Obtenga la serie de Fourier de la función

$f(t) = t^2 + t \quad -\pi < t < \pi \qquad f(t) = f(t+2\pi)$

de periodo $2\pi = T \quad \pi = p$

La serie de Fourier es $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos \frac{n\pi}{p} t + b_n \operatorname{sen} \frac{n\pi}{p} t\right]$

$$a_0 = \frac{1}{p}\int_{-p}^{p} f(t)\,dt = \frac{1}{\pi}\int_{-\pi}^{\pi} t^2 + t\,dt = \frac{1}{\pi}\left[\frac{1}{3}t^3 + \frac{1}{2}t^2\right]_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{\pi}\left[\frac{\pi^3}{3} + \frac{\pi^2}{2} - \left(\frac{(-\pi)^3}{3} + \frac{(-\pi)^2}{2}\right)\right] = \frac{1}{\pi}\left[\frac{\pi^3}{3} + \cancel{\frac{\pi^2}{2}} + \frac{\pi^3}{3} - \cancel{\frac{\pi^2}{2}}\right]$$

$$a_0 = \frac{1}{\pi}\left[\frac{2}{3}\pi^3\right] = \frac{2}{3}\pi^2$$

$$a_n = \frac{1}{p}\int_{-p}^{p} f(t)\cos\left(\frac{n\pi}{p}\right)t\,dt$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(t^2+t)\cos\left(\frac{n\pi}{\pi}\right)t\,dt = \frac{1}{\pi}\int_{-\pi}^{\pi}(t^2+t)\cos(nt)\,dt$$

$u = t^2 + t \qquad \int dv = \int \cos(nt)\,dt$

$\frac{du}{dt} = 2t + 1 \qquad v = \frac{1}{n}\operatorname{sen}(nt)$

$du = 2t + 1\,dt$

$$a_n = \frac{1}{\pi}\left[\frac{t^2+t}{n}\operatorname{sen}(nt)\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi}\frac{1}{n}\operatorname{sen}(nt)(2t+1)\,dt\right]$$

donde $\operatorname{sen}(n\pi) = 0 \quad n = 1, 2, 3, \ldots$

$$a_n = -\frac{1}{n\pi}\int_{-\pi}^{\pi}(2t+1)\operatorname{sen}(nt)\,dt$$

$u = 2t + 1 \qquad \int dv = \int \operatorname{sen}(nt)\,dt$

$\frac{du}{dt} = 2 \qquad v = -\frac{1}{n}\cos(nt)$

$du = 2\,dt$

$$a_n = -\frac{1}{n\pi}\left[-\frac{(2t+1)}{n}\cos(nt)\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} -\frac{1}{n}\cos(nt)(2\,dt)\right]$$

$$a_n = -\frac{1}{n\pi}\left[-\frac{(2\pi+1)}{n}\cos(n\pi) + \frac{(2(-\pi)+1)}{n}\cos(-n\pi) + \frac{2}{n}\left(\frac{1}{n}\operatorname{sen}(nt)\Big|_{-\pi}^{\pi}\right)\right] \quad n=1,2,3,\ldots$$

donde $\cos(n\pi) = \cos(n\pi) = (-1)^n$ y $\operatorname{sen}(n\pi) = 0$

$$a_n = -\frac{1}{n\pi}\left[\frac{-2\pi-1-2\pi+1}{n}\cos(n\pi)\right] = -\frac{1}{n\pi}\left[\frac{-4\pi}{n}(-1)^n\right]$$

$$a_n = \frac{4}{n^2}(-1)^n$$

$$b_n = \frac{1}{p}\int_{-p}^{p} f(t)\operatorname{sen}\left(\frac{n\pi}{p}t\right)dt$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(t^2+t)\operatorname{sen}\left(\frac{n\pi}{\pi}t\right)dt = \frac{1}{\pi}\int_{-\pi}^{\pi}(t^2+t)\operatorname{sen}(nt)\,dt$$

