

Queen's University  
CISC/CMPE 457  
Test 1

October 6, 2016  
Duration: 50 minutes

Closed book

Initial of Family Name: \_\_\_\_

Student Number: SOLUTIONS  
(Write this at the top of every page.)

There are 4 questions and 25 marks total.

Answer all questions.

This exam paper should have 6 pages,  
including this cover page.

1 – Image Basics	/ 10
2 – Transformation and Interpolation	/ 5
3 – Shift Invariance of Convolution	/ 4
4 – Filters	/ 6
<hr/>	
Total	/ 25

The candidate is urged to submit with the answer paper a clear statement of any assumptions made if doubt exists as to the interpretations of any question that requires a written answer.

# 1 Image Basics — 10 points

**Part A — 4 points** Define each of these terms as they are used in image processing:

- sampling EVALUATION OF AN IMAGE (TYPICALLY A CONTINUOUS FIELD) AT DISCRETE POINTS
- quantization DISCRETIZATION OF PIXEL VALUES
- dyanamic range RATIO OF BRIGHTEST TO DIMMEST INTENSITIES A SENSOR CAN DETECT
- contrast DIFFERENCE BETWEEN BRIGHTEST AND DIMMEST VALUES IN AN IMAGE

**Part B — 2 points** Name and describe two sources of sensor noise.

- QUANTUM INEFFICIENCY: NOT ALL PHOTONS THAT ARRIVE AT THE SENSOR ARE DETECTED
- CHARGE SHIFTING: ELECTRONS ARE LOST/GAINED AS THEY ARE SHIFTED OFF AN IMAGE ROW
- A/D: ANALOG-TO-DIGITAL CONVERSION IS NOT ACCURATE
- THERMAL CURRENT: HEAT INDUCES ELECTRON FLOW

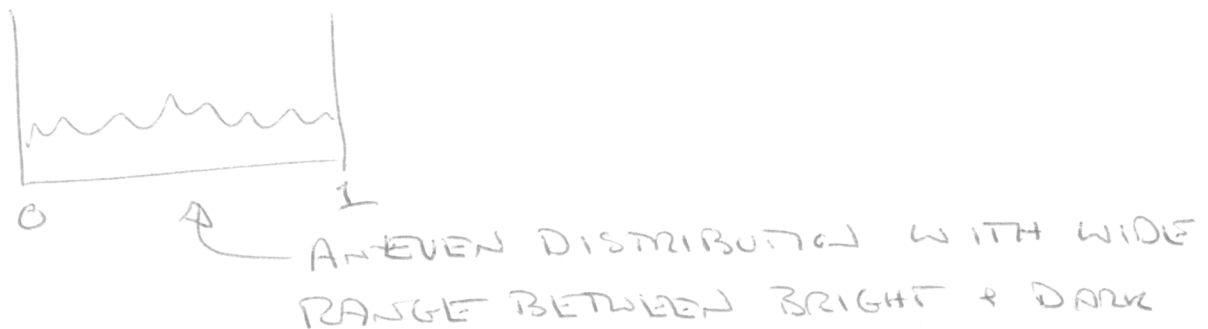
... continued

**Part C — 2 points** What is one value of gamma ( $\gamma$ ) that will increase the contrast of dark pixels with the mapping  $s = r^\gamma$  for original pixel luminance  $r$  and new pixel luminance  $s$ . Draw this graph and label the ranges of the axes.

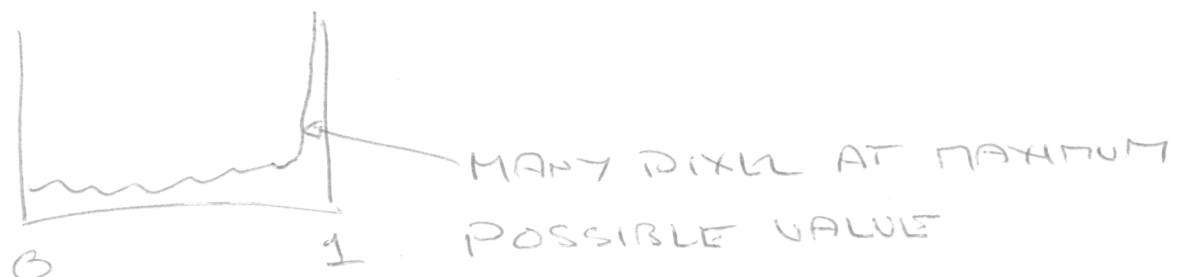
Any  $\gamma \in \{0, 1\}$



**Part D — 1 point** Sketch a histogram of a high-contrast image.



**Part E — 1 point** Sketch a histogram of an image with many saturated pixels.



## 2 Transformation and Interpolation — 5 points

**Part A — 3 points** Find a  $3 \times 3$  homogeneous transform that first rotates by  $45^\circ$ , then translates by  $(1, 2)$ . Show all of your work. Note that  $\sin 45^\circ = \cos 45^\circ = 0.707$ .

