Queen's University
CISC/CMPE 457
Test 3

November 26, 2019
Duration: 50 minutes

Closed book

Initial of Family Name: ___

Student Number: _____
(Write this at the top of every page.)

There are 3 questions and 11 marks total.

Answer all questions.

This exam paper should have 8 pages,
including this cover page.

1 – Feature Detection	/ 4
2 – Compression	/ 4
3 – CT imaging	/ 3
Total	/ 11

The candidate is urged to submit with the answer paper a clear statement of any assumptions made if doubt exists as to the interpretations of any question that requires a written answer.

1   Feature Detection — 4 points

Part A — 1 point   What feature of Otsu's method makes it an efficient algorithm? Explain what the feature does and why it makes the algorithm efficient.

> Otsu's algorithm incrementally updates the variables needed to compute the within-class sum-of-squares. Since each update is done in constant time, the algorithm can efficiently test all possible threshold values.

Part B — 2 points (Canny edge detection) 25 pixels are shown below with their *intensities*.

$$
\begin{array}{ccccc}
0 & 0 & 0 & 0 & 0 \\
0 & 3 & 2 & 3 & 0 \\
0 & 2 & 3 & 2 & 0 \\
0 & 3 & 3 & 5 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{array}
$$

What are the gradient *magnitude* and *quantized direction* of the middle pixel using the Sobel operator in the Canny edge detection algorithm? Show your work.

Gx = 2, Gy = -4

magnitude = sqrt(20)

direction is 4 down and 2 to the right, which is close to the "down and right" quantized direction.

Will the middle pixel be suppressed in the Canny algorithm's "non-maximum supression" step? Show your work.

In the down-and-right direction (at the pixel '5'), the gradient is Gx = -9, Gy = 7, which is a larger magnitude (sqrt 130) than the magnitude at the centre pixel (sqrt 20), so the centre pixel is suppressed.

Part C — 1 point   For Hough circle detection, what is stored in the bin at location (a, b, c) in the Hough parameter space, where a and b are x and y Cartesian coordinates, respectively, and c is a radius? Be very specific with as much detail as possible.

The bin at (a, b, c) stores the number of votes for circles with equation

$$(x - a)^2 + (y - b)^2 = c^2.$$

2   Compression — 4 points

Part A — 1 point Give an *optimal* base-10 arithmetic encoding of the token string ac where $P(a) = \frac{1}{4}$, $P(b) = \frac{1}{4}$, and $P(c) = \frac{1}{2}$. (There is no b in the ac string.) Show your work.

> Split $[0,1]$ range into $[0, \frac{1}{4}]$ for a, $[\frac{1}{4}, \frac{1}{2}]$ for b, and $[\frac{1}{2}, 1]$ for c.
>
> Then $a \in [0, \frac{1}{4}]$ of $[0, 1]$, which is $[0, \frac{1}{4}]$.
>
> Then $ac \in [\frac{1}{2}, 1]$ of $[0, \frac{1}{4}]$, which is $[\frac{1}{8}, \frac{1}{4}]$.
>
> Use the shortest base-10 number in the ac range of $[\frac{1}{8}, \frac{1}{4}] = [0.125, 0.250]$. Choose 0.2 in that range.

Part B — 1 point Under what condition does the entropy of an image provide a lower bound on the compressed bit-rate of that image? Explain.

> Entropy provides a lower bound only when each pixel is a randomly chosen independently of every other pixel, because that required by the definition of entropy.
>
> (Note that the random distribution does not have to be uniform.)

Part C — 1 point With LZW decoding, the index received by the decoder might not yet exist in the decoder's dictionary. In this case, what is the string that is being represented by this index? Assume that the previous string was S.

S + firstchar(S)

Part D — 1 point For JPEG encoding, the quantization table contains larger values in some entries. Where are those entries (state this with respect to frequency) and why are they larger?

Larger entries are in the high-frequency locations (i.e. far from the table's origin). These larger entries cause more quantization at higher frequency, where errors are less perceptible to humans.

3   CT imaging — 3 points

Part A — 1 point   Explain why the Ram-Lak filter (equal to $|\omega|$ in the Fourier domain) results in better images. It is not sufficient to describe *how* the image is improved; you must describe *why* the image is improved.

> The Ram-Lak filter accentuates high frequency parts of the backprojection, so edges are more apparent than without this filter and the "halo" or blurring of ordinary backprojection is removed.
>
> Or: It accounts for the reparameterization of the Fourier space from (ρ, θ) to (u, v).

Part B — 1 point   What is the shape of the function that is the perpendicular projection of a point, $f(x, y)$, onto the CT detector as the detector rotates through angles θ around the centre of the subject, f? Explain.

> The shape is **sinusoidal** because the projection of a point (x, y) onto a line perpendicular to angle θ is $x \cos\theta + y \sin\theta = \rho$, for some ρ.

Part C — 1 point For a subject, $f(x, y)$, of 100×200 pixels, a CT detector of 5 elements, and 10 different detector angles around the subject, what are the dimensions of the components of the linear system

$$W f = \rho$$

that describes the attenuation of rays through the subject? Explain how you got the dimensions.

W has 50 rows for the 5×10 different rays.

W has 20,000 columns for the 100×200 pixels of the subject.

f has 20,000 rows (= number of columns in W).

ρ has 50 rows (= number of rows in W).