MA

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Serie Integral de Fourier

1. Obtener la integral de fourier trigonometrica de la función

$f(t) = k[H(t) - H(t-a)]$ donde $k \neq 0$ y $a > 0$.

$$f(x) = \frac{1}{\pi}\int_0^{\infty} [A(\alpha)\cos\alpha x + B(\alpha)\,\text{Sen}\,\alpha x]\,dx \quad \text{donde}$$

$$A(\alpha) = \int_{-\infty}^{\infty} f(x)\cos\alpha x\,dx = \int_0^{a} K\cos\alpha t\,dt$$

$$A(\alpha) = K\left[\frac{1}{\alpha}\,\text{Sen}\,\alpha t\right]_0^a = \frac{K}{\alpha}\,\text{Sen}\,(\alpha t)$$

$$B(\alpha) = \int_{-\infty}^{\infty} f(x)\,\text{sen}(\alpha x)\,dx = \int_0^{a} K\,\text{Sen}\,\alpha t\,dt = \frac{-K}{\alpha}\cos\alpha t\Big]_0^a =$$

$$B(\alpha) = -\frac{K}{\alpha}[\cos a\alpha - \cos 0] = -\frac{K}{\alpha}[\cos\alpha a - 1]$$

$$\therefore f(x) = \frac{K}{\alpha\pi}\int_0^{\infty}[\,\text{Sen}\,\alpha t\cos\alpha t - (\cos\alpha a + 1)\,\text{sen}\,\alpha x]\,dx.$$

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3. Obtener la integral de Fourier trigonometrica de la función.

$f(t) = \begin{cases} t^3 & \text{si } |t|<a, \\ 0 & \text{si } |t|\geq a \end{cases}$ donde $a>0$. Escribir la forma de la integral cuando a) $a=1$ b) $a=\pi$

a) $f(t) = \begin{cases} t^3 & \text{si } |t|<1 \\ 0 & \text{si } |t|\geq 1 \end{cases}$ $\therefore$ se trata de 1 integral de seno

$f(x) = \frac{2}{\pi}\int_0^\infty B(\alpha)\,\text{sen}\,\alpha x\,dx.$ donde $B(\alpha)=\int_0^\infty f(t)\,\text{sen}(\alpha x)\,dx.$

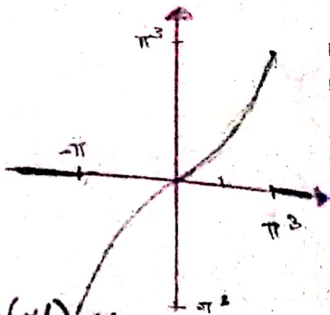
$B(\alpha)=\int_0^1 t^3\,\text{sen}(\alpha t)\,dt + \int_1^\infty \cancel{0\,\text{sen}(\alpha t)\,dt} = \int_0^1 t^3\,\text{sen}(\alpha t)\,dt =$

t^3	+	$\text{sen}(\alpha t)$
$3t^2$	−	$-\frac{1}{\alpha}\cos(\alpha t)$
$6t$	+	$-\frac{1}{\alpha^2}\text{sen}(\alpha t)$
6	−	$\frac{1}{\alpha^3}\cos(\alpha t)$
0		$\frac{1}{\alpha^4}\text{sen}(\alpha t)$

$\left[-\frac{t^3}{\alpha}\cos(\alpha t) + \frac{3t^2}{\alpha^2}\text{sen}(\alpha t) + \frac{6t}{\alpha^3}\cos(\alpha t) - \frac{6}{\alpha^4}\text{sen}(\alpha t)\right]_0^1 =$

$\frac{2}{\pi}\int_0^\infty \left[-\frac{1}{\alpha}\cos(\alpha) + \frac{3}{\alpha^2}\text{sen}(\alpha) + \frac{6}{\alpha^3}\cos(\alpha) - \frac{6}{\alpha^4}\text{sen}(\alpha)\right]\text{Sen}(\alpha t)\,dt.$

b) $f(t) = \begin{cases} t^3 & \text{si } |t|<\pi \\ 0 & \text{si } |t|\geq \pi \end{cases}$



$B(\alpha) = \int_0^\pi t^3\,\text{Sen}(\alpha t\,dt) + \int_\pi^\infty \cancel{0\,\text{Sen}(\alpha t)\,dt} = \int_0^\pi \text{Sen}(\alpha t)\,dt =$

