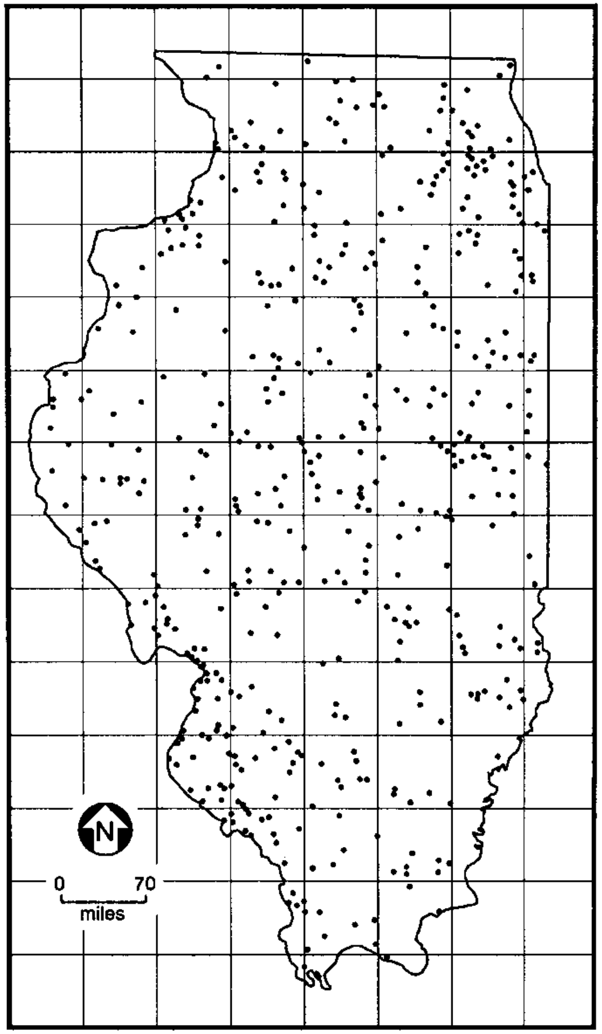
- **Point Pattern Analysis III & IV**
  - Exploring point patterns
    - Quadrat Analysis and Moving Windows
    - Kernel Estimation
    - Second Order Properties
    - Nearest Neighbor Analysis
    - The K Function

First order properties

- Intensity –

  Mean number of events per unit area at point *s*

Figure 5.5 Illinois Tornado Pattern with Quadrats Superimposed



First order properties

- Quadrat analysis
- (Moving windows)
- Kernel estimation

Kernel Estimation

- Smooth estimate of intensity

$$\hat{\lambda}_{\tau}(\mathbf{s}) = \sum_{i=1}^{\overbrace{n}^{\text{Number of events in } R}} \frac{1}{\tau^2} \underbrace{k\left(\frac{(\mathbf{s}-\mathbf{s}_i)}{\tau}\right)}_{\text{Kernel}}$$

Homogeneity, isotropy:
s-**s**$_i$ = h$_i$ (distance between point **s** and event **s**$_i$)

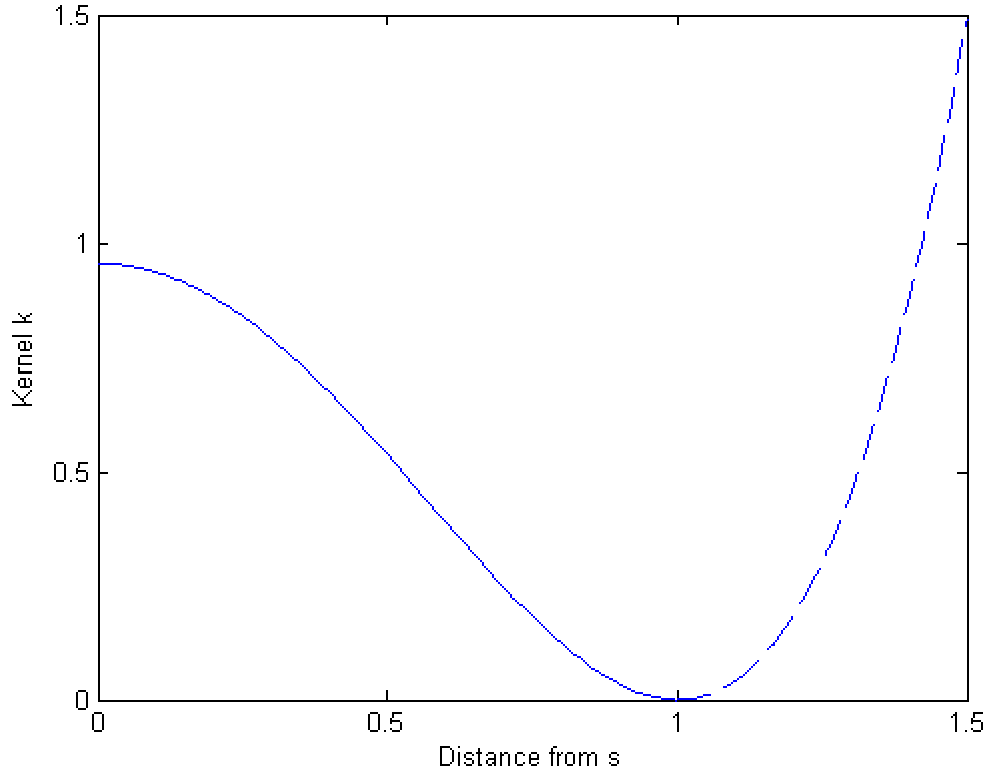
Kernel Estimation

- Kernel function

$$k\left(\frac{h}{\tau}\right) = \begin{cases} \dfrac{3}{\pi}\left(1 - \dfrac{h^2}{\tau^2}\right)^2 & \text{if } \dfrac{h^2}{\tau^2} \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

