MA2.

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① Obtenga la integral de Fourier de f(x) $\begin{cases} 0 & x<0 \\ x & 0<x<3 \\ 0 & x>3 \end{cases}$

$$f(x) = \frac{1}{\pi}\int_0^\infty [A(\alpha)\cos\alpha x + B(\alpha)\operatorname{sen}\alpha x]\,dx.$$

Donde $A(\alpha) = \int_{-\infty}^{\infty} f(x)\cos\alpha x\,dx$ $\quad B(\alpha) = \int_{-\infty}^{\infty} f(x)\operatorname{sen}\alpha x\,dx$

$$A(\alpha) = \int_{-\infty}^{0} \cancel{f(x)}^{0}\cos\alpha x\,dx + \int_0^3 f(x)\cos\alpha x\,dx + \int_3^\infty \cancel{f(x)}^{0}\cos\alpha x\,dx.$$

$$= \int_0^3 x\cos\alpha x\,dx = \frac{x}{\alpha}\operatorname{sen}\alpha x + \frac{1}{\alpha^2}\cos\alpha x.\Big]_0^3 = \left[\frac{3}{\alpha}\operatorname{sen}3\alpha + \frac{1}{\alpha^2}\cos 3\alpha\right] -$$

$x \;+\; \cos\alpha x.$
$1 \;-\; \frac{1}{\alpha}\operatorname{sen}\alpha x.$
$0 \;-\; \frac{1}{\alpha^2}\cos\alpha x.$

$$-\frac{1}{\alpha^2}\cancel{\cos 0}^{1} = \frac{3}{\alpha}\operatorname{sen}3\alpha + \frac{1}{\alpha^2}\cos 3\alpha - \frac{1}{\alpha^2}.$$

$$B(\alpha) = \int_{-\infty}^{0} f(x)\operatorname{sen}\alpha x\,dx + \int_0^3 f(x)\operatorname{sen}\alpha x\,dx + \int_3^\infty f(x)\operatorname{sen}\alpha x\,dx =$$

$$\int_0^3 x\operatorname{sen}\alpha x\,dx = -\frac{x}{\alpha}\cos\alpha x + \frac{1}{\alpha^2}\operatorname{sen}\alpha x\Big]_0^3 = \left[-\frac{3}{\alpha}\cos 3\alpha + \frac{1}{\alpha^2}\operatorname{sen}3\alpha\right]$$

$x \;+\; \operatorname{sen}\alpha x.$
$1 \;-\; \frac{1}{\alpha}\cos\alpha x.$
$0 \;-\; \frac{1}{\alpha^2}\operatorname{sen}\alpha x.$

$$-\frac{1}{\alpha^2}\cancel{\operatorname{sen}\emptyset}^{0} = \left[-\frac{3}{\alpha}\cos 3\alpha + \frac{1}{\alpha^2}\operatorname{sen}3\alpha\right].$$

$$f(x) = \int_0^\infty \frac{1}{\pi}\left[\left(\frac{3\alpha\operatorname{sen}3\alpha + \cos\alpha 3 - 1}{\alpha^2}\right)\cos\alpha x + \left(\frac{-3\alpha\cos 3\alpha + \operatorname{sen}3\alpha}{\alpha^2}\right)\operatorname{sen}\alpha x\right]dx.$$

Utilizar la integral seno y coseno de Fourier para representar $f(x)$.

a) $f(x) \begin{cases} |x| & |x| < \pi \\ 0 & |x| > \pi \end{cases}$

$f(x) = \frac{2}{\pi}\int_0^\infty A(\alpha)\cos\alpha x \, dx.$ donde $A(\alpha) = \int_0^\infty f(x)\cos\alpha x\, dx.$ Int de cos.

$$A(\alpha) = \int_0^\pi f(x)\cos\alpha x\, dx + \int_\pi^\infty f(x)\cos\alpha x\, dx. = \int_0^\pi |x|\cos\alpha x\, dx.$$

$$\int_0^\pi x\cos\alpha x\, dx = +\frac{x}{\alpha}\operatorname{sen}\alpha x + \frac{1}{\alpha^2}\cos\alpha x.\Big]_0^\pi$$

$$\begin{array}{lll} x & + & \cos\alpha x \\ 1 & + & \frac{1}{\alpha}\operatorname{sen}\alpha x. \\ 0 & & \frac{-1}{\alpha^2}\cos\alpha x \end{array}$$

$$= +\frac{\pi}{\alpha}\operatorname{sen}\alpha\pi + \frac{1}{\alpha^2}\cos\alpha\pi - \frac{1}{\alpha^2}\cancel{\cos 0}^{\,1}$$

