MA.

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Serie Transformadas de Fourier

2. Comprobar que la función $f(t)=\begin{cases} 0 & \text{si } t<0 \\ kt & \text{si } 0\le t\le \pi \\ 0 & \text{si } t>\pi \end{cases}$

donde $k\neq 0$, es absolutamente integrable y obtener su transformada de Fourier mediante la definición.

$$F\{f(t)\}=\int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}dt=\int_{-\infty}^{0} 0\, e^{-j\omega t}dt+\int_{0}^{\pi} kt\, e^{-j\omega t}dt+\int_{\pi}^{\infty} 0 e^{-j\omega t}dt=$$

$$F\{f(t)\}=\int_{0}^{\pi} kt e^{-j\omega t}dt=-\frac{kt}{j\omega}e^{-j\omega t}+\frac{k}{\omega^2}e^{-j\omega t}\Big]_0^{\pi}$$

$kt \quad + \quad e^{-j\omega t}$

$k \quad - \quad -\frac{1}{j\omega}e^{-j\omega t}$

$0 \quad \quad -\frac{1}{\omega^2}e^{-j\omega t}$

$$F\{f(t)\}=-\frac{k\pi}{j\omega}e^{-j\omega\pi}+\frac{k}{\omega^2}e^{-j\omega\pi}-\frac{k}{\omega^2}$$

$$-\frac{k}{\omega^2}+e^{-j\omega\pi}\left[-\frac{k\pi}{j\omega}+\frac{k}{\omega^2}\right]$$

$$e^{-j\omega\pi}=\cos(\omega\pi)-j\,\mathrm{sen}(\omega\pi)$$

$$\therefore\ -\frac{k}{\omega^2}+[\cos(\omega\pi)-j\,\mathrm{sen}(\omega\pi)]\left[-\frac{k\pi}{j\omega}+\frac{k}{\omega^2}\right]$$

4. Comprobar que la función $f(t)=\begin{cases} -e^{t} & \text{si } t\le 0 \\ -e^{-t} & \text{si } t>0 \end{cases}$

Es absolutamente integrable y obtener mediante la definición su transformada de Fourier.

$$F\{f(t)\}=\int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}dt=\int^{0} -e^{t}(e^{-j\omega t})dt-\int_{0} e^{-t}(e^{-j\omega t})dt$$

$$F\{f(t)\}=-\int_{-\infty}^{0} e^{t(1-j\omega)}dt-\int_{0}^{\infty} e^{-t(1+\omega j)}dt=-\lim_{a\to-\infty}\left[\frac{1}{1-j\omega}e^{t(1-j\omega)}\right]_a^0$$

$$+\lim_{b\to\infty}\left[\frac{1}{(1+\omega j)}e^{-t(1+j\omega)}\right]_0^b=\left[-\frac{1}{1-j\omega}+\cancel{\frac{1}{1-j\omega}e^{a(1-j\omega)}}\right]+$$

$$\left[\frac{-1}{1+\omega j}+\cancel{\frac{1}{1+\omega j}e^{-b(1+j\omega)}}\right]=-\frac{1}{1+j\omega}-\frac{1}{1-j\omega}$$

$$\therefore\ -\frac{1}{1+j\omega}-\frac{1}{1-j\omega}$$

6 Obtener la tansformada de fourier de la función

$$f(t) = \begin{cases} 0 & \text{Si } t<0 \\ 2 & \text{Si } 0\leq t\leq a \\ 0 & \text{Si } t>a. \end{cases}$$

donde $a>0$. Escribir la transformada de Fourier de la función cuando a) $a=1$ b) $a=\pi$.

Sea $F\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = \int_{-\infty}^{0} 0 e^{-j\omega t} dt + \int_{0}^{a} 2e^{-j\omega t} dt + \int_{a}^{\infty} 0 e^{-j\omega t} dt$

$$F\{f(t)\} = \int_0^a 2e^{-j\omega t} dt = \frac{-2}{j\omega} e^{-j\omega t}\Big]_0^a$$

Si $a=1$

$$-\frac{2}{j\omega} e^{-j\omega t}\Big]_0^1 = -\frac{2}{j\omega}\left[e^{-j\omega} - 1\right] \text{ ó } -\frac{2}{j\omega}\left[\cos(j\omega) - j\,\text{sen}(j\omega) - 1\right]$$

$$e^{-j\omega} = \cos(j\omega) - j\,\text{Sen}(j\omega)$$

ahora $a=\pi$

$$-\frac{2}{j\omega} e^{-j\omega t}\Big]_0^\pi = -\frac{2}{j\omega}\left[e^{-j\omega\pi} - 1\right] \text{ o } -\frac{2}{j\omega}\left[\cos(j\omega) - j\,\text{Sen}(j\omega) - 1\right]$$

18 Obtener la transformada de Fourier de la función.

$$f(t) = \begin{cases} 0 & \text{Si: } t<-1 \\ -\pi & \text{Si: } -1\leq t\leq 0 \\ 0 & \text{Si: } t>0 \end{cases}$$

a) Mediante la definición de la transformada de fourier.