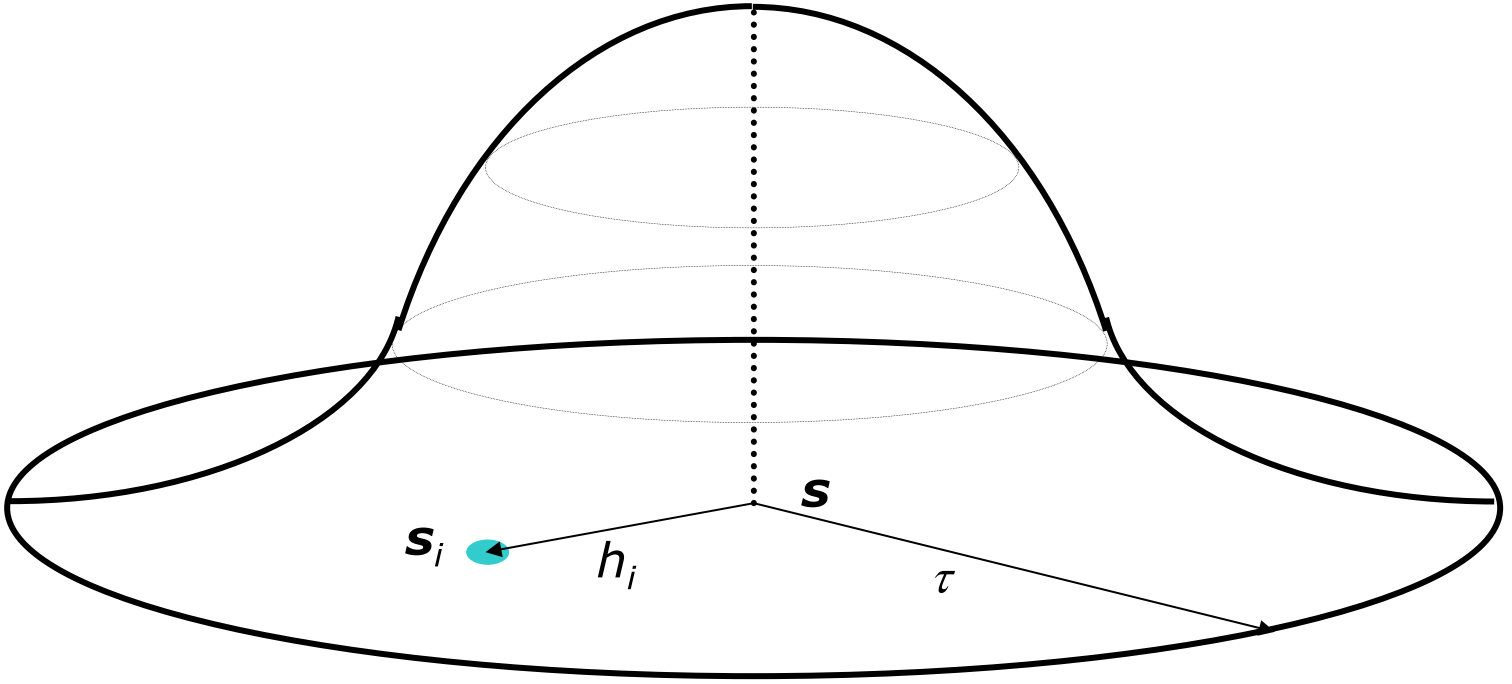
Kernel Estimation

$\tau = 1$



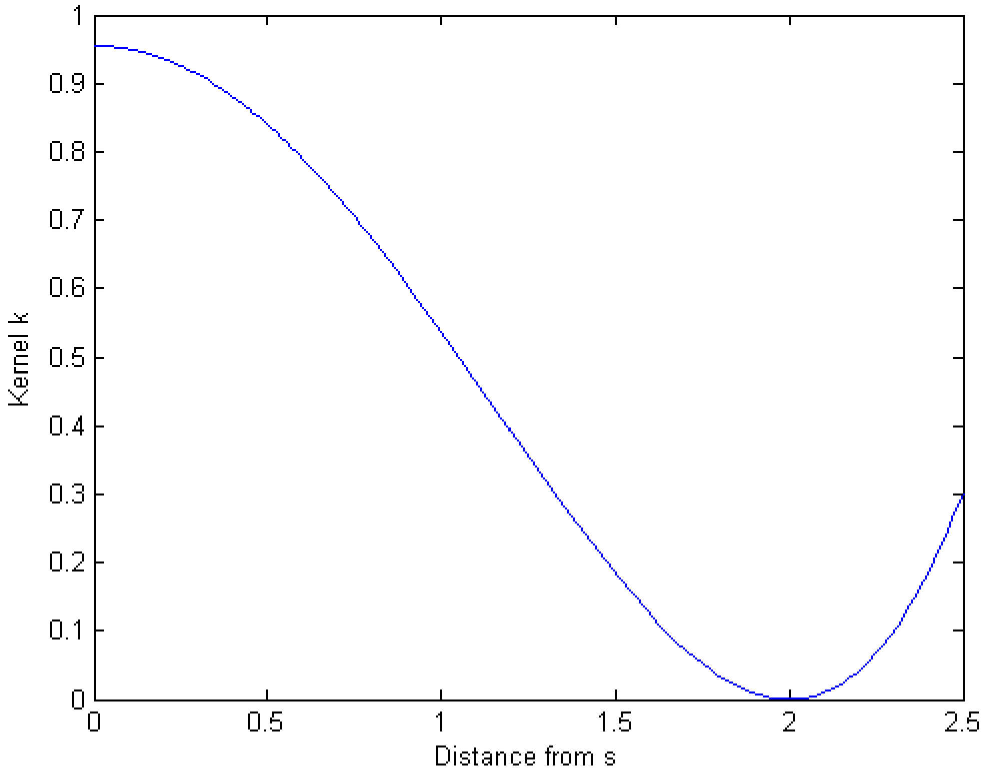
Kernel Estimation

- Kernel function



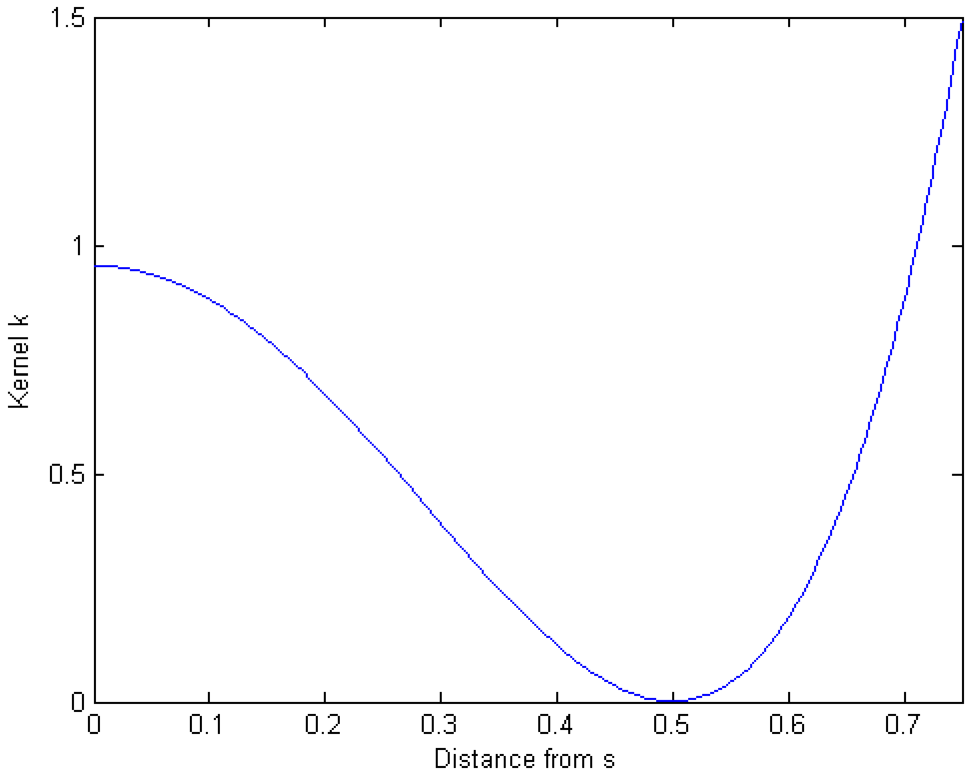
Kernel Estimation

$\tau = 2$

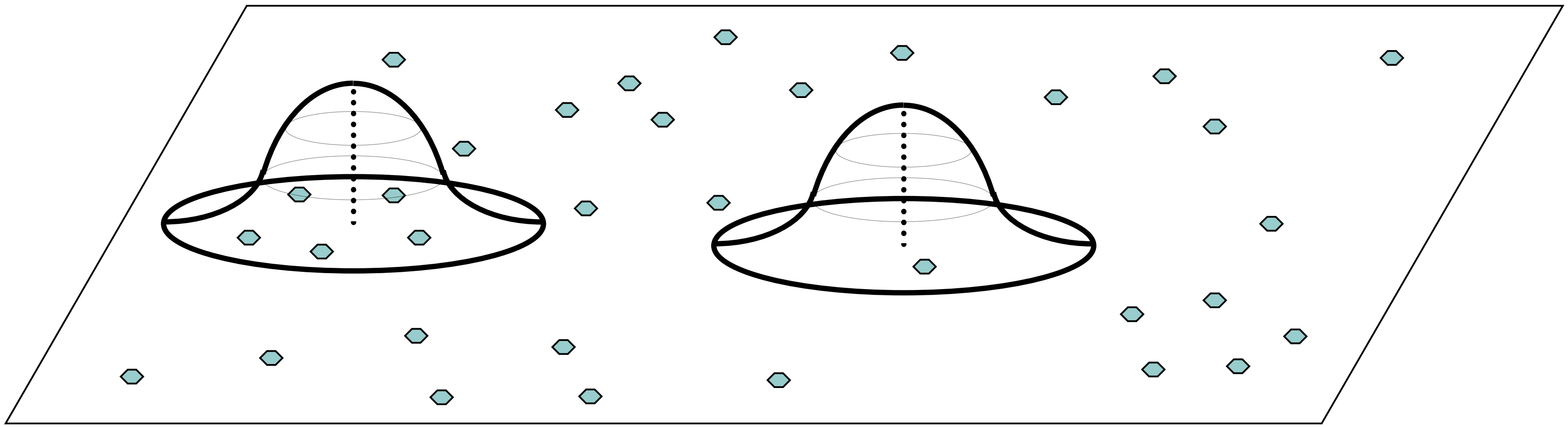


Kernel Estimation

$\tau = 0.5$



Kernel Estimation

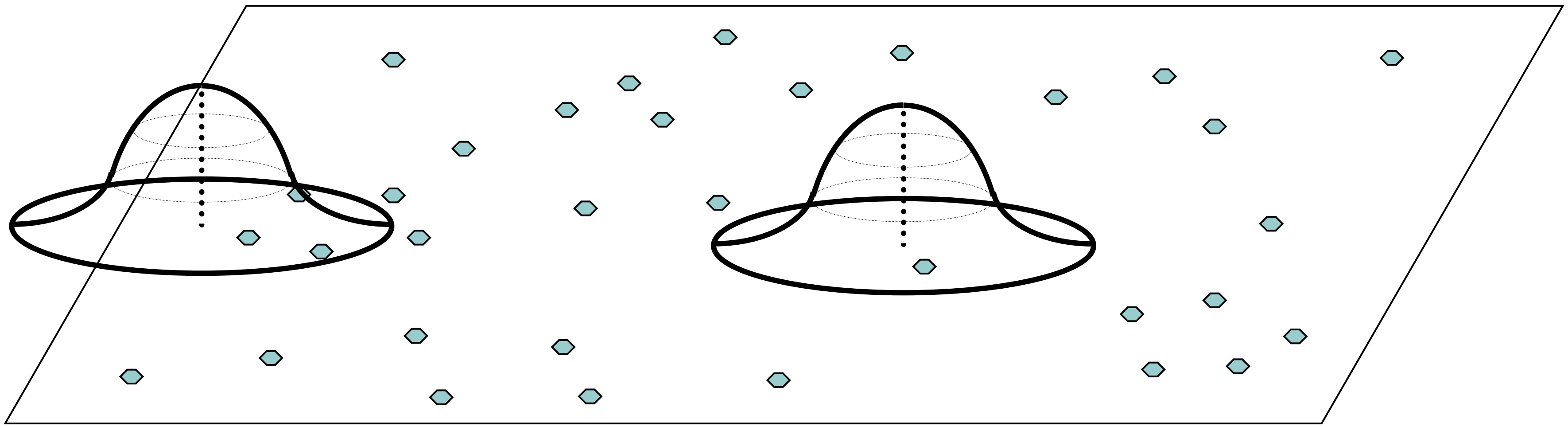


Kernel Estimation

- Relative location of points within window
- Edge effects
- Window size

Kernel estimation

- Edge effects?