$\left[-\frac{t^3}{\alpha}\cos(\alpha t) + \frac{3t^2}{\alpha^2}\text{Sen}(\alpha t) + \frac{6t}{\alpha^3}\cos(\alpha t) - \frac{6}{\alpha^4}\text{Sen}(\alpha t)\right]_0^\pi =$

$-\frac{\pi^3}{\alpha}\cos(\alpha\pi) + \frac{3\pi^2}{\alpha^2}\text{Sen}(\alpha\pi) + \frac{6\pi}{\alpha^3}\cos(\pi\alpha) - \frac{6}{\alpha^4}\text{Sen}(\alpha\pi)$

$\frac{2}{\pi}\left[-\frac{\pi^3}{\alpha}\cos(\alpha\pi) + \frac{3\pi^2}{\alpha^2}\text{sen}(\alpha\pi) + \frac{6\pi}{\alpha^3}\cos(\pi\alpha) - \frac{6}{\alpha^4}\text{Sen}\,\alpha\pi\right]\text{Sen}(\alpha t)\,dt$

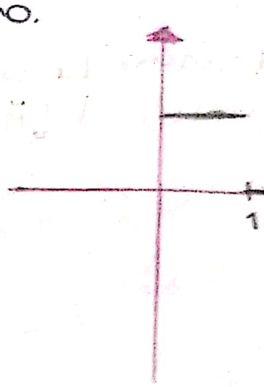
5) Obtener la integral de fourier en cosenos de la función.

$f(t) = \begin{cases} 1 & \text{si } 0 \le t < a \\ 0 & \text{si } t \ge a \end{cases}$ donde $a > 0$. Escribir la integral de fourier en cosenos de la función cuando a) $a = 1$ b) $a = \pi$

a) $F(t) = \begin{cases} 1 & \text{si } 0 \le t \le 1 \\ 0 & \text{si } t \ge 1 \end{cases}$ Donde la Integral de coseno.

$$f(x) = \frac{2}{\pi} \int_0^\infty A(\alpha) \cos \alpha x \, d\alpha.$$

donde $A(\alpha) = \int_0^\infty f(x) \cos \alpha x \, dx$



$$A(\alpha) = \int_0^1 1 \cos(\alpha t) dt + \cancel{\int_1^\infty 0 \cos(\alpha t) dt}^{\,0} = \int_0^1 \cos(\alpha t) dt =$$

$$I = \frac{1}{\alpha} \operatorname{Sen}(\alpha x) \Big|_0^1 = \frac{1}{\alpha} \operatorname{Sen}(\alpha)$$

$$\therefore f(x) = \frac{2}{\pi} \int_0^\infty \frac{1}{\alpha} \operatorname{Sen}(\alpha) \cos \alpha x \, d\alpha.$$

b). $F(t) = \begin{cases} 1 & \text{si } 0 \le t \le \pi \\ 0 & \text{si } t \ge \pi \end{cases}$

$$A(\alpha) = \int_0^\pi 1 \cos(\alpha t) dt + \cancel{\int_\pi^\infty 0 \cos(\alpha t) dt} = \int_0^\pi \cos(\alpha t) dt = \frac{1}{\alpha} \operatorname{Sen} \alpha t \Big|_0^\pi$$

$$A(\alpha) = \frac{1}{\alpha} \operatorname{Sen}(\alpha \pi)$$

$$\therefore f(x) = \frac{2}{\pi} \int_0^\infty \frac{1}{\alpha} \operatorname{sen}(\alpha \pi) \cos(\alpha x) d\alpha.$$

4 Obtener la integral compleja de Fourier de la funcion

$$f(t) = \begin{cases} 0 & si \quad t < -a \\ kt & si \quad -a \le t \le 0 \\ 0 & si \quad t > 0 \end{cases}$$

donde $k \ne 0$ y $a > 0$. Escribir la integral compleja de Fourier de la función cuando

a) $k = -1$ y $a = 1$ b) $k = -1$ y $a = \pi$ c) $k = a$.