Sea $F\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = \int_{-\infty}^{-1} 0 e^{-j\omega t} dt + \int_{-1}^{0} -\pi e^{-j\omega t} dt + \int_{0}^{\infty} 0 e^{-j\omega t} dt$

$$= -\int_{-1}^{0} \pi e^{-j\omega t} dt = \frac{+\pi}{j\omega} e^{-j\omega t}\Big]_{-1}^{0} = \frac{\pi}{j\omega}\left[1 - e^{j\omega}\right]$$

b). con la transformada de Fourier de la función pulso unitario y. Propiedades

$$F\{-\pi p(t+\tfrac{1}{2})] = -\pi F\{p(t+\tfrac{1}{2})\} = -\pi e^{\frac{i\omega}{2}} = -\pi e^{-\frac{i\omega}{2}} \frac{\text{Sen}\frac{\omega}{2}}{\frac{\omega}{2}} =$$

$$= \frac{\pi}{i\omega}(1 - e^{i\omega}).$$

12 Obtener la transformada inversa de Fourier de la función

$F(\omega) = \frac{-2}{(1+3i\omega)(3+3i\omega)}$ Sugerencia. Realiza una descomposición en fracciones parciales.

$$F(\omega) = \left[\frac{-2}{3(\frac{1}{3}+i\omega)(1+i\omega)}\right] = \frac{A}{\frac{1}{3}+i\omega} + \frac{B}{1+i\omega}$$

$A + \frac{1}{3}B = 1 \quad A\omega + B\omega = 0 \quad A\omega = -B\omega \quad A = -B$

$A - \frac{1}{3}A = 1 \quad +\frac{2}{3}A = 1 \quad A = \frac{3}{2} \quad B = -\frac{3}{2}$

$$F(t) = -\frac{2}{3}F^{-1}\left\{\frac{\frac{3}{2}}{i\omega+\frac{1}{3}}\right\} + \frac{3}{2}F^{-1}\left\{\frac{\frac{3}{2}}{i\omega+1}\right\} =$$

$$F(t) = -e^{-\frac{2}{3}t}H(t) + e^{-t}H(t)$$

14. Resolver mediante transformada de Fourier la ecuación. Donde $H(t)$ es la función escalón unitario o de Heaviside.

$y' - y = H(t)e^{-(t-1)} \quad -\infty < t < \infty$

$$F\{y'\} - F\{y\} = F\{H(t)e^{-(t-1)}\}$$

$$S\omega F(\omega) - F(\omega) = \frac{e}{i\omega+1}$$

$$F(\omega) = \frac{e}{(i\omega+1)(i\omega-1)} = e\left[\frac{a}{i\omega-1} + \frac{b}{i\omega+1}\right]$$

$$F(\omega) = e\left[\frac{\frac{1}{2}}{i\omega-1} - \frac{\frac{1}{2}}{i\omega+1}\right\}$$

$$y(t) = \frac{e}{2}F^{-1}\left\{\frac{\frac{1}{2}}{i\omega+1}\right\} - \frac{e}{2}F^{-1}\left\{\frac{1}{i\omega+1}\right\}$$

$$y(t) = \frac{e}{2}e^{-t}u(t) + \frac{e}{2}e^{t}u(t) \text{ ó } \left[\frac{e^{t}-e^{-t}}{2}\right]e\,u(t)$$

10 Obtener la transformada de Fourier de la función $f(t) = -H(-t)e^{2t}$ donde $H(t)$ es la función de escalon unitario o de Heaviside. Trazar la grafica de su espectro de amplitud y frecuencia.

$$\mathcal{F}\{f(t)\} = \mathcal{F}\{-H(-t)e^{2t}\} = -\left[\frac{1}{-(2+i\omega)}\right] = \frac{1}{2+i\omega}\left[\frac{2-i\omega}{2-i\omega}\right] =$$