Kernel estimation

- Edge correction

$$\hat{\lambda}_\tau(\mathbf{s}) = \frac{1}{\delta_\tau(\mathbf{s})} \sum_{i=1}^{n} \frac{1}{\tau^2} k\left(\frac{(\mathbf{s} - \mathbf{s}_i)}{\tau} \right)$$

$$\delta_\tau(\mathbf{s}) = \underbrace{\int_{\mathcal{R}} \frac{1}{\tau^2} k\left(\frac{(\mathbf{s} - \mathbf{s}_i)}{\tau} \right) \mathrm{d}\mathbf{s}}$$

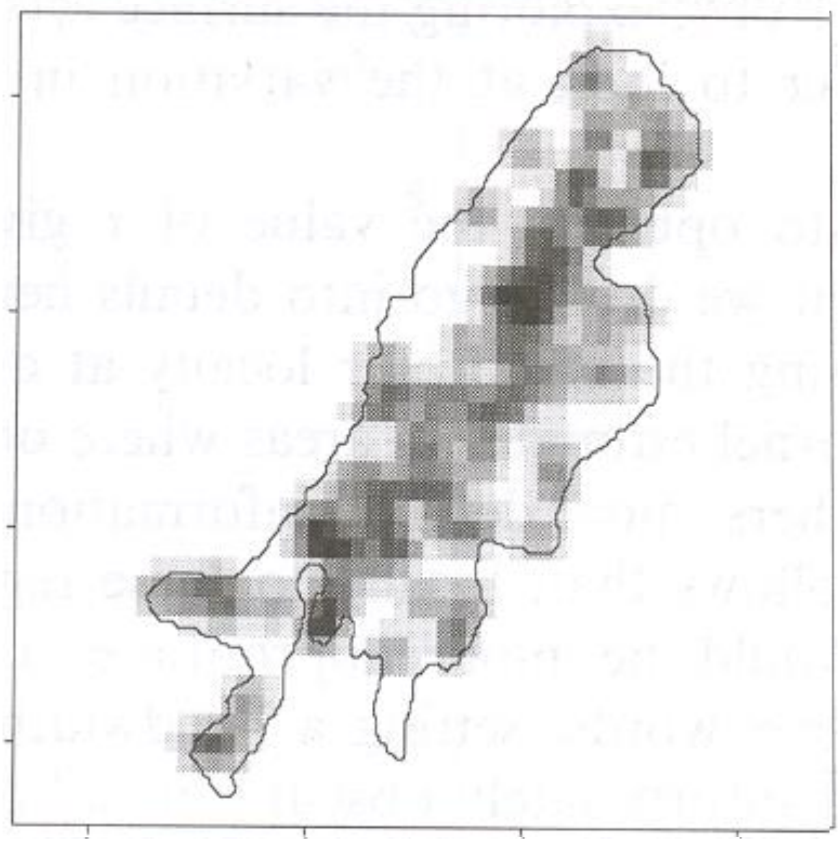
Volume of kernel that is within $\mathcal{R}$

Kernel estimation

- Window size?
  - Visualization and exploration
  - Distribution of events

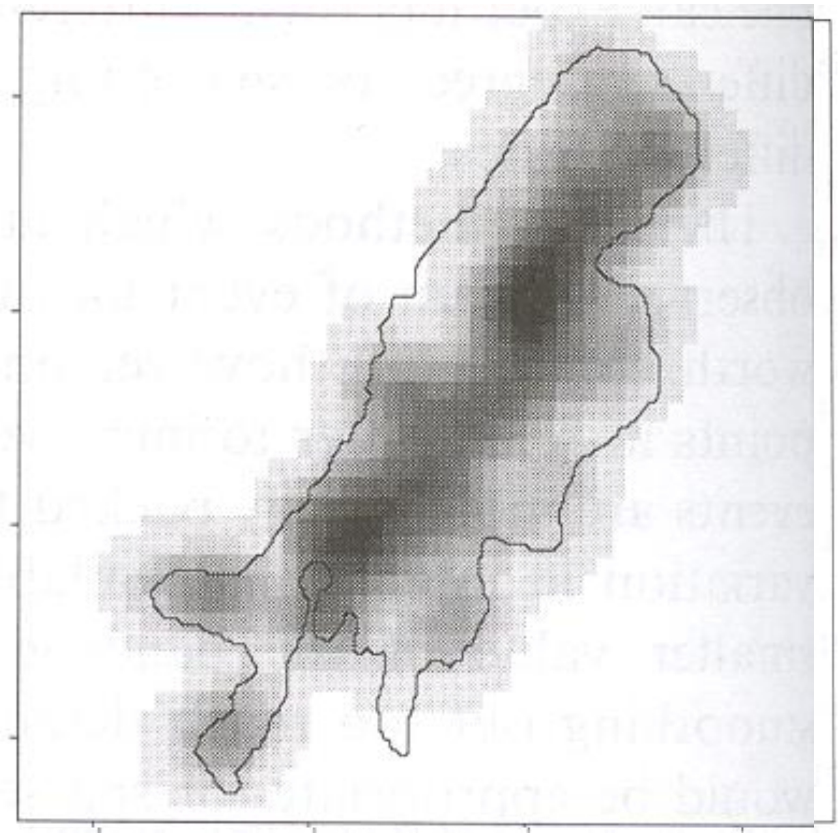
Kernel estimation

- Example: Volcanic craters in Uganda (τ=100)



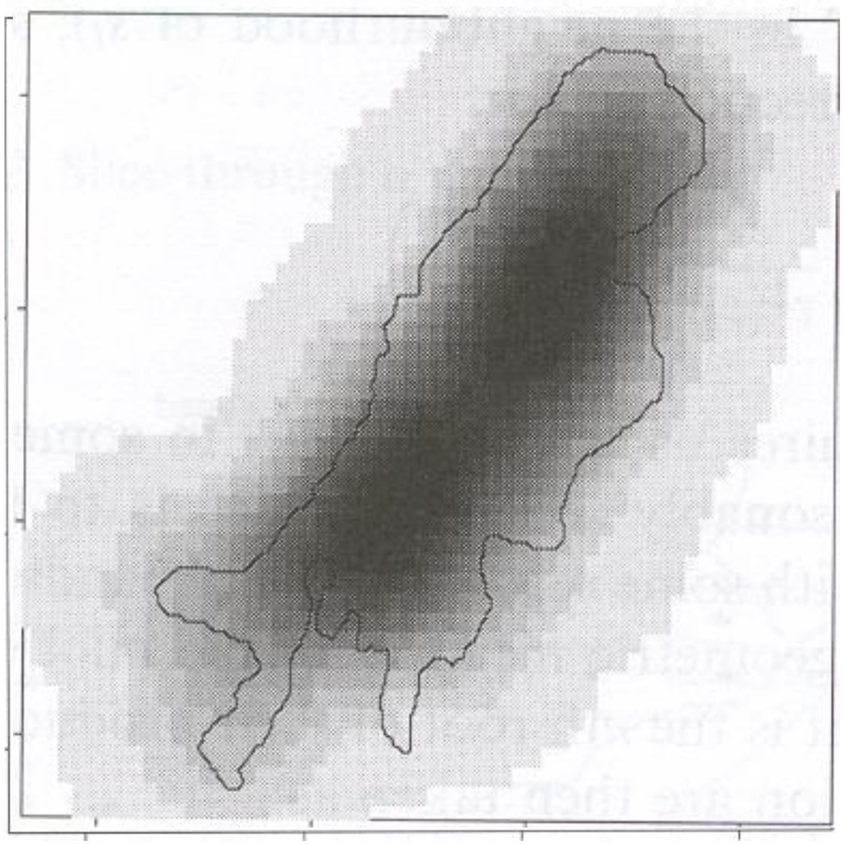
Kernel estimation

- Example: Volcanic craters in Uganda (τ=220)



Kernel estimation

- Example: Volcanic craters in Uganda (τ=500)



Kernel estimation

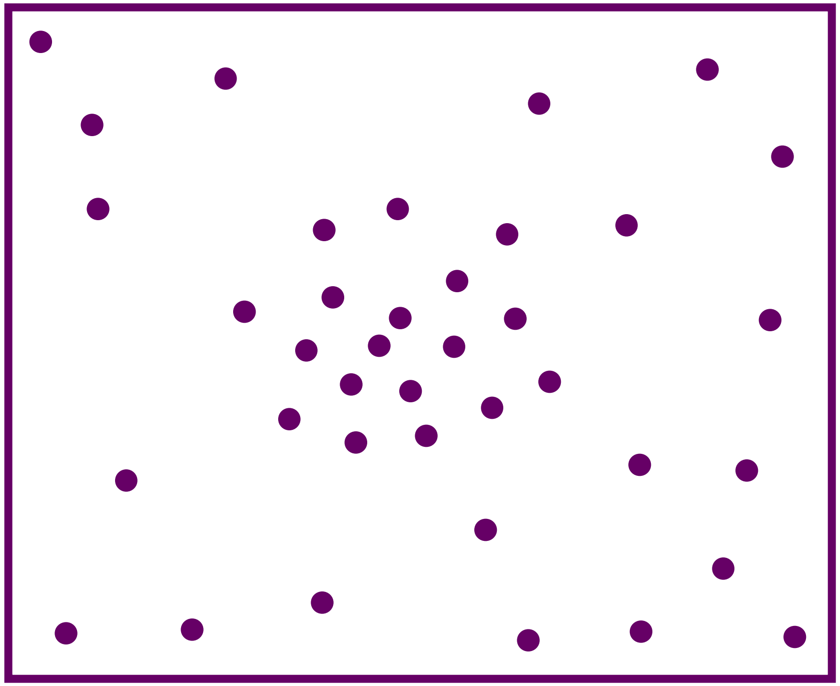
- Window size (Visualization and exploration)
  - Rule of thumb (for $\mathcal{R}$ square unit)

$$\tau = 0.68n^{-0.2}$$

(This must be scaled appropriately)