$u = t^2+t$ $\quad \int dv = \int \operatorname{sen}(nt)\,dt$

$\frac{du}{dt} = 2t+1$

$du = 2t+1\,dt$ $\quad v = -\frac{1}{n}\cos(nt)$

$$b_n = \frac{1}{\pi}\left[-\frac{(t^2+t)}{n}\cos(nt)\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} -\frac{1}{n}\cos(nt)(2t+1)\,dt\right]$$

$$b_n = \frac{1}{\pi}\left[-\frac{(\pi^2+\pi)}{n}\cos(n\pi) + \frac{(-\pi)^2+(-\pi)}{n}\cos(-n\pi) + \frac{1}{n}\int_{-\pi}^{\pi}(2t+1)\cos(nt)\,dt\right] ; \quad \cos(n\pi) = \cos(-n\pi)$$

$$b_n = \frac{1}{\pi}\left[\frac{-\pi^2-\pi+\pi^2-\pi}{n}\cos(n\pi) + \frac{1}{n}\int_{-\pi}^{\pi}(2t+1)\cos(nt)\,dt \right]$$

$u = 2t+1 \qquad \int dv = \int \cos(nt)\,dt \qquad \operatorname{sen}(n\pi) = 0$

$\frac{du}{dt} = 2 \qquad v = \frac{1}{n}\operatorname{sen}(nt)$

$du = 2\,dt$

$$b_n = \frac{1}{\pi}\left[-\frac{2\pi}{n}(-1)^n + \frac{1}{n}\left(\frac{2t+1}{n}\operatorname{sen}(nt) \Big|_{-\pi}^{\pi} + \int_{-\pi}^{\pi} \frac{1}{n}\operatorname{sen}(nt)(2\,dt) \right]$$

$$b_n = \frac{1}{\pi}\left[-\frac{2\pi}{n}(-1)^n + \frac{2}{n}\left(-\frac{1}{n}\cos(nt) \Big|_{-\pi}^{\pi} \right)\right]$$

$$b_n = \frac{1}{\pi}\left[-\frac{2\pi}{n}(-1)^n + \frac{2}{n}\left(-\frac{1}{n}\cancel{\left(\cos(n\pi) - \cos(-n\pi) \right)}\right)\right]$$

cero $\quad \cos(n\pi) = \cos(-n\pi)$

$$b_n = -\frac{2}{n}(-1)^n$$

La serie de Fourier es

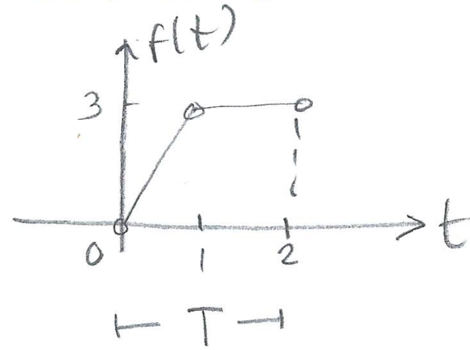
$$f(t) = \frac{\frac{2}{3}\pi^2}{2} + \sum_{n=1}^{\infty}\left[\frac{4}{n^2}(-1)^n\cos(nt) + \frac{-2}{n}(-1)^n\operatorname{sen}(nt) \right]$$

$$f(t) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos(nt) - 2\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\operatorname{sen}(nt)$$

Obtenga la serie de Fourier de la función

$$f(t) = \begin{cases} 3t & 0 < t < 1 \\ 3 & 1 < t < 2 \end{cases}$$

con periodo $T = 2$



La serie de Fourier es:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega t) + b_n \operatorname{sen}(n\omega t) \right]$$

$$a_0 = \frac{2}{T} \int_d^{d+T} f(t)\, dt \;; \quad \omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$$

$$a_0 = \frac{2}{2} \left[\int_0^1 3t\, dt + \int_1^2 3\, dt \right] = \frac{3}{2} t^2 \Big|_0^1 + 3t \Big|_1^2 = \frac{3}{2} + 3 = \frac{9}{2}$$

$$a_n = \frac{2}{T} \int_d^{d+T} f(t) \cos(n\omega t)\, dt$$

$$a_n = \frac{2}{2} \left\{ \left[\int_0^1 3t \cos(n\pi t)\, dt \right] + \int_1^2 3 \cos(n\pi t)\, dt \right\}$$

