UNDERWATER ACOUSTICS

HW1 - RAY TRACING

Done by:
ELKHOLY HASSAN
NASIR AMNEH

MIR Master program

Université de Toulon

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Contents

1 Question 1

Figure 5 illustrates the sound speed profile in the arctic region that can be modeled by equation 1 where $\gamma_0 = 1.63 \times 10^{-2} s^{-1}$ and $c_0 = 1450\, ms^{-1}$. Since sound changes its speed in the vertical plane only, its speed will stay constant if it is pointing exactly toward the horizontal direction "$\theta_0 = 0$" as shown in figure 2. Figure 3 illustrates the expected ray trajectory if the initial angle is very close to zero "almost horizontal". However after running the simulation with angle 0, it worked and there were reflections and refractions. The sound wave refracted right away after turning on the transducer and went up to the surface and then reflected and the cycle continues. This is may be perhaps due the fact that these simulations run based on numerical methods. Figure 4 illustrates the sound ray trajectory, obtained from the MATLAB simulation, for a time period of 30s, transducer depth of 750m, and $\theta_0 = 0$. It is clear from the figure that the waves refracted upward right away and the distance between each two reflections "at the water-air interface" or each two refractions "underwater" is 20 km. And to answer the question clearly, the max depth reached by the simulation is 750m because it went straight up but in theory it go down a little bit and then do the same behaviour as the in the simulation.

$$c(z) = c_0 + \gamma_0 \, |z| \tag{1}$$

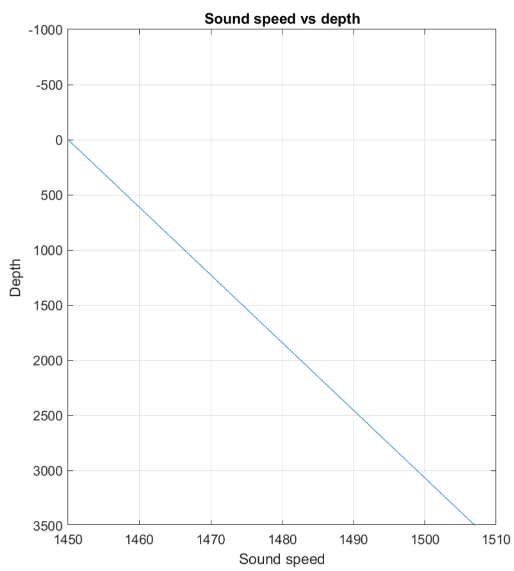

Figure 1: Sound speed profile in the Arctic region

If the sound speed gradient is steep or the surface duct is sufficiently deep (which depends on the frequency of the sound), the sound will be refracted upward. Once the sound reaches the surface, it is reflected, only to be refracted upward again by the gradient. This effectively "traps" the sound in the surface duct.

as per figure 5 we can see that the polar water layer has a somewhat similar linear increase in speed. the same layer results in a similar behaviour at which the horizontal ray's depth does not exceed the depth at which the source is at, instead the ray goes up, gets reflected at the surface and then starts going down (while gradually getting refracted upwards) until it reaches the depth from which it was emitted and it starts going up again.

2 Question 2

After running the MATLAB simulation, the sound ray reached the sea floor if the initial angle of the transducer is smaller than 74.17. At exactly $\theta_0 = 74.17$, the sound will refract before reaching the sea floor as illustrated in figure 6

