MA.

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Serie Transformadas de Fourier

2. Comprobar que la función $f(t)=\begin{cases} 0 & \text{si } t<0 \\ kt & \text{si } 0\leq t\leq \pi \\ 0 & \text{si } t>\pi \end{cases}$

donde $k\neq 0$, es absolutamente integrable y obtener su transformada de Fourier mediante la definición.

$$F\{f(t)\}=\int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}dt=\int_{-\infty}^{0} 0\, e^{-j\omega t}dt+\int_{0}^{\pi} kt\, e^{-j\omega t}dt+\int_{\pi}^{\infty} 0 e^{-j\omega t}dt=$$

$$F\{f(t)\}=\int_{0}^{\pi} kt\, e^{-j\omega t}dt=-\frac{kt}{j\omega}e^{-j\omega t}+\frac{k}{\omega^2}e^{-j\omega t}\Big]_0^{\pi}$$

$kt \quad + \quad e^{-j\omega t}$

$k \quad - \quad -\frac{1}{j\omega}e^{-j\omega t}$

$0 \quad \quad -\frac{1}{\omega^2}e^{-j\omega t}$

$$F\{f(t)\}=-\frac{k\pi}{j\omega}e^{-j\omega\pi}+\frac{k}{\omega^2}e^{-j\omega\pi}-\frac{k}{\omega^2}$$

$$-\frac{k}{\omega^2}+e^{-j\omega\pi}\left[-\frac{k\pi}{j\omega}+\frac{k}{\omega^2}\right]$$

$$e^{-j\omega\pi}=\cos(\omega\pi)-j\,\text{sen}(\omega\pi)$$

$$\therefore\ -\frac{k}{\omega^2}+\left[\cos(\omega\pi)-j\,\text{sen}(\omega\pi)\right]\left[-\frac{k\pi}{j\omega}+\frac{k}{\omega^2}\right]$$

4. Comprobar que la función $f(t)=\begin{cases} -e^{t} & \text{si } t\leq 0 \\ -e^{-t} & \text{si } t>0 \end{cases}$

Es absolutamente integrable y obtener mediante la definición su transformada de Fourier.

$$F\{f(t)\}=\int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}dt=\int_{-\infty}^{0}-e^{t}(e^{-j\omega t})dt-\int_{0}^{\infty}e^{-t}(e^{-j\omega t})dt$$

$$F\{f(t)\}=-\int_{-\infty}^{0}e^{t(1-j\omega)}dt-\int_{0}^{\infty}e^{-t(1+\omega j)}dt=-\lim_{a\to-\infty}\left[\frac{1}{1-j\omega}e^{t(1-j\omega)}\right]_a^0$$

$$+\lim_{b\to\infty}\left[\frac{1}{(1+\omega j)}e^{-t(1+j\omega)}\right]_0^b=\left[-\frac{1}{1-j\omega}+\cancel{\frac{1}{1-j\omega}e^{a(1-j\omega)}}\right]+$$

$$\left[\frac{-1}{1+\omega j}+\cancel{\frac{1}{1+\omega j}e^{-b(1+j\omega)}}\right]=-\frac{1}{1+j\omega}-\frac{1}{1-j\omega}$$

$$\therefore\ -\frac{1}{1+j\omega}-\frac{1}{1-j\omega}$$

6 Obtener la transformada de fourier de la función

$$f(t)=\begin{cases}0 & \text{Si } t<0 \\ 2 & \text{Si } 0\le t\le a \\ 0 & \text{Si } t>a.\end{cases}$$

donde $a>0$. Escribir la transformada de Fourier de la función Cuando a) $a=1$ b) $a=\pi$.

Sea $\mathcal{F}\{f(t)\}=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt=\int_{-\infty}^{0} 0\,e^{-j\omega t}dt+\int_{0}^{a} 2e^{-j\omega t}dt+\int_{a}^{\infty} 0\,e^{-j\omega t}dt$

$$\mathcal{F}\{f(t)\}=\int_0^a 2e^{-j\omega t}dt=\frac{-2}{j\omega}e^{-j\omega t}\Big]_0^a$$

Si $a=1$

$$-\frac{2}{j\omega}e^{-j\omega t}\Big]_0^1=-\frac{2}{j\omega}\left[e^{-j\omega}-1\right] \text{ ó } -\frac{2}{j\omega}\left[\cos(j\omega)-j\,\text{sen}(j\omega)-1\right]$$

$$e^{-j\omega}=\cos(j\omega)-j\,\text{Sen}(j\omega)$$

ahora $a=\pi$

$$-\frac{2}{j\omega}e^{-j\omega t}\Big]_0^{\pi}=-\frac{2}{j\omega}\left[e^{-j\omega\pi}-1\right] \text{ ó } -\frac{2}{j\omega}\left[\cos(j\omega)-j\,\text{Sen}(j\omega)-1\right]$$

18 Obtener la transformada de Fourier de la función.

$$f(t)=\begin{cases}0 & \text{Si: } t<-1 \\ -\pi & \text{Si: } -1\le t\le 0 \\ 0 & \text{Si: } t>0\end{cases}$$

a) Mediante la definición de la transformada de Fourier.