$f(-t) = F(-\omega)$

$U(t)e^{-at}$ $a>0$ $= \frac{1}{a+i\omega}$

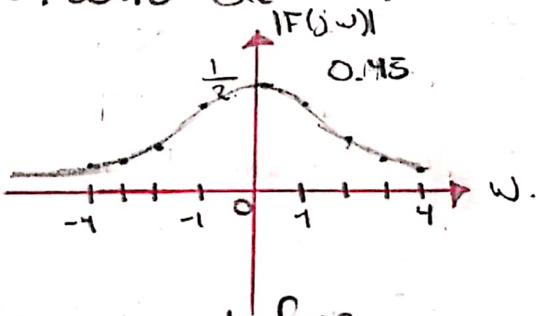
$$= \frac{2-i\omega}{4+\omega^2} = F(j\omega) = \frac{2}{4+\omega^2} - \frac{\omega}{4+\omega^2} j$$

$$|F(j\omega)| = \sqrt{\left(\frac{+2}{4+\omega^2}\right)^2 + \left(\frac{-\omega}{4+\omega^2}\right)^2} = \sqrt{\frac{4+\omega^2}{(4+\omega^2)^2}} = \frac{1}{\sqrt{4+\omega^2}}$$

$$\theta = \arg F(j\omega) = \arctan\left(-\frac{\omega}{2}\right) = -\arctan\left(\frac{\omega}{2}\right)$$

$$\tan\theta = \left(\frac{\frac{-\omega}{4+\omega^2}}{\frac{2}{4+\omega^2}}\right) = \frac{-\omega}{2}$$

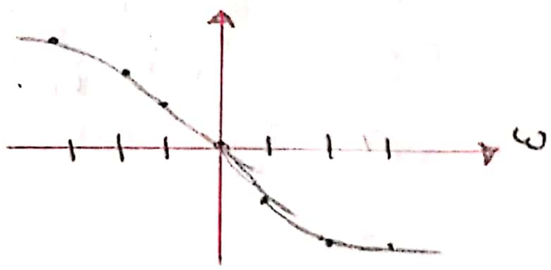
Espectro de amplitud.



ω	$F\|(j\omega)\|$
0	$\frac{1}{2}$
± 1	$0.447 \simeq 0.45$
± 2	$0.353 \simeq 0.35$
± 3	$0.277 \simeq 0.28$
± 4	$0.224 \simeq 0.22$.

Espectro de fase.

ω	$\arg F(j\omega)$
-3	56.31
-2	45
-1	26.57
0	0
1	-26.57
2	45
3	56.31.



$-\frac{\pi}{2} < \arg F(j\omega) \le \frac{\pi}{2}$.

16. Obtener, mediante el teorema de convolución, la transformada inversa de Fourier de la función

$$F(\omega) = \frac{1}{(-2-i\omega)(3+i\omega)}$$

$$\mathcal{F}^{-1}\left\{\frac{1}{3+i\omega}\right\} = e^{-3t}\,u(t)$$

$$\mathcal{F}^{-1}\left\{\frac{1}{-2-i\omega}\right\} = -\mathcal{F}^{-1}\left\{\frac{1}{2+i\omega}\right\} = -e^{-2t}\,u(t)$$

$$y(t) = x(t) * h(t) = -\int_{-\infty}^{\infty} e^{-2x}u(t)\cdot e^{-3(t-x)}\,u(t-x)\,dx$$

$$\int_0^t e^{-2x}\,e^{-4(t-x)}\,dx = -e^{-4t}\int_0^t e^{-2x}e^{4x}\,dx$$

$$= -e^{-4t}\int_0^4 e^{2x}\,dx = -\frac{e^{4t}}{2}\,e^{2x}\Big|_0^t = -\frac{e^{-2t}}{2} + \frac{e^{-4t}}{2}$$

18 Obtener la transformada de Forier de la función $F(\omega) = \frac{1}{(-5-i\omega)^2}$.

$$f(t) = -\mathcal{F}^{-1}\left\{\frac{1}{-(5+i\omega)^2}\right\} = -t\,e^{at}\,u(t).$$

$$F(t) = t\,e^{-at}\,ut \Rightarrow F(\omega) = \frac{1}{(a+i\omega)^2}$$

la función.

