Application exercises on Simscape and Multibody Toolbox of Matlab / Simulink environment

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Ex I- Modelling of a robot PUMA 560 "First 3 joints"

The transformation matrices for the PUMA 560 robot:

$$T_1^0 = \begin{bmatrix} cos\theta_1 & 0 & -sin\theta_1 & 0 \\ sin\theta_1 & 0 & cos\theta_1 & 0 \\ 0 & -1 & 0 & d_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$T_2^1 = \begin{bmatrix} -sin\theta_2 & cos\theta_2 & 0 & d_3 cos\theta_2 \\ cos\theta_2 & sin\theta_2 & 0 & d_3 sin\theta_2 \\ 0 & 0 & -1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$T_3^2 = \begin{bmatrix} sin\theta_3 & 0 & -cos\theta_3 & -d_5 cos\theta_3 \\ -cos\theta_3 & 0 & -sin\theta_3 & -d_5 sin\theta_3 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

From the previous matrices (1)(2)(3), the Homogeneous transformation matrix that express the coordinates for the first 3 dof joints to the base frame can be obtained as:

$$T_3^0 = T_1^0 T_2^1 T_3^2 \quad (4)$$

$$T_3^0 = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

Where:

$m_{11} = -cos\theta_1 * sin\theta_2 * sin\theta_3 - cos\theta_1 * cos\theta_2 * cos\theta_3$

$m_{12} = sin\theta_1$

$m_{13} = cos\theta_1 * cos\theta_3 * sin\theta_2 - cos\theta_1 * cos\theta_2 * sin\theta_3$

$m_{14} = d_4 * sin\theta_1 - d_1 * sin\theta 1 + d_3 * cos\theta_1 * cos\theta_2 - d_5 * cos\theta_1 * cos\theta_2 * cos\theta_3 + d_5 * cos\theta_1 * cos\theta_3 * sin\theta_2$

$m_{21} = -sin\theta_1 * sin\theta_2 * sin\theta_3 - cos\theta_2 * cos\theta_3 * sin\theta_1$

$m_{22} = -cos\theta_1$

$m_{23} = cos\theta_3 * sin\theta_1 * sin\theta_2 - cos\theta_2 * sin\theta_1 * sin\theta_3$

$m_{24} = d_1 * cos\theta_1 - d_4 * cos\theta 1 + d_3 * cos\theta_2 * sin\theta_1 - d_5 * cos\theta_2 * sin\theta_1 * sin\theta_3 + d_5 * cos\theta_3 * sin\theta_1 * sin\theta_2$

$m_{31} = cos\theta_3 * sin\theta_2 - cos\theta_2 * sin\theta_3$

$m_{32} = 0$

$m_{33} = cos\theta_2 * cos\theta_3 + sin\theta_2 * sin\theta_3$

$m_{34} = d_0 - d_3 * sin\theta 2 + d_5 * cos\theta_2 * cos\theta_3 + d_5 * sin\theta_2 * sin\theta_3$

After computing the Homogeneous transformation matrix for the first 3 dof joints to the base frame, the Jacobian matrix can be obtained as:

$$J = \begin{bmatrix} \frac{\partial f_x}{\partial \theta_1} & \frac{\partial f_x}{\partial \theta_2} & \frac{\partial f_x}{\partial \theta_3} \\ \frac{\partial f_y}{\partial \theta_1} & \frac{\partial f_y}{\partial \theta_2} & \frac{\partial f_y}{\partial \theta_3} \\ \frac{\partial f_z}{\partial \theta_1} & \frac{\partial f_z}{\partial \theta_2} & \frac{\partial f_z}{\partial \theta_3} \end{bmatrix} \tag{6}$$

$$J = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \tag{7}$$

Where:

$m_{11} = d_4 * cos\theta_1 - d_1 * cos\theta 1 - d_3 * cos\theta_2 * sin\theta_1 + d_5 * cos\theta_2 * sin\theta_1 * sin\theta_3 - d_5 * cos\theta_3 * sin\theta_1 * sin\theta_2$

$m_{12} = d_5 * cos\theta_1 * cos\theta_2 * cos\theta_3 - d_3 * cos\theta_1 * sin\theta_2 + d_5 * cos\theta_1 * sin\theta_2 * sin\theta_3$

$m_{13} = -d_5 * cos\theta_1 * cos\theta_2 * cos\theta_3 - d_5 * cos\theta_1 * sin\theta_2 * sin\theta_3$

