Application exercises on Simscape and Multibody Toolbox of Matlab / Simulink environment

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Ex 1: Scara robot

1-

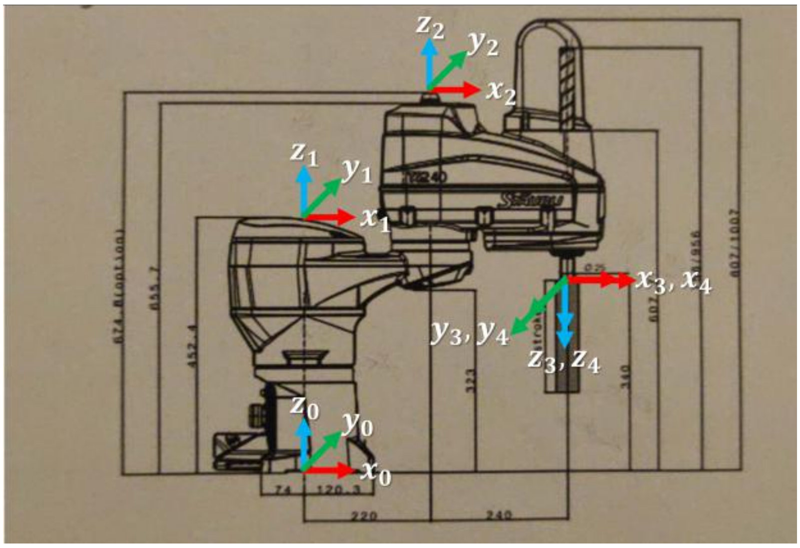

Figure 1: Scara Robot arm

Joint no.	Type of joint σ	$\theta_i(rad)$	$d_i(mm)$	$r(mm)$	$\alpha_i(rad)$
1	Revolute(0)	$\theta_1(q_1)$	0	452.4	0
2	Revolute(0)	$\theta_2(q_2)$	220	222.4	0
3	Revolute(0)	$\theta_3(q_3)$	240	-334	π
4	Prismatic(1)	0	0	q_4	0

Table 1: DH Parameters for the Scara Robot

2-

From the DH parameters table, the transformation matrices for the Scara robot:

$$T_1^0 = \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 & 0 \\ sin\theta_1 & cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 452.4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$T_2^1 = \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 & 220 \\ sin\theta_2 & cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 222.4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$T_3^2 = \begin{bmatrix} cos\theta_3 & -sin\theta_3 & 0 & 240 \\ sin\theta_3 & cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & -334 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$T_4^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

3-

From the previous matrices (1)(2)(3), the Homogeneous transformation matrix that express the coordinates from frame #3 to the base frame can be obtained as:

$$T_3^0 = T_1^0 T_2^1 T_3^2 \tag{5}$$

$$T_3^0 = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

Where:

$m_{11} = cos\theta_3 * (cos\theta_1 * cos\theta_2 - sin\theta_1 * sin\theta_2) - sin\theta_3 * (cos\theta_1 * sin\theta_2 + cos\theta_2 * sin\theta_1)$

$m_{12} = -cos\theta_3 * (cos\theta_1 * sin\theta_2 + cos\theta_2 * sin\theta_1) - sin\theta_3 * (cos\theta_1 * cos\theta_2 - sin\theta_1 * sin\theta_2)$

$m_{13} = 0$

$m_{14} = 220 * cos\theta_1 + 240 * cos\theta_1 * cos\theta_2 - 240 * sin\theta_1 * sin\theta_2$

$m_{21} = cos\theta_3 * (cos\theta_1 * sin\theta_2 + sin\theta_1 * cos\theta_2) + sin\theta_3 * (cos\theta_1 * cos\theta_2 - sin\theta_2 * sin\theta_1)$

$m_{22} = cos\theta_3 * (cos\theta_1 * cos\theta_2 - sin\theta_1 * sin\theta_2) - sin\theta_3 * (cos\theta_1 * sin\theta_2 + cos\theta_2 * sin\theta_1)$

$m_{23} = 0$

$m_{24} = 220 * sin\theta_1 + 240 * cos\theta_1 * sin\theta_2 + 240 * cos\theta_2 * sin\theta_1$

