Queen's University
CISC/CMPE 457
Test 3

November 27, 2017
Duration: 50 minutes

Closed book

Initial of Family Name: ____

Student Number: ______________________
(Write this at the top of every page.)

There are 4 questions and 13 marks total.

Answer all questions.

This exam paper should have 7 pages,
including this cover page.

1 – CT Reconstruction	/ 5
2 – Canny Edge Detection	/ 2
3 – Hough Transform	/ 2
4 – Compression	/ 4
Total	/ 13

The candidate is urged to submit with the answer paper a clear statement of any assumptions made if doubt exists as to the interpretations of any question that requires a written answer.

1 CT Reconstruction — 5 points

Part A — 1 point What is the line $y = x - 3$ in the form $x \cos\theta + y \sin\theta = \rho$? That is, what are θ and ρ ?

The line has slope 1 and intercept -3, so the normal to the line is at -45 degrees (or $\frac{7\pi}{4}$) and its distance from the origin is $\frac{3}{\sqrt{2}}$, so $x\cos(-45) + y\sin(-45) = \frac{3}{\sqrt{2}}$.

Part B — 1 point Explain why a sinogram does not need to store the full range $[0, 2\pi] \times [-\rho_{max}, +\rho_{max}]$, where ρ_{max} is the maximum distance of a ray from the origin.

The attenuation along a ray is the same in the forward direction as in the backward direction. If ray(θ,ρ) is the forward direction, then ray($\theta + \pi$,$-\rho$) is the corresponding ray in the backward direction, and does not need to be stored.

Part C — 1 point What goes $g(\theta, \rho)$ (i.e. the attenuation along rays in direction $\theta + \frac{\pi}{2}$) correspond to in the frequency domain, for a fixed θ?

The Fourier Slice Theorem says that this corresponds to a line in the frequency domain passing through the origin at angle θ.

... continued

Part D — 1 point The filtered backprojection convolves $h(\rho) * g(\theta, \rho)$, where the Fourier transform of $h(\rho)$ is $H(\omega) = |\,\omega\,|$. What is the effect of this convolution on the backprojection? Explain.

`The reconstructed image is sharper because |ω| enhances high frequency components and suppresses low frequency components.`

Part E — 1 point In the algebraic formulation of CT reconstruction, the total attenuation on each ray, i, through the volume is computed as

$$p_i = \sum_j w_{ij}\, f_j$$

for weights w_{ij} and voxel attenuations f_j. The error between the observed attenuation, $\hat{p}_i$, and the computed attenuation is

$$e_i = \hat{p}_i - p_i$$

Explain how the voxel attenuations are modified to incorporate e_i.

`The error,` e_i`, is distributed across the voxel attenuations in proportion to their weight on ray` i`:`

$$f_j \longleftarrow f_j + e_i \;\frac{w_{ij}}{\sum_j w_{ij}}$$

2 Canny Edge Detection — 2 points

Part A — 1 point For the middle pixel below, the gradients in the x and y directions are $G_x = -6$ and $G_y = 8$, respectively.

$$\begin{matrix} 3 & 5 & 2 \\ 4 & 5 & 2 \\ 3 & 1 & 2 \end{matrix}$$

Show expressions for the magnitude and direction of the gradient at the middle pixel. Do not solve for the values. In which of the eight Canny directions is the gradient pointed?

`magnitude =` $\sqrt{G_x^2 + G_y^2} = \sqrt{36+64}$

`direction =` $\arctan(\frac{G_y}{G_x}) = \arctan(\frac{8}{-6})$

`direction is 8 up and 6 left, which is more up/left than it is up, so the direction is up/left.`

Part B — 1 point Suppose that a pixel has a gradient magnitude between the low and high thresholds of the Canny algorithm. When is such a pixel considered to be an edge pixel?

`When it is connected to a pixel with a gradient magnitude above the high threshold.`

... continued

3 Hough Transform — 2 points

Part A — 1 point What does each bin in the parameter space of the Hough Transform represent? What is stored in each bin?

```
A set of the objects being searched for.  Or a range of object
positions.

The number of votes given to objects in that bin.
```

Part B — 1 point For the Generalized Hough Transform (without rotation or scaling), what is the relation between the shape being searched for in the image and the pattern of votes a pixel makes in the parameter space?

```
The pattern of votes in the parameter space is the reflection through
the reference point of the pixels of the object in the image.
```

4 Compression — 4 points

Part A — 2 points Give an arithmetic encoding for the token string `xyz` if each token has equal probability. Show your work, including the interval corresponding to each token.

```
Assign [0,1/3) to x, [1/3,2/3) to y, and [2/3,1) to z

[0,1) -> [0,1/3) -> [1/9,2/9) -> [5/27,6/27)

Use any number in [5/27,6/27).
```

... continued

Part B — 2 points For LZW compression, let there be a single token, `a`, that is initially stored in the LZW dictionary: `dict[1] = a`. What is the LZW encoding of `aaaa`? Show your work.

```
dictionary:  1:a

(a) a known:  keep going
(aa) aa not known:  output 1 for a; dict[2]=aa; start with a
a(aa) aa known:  keep going
a(aaa) aaa not known:  output 2 for aa; dict[3]=aaa; start with a
a last token:  output 1 for a

output is 1, 2, 1
```