$u = 3t \quad \int dv = \int \cos(n\pi t)\, dt$

$\frac{du}{dt} = 3 \quad v = \frac{1}{n\pi} \operatorname{sen}(n\pi t)$

$du = 3\, dt$

$$a_n = \frac{3t}{n\pi} \operatorname{sen}(n\pi t) \Big|_0^1 - \int_0^1 \frac{1}{n\pi} \operatorname{sen}(n\pi t)(3\, dt) + \cancel{\frac{3}{n\pi} \operatorname{sen}(n\pi t) \Big|_1^2}$$

cero

$\operatorname{sen}(n\pi) = 0$

$n = 1, 2, 3, \ldots$

$$a_n = -\frac{3}{n\pi} \left(-\frac{1}{n\pi} \cos(n\pi t) \Big|_0^1 \right)$$

$$a_n = \frac{3}{n^2 \pi^2} \left(\cos(n\pi) - \cos 0 \right) \;; \quad \cos(n\pi) = (-1)^n$$

$$a_n = \frac{3}{\pi^2} \left(\frac{(-1)^n - 1}{n^2} \right) \quad n \neq 0$$

$$b_n = \frac{2}{T}\int_d^{d+T} f(t)\,\text{sen}(n\omega t)\,dt \qquad T=2 \qquad \omega = \frac{2\pi}{T} = \pi$$

$$b_n = \frac{2}{2}\left[\int_0^1 3t\,\text{sen}(n\pi t)\,dt + \int_1^2 3\,\text{sen}(n\pi t)\,dt\right]$$

$$u = 3t \qquad \int dv = \int \text{sen}(n\pi t)\,dt$$

$$\frac{du}{dt} = 3 \qquad v = -\frac{1}{n\pi}\cos(n\pi t) \qquad \cos(n\pi) = (-1)^n$$

$$du = 3\,dt$$

$$b_n = -\frac{3t}{n\pi}\cos(n\pi t)\Big|_0^1 - \int_0^1 -\frac{1}{n\pi}\cos(n\pi t)(3\,dt) - \frac{3}{n\pi}\cos(n\pi t)\Big|_1^2$$

$$b_n = -\frac{3}{n\pi}(-1)^n + \cancel{\frac{3}{n\pi}\left(\frac{1}{n\pi}\,\text{sen}(n\pi t)\Big|_0^1\right)} - \frac{3}{n\pi}\left(\cos(2n\pi) - (-1)^n\right)$$

cero

$$b_n = -\frac{3}{n\pi}(-1)^n - \frac{3}{n\pi}\left(1 - (-1)^n\right) \qquad \cos(2n\pi) = 1$$

$$n = 1, 2, \ldots$$

$$b_n = -\frac{3}{n\pi} \qquad n \neq 0$$

Así, la serie de Fourier es:

$$f(t) = \frac{\frac{9}{2}}{2} + \sum_{n=1}^{\infty}\left[\frac{3}{\pi^2}\left(\frac{(-1)^n - 1}{n^2}\right)\cos(n\pi t) - \frac{3}{n\pi}\,\text{sen}(n\pi t)\right]$$

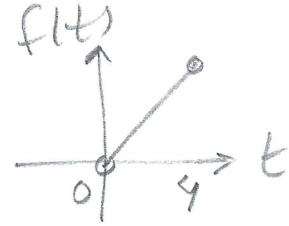
$$f(t) = \frac{9}{4} + \frac{3}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n - 1}{n^2}\cos(n\pi t) - \frac{3}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\,\text{sen}(n\pi t)$$