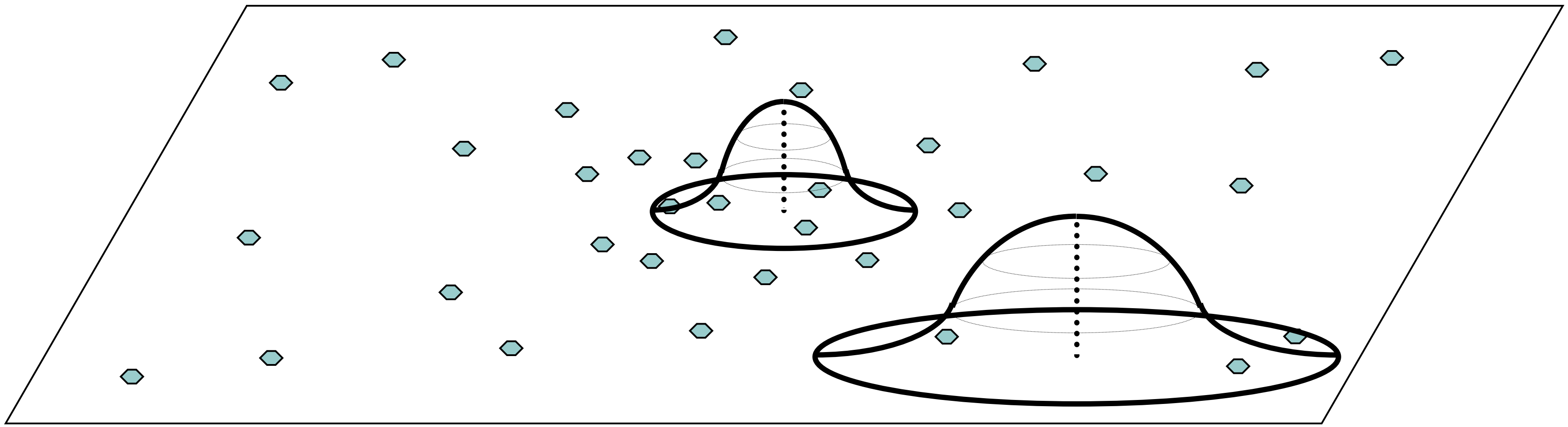
Kernel estimation

- Window size



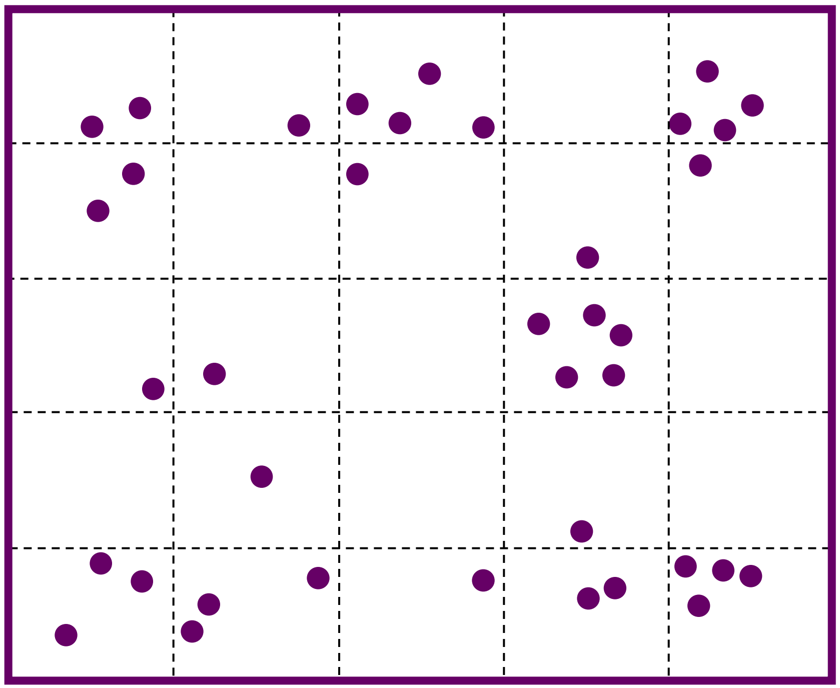
Kernel estimation

- Adaptive kernel bandwidths



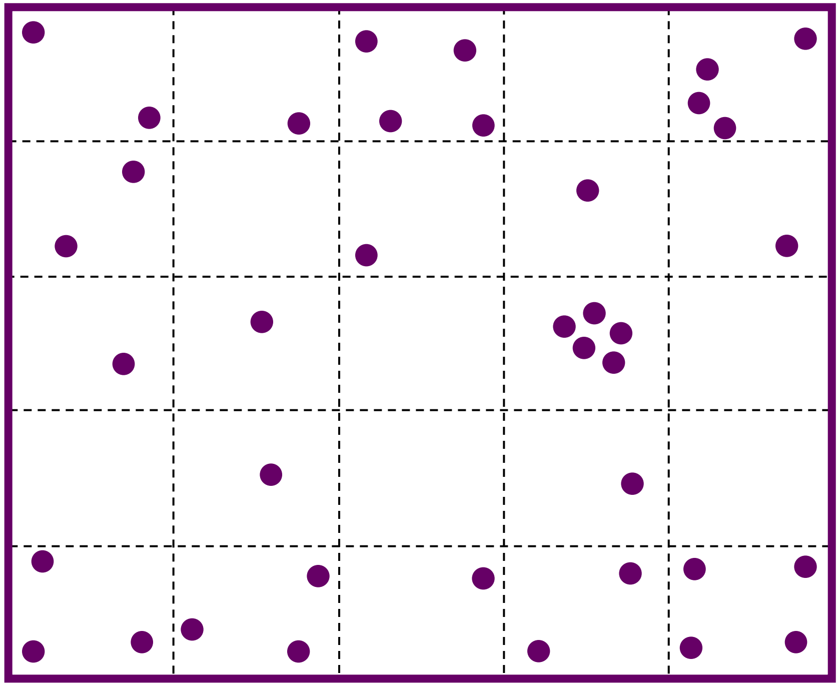
Second order properties

- Small scale variation



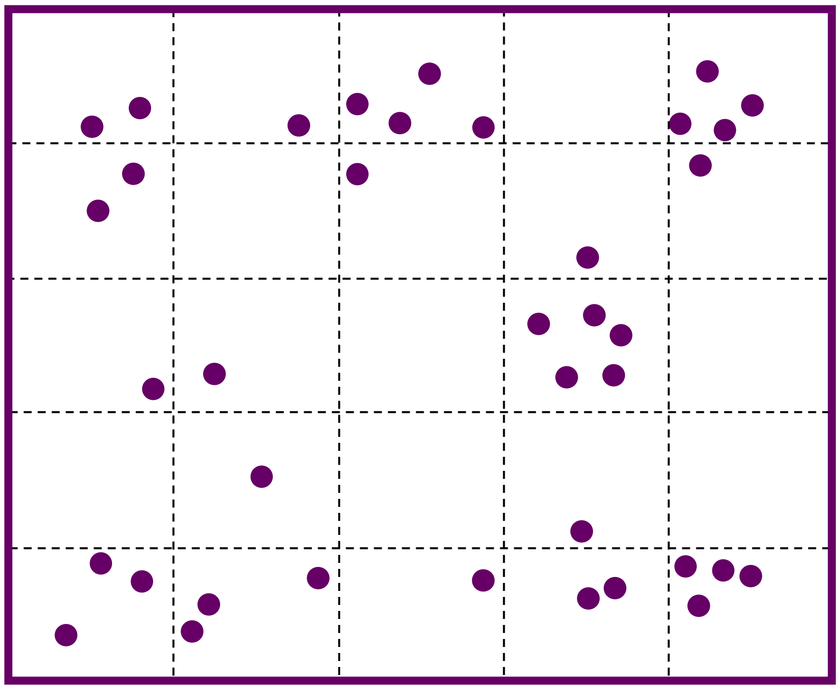
Second order properties

- Small scale variation



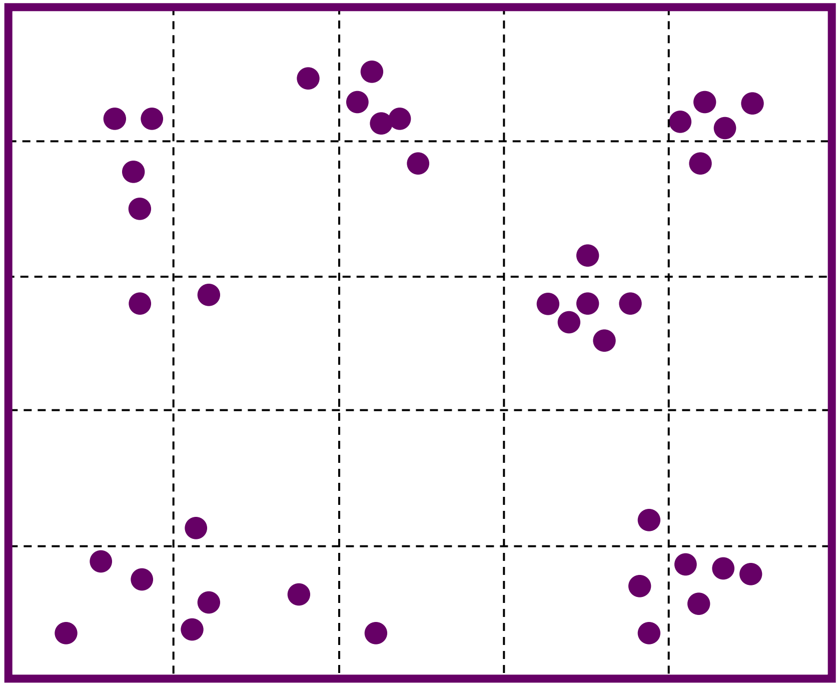
Second order properties

- Small scale variation



Second order properties

- Small scale variation



Second order properties

- Nearest neighbor analysis
- K functions

Nearest neighbor analysis

- Event-Event nearest neighbor analysis
  - Applicable only to *mapped* point patterns
- Point-Event nearest neighbor analysis
  - Applicable to *mapped* point patterns or for sampling purposes

Nearest neighbor analysis

- Event-Event nearest neighbor analysis (distribution function)

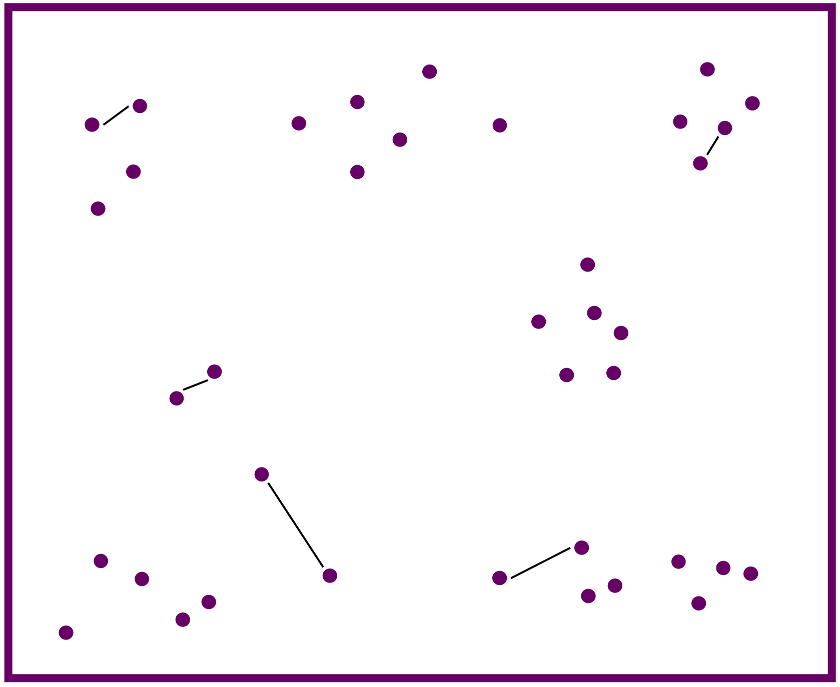
$$\hat{G}(w) = \frac{\#(w_i \leq w)}{n}$$

w_i : Distance from event *i* to nearest neighbor
w : Distance
n : Number of events

Q: What is the range of possible values for $\hat{G}(w)$?

