Queen's University
CISC/CMPE 457
Test 1

September 24, 2020
noon
Duration: 24 hours

Closed book

Initial of Family Name: ____

Student Number: ______________________________
(Write this at the top of every page.)

There are 8 questions and 32 marks total.

Answer all questions.

This exam paper should have 12 pages,
including this cover page.

1 – Metrics	/ 3
2 – Sensors	/ 6
3 – Pixel Transforms	/ 3
4 – Vignetting	/ 3
5 – Histograms	/ 2
6 – Homogeneous Transforms	/ 3
7 – Distributive Convolution	/ 6
8 – Filters	/ 6
Total	/ 32

The candidate is urged to submit with the answer paper a clear statement of any assumptions made if doubt exists as to the interpretations of any question that requires a written answer.

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1 Metrics — 3 points

P-norms with $p < 1$ are sometimes used (especially in machine learning in higher-dimensional spaces). For the p-norm with $p = 0.5$ in two dimensions, draw the 2-neighbourhood and draw the 4-neighbourhood. (You might have to draw these two neighbourhoods on paper, then submit a photo to ONQ.)

The 2-neighbourhood with $p = 0.5$ has $(\sqrt{|x|} + \sqrt{|y|})^2 \leq 2$. Try different values of x and y:

x, y	0.5-norm
$0, 0$	0
$1, 0$	1
$1, 1$	4
$2, 0$	2
$2, 1$	5.8
$2, 2$	7.8

Note that negative coordinates have their absolute value taken, so there's symmetry through the origin. And (x, y) has the same 0.5-norm as (y, x).

So the 2-neighbourhood is the 0,1,2 digits below, around the 0 digit.

```
.  .  2  .  .
.  .  1  .  .
2  1  0  1  2
.  .  1  .  .
.  .  2  .  .
```

Similarly, the 4-neighbourhood is

```
.  .  .  .  4  .  .  .  .
.  .  .  .  3  .  .  .  .
.  .  .  .  2  .  .  .  .
.  .  .  4  1  4  .  .  .
4  3  2  1  0  1  2  3  4
.  .  .  4  1  4  .  .  .
.  .  .  .  2  .  .  .  .
.  .  .  .  3  .  .  .  .
.  .  .  .  4  .  .  .  .
```

2 Sensors — 6 points

Suppose you have a square CCD array of pixels and only 4 analog-to-digital (A/D) converters. Each pixel must be shifted to an edge of the array, then out of the array into another charge shifter that sits along the edge of the array, then along the charge shifter to the A/D converter (as was discussed in the video lecture). Each pixel can be shifted through the array in only one direction: upward, downward, leftward, or rightward.

Part A — 2 points For an $n \times n$ array with A/D converters positioned on the corners of the array (as discussed in the video lecture), what is the maximum number of shifts that a pixel could undergo, including the shifts in the charge shifter that sits along the edge of the array? Explain.

The middle pixel of the array would have to be shifted n/2 times to get into the charge shifter, then n/2 to get into the A/C converter on the corner, for a total of n shifts.

Part B — 2 points The A/D converters could, alternatively, be positioned at the *middles* of the edges of the array, with charge shifters that bring pixels toward the middle converter along each edge. In this configuration, what is the maximum number of shifts that a pixel could undergo, including the shifts in the charge shifter that sits along the edge of the array? Explain.

A pixel gets shifted horizontally or vertically. The number of shifts is the L_1 (Manhattan) distance that the pixel moves. The L_1 distance to the closest middle of an edge is at most $\frac{n}{2}$ for any pixel in the array. To see this, draw the $\frac{n}{2}$-neighbourhood around the middle position on each edge. (Note that, for A/D converters on the *corners*, the maximum L_1 distance is n ... to the centre pixel of the array.)

Part C — 2 points What effect does the number of shifts have on the image quality? Explain.

More shifts result in more noise because each shift may cause electrons to be left behind, or added from free (thermal) electrons.