Sea $f(t) = t \quad t \in (0,4)$

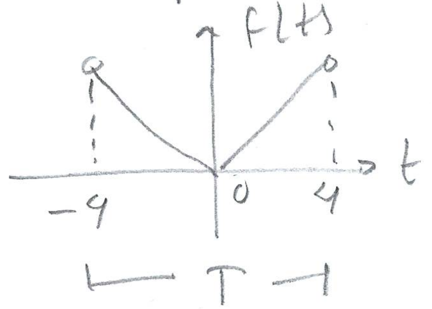
a) Obtenga la serie de Fourier de medio recorrido en cosenos

b) Obtenga la serie de Fourier de medio recorrido en senos

a) $f(t) = t \quad 0<t<4$



se hace una función par. Como:



donde el periodo es $T = 8$

La serie de cosenos es:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t), \quad \text{donde } b_n = 0$$

Así

$$a_0 = \frac{2}{T}\int_d^{d+T} f(t)\,dt = \frac{2}{8}\left[\int_{-4}^{0} -t\,dt + \int_0^4 t\,dt\right]$$

$$a_0 = \frac{1}{4}\left[-\frac{1}{2}t^2\Big|_{-4}^{0} + \frac{1}{2}t^2\Big|_0^4\right] = \frac{1}{4}\left[\frac{1}{2}(-4)^2 + \frac{4^2}{2}\right]$$

$$a_0 = \frac{1}{4}\left[\frac{16}{2} + \frac{16}{2}\right] = 4$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$a_n = \frac{2}{T}\int_d^{d+T} f(t)\cos(n\omega t)\,dt$$

$$a_n = \frac{2}{8}\left[\int_{-4}^{0} -t\cos\left(\frac{n\pi}{4}t\right)dt + \int_0^4 t\cos\left(\frac{n\pi}{4}t\right)dt\right]$$

$$a_n = \frac{1}{4}\left[-\int_{-4}^{0} t\cos\left(\frac{n\pi}{4}t\right)dt + \int_{0}^{4} t\cos\left(\frac{n\pi}{4}t\right)dt\right]$$

$u = t \qquad \int dv = \int \cos\left(\frac{n\pi}{4}t\right)dt$

$\frac{du}{dt} = 1 \qquad v = \frac{4}{n\pi}\operatorname{sen}\left(\frac{n\pi}{4}t\right)$

$du = dt$

$$a_n = \frac{1}{4}\left[-\left(\underbrace{\cancel{\frac{4t}{n\pi}\operatorname{sen}\left(\frac{n\pi}{4}t\right)\Big|_{-4}^{0}}}_{\text{cero}} - \int_{-4}^{0}\frac{4}{n\pi}\operatorname{sen}\left(\frac{n\pi}{4}t\right)dt\right)\right.$$

$$\left. + \underbrace{\cancel{\frac{4t}{n\pi}\operatorname{sen}\left(\frac{n\pi}{4}t\right)\Big|_{0}^{4}}}_{\text{cero}} - \int_{0}^{4}\frac{4}{n\pi}\operatorname{sen}\left(\frac{n\pi}{4}t\right)dt\right]$$

$$a_n = \frac{1}{4}\left[\frac{4}{n\pi}\left(-\frac{4}{n\pi}\cos\left(\frac{n\pi}{4}t\right)\Big|_{-4}^{0}\right) - \frac{4}{n\pi}\left(-\frac{4}{n\pi}\cos\left(\frac{n\pi}{4}t\right)\Big|_{0}^{4}\right)\right]$$

$$a_n = \frac{1}{4}\left[-\frac{16}{n^2\pi^2}\left(\cos(0) - \cos(-n\pi)\right) + \frac{16}{n^2\pi^2}\left(\cos(n\pi) - \cos 0\right)\right]$$

