# Project: Discrete logistic growth

Suppose a rabbit colony has a population $x(n)$ at month $n$, where $x$ is measured in thousands. If the population were growing in an unbounded environment, the population obeys

$$x(n+1) = x(n) + r \cdot x(n) \tag{1}$$

where $r$ is the per-capita growth rate. Suppose if instead the population is in a bounded environment (like an island), growth is limited, and the population obeys

$$x(n+1) = x(n) + r\left(1 - \frac{x(n)}{K}\right)x(n) \tag{2}$$

where $K$ is a parameter we refer to as the carrying capacity.

Suppose $r = 0.1$ and $K = 0.6$.

a. Try to state the assumptions underlying models (1) and (2).

b. According to your intuition, what population sizes are *steady states*, meaning that if the population had that value at time $n = 0$, then it would remain at that value?

c. Sketch your intuition for the population $x(t)$ from a starting population $x(1) = 0.2$.

Write code to solve the dynamical system, and answer the following questions:

d. Suppose $r = 0.1$ and $K = 0.6$. Generate time series of the populations for a few starting populations $x(1)$. Does it match your intuition?

e. Suppose $r = 2.1$ and $K = 0.6$. Generate time series of the populations for a few starting populations $x(1)$.

In a discrete-time dynamical system, if the population cycles between two values, the solution is called a two-cycle. Cycling between $N$ values is called an $N$-cycle.

f. Check that at $r = 2.5$ and $K = 0.6$ there is a 4-cycle.

g. (Optional) Can you find a value of $r, K$ and $x(1)$ that gives a 3-cycle?

h. In this part, we will do a parameter sweep for $0 < r < 3.0$, with fixed $K = 0.6$. The goal is to generate a diagram where the horizontal axis is the parameter value $r$. On the vertical axis, if there is a stable steady state, plot the steady-state population. If there is an $N$-cycle, plot the $N$ values of $x$ that it cycles through. [1]

- Hint: One way to plot the steady state or the $N$-cycle is to simulate the system until $n_{\max}$, and plot the last half values of $x(n)$. You need to choose $n_{\max}$ large enough so that the dynamics have settled into their steady state (or steady cycle) by $n_{\max}/2$.
- Hint: How many $r$ values should you explore?

i. (Time permitting) The continuous-time model of logistic growth looks like

$$\frac{dx}{dt} = r\left(1 - \frac{x}{K}\right)x. \tag{3}$$

Can you predict the behavior of this model? How is it similar/different to the discrete-time version? (You could also numerically explore this model with an ODE solver).

[1] The pattern might look erratic. This dynamical system is known to exhibit *chaos*! This particular type of chaos is called period-doubling chaos.