Queen's University
CISC/CMPE 457
Test 3 SOLUTIONS

November 29, 2016
Duration: 50 minutes

Closed book

Initial of Family Name: ____

Student Number: ________________________
(Write this at the top of every page.)

There are 4 questions and 19 marks total.

Answer all questions.

This exam paper should have 7 pages,
including this cover page.

1 – Fourier Applications	/ 6
2 – CT Imaging	/ 6
3 – Feature detection	/ 5
4 – Compression	/ 2
Total	/ 19

The candidate is urged to submit with the answer paper a clear statement of any assumptions made if doubt exists as to the interpretations of any question that requires a written answer.

1 Fourier Applications — 6 points

Part A — 2 points How can you construct a band-pass filter that keeps only frequencies between L and H (with $L < H$), using only a low-pass filter and a high-pass filter? Specify the cutoff frequencies of the low-pass filter and the high-pass filter.

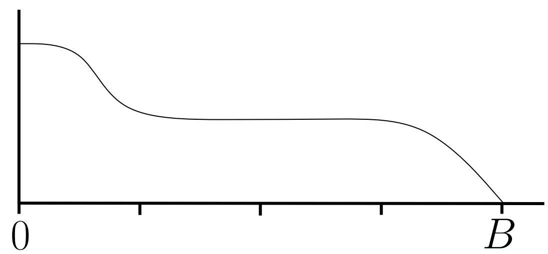
```
[1 mark] use high-pass with cutoff L
[1 mark] also use low-pass with cutoff H
```

Part B — 2 points The Butterworth low-pass filter is $H(x) = \dfrac{1}{1 + (\frac{x}{D_0})^{2n}}$. What happens at frequency D_0? What happens as n increases?

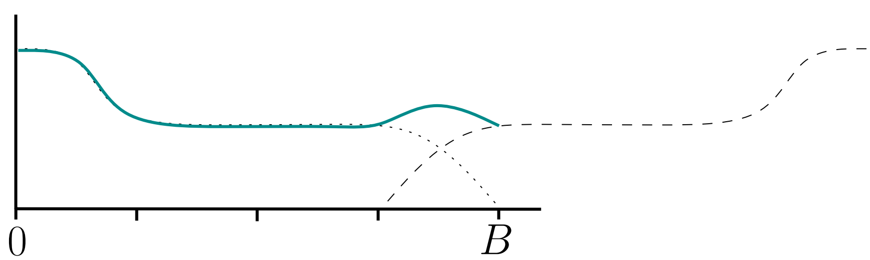
```
[1 mark] At D_0, H(x) is 0.5
[1 mark] H(x) becomes steeper at D_0 as n increases
```

... continued

Part C — 2 points Given a signal that has maximum frequency B with the spectrum shown on the first graph below, draw on the second graph below the spectrum of the reconstructed signal if the original signal is sampled at rate $1.5B$.



spectrum of original signal



spectrum of reconstructed signal

```
Should have spectrum mirrored at 1/2 the sample rate (i.e. 0.75 B), then
added to the original spectrum to generate the aliased signal.

[1 mark] mirrored
[1 mark] at correct position (0.75 B) and added to non-mirrored spectrum
```

... continued

2 CT Imaging — 6 points

Part A — 1 point The Radon transform is

$$g(\rho, \theta) = \int\int f(x, y)\ \delta(x \cos\theta + y \sin\theta - \rho)\ dx\ dy$$

For a 2D object with x-ray attenuation $f(x, y)$, what is the geometric interpretation of the Radon transform? Answer in terms of attenuation and the parameters ρ and θ.

```
[1 mark] The total attenuation along the line of x cos theta + y sin theta = rho
```

Part B — 1 point The Fourier Slice Theorem states that $G(\omega, \theta) = \int g(\rho, \theta)\ e^{-2\pi i\omega\rho}\ d\rho$. Draw $G(\omega, \theta)$ in the Fourier domain for a fixed θ and varying ω. Label θ and ω.

```
[1 mark] Line through G(u,v) origin at angle theta.
[1 mark] omega is distance along the line from the origin.
```

... continued

Part C — 2 points What is the geometric interpretation of $G(\omega, \theta)$ as defined in Part B above? Answer in terms of the parameter θ and the original function, $f(x, y)$.

```
G is the Fourier transform of the [1 mark] **projection** of f(x,y)
[1 mark] **perpendicular** to direction theta.
```

Part D — 2 points The Ram-Lak filter is $H(\omega) = |\,\omega\,|$. Explain, with reference to the reparameterization of the Fourier domain from (u, v) to (θ, ω), how the Ram-Lak filter is derived.

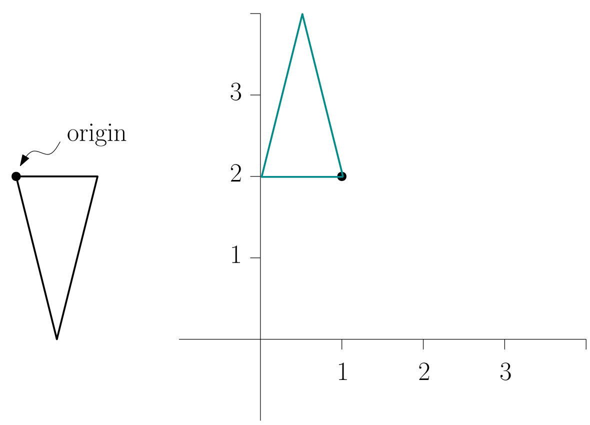
```
In reparameterizing from u,v to theta,omega, [1 mark] infinitesimal
area du dv becomes area domega omega dtheta, so [1 mark] the omega
term multiplies the G(omega,theta) value inside the integral
```

3 Feature detection — 5 points

Part A — 2 points Describe the method by which Canny edge detection removes pixels that are not at the centre of a wide edge.

```
Only select a pixel if it is [1 mark] a local maximum in the
(quantized) [1 mark] direction perpendicular to the gradient
```

Part B — 1 point For detecting the shape below with its indicated origin, draw the Generalized Hough Transform for the point $(1, 2)$. The shape is one unit wide and two units high.



Part C — 2 points Name one disadvantage of the k-means segmentation method. Name one advantage of Otsu's method.

```
[1 mark] k-means has different segmentations depending upon the initial
         choices of means

[1 mark] Otsu takes time linear in the number of pixel values (not in
         the number of pixels)
```

4 Compression — 2 points

Part A — 1 point Calculate the entropy of the following 2×2 image containing pixels with values 0 and 1. Show your work

1	0
0	1

```
[1 mark] for formula
[1 mark] for correct use of formula and calculation

H = - sum_x P(x) log P(x)   for all possible values, x (only 0 and 1 in this case)
  = - (P(0) log P(0) + P(1) log P(1))
  = - (0.5 (-1) + 0.5 (-1))
  = 1
```

Part B — 1 point What does the entropy tell about image encoding?

```
[1 mark] Its equal to the average minimum number of bits required
per pixel, assuming random pixels.
```