Sea $\mathcal{F}\{f(t)\}=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt=\int_{-\infty}^{-1} 0\,e^{-j\omega t}dt+\int_{-1}^{0} -\pi e^{-j\omega t}dt+\int_{0}^{\infty} 0\,e^{-j\omega t}dt$

$$=-\int_{-1}^{0}\pi e^{-j\omega t}dt=\frac{+\pi}{j\omega}e^{-j\omega t}\Big]_{-1}^{0}=\frac{\pi}{j\omega}\left[1-e^{j\omega}\right]$$

b). con la transformada de Fourier de la función pulso unitario y Propiedades

$$\mathcal{F}\{-\pi P(t+\tfrac{1}{2})\}=-\pi\,\mathcal{F}\{P(t+\tfrac{1}{2})\}=-\pi e^{i\frac{\omega}{2}}=-\pi e^{-\frac{i\omega}{2}}\frac{\text{Sen}\frac{\omega}{2}}{\frac{\omega}{2}}=$$

$$=\frac{\pi}{i\omega}(1-e^{i\omega}).$$

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12 Obtener la transformada inversa de Fourier de la función $F(\omega)=\frac{-2}{(1+3i\omega)(3+3i\omega)}$ Sugerencia. Realiza una descomposición en fracciones parciales.

$$F(\omega)=\left[\frac{-2}{3(\frac{1}{3}+i\omega)(1+i\omega)}\right]=\frac{A}{\frac{1}{3}+i\omega}+\frac{B}{1+i\omega}$$

$A+\frac{1}{3}B=1$ $\quad A\omega+B\omega=0$ $\quad A\omega=-B\omega$ $\quad A=-B$

$A-\frac{1}{3}A=1$ $\quad +\frac{2}{3}A=1$ $\quad A=\frac{3}{2}$ $\quad B=-\frac{3}{2}$

$$F(t)=-\frac{2}{3}F^{-1}\left\{\frac{\frac{3}{2}}{i\omega+\frac{1}{3}}\right\}+\frac{3}{2}F^{-1}\left\{\frac{\frac{3}{2}}{i\omega+1}\right\}=$$

$$F(t)=-e^{-\frac{2}{3}t}H(t)+e^{-t}H(t)$$

14. Resolver mediante transformada de Fourier la ecuación $y'-y=H(t)e^{-(t-1)}$ $\quad -\infty<t<\infty$ Donde $H(t)$ es la función escalón unitario o de Heaviside.

$$F\{y'\}-F\{y\}=F\{H(t)e^{-(t-1)}\}$$

$$F(\omega)=\frac{e}{(i\omega+)(i\omega-1)}=e\left[\frac{a}{i\omega-1}+\frac{b}{i\omega+1}\right]$$

$$F(\omega)=e\left[\frac{\frac{1}{2}}{i\omega-1}-\frac{\frac{1}{2}}{i\omega+1}\right]$$

$$y(t)=\frac{e}{2}F^{-1}\left\{\frac{\frac{1}{2}}{i\omega+1}-\frac{e}{2}F^{-1}\left\{\frac{1}{i\omega+1}\right\}\right.$$

$$y(t)=\frac{e}{2}e^{-t}u(t)+\frac{e}{2}e^{t}u(t)\ \text{ó}\ \left[\frac{e^{(t-1)}-e^{-(t-1)}}{2}\right]u(t).$$

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10 Obtener la transformada de Fourier de la función $f(t) = -H(-t)e^{2t}$ donde $H(t)$ es la función de escalón unitario o de Heaviside. Trazar la gráfica de su espectro de amplitud y frecuencia.

$$F\{f(t)\} = F\{-H(-t)e^{2t}\} = -\left[\frac{1}{-(2+i\omega)}\right] = \frac{1}{2+i\omega}\left[\frac{2-i\omega}{2-i\omega}\right] =$$