$\cos(n\pi) = \cos(-n\pi) = (-1)^n$

$$a_n = \frac{1}{4}\left[\frac{-16}{n^2\pi^2}\left(1 - (-1)^n\right) + \frac{16}{n^2\pi^2}\left((-1)^n - 1\right)\right]$$

$$a_n = \frac{1}{4}\left[\frac{-16 + 16(-1)^n + 16(-1)^n - 16}{n^2\pi^2}\right] = \frac{1}{4}\left[\frac{-32 + 32(-1)^n}{n^2\pi^2}\right]$$

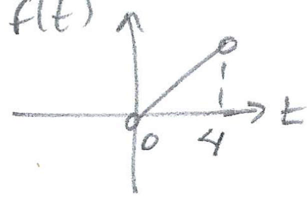
$$a_n = \frac{1}{4}\left[\frac{4(-8 + 8(-1)^n)}{n^2\pi^2}\right] = \frac{8(-1 + (-1)^n)}{n^2\pi^2}, \quad n \neq 0$$

La serie de Fourier de medio recorrido en cosenos es:

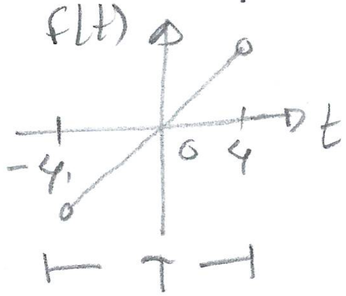
$$f(t) = \frac{4}{2} + \sum_{n=1}^{\infty}\frac{8(-1 + (-1)^n)}{n^2\pi^2}\cos\left(\frac{n\pi}{4}t\right), \text{ o bien}$$

$$f(t) = 2 + \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{-1 + (-1)^n}{n^2}\cos\left(\frac{n\pi}{4}t\right)$$

b) $f(t) = t \quad 0<t<4$



se hace una función impar como:



donde el periodo $T=8$

Entonces la serie de senos se calcula como:

$$f(t) = \sum_{n=1}^{\infty} b_n \operatorname{sen}\left(\frac{n\pi}{4}t\right)$$ donde $a_0=0$, $a_n=0$ y

$$b_n = \frac{2}{p}\int_0^p f(t)\operatorname{sen}\left(\frac{n\pi}{4}t\right)dt$$ con $p=4$

$$b_n = \frac{2}{4}\int_0^4 t\operatorname{sen}\left(\frac{n\pi}{4}t\right)dt$$

$u=t$ $\qquad \int dv = \int \operatorname{sen}\left(\frac{n\pi}{4}t\right)dt$

$\frac{du}{dt}=1$

$du=dt$ $\qquad v = -\frac{4}{n\pi}\cos\left(\frac{n\pi}{4}t\right)$

$$b_n = \frac{1}{2}\left[-\frac{4t}{n\pi}\cos\left(\frac{n\pi}{4}t\right)\Big|_0^4 - \int_0^4 -\frac{4}{n\pi}\cos\left(\frac{n\pi}{4}t\right)dt\right]$$

$$b_n = \frac{1}{2}\left[-\frac{16}{n\pi}\left(\cos(n\pi) - 0\right) + \cancel{\frac{4}{n\pi}\left(\frac{4}{n\pi}\operatorname{sen}\left(\frac{n\pi}{4}t\right)\Big|_0^4\right)}\right]$$

cero

$$b_n = \frac{1}{2}\left[-\frac{16}{n\pi}\left((-1)^n - 0\right)\right] = -\frac{8}{n\pi}(-1)^n \quad ; \quad n \neq 0$$

La serie de Fourier de medio recorrido en senos es:

$$f(t) = \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\operatorname{sen}\left(\frac{n\pi}{4}t\right)$$

Forma compleja de la serie de Fourier de una función periódica $f(t)$ de periodo T

$$f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n\cos(nwt)+b_n\,\mathrm{sen}(nwt)\right]$$

$$\cos(nwt)=\frac{e^{nwti}+e^{-nwti}}{2}\quad;\quad \mathrm{sen}(nwt)=\frac{e^{nwti}-e^{-nwti}}{2i}$$

$$f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\left[\frac{e^{nwti}+e^{-nwti}}{2}\right]+b_n\left[\frac{e^{nwti}-e^{-nwti}}{2i}\right]$$

$$f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[\left(\frac{a_n}{2}+\frac{b_n}{2i}\right)e^{nwti}+\left(\frac{a_n}{2}-\frac{b_n}{2i}\right)e^{-nwti}\right]$$

