MA2

Mendoza de la Vega Duke Elizabeth.

Tarea.

Obtenga la transformada de Fourier del pulso exponencal bilateral dada por.

$$f(t) \begin{cases} e^{at} & t \leq 0 \\ e^{-at} & t > 0 \end{cases} \qquad a > 0$$

$$F\{f(t)\} = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t} dt = \int_{-\infty}^{0} e^{at} e^{-j\omega t} dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t} dt =$$

$$\int_{-\infty}^{t} e^{(a-j\omega)t} dt + \int_{0}^{\infty} e^{-(a+j\omega)t} dt = \frac{1}{(a-j\omega)} e^{(a-j\omega)t} \Big]_{-\infty}^{0} -$$

$$- \frac{1}{(a+j\omega)} e^{-(a+j\omega)t} \Big]_{t}^{\infty} = \frac{1}{(a+j\omega)} \Big[1 - 0 \Big] - \frac{1}{(a-j\omega)} \Big[0 - 1 \Big] =$$

$$\frac{1}{(a+i\omega)} + \frac{1}{(a-i\omega)} = \frac{(a-i\omega)}{a^2+\omega^2} + \frac{(a+i\omega)}{a^2+\omega^2} = \frac{2a}{a^2+\omega^2}$$

Obtengo la transformada de fourier del pulso "encendido - apagado" mostrado en la figura

$$f(t)\begin{cases} A & -T \le t < 0 \\ -A & 0 \le t \le T \end{cases}$$

$$F\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt = \int_{-T}^{0} A\, e^{-j\omega t}\,dt + \int_{0}^{T} -A\, e^{-j\omega t}\,dt =$$

$$-\frac{A}{j\omega} e^{-j\omega t}\Big]_{-T}^{0} + \frac{A}{j\omega} e^{-j\omega t}\Big]_{0}^{T} = -\frac{A}{j\omega} \cancel{e^{0}}^{1} + \frac{A}{j\omega} e^{j\omega T} +$$

$$+ \frac{A}{j\omega} e^{-j\omega T} - \frac{A}{j\omega} \cancel{e^{0}}^{1} = -\frac{2A}{j\omega} + \frac{A}{j\omega}\left[e^{j\omega T} + e^{-j\omega T}\right] =$$

$$e^{-j\omega T} = \cos(-\omega T) + j\,\mathrm{Sen}(-\omega T) = \cos(\omega T) - j\,\mathrm{Sen}(\omega T)$$

$$e^{+j\omega T} = \cos(\omega T) + j\,\mathrm{Sen}(\omega T)$$

$$= -\frac{2A}{j\omega} + \frac{A}{j\omega}\left[\cos(\omega T) + \cancel{j\,\mathrm{Sen}(\omega T)} + \cos(\omega T) - \cancel{j\,\mathrm{Sen}(\omega T)}\right]$$

$$= -\frac{2A}{j\omega} + \frac{A}{j\omega}\left[2\cos(\omega T)\right] = -\frac{2A}{j\omega} + \frac{2A}{j\omega}\cos(\omega T)$$

Un Pulso triangular esta definido por.

$$f(t)\begin{cases}\frac{A}{T}t+A & T\leq t\leq 0\\ -\frac{A}{T}t+A. & 0\leq t\leq T\end{cases}$$

Obtengo la transformada de fourier de f(t)

$$F\{f(t)\}=\int_{-\infty}^{\infty}f(t)e^{-j\omega t}dt=\underbrace{\int_{-T}^{0}\left(\frac{A}{T}t+A\right)e^{-j\omega t}dt}+\int_{0}^{T}\left(-\frac{A}{T}t+A\right)e^{-j\omega t}dt$$

$\frac{A}{T}t+A \qquad e^{-j\omega t}$

$\frac{A}{T} \qquad \frac{-1}{j\omega}e^{-j\omega t}$

$0 \qquad -\frac{1}{\omega^2}e^{-j\omega t}$

$$I_1=\left(-\frac{At}{Tj\omega}e^{-j\omega t}-\frac{A}{j\omega}e^{-j\omega t}\right)+\frac{A}{T\omega^2}e^{-j\omega T}\Big]_{-T}^{0}$$

$$=0-\frac{A}{j\omega}e^0+\frac{A}{T\omega^2}e^0-\left(\cancel{\frac{A}{j\omega}e^{j\omega T}}-\cancel{\frac{A}{j\omega}e^{j\omega T}}+\frac{A}{T\omega^2}e^{j\omega T}\right)=-\frac{A}{j\omega}+\frac{A}{T\omega^2}-\frac{A}{T\omega^2}e^{j\omega T}$$

$$I_2=\frac{tA}{Tj\omega}e^{-j\omega t}-\frac{A}{j\omega}e^{-j\omega t}-\frac{A}{T\omega^2}e^{-j\omega t}\Big]_0^T=$$

$$\cancel{\frac{A}{j\omega}e^{-j\omega T}}-\cancel{\frac{A}{j\omega}e^{-j\omega T}}-\frac{A}{T\omega^2}e^{-j\omega T}-\left(0-\frac{A}{j\omega}e^0-\frac{A}{T\omega^2}e^0\right)=$$

$$-\frac{A}{T\omega^2}e^{-j\omega T}+\frac{A}{j\omega}+\frac{A}{T\omega^2}$$

$$I=-\cancel{\frac{A}{j\omega}}+\frac{A}{T\omega^2}-\frac{A}{T\omega^2}e^{-j\omega T}-\frac{A}{T\omega^2}e^{+j\omega T}+\cancel{\frac{A}{j\omega}}+\frac{A}{T\omega^2}$$

$$=-\frac{A}{T\omega^2}\left[\cos(\omega T)-\cancel{j\,sen(\omega T)}+\cos(\omega T)\cancel{+j\,sen(\omega T)}\right]+\frac{2A}{T\omega^2}$$

$$=-\frac{A2}{T\omega^2}\left[\cos(\omega T)-1\right]$$

$$e^{-j\omega T}=\cos(-\omega T)+j\,sen(-\omega T)=\cos(\omega T)-j\,sen(\omega T)$$

$$e^{j\omega T}=\cos(\omega T)+j\,sen(\omega T)$$