Obtenga la forma compleja de la serie de Fourier de la función diente de sierra

$$f(t) = \frac{2t}{T} \qquad 0 < t < 2T \qquad f(t+2T) = f(t)$$

Se pide

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{in\omega t} \; ; \; C_n = \frac{1}{T}\int_{d}^{d+T} f(t) e^{-in\omega t} dt$$

$$C_n = \frac{1}{2T}\int_0^{2T} \frac{2t}{T} e^{-i\frac{n\pi}{T}t} dt \qquad \omega = \frac{2\pi}{T} = \frac{2\pi}{2T} = \frac{\pi}{T}$$

$$C_n = \frac{1}{T^2}\int_0^{2T} t e^{-\frac{in\pi}{T}t} dt \qquad u = t \quad \int dv = \int e^{-\frac{in\pi}{T}t} dt$$

$$\frac{du}{dt} = 1$$

$$du = dt \qquad v = -\frac{T}{in\pi} e^{-\frac{in\pi}{T}t}$$

$$C_n = \frac{1}{T^2}\left[-\frac{T}{in\pi} t e^{-\frac{in\pi}{T}t}\Big|_0^{2T} - \int_0^{2T} -\frac{T}{in\pi} e^{-\frac{in\pi}{T}t} dt \right]$$

$$C_n = \frac{1}{T^2}\left[-\frac{2T^2}{in\pi} e^{-2in\pi} + \frac{T}{in\pi}\left(-\frac{T}{in\pi} e^{-\frac{in\pi}{T}t}\Big|_0^{2T} \right)\right]$$

$$e^{-2in\pi} = \cos(-2n\pi) + i\,\text{sen}(-2n\pi) = 1 \quad , \; n = 0, \pm 1, \pm 2, \ldots$$

$$C_n = \frac{1}{T^2}\left[-\frac{2T^2}{in\pi} + \underbrace{\cancel{\frac{T^2}{n^2\pi^2}\left(e^{-2in\pi} - 1 \right)}}_{\text{cero}} \right] = -\frac{2}{in(\pi)}\left[\frac{-i}{-i}\right] = \frac{2i}{n\pi}$$

$$C_n = \frac{2}{\pi}\left[\frac{1}{n}\right] i \; : \; n \neq 0 \; ; \; \text{Si } n = 0 \quad C_0 = \frac{1}{2T}\int_0^{2T} \frac{2t}{T} dt = \frac{1}{T^2}\left[\frac{1}{2}t^2\right]_0^{2T}$$

$$C_0 = \frac{1}{T^2}\left[\frac{1}{2}(2T)^2\right] = 2$$

Así

$$f(t) = 2 + \frac{2}{\pi}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \left(\frac{1}{n}\right) i e^{\frac{in\pi}{T}t}$$

La serie en su forma trigonométrica depende de a_0, a_n y b_n, donde

$$a_0 = 2c_0 \qquad a_n = c_n + c_n^* \qquad b_n = j(c_n + c_n^*)$$

$$a_0 = 2(2) = 4 \qquad a_n = \frac{2}{\pi n} i - \frac{2}{\pi n} i = 0 \qquad b_n = i\left[\frac{2i}{\pi n} + \frac{2i}{\pi n}\right] = -\frac{4}{\pi n}$$

$$f(t) = \frac{4}{2} + \sum_{n=1}^{\infty} -\frac{4}{\pi n} \operatorname{sen}\left(\frac{n\pi}{T} t\right)$$

$$f(t) = 2 - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \operatorname{sen}\left(\frac{n\pi}{T} t\right)$$

Dibuje los espectros de amplitud y fase de la función periódica

$$f(t) = \frac{2t}{T} \qquad 0 < t < 2T \qquad f(t+2T) = f(t)$$

Considerar las formas compleja y real

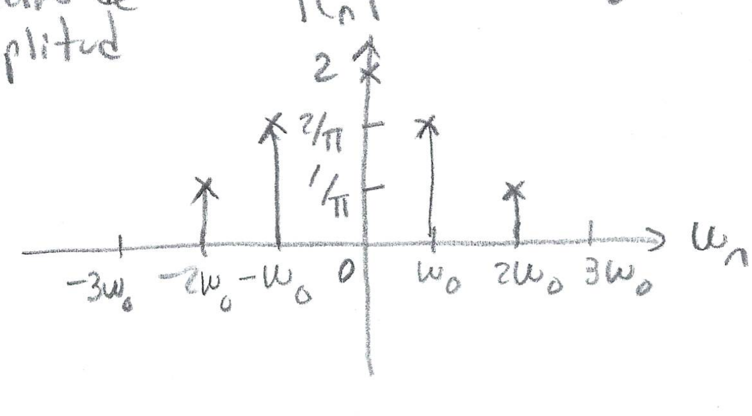
Se sabe que los coeficientes complejos son