Haciendo $\frac{a_0}{2}=C_0$, $C_n=\frac{a_n}{2}+\frac{b_n}{2i}=\frac{1}{2}\left[a_n-b_ni\right]$,

$C_n^*=\frac{a_n}{2}-\frac{b_n}{2i}=\frac{1}{2}\left[a_n+b_ni\right]$, se escribe

$$f(t)=C_0+\sum_{n=1}^{\infty}\left[C_ne^{nwti}+C_n^*e^{-nwti}\right],$$ reescribiendo $C_n^*=C_{-n}$

$$f(t)=C_0+\sum_{n=1}^{\infty}C_ne^{nwti}+\sum_{n=1}^{\infty}C_{-n}e^{-nwti}$$

$$f(t)=C_0+\sum_{n=1}^{\infty}C_ne^{nwti}+\sum_{n=-1}^{-\infty}C_ne^{nwti}$$

$$f(t)=\sum_{n=-\infty}^{\infty}C_ne^{nwti}\quad;\quad C_0e^{0}=C_0$$

Usando $C_0=\frac{1}{2}a_0=\frac{1}{T}\int_d^{d+T}f(t)\,dt$

$$C_n=\frac{1}{2}(a_n-b_ni)=\frac{1}{T}\left[\int_d^{d+T}f(t)\cos(nwt)\,dt-i\int_d^{d+T}f(t)\,\mathrm{sen}(nwt)\,dt\right]$$

$$C_n = \frac{1}{2}(a_n - b_n i) = \frac{1}{T}\int_d^{d+T} f(t)\left[\cos(nwt) - i\,\text{sen}(nwt)\right] dt$$

$$C_n = \frac{1}{2}(a_n - b_n i) = \frac{1}{T}\int_d^{d+T} f(t)\, e^{-inwt}\, dt$$

$$C_n^* = C_{-n} = \frac{1}{2}(a_n + b_n i) = \frac{1}{T}\left[\int_d^{d+T} f(t)\cos(nwt)\, dt + i\int_d^{d+T} f(t)\,\text{sen}(nwt)\, dt\right]$$

$$C_n^* = C_{-n} = \frac{1}{2}(a_n + b_n i) = \frac{1}{T}\int_d^{d+T} f(t)\left[\cos(nwt) + i\,\text{sen}(nwt)\right] dt$$

$$C_n^* = C_{-n} = \frac{1}{2}(a_n + b_n i) = \frac{1}{T}\int_d^{d+T} f(t)\, e^{nwti}\, dt$$

Entonces:

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{nwti} \; ; \; C_n = \frac{1}{T}\int_d^{d+T} f(t)\, e^{-jnwt}\, dt \, , \; n = 0, \pm 1, \pm 2, \ldots$$

$$C_n = |C_n| e^{nwti} \; ; \; |C_n| = \sqrt{\left(\frac{1}{2}a_n\right)^2 + \left(\frac{1}{2}b_n\right)^2}$$

$$|C_n| = \sqrt{\frac{a_n^2}{4} + \frac{b_n^2}{4}} = \sqrt{\frac{a_n^2 + b_n^2}{4}} = \frac{1}{2}\sqrt{a_n^2 + b_n^2}$$

Obtenga la forma compleja de la expansión de la serie de Fourier de la función periódica