$m_{21} = d_4 * sin\theta_1 - d_1 * sin\theta 1 + d_3 * cos\theta_1 * cos\theta_2 - d_5 * cos\theta_1 * cos\theta_2 * sin\theta_3 + d_5 * cos\theta_1 * cos\theta_3 * sin\theta_2$

$m_{22} = d_5 * cos\theta_2 * cos\theta_3 * sin\theta_1 - d_3 * sin\theta_1 * sin\theta_2 + d_5 * sin\theta_1 * sin\theta_2 * sin\theta_3$

$m_{23} = -d_5 * cos\theta_2 * cos\theta_3 * sin\theta_1 - d_5 * sin\theta_1 * sin\theta_2 * sin\theta_3$

$m_{31} = 0$

$m_{32} = d_5 * cos\theta_2 * sin\theta_3 - d_3 * cos\theta_2 - d_5 * cos\theta_3 * sin\theta_2$

$m_{33} = d_5 * cos\theta_3 * sin\theta_2 - d_5 * cos\theta_2 * sin\theta_3$

Fig. 1 shows the constructed Simulink model of first 3 dof joints of the PUMA 560 Robot Arm.

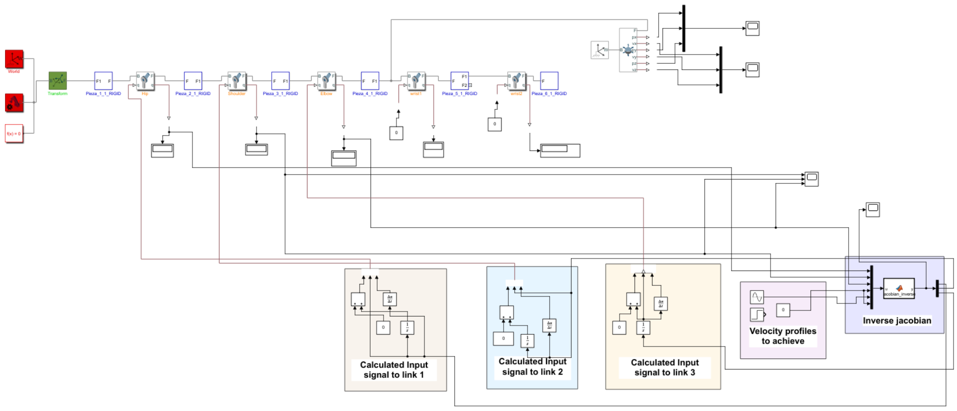

Figure 1: Simulink model of first 3 dof joints of the PUMA 560 Robot

The position trajectory generated from the Simulink model of the PUMA robot is shown in Fig. 2.

The velocity trajectory generated from the Simulink model of the PUMA robot is shown in Fig. 3.