$f(-t) = F(-\omega)$

$U(t)\,e^{-at} \quad a>0 = \frac{1}{a+i\omega}$

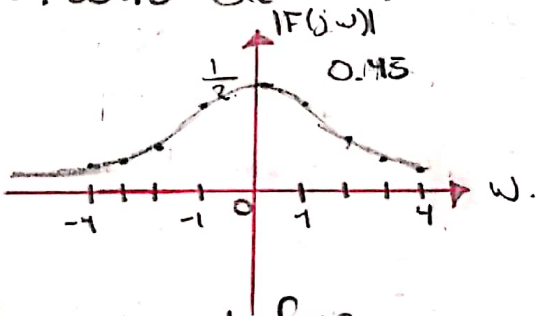
$$= \frac{2-i\omega}{4+\omega^2} = F(j\omega) = \frac{2}{4+\omega^2} - \frac{\omega}{4+\omega^2}\,j$$

$$|F(j\omega)| = \sqrt{\left(\frac{+2}{4+\omega^2}\right)^2 + \left(\frac{-\omega}{4+\omega^2}\right)^2} = \sqrt{\frac{4+\omega^2}{(4+\omega^2)^2}} = \frac{1}{\sqrt{4+\omega^2}}$$

$$\theta = \arg F(j\omega) = \arctan\left(-\frac{\omega}{2}\right) = -\arctan\left(\frac{\omega}{2}\right)$$

$$\tan\theta = \left(\frac{\frac{-\omega}{4+\omega^2}}{\frac{2}{4+\omega^2}}\right) = \frac{-\omega}{2}$$

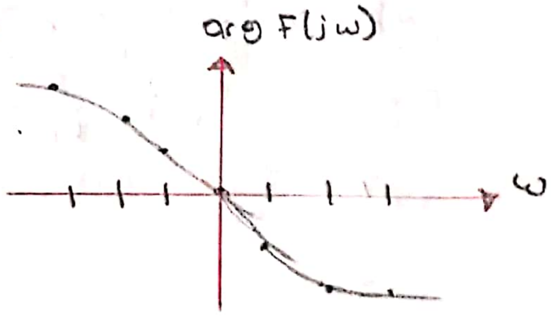
Espectro de amplitud.



ω	$F\|(j\omega)\|$
0	$\frac{1}{2}$
± 1	$0.447 \approx 0.45$
± 2	$0.353 \approx 0.35$
± 3	$0.277 \approx 0.28$
± 4	$0.224 \approx 0.22$

Espectro de fase.

ω	$\arg F(j\omega)$
-3	56.31
-2	45
-1	26.57
0	0
1	-26.57
2	45
3	56.31



$$-\frac{\pi}{2} < \arg F(j\omega) \leq \frac{\pi}{2}$$

16. Obtener, mediante el teorema de convolución, la transformad inversa de Fourier de la función

$$F(\omega) = \frac{1}{(-2-i\omega)(3+i\omega)}$$

$$\mathcal{F}^{-1}\left\{\frac{1}{3+i\omega}\right\} = e^{-3t}\,u(t).$$

$$\mathcal{F}^{-1}\left\{\frac{1}{-2-i\omega}\right\} = -\mathcal{F}^{-1}\left\{\frac{1}{2+i\omega}\right\} = -e^{-2t}\,u(t).$$

$$y(t) = x(t) * h(t) = -\int_{-\infty}^{\infty} e^{-2x}u(t)\cdot e^{-3(t-x)}\,u(t-x)\,dx.$$

$$\int_0^t e^{-2x}\,e^{-4(t-x)}\,dx = -e^{-4t}\int_0^t e^{-2x}e^{4x}\,dx$$

$$= -e^{-4t}\int_0^4 e^{2x}\,dx = -\frac{e^{-4t}}{2}\,e^{2x}\Big|_0^t = -\frac{e^{-2t}}{2} + \frac{e^{-4t}}{2}.$$

18 Obtener la transformada de Forier de la función $F(\omega) = \frac{1}{(-5-i\omega)^2}$.

$$f(t) = -\mathcal{F}^{-1}\left\{\frac{1}{-(5+i\omega)^2}\right\} = -t\,e^{5t}\,u(t).$$

$$F(t) = te^{-at}ut \Rightarrow F(\omega) = \frac{1}{(a+i\omega)^2}$$

Escaneado con CamScanner

Obtener la transformada de coseno de la función

$f(t) = \begin{cases} kt^3 & \text{Si } 0 \le t < a \\ 0 & \text{Si } t \ge a. \end{cases}$ donde $k \neq 0$, y $a > 0$. Escribir la transformada seno de Fourier cuando:

a) $k = 1$ y $a = 1$ b) $k = -1$ y $a = 1$ c) $k = a$

Usando las formulas encontradas en el zill

Transformada de Cos de Fourier es $\mathcal{F}\{F(x)\} = \int_0^\infty f(x)\cos(\alpha x)dx = F(\alpha)$

$$\mathcal{F}\{F(x)\} = \int_0^a kt^3\cos(\alpha t)dt = \frac{kt^3}{\alpha}\operatorname{Sen}(\alpha t) + \frac{3kt^2}{\alpha^2}\cos(\alpha t) - \frac{6kt}{\alpha^3}\operatorname{Sen}(\alpha t) - \frac{6k}{\alpha^4}\cos(\alpha t)\Big]_0^a$$

kt^3 $+$ $\cos(\alpha t)$.
$3kt^2$ $-$ $\frac{1}{\alpha}\operatorname{Sen}(\alpha t)$.
$6kt$ $+$ $-\frac{1}{\alpha^2}\cos(\alpha t)$.
$6k$ $-$ $-\frac{1}{\alpha^3}\operatorname{Sen}(\alpha t)$.
0 $+\frac{1}{\alpha^4}\cos(\alpha t)$.

Transformada de Seno de Fourier. $\mathcal{F}\{F(x)\} = \int_0^\infty f(x)\operatorname{Sen}\alpha x\,dx.$

$$\mathcal{F}\{F(t)\} = \int_0^a kt^3\operatorname{Sen}(\alpha t)dt = \frac{-kt^3}{\alpha}\cos(\alpha t) + \frac{3kt^2}{\alpha^2}\operatorname{Sen}(\alpha t) + \frac{6kt}{\alpha^3}\cos(\alpha t) - \frac{6k}{\alpha^4}\operatorname{Sen}(\alpha t)\Big]_0^a$$

kt^3 $+$ $\operatorname{Sen}(\alpha t)$.
$3kt^2$ $-$ $-\frac{1}{\alpha}\cos(\alpha t)$
$6kt$ $+$ $-\frac{1}{\alpha^2}\operatorname{Sen}(\alpha t)$
$6k$ $-$ $+\frac{1}{\alpha^3}\cos(\alpha t)$
$0.$ $\frac{1}{\alpha^4}\operatorname{Sen}(\alpha t)$

Para a) $k=1$ y $a=1$.

$$\int_0^1 t^3\operatorname{Sen}(\alpha t)dt = \frac{-t^3}{\alpha}\cos(\alpha t) + \frac{3t^2}{\alpha^2}\operatorname{Sen}(\alpha t) + \frac{6t}{\alpha^3}\cos(\alpha t) - \frac{6}{\alpha^4}\operatorname{Sen}(\alpha t)\Big]_0^1 = -\frac{1}{\alpha}\cos(\alpha) + \frac{3}{\alpha^2}\operatorname{Sen}(\alpha) + \frac{6}{\alpha^4}\cos(\alpha) - \frac{6}{\alpha^4}\operatorname{Sen}(\alpha)$$

ahora b). $k=-1$ y $a=1$.

$$\int_0^1 -t^3\operatorname{Sen}(\alpha t)dt = \frac{t^3}{\alpha}\cos(\alpha t) - \frac{3t^2}{\alpha^2}\operatorname{Sen}(\alpha t) - \frac{6t}{\alpha^3}\cos(\alpha t) + \frac{6}{\alpha^4}\operatorname{Sen}\alpha t\Big]_0^1$$

$$= \frac{1}{\alpha}\cos(\alpha) - \frac{3}{\alpha^2}\operatorname{Sen}(\alpha) - \frac{6}{\alpha^3}\cos(\alpha) + \frac{6}{\alpha^4}\operatorname{Sen}(\alpha)$$

ahora c) $k=a$.

$$\int_0^a at^3\operatorname{Sen}(\alpha t)dt = -\frac{at^3}{\alpha}\cos(\alpha t) + \frac{3at^2}{\alpha^2}\operatorname{Sen}(\alpha t) + \frac{6at}{\alpha^3}\cos(\alpha t) - \frac{6a}{\alpha^4}\operatorname{Sen}(\alpha t)\Big]_0^a$$

$$= -\frac{a^4}{\alpha}\cos(\alpha a) + \frac{3a^3}{\alpha^2}\operatorname{Sen}(\alpha a) + \frac{6a^2}{\alpha^3}\cos(\alpha a) - \frac{6a}{\alpha^4}\operatorname{Sen}(\alpha a).$$