$m_{31} = 0$

$m_{32} = 0$

$m_{33} = 1$

$m_{34} = \frac{1704}{5}$

4-

After computing the Homogeneous transformation matrix from frame #3 to the base frame, the Jacobian matrix can be obtained as:

$$J = \begin{bmatrix} \frac{\partial f_x}{\partial \theta_1} & \frac{\partial f_x}{\partial \theta_2} & \frac{\partial f_x}{\partial \theta_3} \\ \frac{\partial f_y}{\partial \theta_1} & \frac{\partial f_y}{\partial \theta_2} & \frac{\partial f_y}{\partial \theta_3} \\ \frac{\partial f_z}{\partial \theta_1} & \frac{\partial f_z}{\partial \theta_2} & \frac{\partial f_z}{\partial \theta_3} \end{bmatrix} \tag{7}$$

$$J = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \tag{8}$$

Where:

$m_{11} = -220 * sin\theta_1 - 240 * cos\theta_1 * sin\theta_2 - 240 * cos\theta_2 * sin\theta_1$

$m_{12} = -240 * cos\theta_1 * sin\theta_2 - 240 * cos\theta_2 * sin\theta_1$

$m_{13} = 0$

$m_{14} = 0$

$m_{21} = 220 * cos\theta_1 + 240 * cos\theta_1 * cos\theta_2 - 240 * sin\theta_1 * sin\theta_2$

$m_{22} = 240 * cos\theta_1 * cos\theta_2 - 240 * sin\theta_1 * sin\theta_2$

$m_{23} = 0$

$m_{24} = 0$

$m_{31} = 0$

$m_{32} = 0$

$m_{33} = 0$

$m_{34} = 1$

5-

After computing the Homogeneous transformation matrix from the end-effector frame to the base frame T_4^0, the Jacobian matrix of the 4 joints(3 revolute and 1 prismatic) can be obtained as:

$$J = \begin{bmatrix} \frac{\partial f_x}{\partial \theta_1} & \frac{\partial f_x}{\partial \theta_2} & \frac{\partial f_x}{\partial \theta_3} & \frac{\partial f_x}{\partial q_4} \\ \frac{\partial f_y}{\partial \theta_1} & \frac{\partial f_y}{\partial \theta_2} & \frac{\partial f_y}{\partial \theta_3} & \frac{\partial f_y}{\partial q_4} \\ \frac{\partial f_z}{\partial \theta_1} & \frac{\partial f_z}{\partial \theta_2} & \frac{\partial f_z}{\partial \theta_3} & \frac{\partial f_z}{\partial q_4} \end{bmatrix} \tag{9}$$

$$J = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \tag{10}$$

Where:

$m_{11} = -220 * sin\theta_1 - 240 * cos\theta_1 * sin\theta_2 - 240 * cos\theta_2 * sin\theta_1$

$m_{12} = -240 * cos\theta_1 * sin\theta_2 - 240 * cos\theta_2 * sin\theta_1$

$m_{13} = 0$

$m_{14} = 0$

$m_{21} = 220 * cos\theta_1 + 240 * cos\theta_1 * cos\theta_2 - 240 * sin\theta_1 * sin\theta_2$

$m_{22} = 240 * cos\theta_1 * cos\theta_2 - 240 * sin\theta_1 * sin\theta_2$

$m_{23} = 0$

$m_{24} = 0$

$m_{31} = 0$

$m_{32} = 0$

$m_{33} = 0$

$m_{34} = 1$

But since we are interested in the horizontal plane only, we can ignore the third row as it expresses the partial derivatives of the velocity along the z-direction with respect to θ_1, θ_2, θ_3 and q_4. Since joint3 rotates around itself, it will not have any effect in changing the velocity along any of the three axes. So, we can ignore the third column from the matrix. Moreover, since joint4 is a prismatic joint along the z-direction that does not change its orientation due to the kinematic modeling of the robot, it will not have any effects on the velocities along the x and y directions. So, we can also ignore the fourth column of the matrix. At the end our final jacobian matrix J_{θ_1,θ_2} will be $2 * 2$ as shown below where the only variables affecting the motion within the horizontal plane are θ_1 and θ_2.