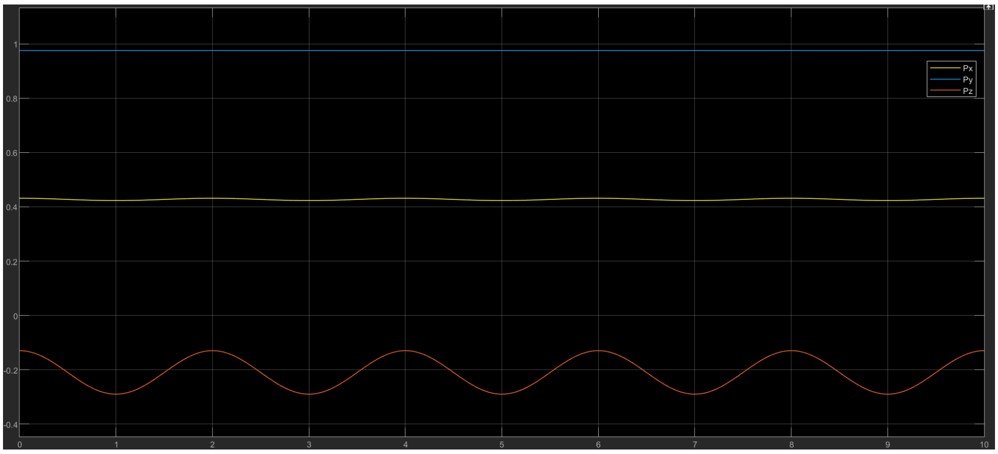

Figure 2: Position trajectory generated from the Simulink model

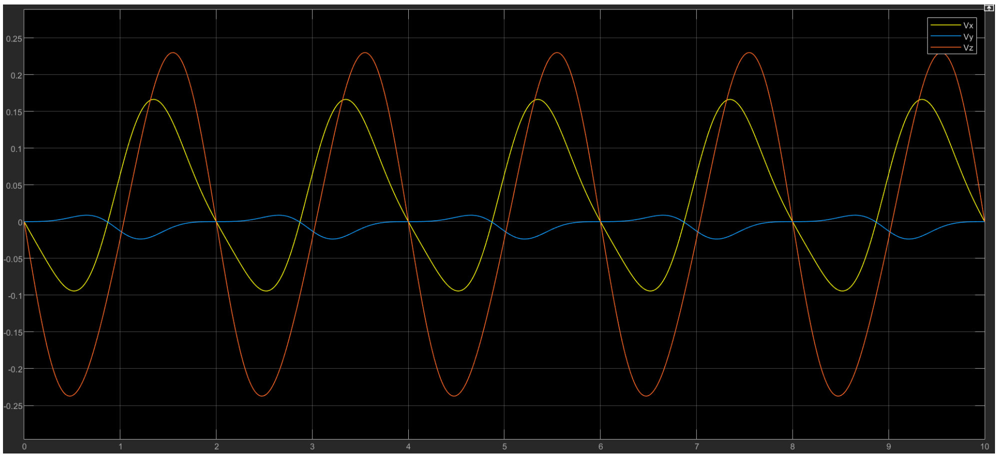

Figure 3: Velocity trajectory generated from the Simulink model

Extra: Modelling the whole PUMA 560 robot

We decided to model the whole 5 dof to see the differences. In order to make things simpler, we changed the frames on SolidWorks to be the same as in Matlab. So, now we will be able to perform both of the position and velocity control.

The Homogeneous transformation matrix that expresses the coordinates for the whole PUMA 560 robot can be obtained as:

$$T_5^0 = T_1^0 T_2^1 T_3^2 T_4^3 T_5^4 \tag{8}$$

After computing the Homogeneous transformation matrix for the first whole PUMA 560 robot, the Jacobian matrix can be obtained bu differentiating the last column of the homogeneous transformation matrix (position vector) with respect to the 5 joints as:

$$J = \begin{bmatrix} \frac{\partial f_x}{\partial \theta_1} & \frac{\partial f_x}{\partial \theta_2} & \frac{\partial f_x}{\partial \theta_3} & \frac{\partial f_x}{\partial \theta_4} & \frac{\partial f_x}{\partial \theta_5} \\ \frac{\partial f_y}{\partial \theta_1} & \frac{\partial f_y}{\partial \theta_2} & \frac{\partial f_y}{\partial \theta_3} & \frac{\partial f_y}{\partial \theta_4} & \frac{\partial f_y}{\partial \theta_5} \\ \frac{\partial f_z}{\partial \theta_1} & \frac{\partial f_z}{\partial \theta_2} & \frac{\partial f_z}{\partial \theta_3} & \frac{\partial f_z}{\partial \theta_4} & \frac{\partial f_z}{\partial \theta_5} \end{bmatrix} \tag{9}$$

Fig. 4 shows the constructed Simulink model of the whole PUMA 560 Robot Arm including the last 2 dof joints (wrist).

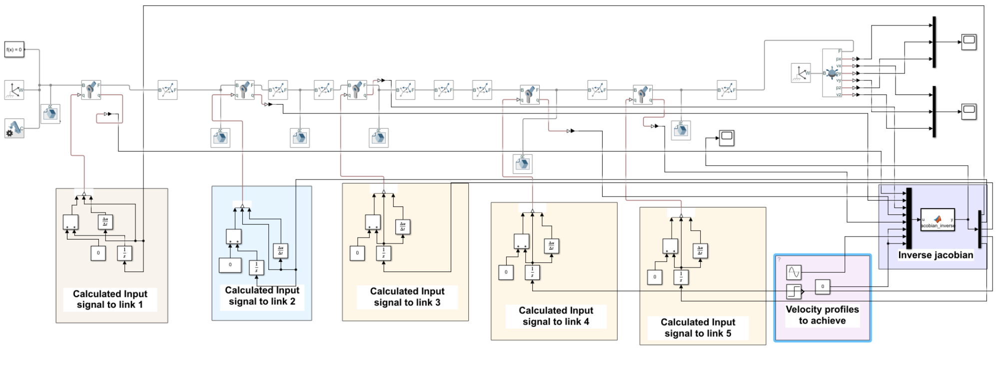

Figure 4: Simulink model of the whole joints of the PUMA 560 Robot

In the first section were we solved for the first 3 dof joints the last 2 dof joints of the wrist were given zero inputs. But, when we solved for the whole PUMA 560 robot joints the end-effector velocity trajectory was close to the commanded input velocity trajectory.