Nearest neighbor analysis

- Event-Event nearest neighbor analysis



Nearest neighbor analysis

- Event-Event nearest neighbor analysis (distribution function)

$$\hat{G}(w) = \frac{\#(w_i \leq w)}{n}$$

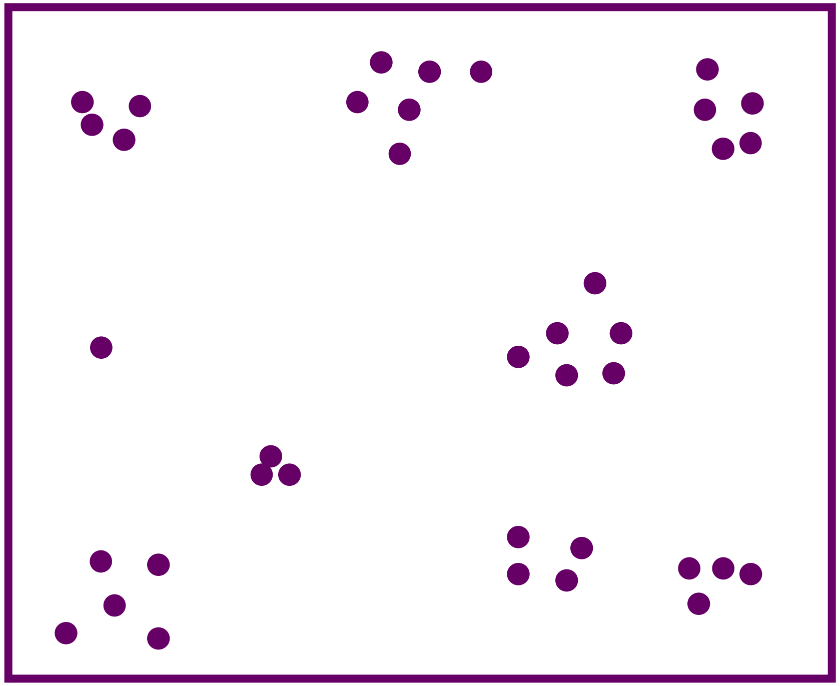
Q: What is the range of possible values for $\hat{G}(w)$?

Nearest neighbor analysis

- Distribution function

Nearest neighbor analysis

- Clustering?

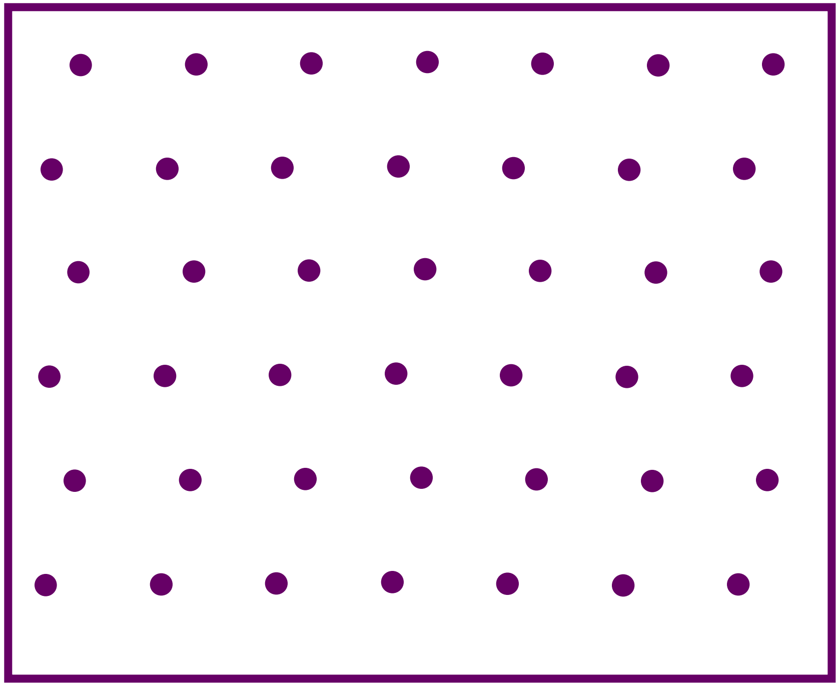


Nearest neighbor analysis

- Clustering?

Definitions: Patterns

- Regularity?

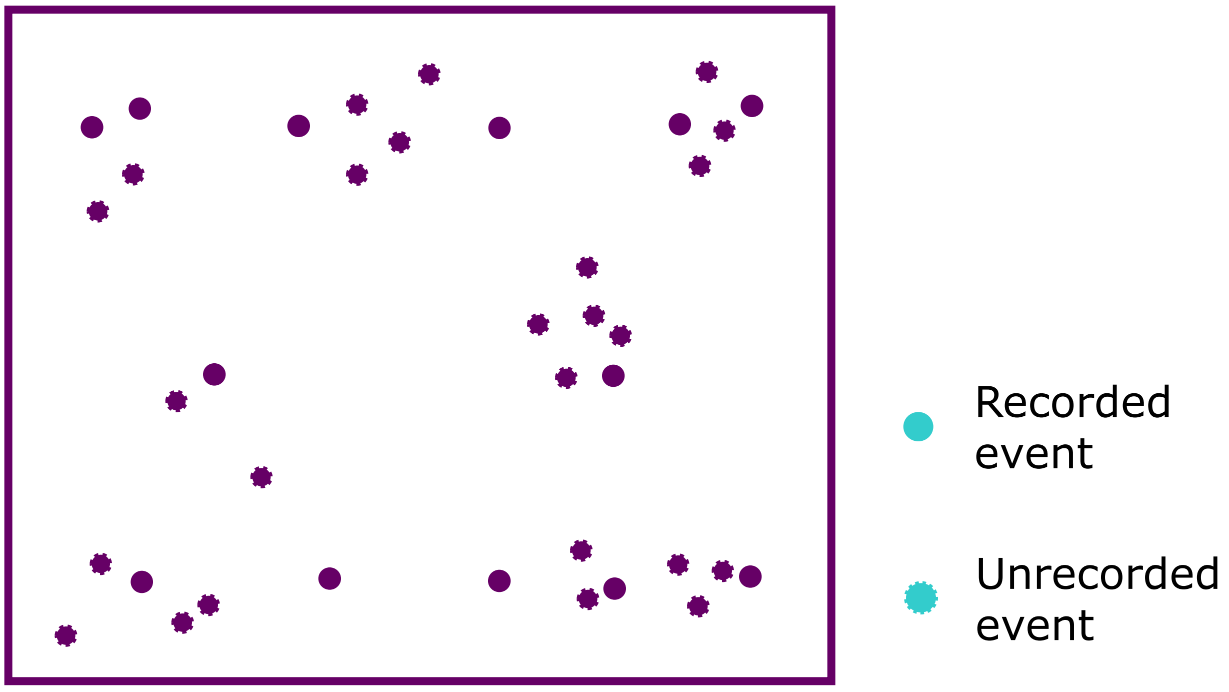


Nearest neighbor analysis

- Regularity?

