Desarrolle $f(z)=\dfrac{1}{(z+1)(z+2)}$ en una serie de Laurent en potencias de $(z-1)$ que sea válida en un dominio anular que contenga el punto $z=\dfrac{7}{2}$. Determine el dominio en el que la serie converge a $f(z)$

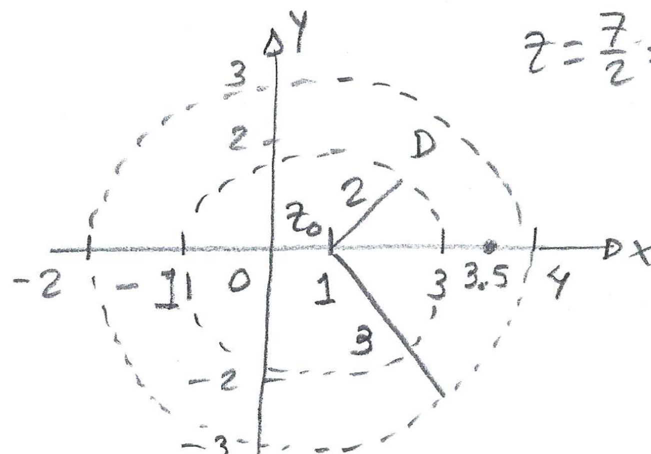
$$f(z)=\sum_{n=-\infty}^{\infty} C_n (z-z_0)^n$$

$$\frac{1}{(z+1)(z+2)}=\sum_{n=-\infty}^{\infty} C_n (z-1)^n \qquad C_n=\frac{1}{2\pi i}\oint_c \frac{1}{(z-z_0)^{n+1}} f(z)\,dz$$

$z+1=0 \quad z+2=0 \qquad z_0=1$

$z=-1 \quad z=-2$

$z=\frac{7}{2}=3.5$



$D: \qquad 2<|z-1|<3$

$$f(z)=\frac{1}{(z+1)(z+2)}=\frac{A}{z+1}+\frac{B}{z+2}$$

$$f(z)=\frac{A(z+2)+B(z+1)}{(z+1)(z+2)}=\frac{(A+B)z+2A+B}{(z+1)(z+2)}$$

$A+B=0 \qquad A=-B$

$2A+B=1 \qquad A=-(-1)$

$2(-B)+B=1 \qquad A=1$

$-2B+B=1$

$-B=1$

$B=-1$

$$f(z)=\frac{1}{z+1}+\frac{-1}{z+2}$$

$$f(z)=\frac{1}{z+1-1+1}+\frac{-1}{z+2-1+1}=\frac{1}{z-1+2}+\frac{-1}{z-1+3}=f_1(z)+f_2(z)$$

i)

$$f_1(z)=\frac{\frac{1}{z-1}}{\frac{z-1}{z-1}+\frac{2}{z-1}}=\frac{1}{z-1}\left[\frac{1}{1+\frac{2}{z-1}}\right] \qquad \left|\frac{2}{z-1}\right|<1$$

$2<|z-1|$

ii)

$$f_1(z)=\frac{\frac{1}{2}}{\frac{z-1}{2}+\frac{2}{2}}=\frac{1}{2}\left[\frac{1}{1+\frac{z-1}{2}}\right] \qquad \left|\frac{z-1}{2}\right|<1$$

$|z-1|<2$

iii) $f_2(z) = \dfrac{-1}{z-1+3}$

$$f_2(z) = \frac{-\frac{1}{z-1}}{\frac{z-1}{z-1} + \frac{3}{z-1}} = -\frac{1}{z-1}\left[\frac{1}{1+\frac{3}{z-1}}\right] \qquad \left|\frac{3}{z-1}\right| < 1$$

$$3 < |z-1|$$

iv)

$$f_2(z) = \frac{-\frac{1}{3}}{\frac{z-1}{3} + \frac{3}{3}} = -\frac{1}{3}\left[\frac{1}{1+\frac{z-1}{3}}\right] \qquad \left|\frac{z-1}{3}\right| < 1$$

$$|z-1| < 3$$

Usando i) y iv)

$$f(z) = \frac{1}{z-1}\left[\frac{1}{1+\frac{2}{z-1}}\right] - \frac{1}{3}\left[\frac{1}{1+\frac{z-1}{3}}\right] \qquad 2 < |z-1| < 3$$

Ayuda $\dfrac{1}{1+u} = 1 - u + u^2 - u^3 + \cdots \qquad |u| < 1$

$$f(z) = \frac{1}{z-1}\left[1 - \frac{2}{z-1} + \left(\frac{2}{z-1}\right)^2 - \left(\frac{2}{z-1}\right)^3 + \cdots\right] - \frac{1}{3}\left[1 - \frac{z-1}{3} + \left(\frac{z-1}{3}\right)^2 - \left(\frac{z-1}{3}\right)^3 + \cdots\right]$$

$$f(z) = (z-1)^{-1} - 2(z-1)^{-2} + 2^2(z-1)^{-3} - 2^3(z-1)^{-4} + \cdots$$
$$-\frac{1}{3} + \frac{z-1}{3^2} - \frac{(z-1)^2}{3^3} + \frac{(z-1)^3}{3^4} - \cdots$$

$$f(z) = \cdots - 2^3(z-1)^{-4} + 2^2(z-1)^{-3} - 2(z-1)^{-2} + (z-1)^{-1} - \frac{1}{3} + \frac{z-1}{3^2} - \frac{(z-1)^2}{3^3} + \frac{(z-1)^3}{3^4} - \cdots$$

$$f(z) = \sum_{n=-\infty}^{\infty} c_n (z-1)^n \qquad c_n = (-1)^{n+1} 2^{-n-1}, \quad n \leq -1$$

$$c_n = \left(-\frac{1}{3}\right)^{n+1}, \quad n \geq 0$$