The results for solving for the whole PUMA 560 robot joints were better as the output velocity profiles were close to the input velocity profiles as now the robot has more dof so it can follow the commanded velocity profile as shown in the comparison of Fig. 5 and Fig. 6

In Fig. 6, we can see that the change of velocity in the x-axis for the 5 dof joints is less than that of the 3 dof joints and the same thing is noticed in the z-axis. This is the result of utilizing all the available degree of freedoms.

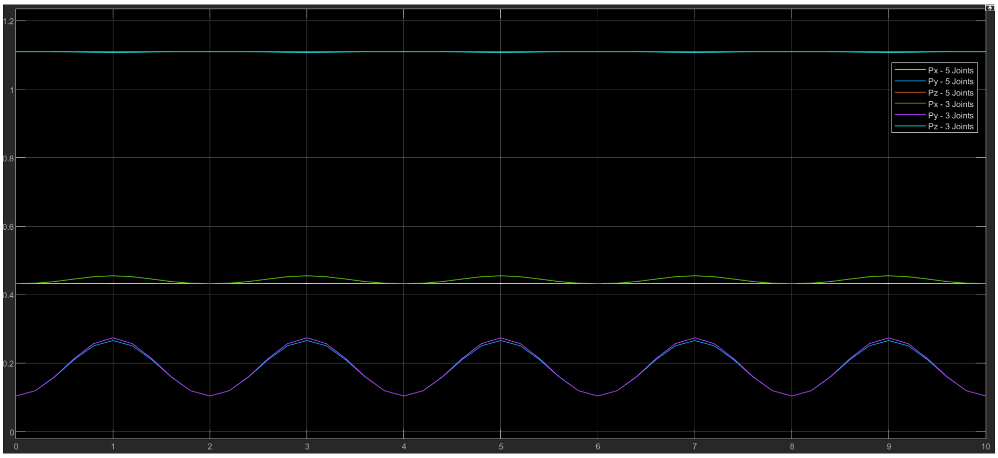

Figure 5: Position trajectory generated from the Simulink model for 3 dof vs 5 dof

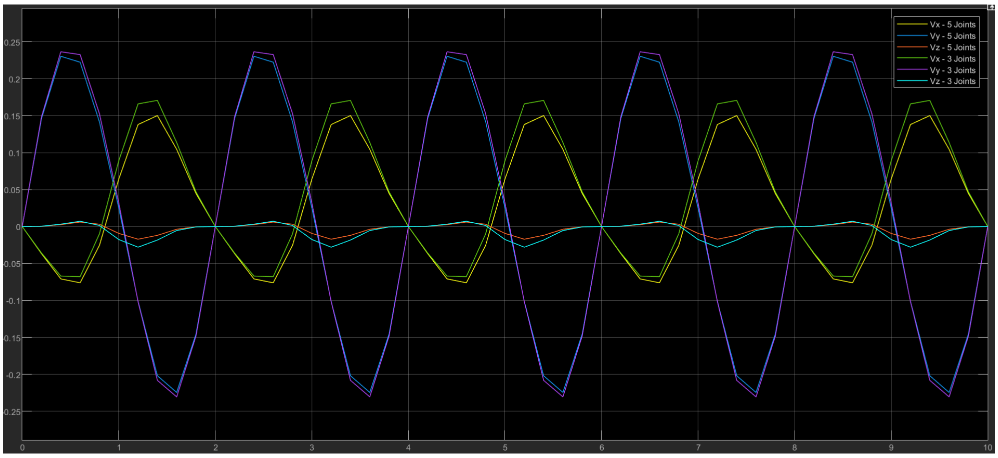

Figure 6: Velocity trajectory generated from the Simulink model for 3 dof vs 5 dof

Finally, we can't command the robot to strictly follow a velocity profile in the y-direction cause there will be other movements along the other 2 axes. But, utilizing all the available degree of freedoms would result in much better performance than utilizing only the first 3 dof joints.