Ejemplo: $\displaystyle\int_C \frac{5z-2}{z(z-1)}\,dz \qquad C: \ |z|=2$ Sentido positivo

Hay dos singularidades $z=0 \quad z=1$ interiores a C

Hallando el residuo en $z=0$

$$f(z)=\frac{5z-2}{z(z-1)}=\left[5-\frac{2}{z}\right]\left[\frac{1}{z-1}\right]=\left[5-\frac{2}{z}\right]\left[\frac{-1}{1-z}\right]$$

$$f(z)=\left(\frac{2}{z}-5\right)\left(\frac{1}{1-z}\right)=\left(\frac{2}{z}-5\right)\left(1+z+z^2+z^3+\cdots\right)$$

$$f(z)=\frac{2}{z}-3-3z-3z^2-\cdots \qquad 0<|z|<1$$

$\therefore$ El coeficiente de $\frac{2}{z}=2z^{-1}$ es $\underline{\underline{2}}$

Hallando el residuo en $z=1$

$$f(z)=\frac{5z-2}{z(z-1)}=\frac{5z-2}{(z+1-1)(z-1}=\frac{5z-2}{(z-1)}\left[\frac{1}{z-1+1}\right]$$

$$f(z)=\left(5+\frac{3}{z-1}\right)\left(\frac{1}{1+z-1}\right)=\left(5+\frac{3}{z-1}\right)\left(1-(z-1)+(z-1)^2-(z-1)^3+\cdots\right)$$

$$0<|z-1|<1$$

$\therefore$ El coeficiente de $\frac{1}{z-1}$ es $\underline{\underline{3}}$

Así

$$\int_C \frac{5z-2}{z(z-1)}\,dz=2\pi i[2+3]=10\pi i$$

Otra alternativa:

$$\int_C \frac{5z-2}{z(z-1)}\,dz=\int_C \frac{2}{z}\,dz+\int_C \frac{3}{z-1}\,dz=4\pi i+6\pi i=10\pi i$$