$$c_0 = 2 \quad y \quad c_n = \frac{2}{\pi n} i, \quad n = \pm 1, \pm 2, \dots$$

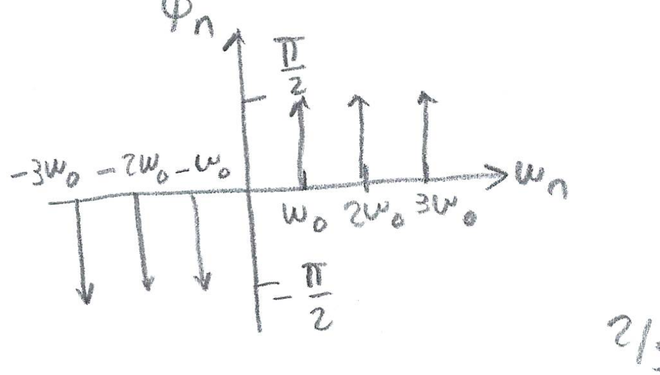
Entonces $|c_n| = \sqrt{\left(\frac{2}{\pi n}\right)^2} = \frac{2}{\pi n}$

$$y \quad \phi_n = \arg c_n = \begin{cases} \frac{\pi}{2} & n = 1, 2, 3, \dots \\ -\frac{\pi}{2} & n = -1, -2, -3, \dots \end{cases}$$

Espectro de amplitud



Espectro de fase



Considerando los coeficientes en forma trigonométrica

$$a_0 = 4 \qquad a_n = 0 \qquad b_n = -\frac{4}{\pi n}$$

Entonces, considerando $f(t) = A_0 + \sum_{n=1}^{\infty} A_n \operatorname{sen}\left(\frac{2n\pi t}{T} + \phi_n\right)$

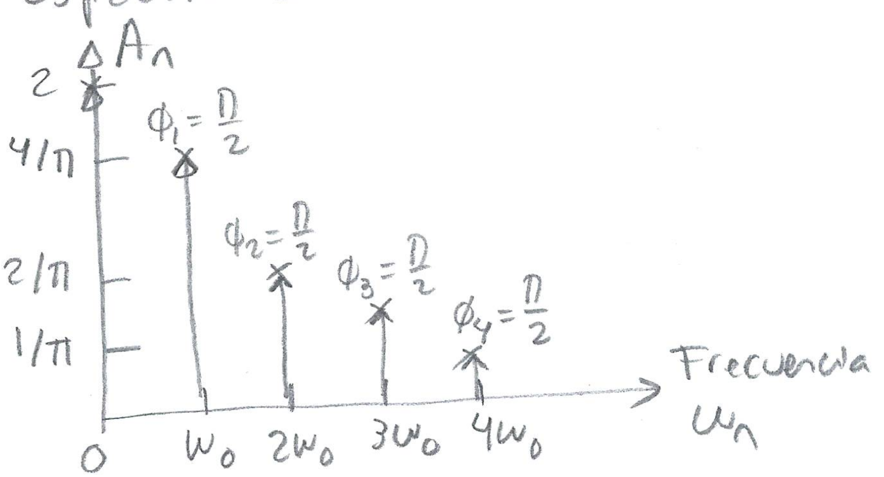
$$A_0 = \frac{1}{2} a_0, \quad A_n = \sqrt{a_n^2 + b_n^2}$$

$$\operatorname{sen} \phi_n = \frac{b_n}{A_n} \quad \text{y} \quad \cos \phi_n = \frac{a_n}{A_n}$$

Entonces

$$A_0 = \frac{4}{2} = 2 \qquad A_n = \sqrt{0^2 + \left(-\frac{4}{\pi n}\right)^2} = \frac{4}{\pi n}, \quad n = 1, 2, 3, \ldots$$