$$J_{\theta_1,\theta_2} = \begin{bmatrix} \frac{\partial f_x}{\partial \theta_1} & \frac{\partial f_x}{\partial \theta_2} \\ \frac{\partial f_y}{\partial \theta_1} & \frac{\partial f_y}{\partial \theta_2} \end{bmatrix} \tag{11}$$

$$J_{\theta_1,\theta_2} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \tag{12}$$

Where:

$m_{11} = -220 * sin\theta_1 - 240 * cos\theta_1 * sin\theta_2 - 240 * cos\theta_2 * sin\theta_1$

$m_{12} = -240 * cos\theta_1 * sin\theta_2 - 240 * cos\theta_2 * sin\theta_1$

$m_{21} = 220 * cos\theta_1 + 240 * cos\theta_1 * cos\theta_2 - 240 * sin\theta_1 * sin\theta_2$

$m_{22} = 240 * cos\theta_1 * cos\theta_2 - 240 * sin\theta_1 * sin\theta_2$

So, to calculate the singularities we need to find which combinations of θ_1 and θ_2 that make the determinant of the jacobian matrix equals to zero. For that, we wrote a Matlab code to calculate these combinations numerically and we found that these combinations are all possible values of θ_1 when θ_2 is equal to zero.

Ex 2: Modeling of a Hexapod Robot leg

1-

The desired controlled dof of the foot is **3**.

2-

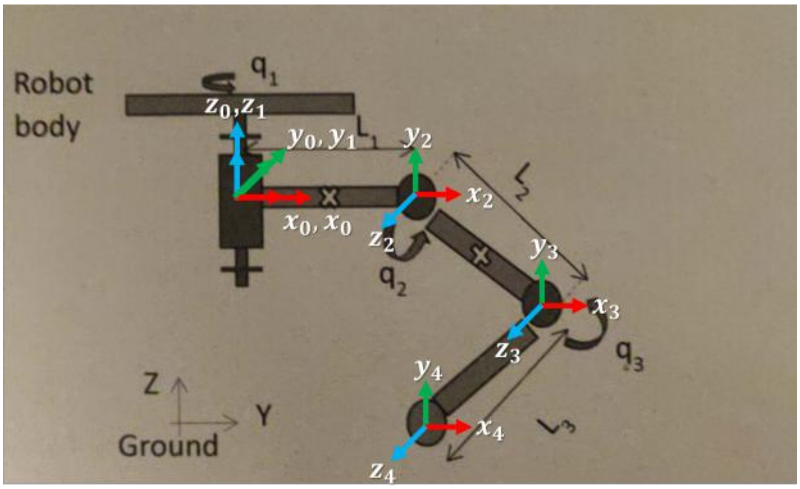

Figure 2: Hexapod Robot leg

Joint no.	σ	$\theta_i(rad)$	$d_i(mm)$	$r(mm)$	$\alpha_i(rad)$
1	0	θ_1	0	0	0
2	0	θ_2	L_1	0	$\pi/2$
3	0	θ_3	L_2	0	0
4	0	0	L_3	0	0

Table 2: DH Parameters for the Hexapod Robot leg

From the DH parameters table, the transformation matrices of the leg:

$$T_1^0 = \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 & 0 \\ sin\theta_1 & cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

$$T_2^1 = \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 & L_1 \\ 0 & 0 & -1 & 0 \\ sin\theta_2 & cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

$$T_3^2 = \begin{bmatrix} cos\theta_3 & -sin\theta_3 & 0 & L_2 \\ sin\theta_3 & cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

$$T_4^3 = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

From the previous 4 matrices, the final Homogeneous transformation matrix can be obtained as:

$$T_4^0 = T_1^0 T_2^1 T_3^2 T_4^3 \tag{17}$$

$$T_4^0 = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

Where:

$m_{11} = cos\theta_1 * cos\theta_2 * cos\theta_3 - cos\theta_1 * sin\theta_2 * sin\theta_3$

$m_{12} = -cos\theta_1 * cos\theta_2 * sin\theta_3 - cos\theta_1 * cos\theta_3 * sin\theta_2$

$m_{13} = sin\theta_1$

$m_{14} = L_1 * cos\theta_1 + L_3 * (cos\theta_1 * cos\theta_2 * cos\theta_3 - cos\theta_1 * sin\theta_2 * sin\theta_3) + L_2 * cos\theta_1 * cos\theta_2$