Nearest neighbor analysis

- Sampled point pattern



Nearest neighbor analysis

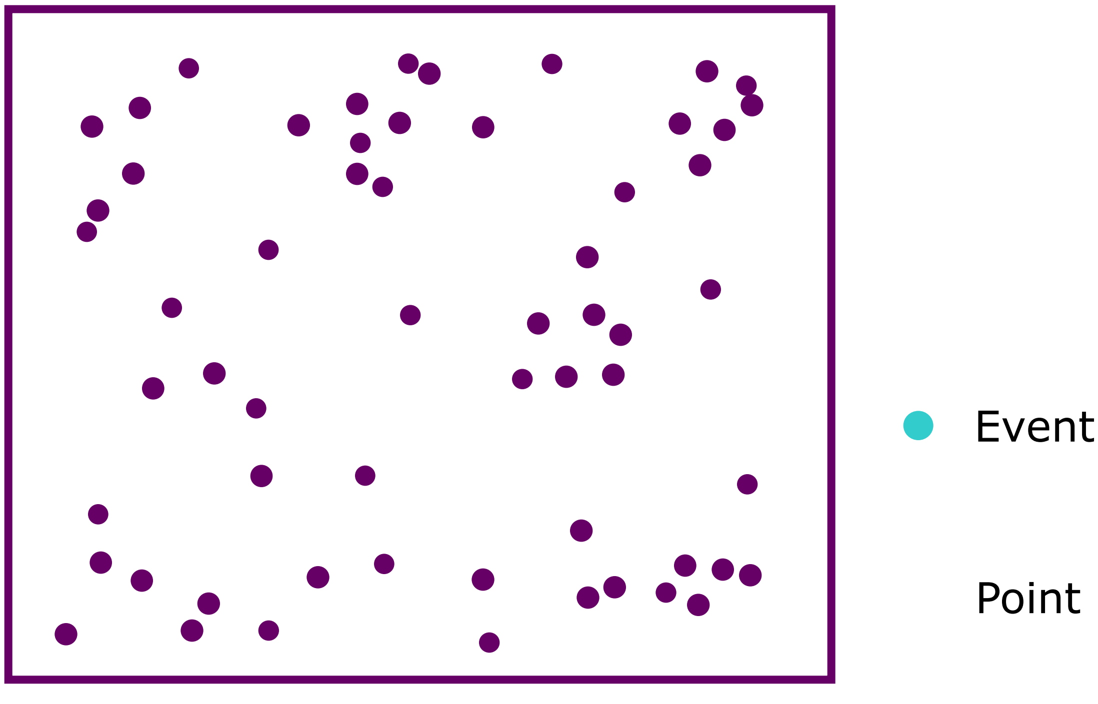
- Point-Event nearest neighbor analysis

$$\hat{F}(x) = \frac{\#(x_i \leq x)}{m}$$

x_i : Distance from point *i* to nearest event
x : Distance
m : Number of points

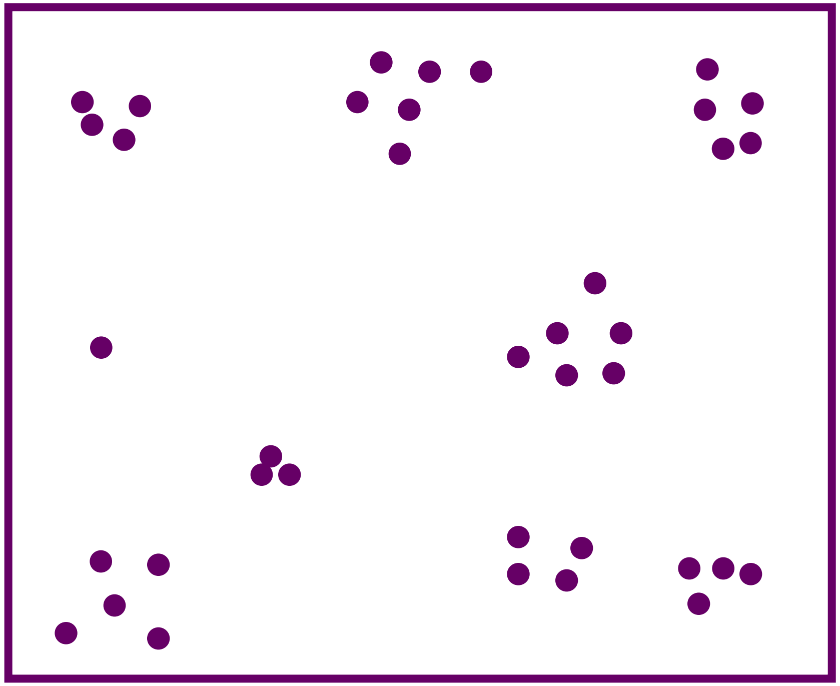
Nearest neighbor analysis

- Point-Event nearest neighbor analysis



Nearest neighbor analysis

- Clustering?

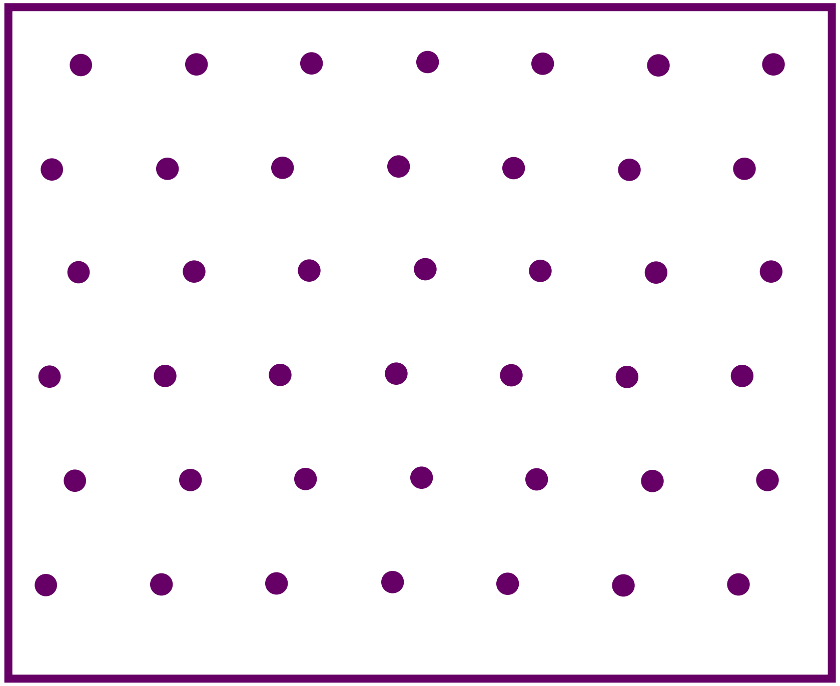


Nearest neighbor analysis

- Clustering?

Definitions: Patterns

- Regularity?

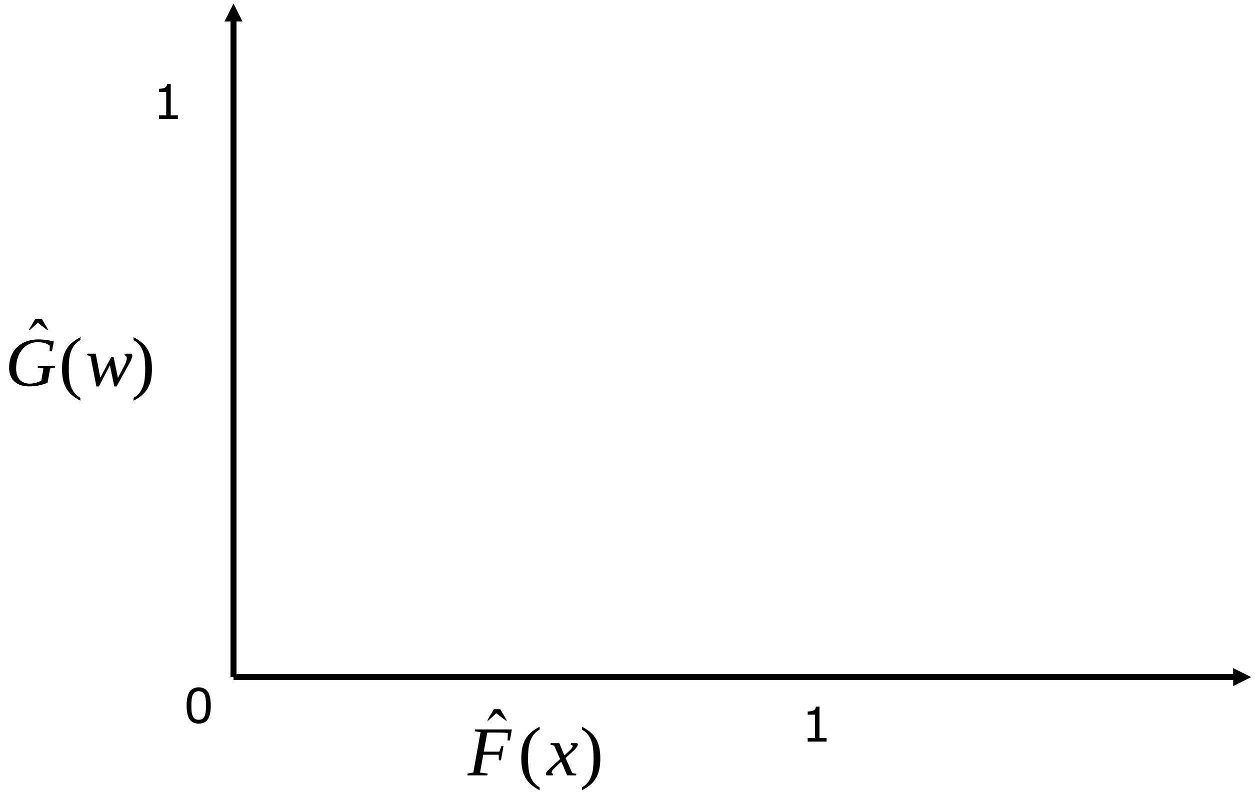


Nearest neighbor analysis

- Regularity?

