Queen's University
CISC/CMPE 457
Test 2

November 2, 2018
Duration: 50 minutes

Closed book

Initial of Family Name: ____

Student Number: ____________________
(Write this at the top of every page.)

There are 5 questions and 17 marks total.

Answer all questions.

This exam paper should have 7 pages,
including this cover page.

1 – Short Answers	/ 7
2 – Aliasing	/ 3
3 – Interpretation of the Fourier Transform	/ 4
4 – Edge Detection	/ 2
5 – Line Parameterizations	/ 1
Total	/ 17

The candidate is urged to submit with the answer paper a clear statement of any assumptions made if doubt exists as to the interpretations of any question that requires a written answer.

1 Short Answers — 7 points

Part A — 1 point Can $\cos^2 t$ be represented as a Fourier Series? Explain.

Yes, because it's periodic and any periodic function can be represented as a Fourier Series.

Part B — 1 point For a function $f(x)$ that has period T, show an expression for the projection of $f(x)$ onto the third Real Fourier basis function, which is $\cos\left(\frac{2\,\pi\,x}{T}\right)$ for period T.

$$\frac{\int_0^T f(x)\cos(\frac{2\pi x}{T})\;dx}{\int_0^T \cos^2(\frac{2\pi x}{T})\;dx} \quad \text{or} \quad \frac{\int_0^T f(x)\cos(\frac{2\pi x}{T})\;dx}{\pi}$$

Part C — 1 point Under what general condition should convolution of two signals *not* be performed as a multiplication in the Fourier domain? Explain.

Do not do this if one of the signals is small (such as a 7x7 filter), since the extra work of moving to and from the Fourier domain would add too much time to the calculation.

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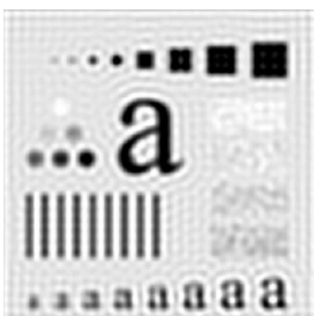
Part D — 2 points If a 1D Fast Fourier Transform (FFT) of a discrete signal of N samples takes T seconds, about how many seconds does a 2D FFT of an $N \times N$ signal take? Explain how the 2D FFT is done and show your calculation.

[1] 2D is done with N 1D FFTs in columns (or rows), followed by N 1D FFTs in rows (or columns).

[1] Calculate: NxT (columns) + NxT (rows) = 2 N T

Part E — 1 point Explain why a filter that sharply cuts off frequencies above a certain threshold results in echos, such as shown below.



The sharp cutoff corresponds to multiplication by an ideal low pass filter in the frequency domain (i.e. a box filter), which corresponds to multiplying by a sinc() function in the spatial domain. The lobes of the sinc() function cause echos in the spatial domain.

It's not sufficient to say "this is called ringing".

Part F — 1 point Otsu's method maximizes the between-class variance, $N_1 N_2 (\mu_1 - \mu_2)^2$, where N_i and μ_i are the size and mean of class i, respectively. *Briefly* explain what makes Otsu's method very efficient.

The method computes the between-class variance incrementally with increasing T. Each increment takes constant time because the N and μ are updated from the previous N and μ.

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2 Aliasing — 3 points

Suppose that we want to sample a continuous function, $f(t)$, so that it can later be perfectly reconstructed. The highest-frequency component of f's Real Fourier Series is $\cos 4t$.

Part A — 1 point What is the highest frequency in $f(t)$? Explain.

$\cos 4t$ has 4 cycles per 2π, or frequency $\frac{4}{2\pi} = \frac{2}{\pi}$.

Part B — 2 points For perfect reconstruction, what must be the minimum distance between samples of $f(t)$? Explain.

[1] The function must be sampled at twice the highest frequency, or with frequency $2 \times \frac{2}{\pi} = \frac{4}{\pi}$.

[1] For frequency $\frac{4}{\pi}$, the period is $\frac{\pi}{4}$, so the minimum spacing is $\frac{\pi}{4}$.

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3 Interpretation of the Fourier Transform — 4 points

For a 1024×512 image containing a grid (like the grids in Assignment 2), suppose that closest-to-the-origin high-magnitude peak in the image's Fourier Transform is at position $(0, 64)$.

Part A — 1 point What features in the image does that peak correspond to? Explain.

> [0.5] horizontal lines
>
> [0.5] Because a peak on the y axis corresponds to a horizontal wave travelling upward. The crests of the wave are horizontal lines.

Part B — 2 points What is the spacing, in pixels, between those features? Show your calculations and explain carefully with reference to the image size and the position of the peak.

> For an image 512 pixels in height, the basis function with wavenumber 1 has period of 512 pixels (i.e. it's a single cycle from top to bottom), and the basis function with wavenumber k has period 512/k pixels. Since position y = 64 corresponds to wavenumber 64, the period of this wave is 512/64 = 8. So the horizontal lines are 8 pixels apart.
>
> [1] The peak at (0, 64) has wavenumber 64 and represents 64 cycles per 512 pixels in the vertical direction.
>
> [1] Calulate the period of the wave, which is 1/freqency, which is 1/(64/512) = 8 pixels.

Part C — 1 point Explain **why** the Fourier Transform will also have peaks at $(0, 128)$, $(0, 192)$, $(0, 256)$, and so on.

> Because the sharp edges of the grid must be represented by a sum of harmonics of the base frequency, and those harmonics are at 2x, 3x, 4x, and so on of the base frequency, corresponding to peaks at the locations listed above.

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4 Edge Detection — 2 points

Consider the following 3×3 pixels:

$$\begin{matrix} 4 & 6 & 9 \\ 0 & 5 & 0 \\ 1 & 4 & 4 \end{matrix}$$

Part A — 1 point If the Canny algorithm uses the central difference gradient operator (which is $\frac{1}{2}(f(x+1,y) - f(x-1,y))$ in the x direction and $\frac{1}{2}(f(x,y+1) - f(x,y-1))$ in the y direction) what is the **discretized gradient direction** computed by the Canny algorithm? Show your work.

Gx = (0-0)/2 = 0

Gy = (6-4)/2 = 1

Direction of gradient = 90 degrees since Gx = 0.

Gradient points in direction of fastest increase, so it points DIRECTLY UPWARD.

Note that gradient calculated by Canny is NOT in the up-and-right direction because Canny combines Gx and Gy, so ignores the four corner values.

Part B — 1 point Based on your gradient direction, is the middle pixel suppressed by the Canny algorithm? Explain.

For a gradient directly upward, the middle pixel IS suppressed because it is not a local maximum in that direction. Answers with gradients in other directions, depend on whether it's a local maximum in those directions.

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5 Line Parameterizations — 1 point

The Hough Transform uses the (θ, ρ) line parameterization:

$$x \cos\theta + y \sin\theta = \rho.$$

Draw a diagram of the line $y = -x + 4$. Determine the θ and ρ for the line. Show your work.

Diagram with line of slope -1 passing through (0,4) and (4,0).
Statement of theta = 45 degrees. calculation of $\rho = \sqrt{2^2 + 2^2}$.