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \pi - x & 0 \leq x < \pi \end{cases} \quad T = 2\pi \quad w = \frac{2\pi}{T} = \frac{2\pi}{2\pi}$$

$$w = 1$$

se pide

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{nwxi}, \quad C_n = \frac{1}{T}\int_d^{d+T} f(x) e^{-inwx} dx$$

$$C_n = \frac{1}{2\pi}\left[\int_{-\pi}^{0} 0 e^{0} dx + \int_0^{\pi} (\pi - x) e^{-inx} dx\right]$$

$$C_n = \frac{1}{2\pi}\left[\int_0^{\pi} (\pi - x) e^{-inx} dx\right]; \quad u = \pi - x \quad \int dv = \int e^{-inx} dx$$

$$\frac{du}{dx} = -1 \qquad v = -\frac{1}{in} e^{-inx}$$

$$du = -dx$$

$$C_n = \frac{1}{2\pi}\left[-\frac{(\pi - x)}{in} e^{-inx}\Big|_0^{\pi} - \int_0^{\pi} -\frac{1}{in} e^{-inx} (-dx)\right]$$

$$C_n = \frac{1}{2\pi}\left[\frac{\pi}{in} - \frac{1}{in}\left(-\frac{1}{in} e^{-inx}\Big|_0^{\pi}\right)\right]$$

$$C_n = \frac{1}{2\pi}\left[\frac{\pi}{in} + \frac{1}{-n^2}\left(e^{-in\pi} - e^{0}\right)\right]; \quad e^{-in\pi} = \cos(n\pi) - i\cancel{\operatorname{sen}(-n\pi)}$$

cero

$$C_n = \frac{1}{2\pi}\left[\frac{\pi}{in} - \frac{1}{n^2}\left((-1)^n - 1\right)\right] = \frac{1}{2\pi}\left[-\frac{\pi}{n} i + \frac{1 - (-1)^n}{n^2}\right]$$

$$C_n = \frac{1 - (-1)^n}{2\pi n^2} - \frac{1}{2n} i \; ; \; n \neq 0$$

donde

$$C_0 = \frac{1}{2\pi}\left[\cancel{\int_{-\pi}^{0} 0 e^{0} dx} + \int_0^{\pi} (\pi - x) e^{0} dx\right] = \frac{1}{2\pi}\left[\frac{\pi^2}{2}\right] = \frac{\pi}{4}$$

cero

Entonces la forma compleja es:

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\omega x} \quad ; \quad T = 2\pi \quad \omega = 1$$

$$f(x) = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \left[\frac{1-(-1)^n}{2\pi n^2} - \frac{1}{2n} i \right] e^{inx} \; ; \; \underset{n=0}{C_0 = \frac{\pi}{4}}$$

Observación, relacionando los coeficientes

$$c_n = \frac{1}{2}(a_n - b_n i) = \frac{1-(-1)^n}{2\pi n^2} - \frac{1}{2n} i \quad \text{se tiene}$$

$$\frac{1}{2} a_n = \frac{1-(-1)^n}{2\pi n^2} \qquad a_n = 2\left[\frac{1-(-1)^n}{2\pi n^2}\right] = \frac{1-(-1)^n}{\pi n^2}$$

$$-\frac{1}{2} b_n = -\frac{1}{2n} \qquad b_n = -2\left(-\frac{1}{2n}\right) = \frac{1}{n}$$

$$C_0 = \frac{1}{2} a_0 \qquad \frac{\pi}{4} = \frac{1}{2} a_0 \qquad a_0 = 2\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$$

La función dada $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \pi - x & 0 \leq x < \pi \end{cases}$ en su forma trigonométrica sería.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \operatorname{sen}\left(\frac{n\pi}{p}x\right)\right] \; ; \; p = \pi$$

$$f(x) = \frac{\frac{\pi}{2}}{2} + \sum_{n=1}^{\infty}\left[\frac{1-(-1)^n}{\pi n^2}\cos(nx) + \frac{1}{n}\operatorname{sen}(nx)\right], \text{ esto es:}$$

$$f(x) = \frac{\pi}{4} + \frac{1}{\pi}\sum_{n=1}^{\infty} \frac{1-(-1)^n}{n^2}\cos(nx) + \sum_{n=1}^{\infty}\frac{1}{n}\operatorname{sen}(nx)$$