Nearest neighbor analysis

- Check for random patterns



Nearest neighbor analysis

- Example

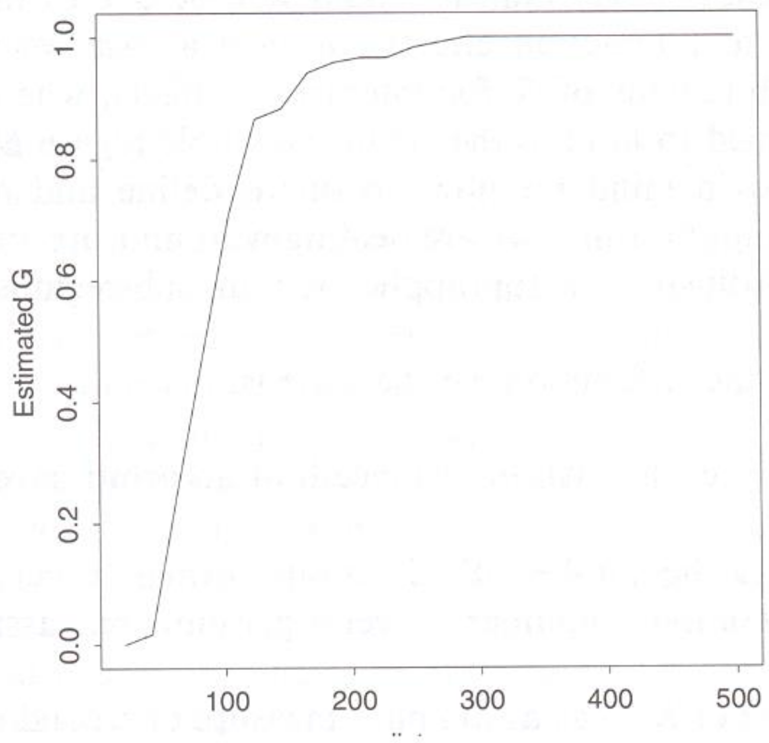
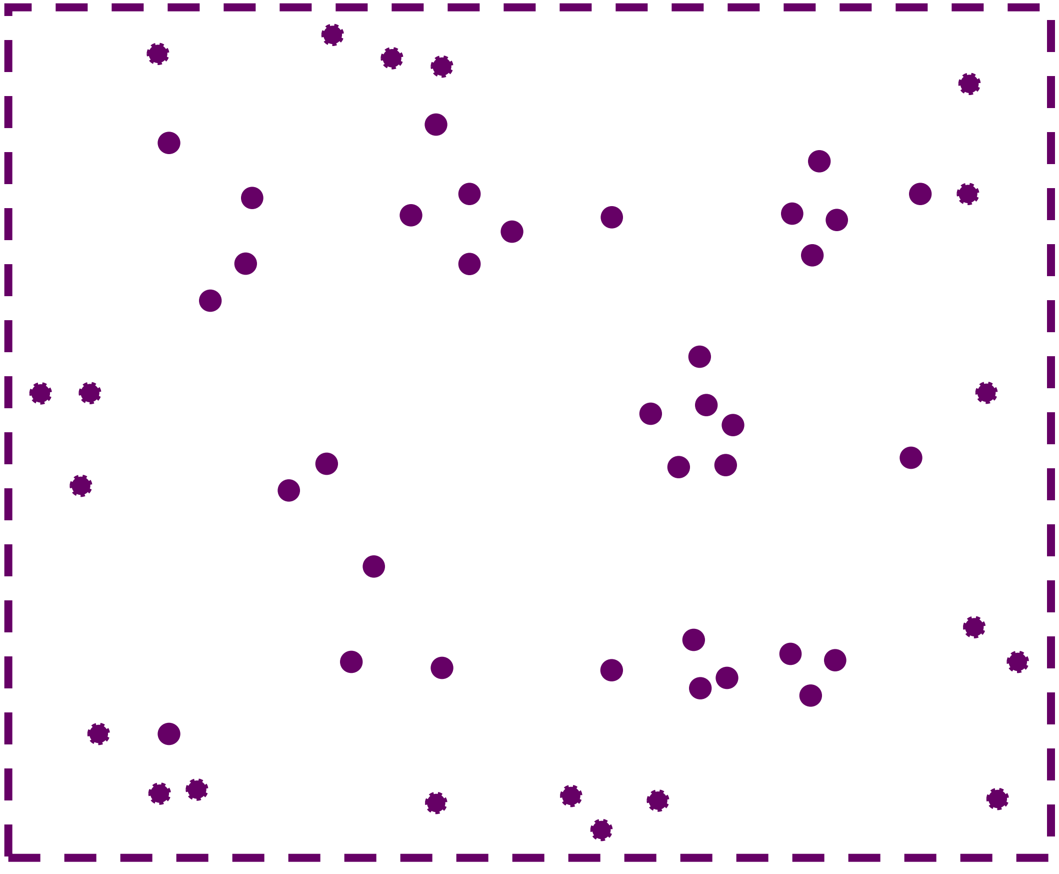


Fig. 3.6 Nearest neighbour distribution function for volcanic craters

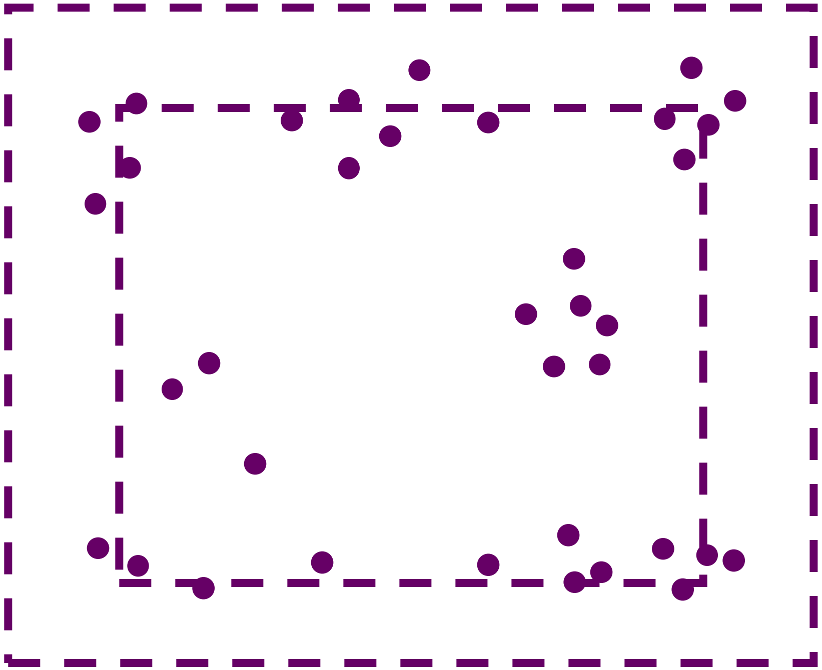
Nearest neighbor analysis

- Edge effects



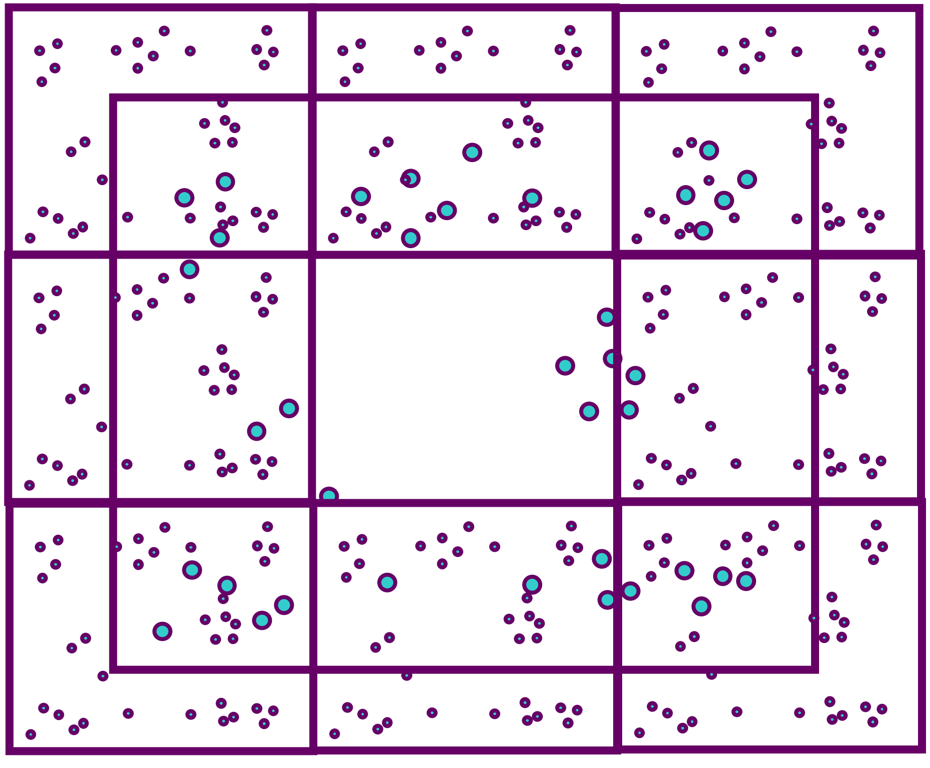
Nearest neighbor analysis

- Edge effects: remedial measures
  - Guard area



Nearest neighbor analysis

- Edge effects: remedial measures
  - Toroidal correction



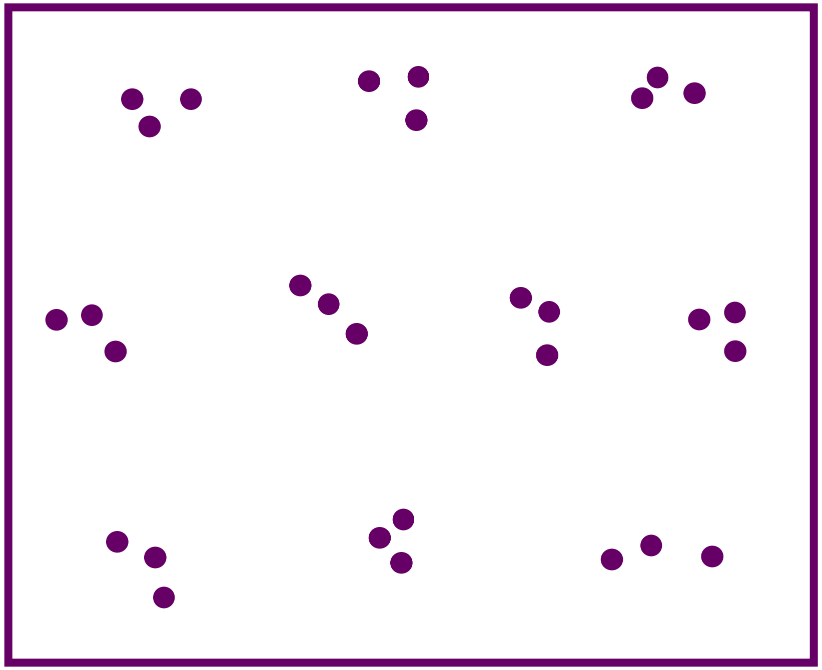
Nearest neighbor analysis

- Event-Event nearest neighbor analysis with edge correction

$$\hat{G}(w) = \frac{\#(b_i > w \geq w_i)}{\#(b_i > w)}$$

Nearest neighbor analysis

- Distance to nearest neighbor *only*
- Small scale analysis – no indication of what happens at other scales