$$C(\alpha) = \int_{-\infty}^{\infty} f(x) e^{i\alpha x} dx \quad y \quad f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} C(\alpha) e^{-i\alpha x} d\alpha$$

$$C(\alpha) = \cancel{\int_{-\infty}^{-a} 0 \, e^{i\alpha t} dt} + \int_{-a}^{0} kt \, e^{i\alpha t} dt + \cancel{\int_{0}^{\infty} 0 \, e^{-i\alpha t} dt} = \int_{-a}^{0} kt \, e^{i\alpha t} dt$$

$kt \quad + \quad e^{i\alpha t}$

$k \quad - \quad \frac{1}{i\alpha} e^{i\alpha t}$

$0 \quad\quad -\frac{1}{\alpha^2} e^{i\alpha t}$

$$C(\alpha) = \frac{kt}{i\alpha} e^{i\alpha t} + \frac{k}{\alpha^2} e^{i\alpha t} \Big]_{-a}^{0}$$

Donde en a $k = -1$ y $a = 1$

$$C(\alpha) = \frac{-t}{i\alpha} e^{i\alpha t} - \frac{1}{\alpha^2} e^{+i\alpha t} \Big]_{-1}^{0} = -\frac{1}{\alpha^2} - \frac{1}{i\alpha} e^{-i\alpha} + \frac{1}{\alpha^2} e^{-i\alpha}$$

$$\therefore f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[-\frac{1}{\alpha^2} - \frac{1}{i\alpha} e^{-i\alpha} + \frac{1}{\alpha^2} e^{-i\alpha} \right] e^{-i\alpha t} d\alpha$$

ahora en b $k = -1$ y $a = \pi$.

$$C(\alpha) = \frac{-t}{i\alpha} e^{i\alpha t} - \frac{1}{\alpha^2} e^{i\alpha t} \Big]_{-\pi}^{0} = -\frac{1}{\alpha^2} - \frac{\pi}{i\alpha} e^{-i\alpha\pi} - \frac{1}{\alpha^2} e^{-\pi i\alpha}$$

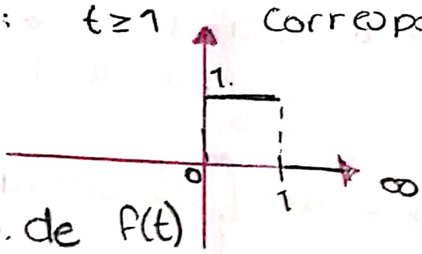
$$\therefore f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[-\frac{1}{\alpha^2} - \frac{\pi}{i\alpha} e^{-i\alpha\pi} - \frac{1}{\alpha^2} e^{-\pi i\alpha} \right] e^{-i\alpha t} d\alpha$$

por ultimo para c. $k = a$

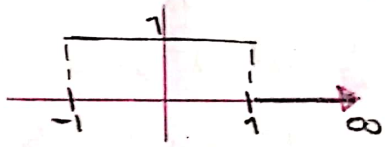
$$C(\alpha) = \frac{at}{i\alpha} e^{i\alpha t} + \frac{a}{\alpha^2} e^{i\alpha t} \Big]_{-a}^{0} = \frac{a}{\alpha^2} + \frac{a^2}{i\alpha} e^{-i\alpha a} - \frac{a}{\alpha^2} e^{-i\alpha a}$$

$$\therefore f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\frac{a}{\alpha^2} + \frac{a^2}{i\alpha} e^{-i\alpha a} - \frac{a}{\alpha^2} e^{-i\alpha a} \right] e^{-i\alpha t} d\alpha.$$

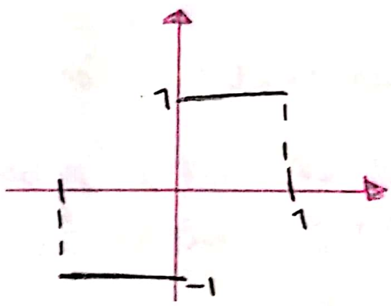
7. Dibujar en el intervalo $(-\infty, \infty)$ los graficos de las funciones a las que convergen las integrales de fourier en senos y cosenos de la función $f(t) \begin{cases} 1 & si \quad 0 \leq t < 1 \\ 0 & si \quad t \geq 1 \end{cases}$ No es necesario obtener las correspondientes integrales de fourier.



Para Integral de coseno de $f(t)$



Para Integral de seno de $f(t)$.



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