$m_{21} = cos\theta_2 * cos\theta_3 * sin\theta_1 - sin\theta_1 * sin\theta_2 * sin\theta_3$

$m_{22} = -cos\theta_2 * sin\theta_1 * sin\theta_3 - cos\theta_3 * sin\theta_1 * sin\theta_2$

$m_{23} = -cos\theta_1$

$m_{24} = L_1 * sin\theta_1 + L_3 * (cos\theta_2 * cos\theta_3 * sin\theta_1 - sin\theta_1 * sin\theta_2 * sin\theta_3) + L_2 * cos\theta_2 * sin\theta_1$

$m_{31} = cos\theta_2 * sin\theta_3 + cos\theta_3 * sin\theta_2$

$m_{32} = cos\theta_2 * cos\theta_3 - sin\theta_2 * sin\theta_3$

$m_{33} = 0$

$m_{34} = L_2 * sin\theta_2 + L_3 * (cos\theta_2 * sin\theta_3 + cos\theta_3 * sin\theta_2)$

3-

After computing the final Homogeneous transformation matrix, the Jacobian matrix that gives the foot motion relative to the ground frame can be obtained as:

$$J = \begin{bmatrix} \frac{\partial f_x}{\partial \theta_1} & \frac{\partial f_x}{\partial \theta_2} & \frac{\partial f_x}{\partial \theta_3} \\ \frac{\partial f_y}{\partial \theta_1} & \frac{\partial f_y}{\partial \theta_2} & \frac{\partial f_y}{\partial \theta_3} \\ \frac{\partial f_z}{\partial \theta_1} & \frac{\partial f_z}{\partial \theta_2} & \frac{\partial f_z}{\partial \theta_3} \end{bmatrix} \tag{19}$$

$$J = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \tag{20}$$

Where:

$$m_{11} = L_3 * (sin\theta_1 * sin\theta_2 * sin\theta_3 - cos\theta_2 * cos\theta_3 * sin\theta_1) - L_1 * sin\theta_1 - L_2 * cos\theta_2 * sin\theta_1$$

$$m_{12} = -L_3 * (cos\theta_1 * cos\theta_2 * sin\theta_3 + cos\theta_1 * cos\theta_3 * sin\theta_2) - L_2 * cos\theta_1 * sin\theta_2$$

$$m_{13} = -L_3 * (cos\theta_1 * cos\theta_2 * sin\theta_3 + cos\theta_1 * cos\theta_3 * sin\theta_2)$$

$$m_{21} = L_1 * cos\theta_1 - L_3 * (cos\theta_1 * sin\theta_2 * sin\theta_3 - cos\theta_1 * cos\theta_2 * cos\theta_3) + L_2 * cos\theta_1 * cos\theta_2$$

$$m_{22} = -L_3 * (cos\theta_2 * sin\theta_1 * sin\theta_3 + cos\theta_3 * sin\theta_1 * sin\theta_2) - L_2 * sin\theta_1 * sin\theta_2$$

$$m_{23} = -L_3 * (cos\theta_2 * sin\theta_1 * sin\theta_3 + cos\theta_3 * sin\theta_1 * sin\theta_2)$$

$$m_{31} = 0$$

$$m_{32} = L_3 * (cos\theta_2 * cos\theta_3 - sin\theta_2 * sin\theta_3) + L_2 * cos\theta_2$$

$$m_{33} = L_3 * (cos\theta_2 * cos\theta_3 - sin\theta_2 * sin\theta_3)$$

4-

Fig. 3 shows the constructed Simulink model of the Hexapod leg.

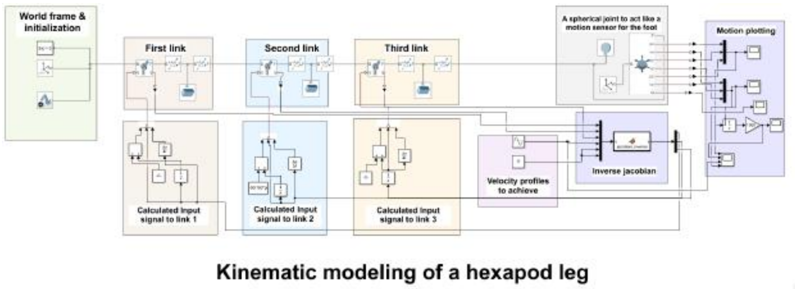

Figure 3: Simulink model of the Hexapod leg

The position trajectory generated from the Simulink model of the Hexapod leg is shown in Fig. 4.