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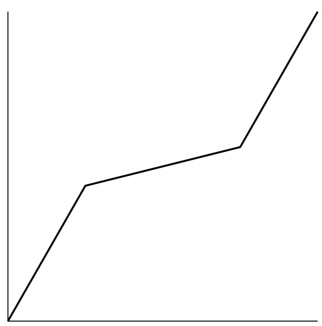
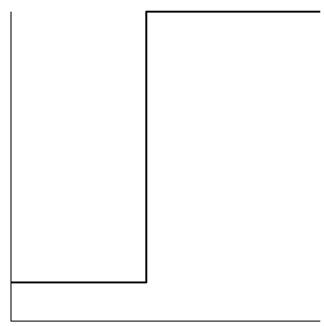
3 Pixel Transforms — 3 points

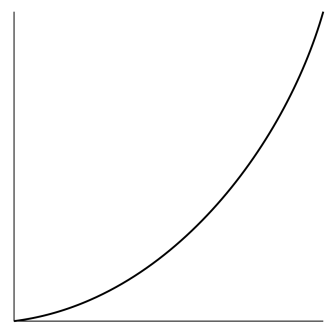
Draw graphs of three intensity transforms that (A) decreases contrast in mid-range intensities, (B) thresholds the image, and (C) smoothly increases contrast in high intensities while smoothly decreasing contrast in low intensities. You'll have to upload an image of your graphs.



(A) slope of curve in middle is less than 1



(B) curve jumps between two constant values (could be 0 and 1, but not necessarily)



(C) curve has smoothly increasing slope

4 Vignetting — 3 points

Suppose you have a camera that does not control vignetting. Explain what you would do to remove vignetting from a photograph.

Take a photo of a neutral white background. Scale the pixel intensities so that the maximum intensity is 1.0. In a photo editor, divide each pixel of the photograph by intensity in the corresponding pixel of the (scaled) neutral white background.

5 Histograms — 2 points

The goal of histogram equalization is to get an equal number of pixels at each intensity. Explain why an equalized histogram of a low-contrast image does *not* have an equal number of pixels at each intensity.

A low contrast image has its pixel intensities in a small range, say $[a, b]$. Histogram equalization will spread that range of intensities to a larger range, $[c, d]$. Since $d - c > b - a$, there will be some intensities with *no* pixels, while the other intensities will have the same (unequal) numbers of pixels that they had before the equalization.

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6 Homogeneous Transforms — 3 points

Derive a 3×3 homogeneous transform that transforms 2D homogeneous pixels, $[x, y, 1]$, by scaling by 0.5, then rotating by 180 degrees, then translating by $[1, 2]$. Show your work.

$$\underbrace{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{translate [1,2]}} \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{rotate 180}} \underbrace{\begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{scale 0.5}} = \underbrace{\begin{bmatrix} -0.5 & 0 & 1 \\ 0 & -0.5 & 2 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{product}}$$

7 Distributive Convolution — 6 points

Prove that convolution is distributive: $A*(B+C) = A*B+A*C$, where $*$ is the convolution operation. Use the same style of proof as in the online notes.

Proof in 1D:

$$
\begin{aligned}
(A*(B+C))(x) &= \textstyle\sum_i A(x-i)(B+C)(i) \\
&= \textstyle\sum_i A(x-i)(B(i)+C(i)) \\
&= \textstyle\sum_i (A(x-i)B(i)+A(x-i)C(i)) \\
&= \textstyle\sum_i A(x-i)B(i) + \sum_i A(x-i)C(i) \\
&= A*B+A*C
\end{aligned}
$$

... continued

8 Filters — 6 points

Consider two filters: $H = [-1 \ \ 0 \ \ 1]$ and $V = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

Part A — 2 points Compute $D = H * V$. Note that the dimensions of D are 3x3.

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

Part B — 2 points Given a 1000×2000 image, I, how many multiplications are required to compute $D * I$? Show your work.

The new image is of size 1002×2002, since an extra row and column appears around each edge of the image where the 3×3 filter overlaps by only one row or column.

For each of the 1002×2002 elements, 9 ($= 3 \times 3$) multiplications are done, for a total of 18,054,036 multiplications.

You might argue that only 6 multiplications are needed because you know that some elements of the 3×3 filter are zero. In this case, you would have a total of $1002 \times 2002 \times 6 = 12,036,024$ multiplications.

Part C — 2 points How can you compute $D * I$ with fewer multiplications? How many multiplications are used by this more efficient method?

$D * I = (H * V) * I = H * (V * I)$

So we can compute $R = V * I$ in one step, then compute $H * R$ in another step.

$V * I$ takes $1000 \times 2002 \times 3$ multiplications since the convolution is one larger in the *vertical* dimension only (or $1000 \times 2002 \times 2$ if you eplicitly ignore zeroes).

Then $H * R$ takes $1002 \times 2002 \times 3$ multiplications since the convolution is one larger in the *horizontal* direction while the R is already 2002 in the vertical direction (or $1002 \times 2002 \times 2$ if zeroes are ignored).

The total is $1000 \times 2002 \times 3 + 1002 \times 2002 \times 2 = 12,024,012$ multiplications (or 8,016,008 if you ignore zeroes).