$$P^l = T R P = M P$$

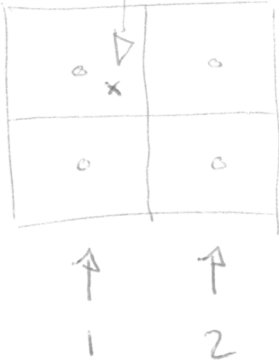
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} .707 & -.707 & 0 \\ .707 & .707 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} .707 & -.707 & 1 \\ .707 & .707 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Part B — 2 points** With bilinear interpolation at location  $(1.2, 5.9)$ , state the integer coordinates of the four pixels that are summed to produce an interpolated pixel and, for each pixel, state its weight in the sum.

6 →

5 →



$p[1.2, 5.9] = \frac{(1-\alpha)(1-\beta)}{1} \times p[1, 5] + \frac{(1-\alpha)\beta}{1} \times p[1, 6] + \frac{\alpha(1-\beta)}{1} \times p[2, 5] + \frac{\alpha\beta}{1} \times p[2, 6]$

CAN SUBSTITUTE ACTUAL VALUE, OR NOT.

$\alpha = 1.2 - \lfloor 1.2 \rfloor = 0.2$

$\beta = 5.9 - \lfloor 5.9 \rfloor = 0.9$

... continued

### 3 Shift Invariance of Convolution — 4 points

The shift invariant property of convolution states that

$$S_a(I) * H = S_a(I * H)$$

where  $S_a(X)$  denotes a shift of  $X$  by  $a$  units to the right.

**Part A — 1 point** Show a convolution filter that performs a shift of 2 units to the right.

$$\begin{array}{c} \{0 \ 0 \ 0 \ 0 \ 1\} \\ \uparrow \\ \text{centre} \end{array}$$

**Part B — 3 points** Let  $R_a$  be a convolution filter that performs a shift of  $a$  units to the right. Prove that  $S_a(I) * H = S_a(I * H)$ , using only the associative, commutative, and identity properties of convolution. Note that  $S_a(X) = R_a * X$ .

$$\begin{aligned} S_a(I) * H &= (R_a * \mathbf{I}) * H \\ &= R_a * (I * H) \quad (\text{Assoc}) \\ &= S_a(I * H) \end{aligned}$$

## 4 Filters — 6 points

**Part A — 1 point** Given a box filter with weights that add up to 2, what effect (other than blurring) does the box filter have when convolved with an image?

IMAGE IS BRIGHTENED BY FACTOR OF 2

**Part B — 3 points** What is the minimum number of multiplications needed to apply a difference-of-Gaussians filter,  $G_1 - G_2$ , to an  $M \times N$  image if  $G_1$  is  $7 \times 7$  and  $G_2$  is  $5 \times 5$ ? Explain and show your work.

SEPARATE EACH GAUSSIAN

$$G_1 = G_{1A} * G_{1B}$$

$$G_2 = G_{2A} * G_{2B}$$

Annotations:  $G_{1A}$  is  $7 \times 7$ ,  $G_{1B}$  is  $7 \times 1$ ,  $G_{2A}$  is  $5 \times 5$ ,  $G_{2B}$  is  $5 \times 1$ .

$G_{1A} * G_{2B} * I$  COSTS  $MN \times (7+7)$   
 $G_{2A} * G_{2B} * I$  COSTS  $MN \times (5+5)$

COST =  $24 MN$

**Part C — 2 points** Convolve the following filter  $F$  and image  $I$ , and show the result. The origin of  $F$  is in its middle.

$$F = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} \quad I = \begin{bmatrix} 10 & 20 \\ 30 & 40 \end{bmatrix}$$

10 20 20 40  
30 40 60 80

This is gotten when  $F$  is NOT reversed (which is the wrong thing to do)

This is gotten by reversing  $F$  (which is the correct thing to do)

~~|    |    |    |    |
|----|----|----|----|
| 20 | 40 | 10 | 20 |
| 60 | 80 | 20 | 40 |~~

E.G.

1	0	2
	10	20
	30	40

→ COULD ALSO CLAMP PIXEL VALUES SO THAT PIXELS OUTSIDE ARE EQUAL TO CLOSEST PIXEL INSIDE

$$1 \times 0 + 0 \times 10 + 2 \times 20 = 20$$

... End Of Test 1