$$= +\frac{\pi}{\alpha}\operatorname{sen}\alpha\pi + \frac{1}{\alpha^2}\cos\alpha\pi - \frac{1}{\alpha^2}.$$

$$f(x) = \frac{2}{\pi}\int_0^\infty \left(+\frac{\pi}{\alpha}\operatorname{sen}\alpha\pi + \frac{1}{\alpha^2}\cos\alpha\pi - \frac{1}{\alpha^2}\right)\cos\alpha x.$$

b) $f(x) \begin{cases} x & |x| < \pi \\ 0 & |x| > \pi \end{cases}$

$f(x) = \frac{2}{\pi}\int_0^\infty B(\alpha)\operatorname{sen}\alpha x\, d\alpha.$ $B(\alpha) = \int_0^\infty f(x)\operatorname{sen}\alpha x\, dx.$

$$B(\alpha) = \int_0^\pi f(x)\operatorname{sen}\alpha x\, dx + \int_\pi^\infty f(x)\operatorname{sen}\alpha x\, dx = \int_0^\pi x\operatorname{sen}\alpha x\, dx =$$

$$\begin{array}{lll} x & + & \operatorname{sen}\alpha x \\ 1 & - & -\frac{1}{\alpha}\cos\alpha x \\ 0 & & -\frac{1}{\alpha^2}\operatorname{sen}\alpha x. \end{array}$$

$$= -\frac{x}{\alpha}\cos\alpha x + \frac{1}{\alpha^2}\operatorname{sen}\alpha x\Big]_0^\pi =$$

$$= -\frac{\pi}{\alpha}\cos\pi\alpha + \frac{1}{\alpha^2}\operatorname{sen}\alpha\pi$$

$$f(x) = \frac{2}{\pi}\int_0^\infty \left(-\frac{\pi}{\alpha}\cos\pi\alpha + \frac{1}{\alpha^2}\operatorname{sen}\alpha\pi\right)\operatorname{sen}\alpha x\, d\alpha.$$

Resolver las ecuaciones integral dada para la función. Para la función $f(x)$.

a) $\int_0^{\infty} f(x) \cos \alpha x \, dx = e^{-\alpha}$.

Si la integral de coseno es par y cota en el intervalo $(-\infty, \infty)$ se puede decir lo sig.

$f(x) = \frac{2}{\pi} \int_0^{\infty} A(\alpha) \cos(\alpha x) d\alpha$ donde $A(\alpha) = \int_0^{\infty} f(x) \cos(\alpha x) dx$

Si $A(\alpha) = e^{-\alpha}$. se tendrá

$f(x) = \frac{2}{\pi} \int_0^{\infty} e^{-\alpha} \cos(\alpha x) . d\alpha \longrightarrow$ Con esto se propone responder el inciso b.

$U = \cos(\alpha x)$ $\quad dv = e^{-\alpha} dx$

$du = -x \operatorname{sen}(\alpha x) d\alpha$. $\quad v = -e^{\alpha}$.

$$f(x) = \frac{2}{\pi} \int_0^{\infty} e^{-\alpha} \cos \alpha x \, d\alpha = -e^{-\alpha} \cos(\alpha x) \Big]_0^{\infty} - x \int_0^{\infty} \operatorname{sen}(\alpha x) e^{-\alpha} dx =$$

$U = \operatorname{sen} \alpha x$. $\quad dv = e^{-\alpha}$.

$du = x \cos(\alpha x) d\alpha$. $\quad v = -e^{\alpha}$.

$$= -e^{-\alpha} \cos(\alpha x) \Big|_0^{\infty} - x \left[-e^{-\alpha} \operatorname{sen}(\alpha x + x^2 \int_0^{\infty} \cos(\alpha x) e^{-\alpha} d\alpha \right] =$$

$$\int_0^{\infty} \cos(\alpha x) e^{-\alpha} + x^2 \int_0 \cos(\alpha x) e^{-\alpha} = -e^{-\alpha} \cos(\alpha x) + x e^{-\alpha} \operatorname{sen}(\alpha x) \Big|_0 =$$

$$= \left(\frac{1}{1+x^2} \right) \left(-e^{-\alpha} \cos(\alpha x) + x e^{-\alpha} \operatorname{sen}(\alpha x) \Big|_0^{\infty} \right) = \frac{1}{(1+x^2)} \left[0 + 0 - (e^0 \cos(0) + 0) \right]$$

$$= \frac{1}{(1+x^2)}$$

$\therefore$ Se comprueba que $f(x) = \frac{2}{\pi} \left[\frac{1}{1+x^2} \right]$ Para $x > 0$.

Porque solo se tomara ∞ positivo.