Espectro de frecuencia real discreta

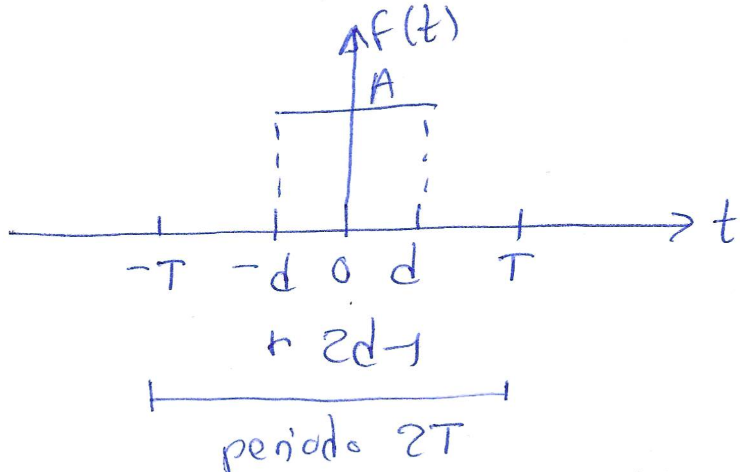


Considerar

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2T} = \frac{\pi}{T}$$

$$\omega_n = \frac{\pi}{T} n$$

Obtenga la forma compleja de la expansión en serie de Fourier del tren infinito periódico de pulsos rectangulares idénticos de magnitud A y duración 2d.



$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2T}$$

$$\omega = \frac{\pi}{T}$$

La función $f(t)$ es:

$$f(t) = \begin{cases} 0 & -T < t < -d \\ A & -d < t < d \\ 0 & d < t < T \end{cases}$$

La serie compleja es:

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{in\omega t} \quad ; \quad C_n = \frac{1}{T}\int_{d}^{d+T} f(t) e^{-in\omega t} dt$$

$$C_n = \frac{1}{2T}\left[\int_{-T}^{-d} 0 e^{-in\omega t} dt + \int_{-d}^{d} A e^{-in\omega t} dt + \int_{d}^{T} 0 e^{-in\omega t}\right]$$

$$C_n = \frac{A}{2T}\left[-\frac{1}{in\omega} e^{-in\omega t}\Big|_{-d}^{d}\right] = -\frac{A}{2Tin\omega}\left[e^{-in\omega d} - e^{in\omega d}\right]$$

$$e^{-in\omega d} = \cos(-n\omega d) + i\,\text{sen}(-n\omega d) = \cos(n\omega d) - i\,\text{sen}(n\omega d)$$

$$e^{in\omega d} = \cos(n\omega d) + i\,\text{sen}(n\omega d)$$

$$C_n = -\frac{A}{2in\pi}\left[-2i\,\text{sen}\left(\frac{n\pi d}{T}\right)\right] = \frac{A}{n\pi}\,\text{sen}\left(\frac{n\pi d}{T}\right) ; \ n \neq 0$$

Si $n=0$

$$C_0 = \frac{1}{2T}\int_{-d}^{d} A\,dt = \frac{A}{2T} t\Big|_{-d}^{d} = \frac{A}{2T}[d+d] = \frac{2Ad}{2T} = \frac{Ad}{T}$$

Entonces, la serie de Fourier en su forma compleja es

$$f(t) = c_0 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} c_n e^{inwt} \; ; \quad c_0 = \frac{Ad}{T} , \quad c_n = \frac{A}{n\pi} \operatorname{sen}\left(\frac{n\pi d}{T}\right)$$

$$w = \frac{\pi}{T}$$

Así

$$f(t) = \frac{Ad}{T} + \frac{A}{\pi} \sum_{\substack{n=\infty \\ n\neq 0}}^{\infty} \frac{1}{n} \operatorname{sen}\left(\frac{n\pi d}{T}\right) e^{\frac{in\pi}{T}t}$$

Por otra parte, si hacemos $t = \frac{n\pi d}{T}$ y establecemos

$$\operatorname{senc} t = \begin{cases} \dfrac{\operatorname{sen} t}{t} & t \neq 0 \\ 1 & t = 0 \end{cases}$$

$$\operatorname{senc} \frac{n\pi d}{T} = \begin{cases} \dfrac{\operatorname{sen}\frac{n\pi d}{T}}{\frac{n\pi d}{T}} & \frac{n\pi d}{T} \neq 0 \\ 1 & \frac{n\pi d}{T} = 0 \end{cases}$$

$$\frac{1}{T}(n\pi d) = 0 \qquad \frac{1}{T} \neq 0$$

$$n\pi d = 0 \qquad \pi \neq 0$$

$$d \neq 0$$

$$\therefore n = 0$$

Para $n = \pm 1, \pm 2, \pm 3, \ldots$

$$\frac{Ad}{T}\left[\frac{1}{\frac{n\pi d}{T}}\right] \operatorname{sen} \frac{n\pi d}{T} = \frac{A}{n\pi} \operatorname{sen}\left(\frac{n\pi d}{T}\right)$$

La expansión en serie de Fourier compleja para el tren infinito de pulsos de $f(t)$ es

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{Ad}{T} \operatorname{senc}\left(\frac{n\pi d}{T}\right) e^{\frac{in\pi}{T}t}$$

nota: La función $\operatorname{senc} t = f(t)$ se conoce como función de muestreo