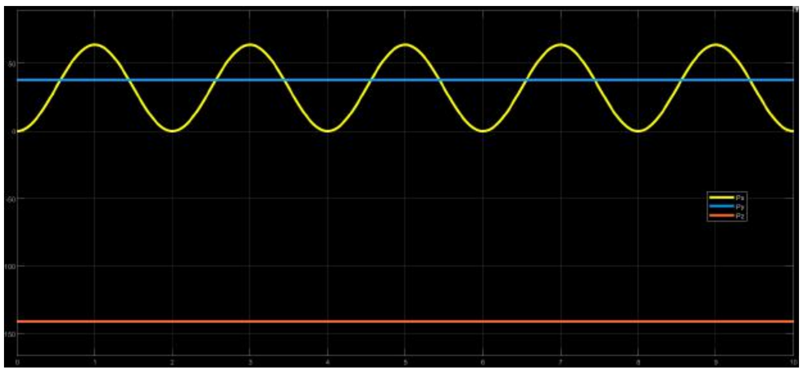

Figure 4: Position trajectory generated from the Simulink model

Yes we can make the leg move in one direction but there are some oscillations happening in the other axes and this is shown in Fig. 5. We tried to input a velocity profile as a *sin* signal in the x-direction only and the inputs for the y and z directions were zeros. However, the scope showed small oscillations in the y and z directions.

Moreover, despite having no oscillations in the position trajectory along y and z directions as shown in Fig. 4, there were some observed oscillations in the velocity trajectory in Fig. 5. This is may be caused by an error in the position sensing unit in the motion sensor "the spherical joint'. So, we integrated the velocity trajectory in Fig. 5 to obtain the position trajectory and indeed there were some oscillations in the position trajectory as well which are shown in Fig. 6. Please note that there are some offset differences between Fig. 4 and Fig. 6 because we did not add the integration constants.

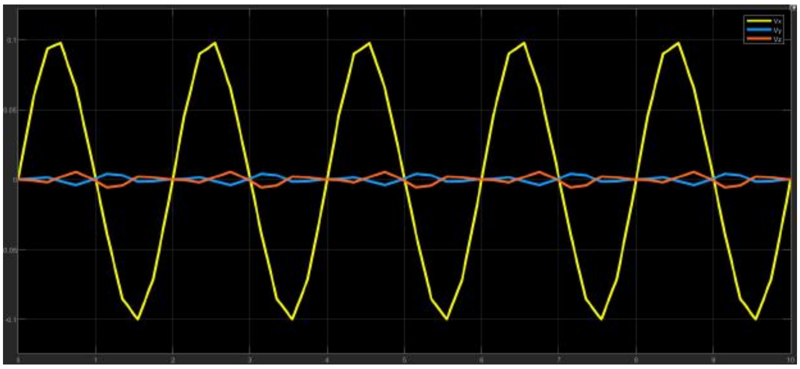

Figure 5: Velocity trajectory generated from the Simulink model

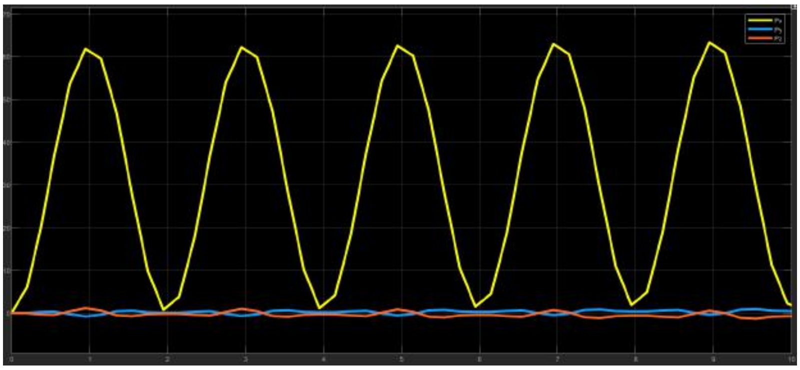

Figure 6: Position trajectory obtained from integrating the Velocity trajectory