Obtener la transformada de coseno de la función

$f(t) = \begin{cases} kt^3 & \text{Si } 0 \le t < a \\ 0 & \text{Si } t \ge a. \end{cases}$ donde $k \ne 0$, y $a > 0$. Escribir la transformada seno de Fourier cuando:

a) $k = 1$ y $a = 1$ b) $k = -1$ y $a = 1$ c) $k = a$

Usando las formulas encontradas en el zill

Transformada de Cos de Fourier es $\mathcal{F}\{F(x)\} = \int_0^\infty f(x)\cos(\alpha x)dx = F(\alpha)$

$$\mathcal{F}\{F(x)\} = \int_0^a kt^3\cos(\alpha t)dt = \frac{kt^3}{\alpha}\operatorname{sen}(\alpha t) + \frac{3kt^2}{\alpha^2}\cos(\alpha t) - \frac{6kt}{\alpha^3}\operatorname{sen}(\alpha t) - \frac{6k}{\alpha^4}\cos(\alpha t)\Big]_0^a$$

kt^3 + $\cos(\alpha t)$

$3kt^2$ - $\frac{1}{\alpha}\operatorname{sen}(\alpha t)$

$6kt$ + $-\frac{1}{\alpha^2}\cos(\alpha t)$

$6k$ - $-\frac{1}{\alpha^3}\operatorname{sen}(\alpha t)$

0 + $\frac{1}{\alpha^4}\cos(\alpha t)$

Transformada de Seno de Fourier. $\mathcal{F}\{F(x)\} = \int_0^\infty f(x)\operatorname{sen}\alpha x\, dx.$

$$\mathcal{F}\{F(t)\} = \int_0^a kt^3\operatorname{sen}(\alpha t)dt = \frac{-kt^3}{\alpha}\cos(\alpha t) + \frac{3kt^2}{\alpha^2}\operatorname{sen}(\alpha t) + \frac{6kt}{\alpha^3}\cos(\alpha t) - \frac{6k}{\alpha^4}\operatorname{sen}(\alpha t)\Big]_0^a$$

kt^3 + $\operatorname{sen}(\alpha t)$

$3kt^2$ - $-\frac{1}{\alpha}\cos(\alpha t)$

$6kt$ + $-\frac{1}{\alpha^2}\operatorname{sen}(\alpha t)$

$6k$ - $+\frac{1}{\alpha^3}\cos(\alpha t)$

0 $\frac{1}{\alpha^4}\operatorname{sen}(\alpha t)$

Para a) $k = 1$ y $a = 1$.

$$\int_0^1 t^3\operatorname{sen}(\alpha t)dt = \frac{-t^3}{\alpha}\cos(\alpha t) + \frac{3t^2}{\alpha^2}\operatorname{sen}(\alpha t) + \frac{6t}{\alpha^3}\cos(\alpha t) - \frac{6}{\alpha^4}\operatorname{sen}(\alpha t)\Big]_0^1 = -\frac{1}{\alpha}\cos(\alpha) + \frac{3}{\alpha^2}\operatorname{sen}(\alpha) + \frac{6}{\alpha^4}\cos(\alpha) - \frac{6}{\alpha^4}\operatorname{sen}(\alpha)$$

ahora b). $k = -1$ y $a = 1$.

$$\int_0^1 -t^3\operatorname{sen}(\alpha t)dt = \frac{t^3}{\alpha}\cos(\alpha t) - \frac{3t^2}{\alpha^2}\operatorname{sen}(\alpha t) - \frac{6t}{\alpha^3}\cos(\alpha t) + \frac{6}{\alpha^4}\operatorname{sen}\alpha t\Big]_0^1$$

$$= \frac{1}{\alpha}\cos(\alpha) - \frac{3}{\alpha^2}\operatorname{sen}(\alpha) - \frac{6}{\alpha^3}\cos(\alpha) + \frac{6}{\alpha^4}\operatorname{sen}(\alpha)$$

ahora c) $k = a$.

$$\int_0^a at^3\operatorname{sen}(\alpha t)dt = -\frac{at^3}{\alpha}\cos(\alpha t) + \frac{3at^2}{\alpha^2}\operatorname{sen}(\alpha t) + \frac{6at}{\alpha^3}\cos(\alpha t) - \frac{6a}{\alpha^4}\operatorname{sen}(\alpha t)\Big]_0^a$$

$$= -\frac{a^4}{\alpha}\cos(\alpha a) + \frac{3a^3}{\alpha^2}\operatorname{sen}(\alpha a) + \frac{6a^2}{\alpha^3}\cos(\alpha a) - \frac{6a}{\alpha^4}\operatorname{sen}(\alpha a).$$