The *K* function

- Various scales
- Implicit assumptions
  - Homogeneity
  - Isotropy

The *K* function

- $\lambda K(h)=$
  $E(\#($events within distance h of an arbitrary event$))$

$$\hat{K}(h)=\frac{1}{\lambda^2 R}\sum_{i\neq j}\sum I_h(d_{ij})$$

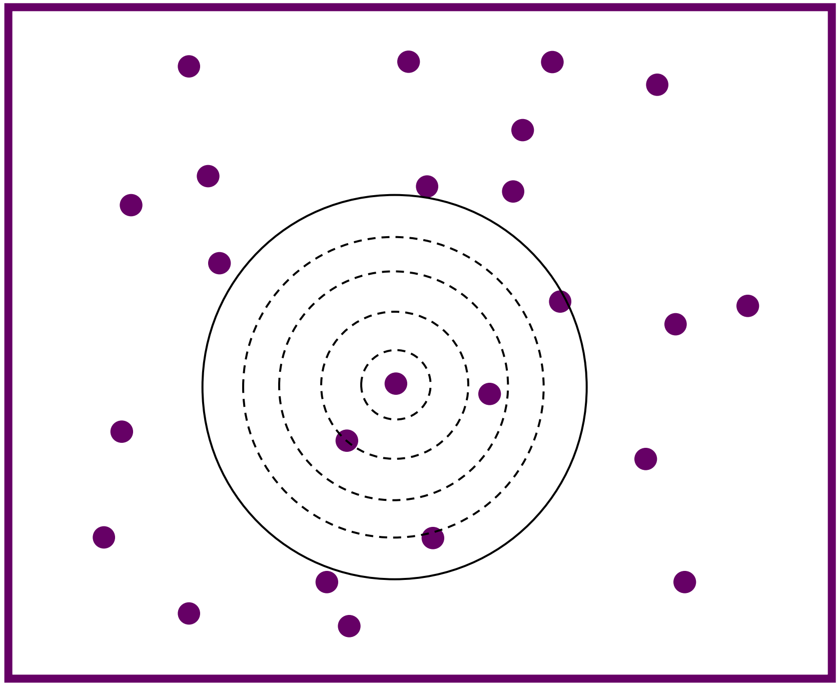
λ : Intensity
R : Area of region $\mathcal{R}$
I_h : Indicator function (1 if $d_{ij}<h$, 0 otherwise)

The *K* function

- λ=n/R
- w_{ij}= *weight to correct for edge effects*

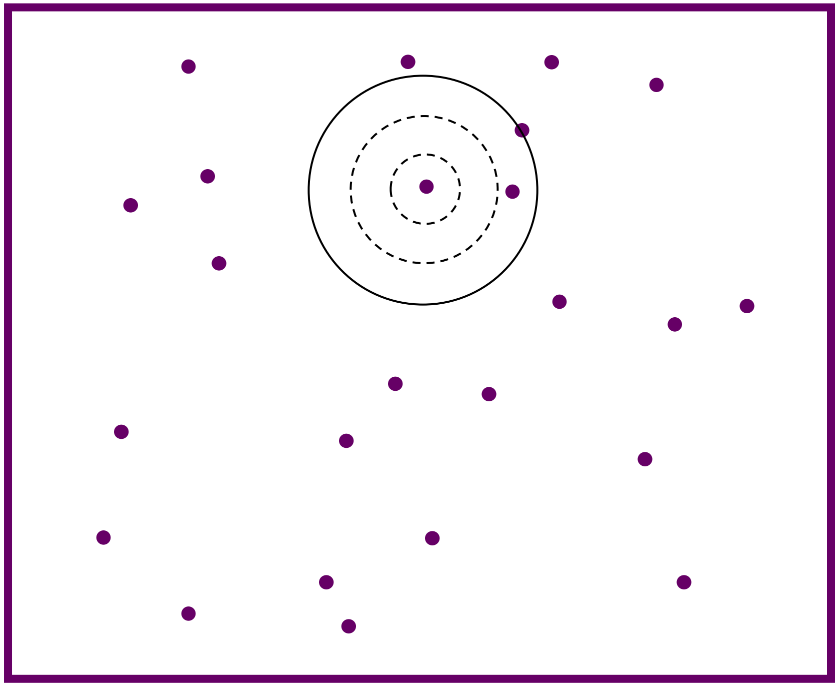
$$\hat{K}(h) = \frac{R}{n^2} \sum_{i \neq j} \sum \frac{I_h(d_{ij})}{w_{ij}}$$

The *K* function



Four events within distance 5 of event *i*

The *K* function



Four events within distance 5 of event *i*

The L function

- If the point pattern is random:

$$K(h) = \pi h^2$$

- Under regularity:

$$K(h) < \pi h^2$$

- Under clustering:

$$K(h) > \pi h^2$$

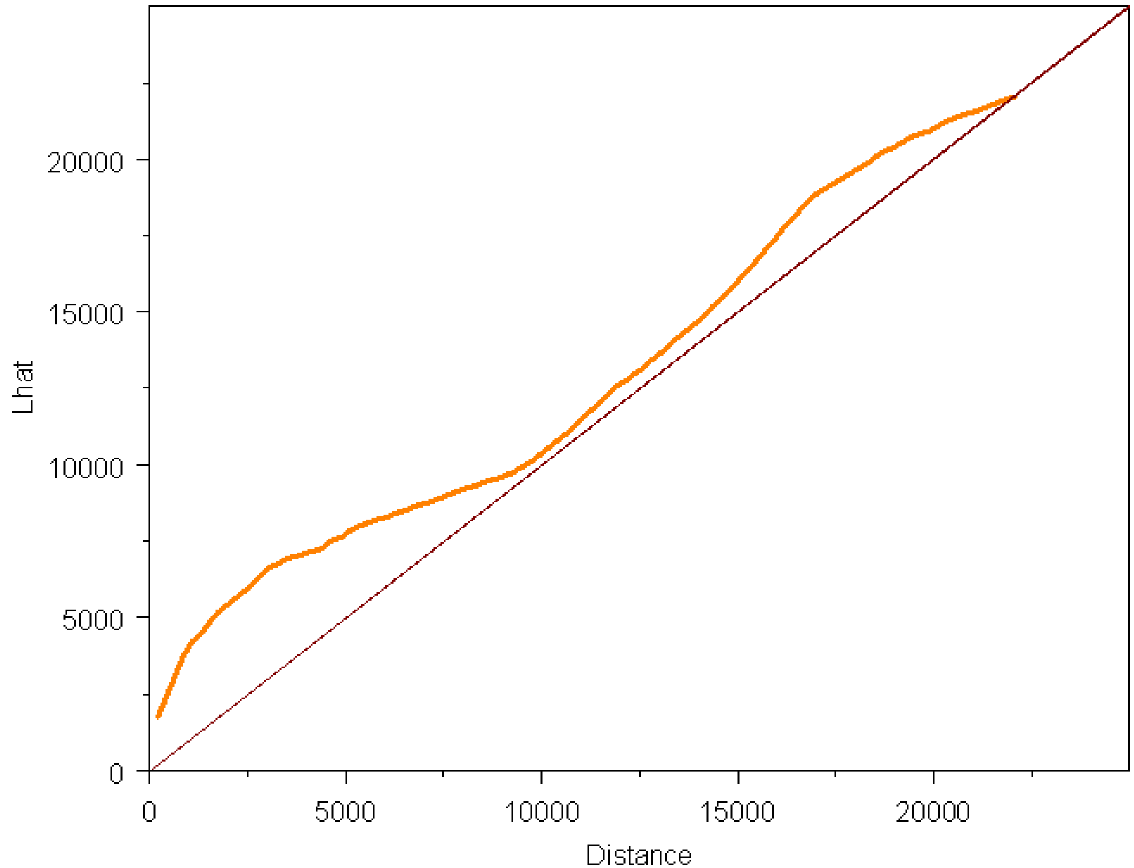
The *L* function

- Compare the K function to the basic random condition

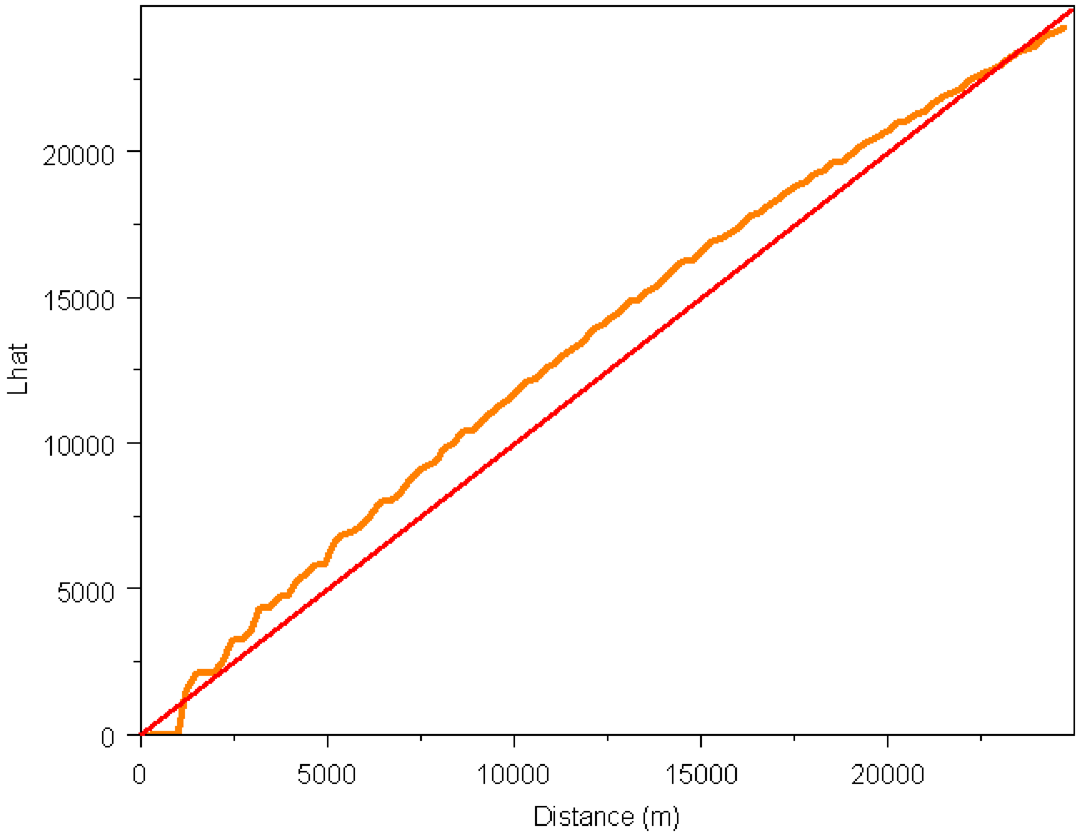
$$\hat{L}(h) = \sqrt{\frac{\hat{K}(h)}{\pi}} - h$$

The *L* function

- Example: The *L* function plotted



L-Function of Regular Pattern



Next …

- Point Pattern V & VI: Simulation and Inference