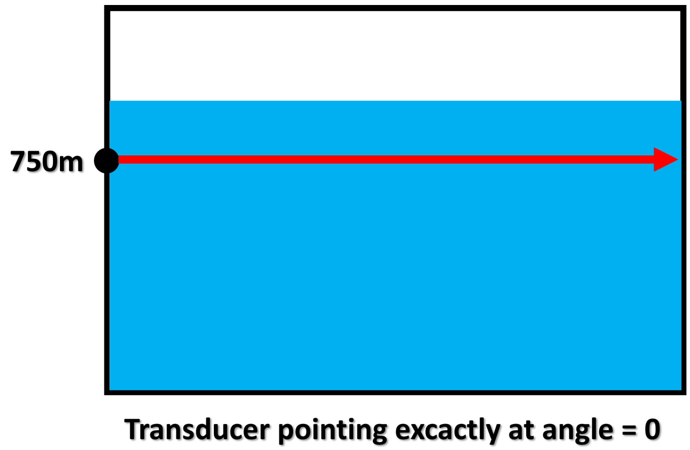

Figure 2: Sound ray if the initial angle is exactly 0

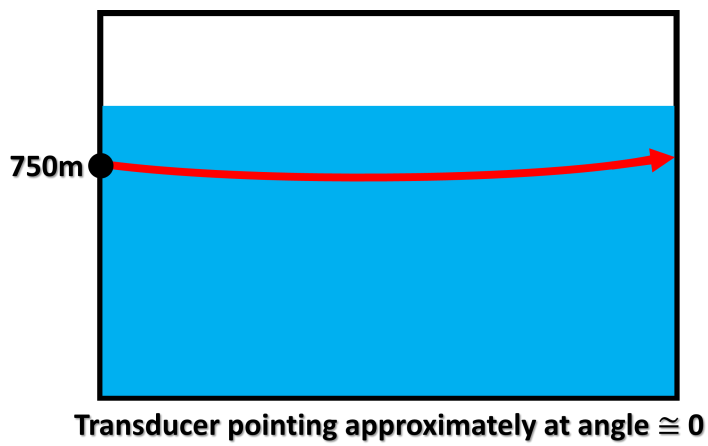

Figure 3: Sound ray if the initial angle is close to 0

if the sound speed varies linearly with depth, the rays follow arcs of circles. and thus according to Snell's Law:

$$\frac{\sin\theta_s}{C_s} = \frac{\sin\theta}{C} \tag{2}$$

in order to make our θ_s a function of z we have to substitute C

$$\theta_s = \arcsin\frac{C_s\sin\theta}{(C_0+\gamma|z|)} \tag{3}$$

when solving for θ_s, this yields:

$$\theta_s = \arcsin\frac{\sin\theta C_s}{C} \tag{4}$$

to find C_s we have to substitute $z = 750$ into equation 1. in addition to that, the depth we aim for, thus Z, is the sea depth which is 3500 meters.when reaching the bottom, θ will be equal to $\frac{\pi}{2}$ this will give the answer:

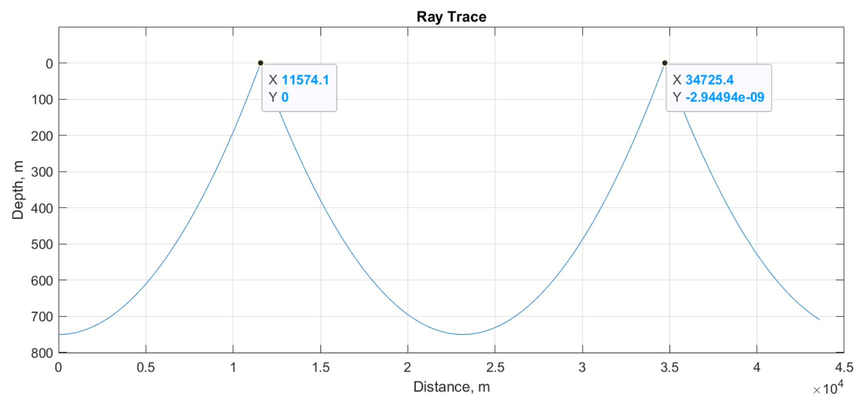

Figure 4: Ray trajectory for 30 second, $\theta_0 = 0$

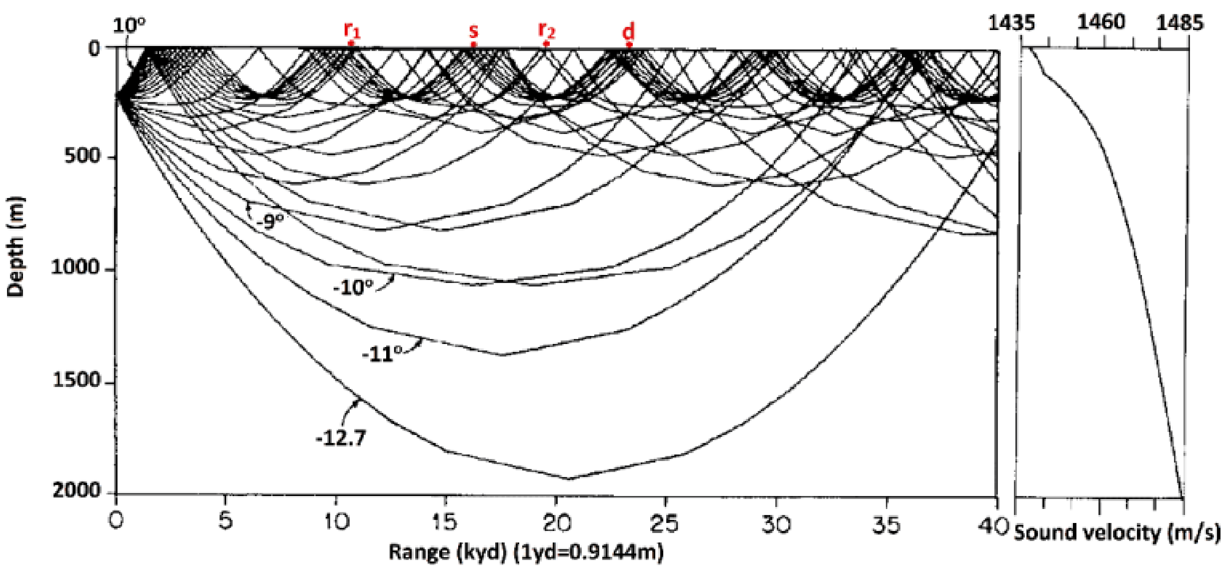

Figure 5: Test result of nonlinear sound propagation in the Arctic region. Sound velocity with depth (right) and the corresponding sound ray (left), [Urick, Sound Propagation in the Sea (1982)].

$$\theta_s = \arcsin \frac{\sin \frac{\pi}{2} 1450}{1507.05} \tag{5}$$

and thus $\theta_s = 74.1845$

3 Question 3

The simple definition of time is distance divided by velocity as illustrated in Eq.6. But since the velocity of sound is not uniform at different depths, we have to consider sound velocity as a function of depth. Eq.7 illustrates the analytical expression for calculating the round trip time where C(z) is a function that provides the sound velocity at a given depth, dz is the small depth covered by the propagating wave. And it was necessary to do this summation operation because the expression $dz/C(z)$ alone calculates the small time taken to cover a small depth of dz. And to calculate the whole round trip we have to do a summation of these time steps from an initial depth of zero "echosounder is assumed to be at the surface" and final depth equals

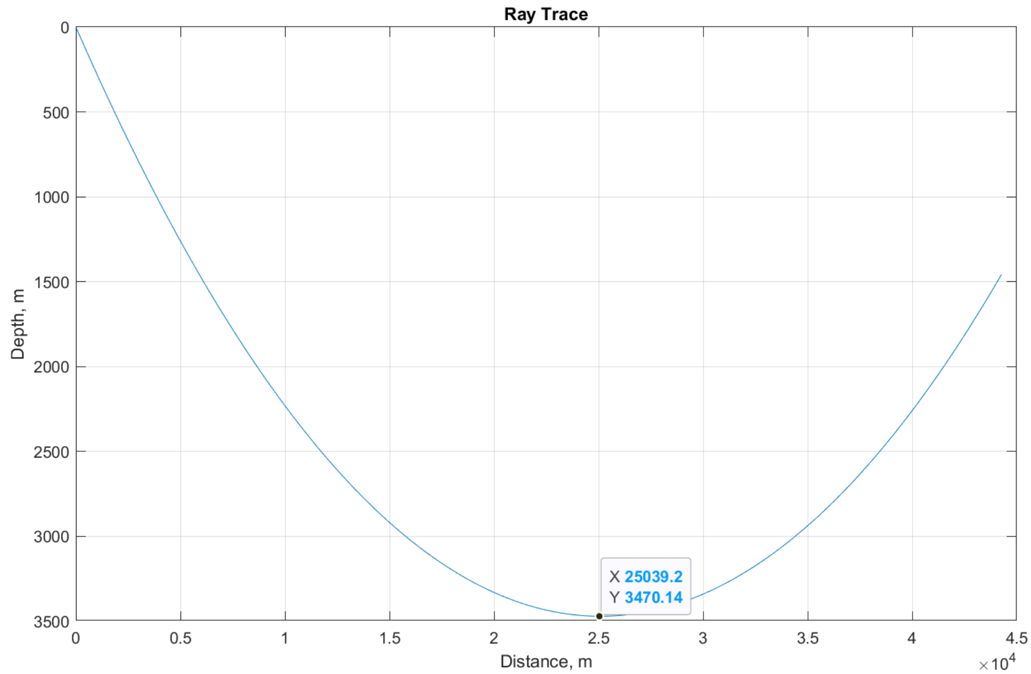

Figure 6: Threshold angle for reaching seafloor

the depth of the ocean which in our case is 3500m. And of course, we had to multiply the result by 2 to get the round trip time of propagating the sound wave back and forth.
Regarding an analytical expression for calculating the depth, a naive approach will be to multiply the sound velocity by half the round trip time as demonstrated in Eq.8. But as explained before, the sound velocity is not uniform and it has to be expressed as a function. Eq.9 illustrates how to calculate the ocean depth by taking into account the non-uniformity of the sound velocity profile.

For the experiment of the echosounder, we have utilized our software and positioned the sound transducer at the surface and initialized a sound wave directed with a vertical incidence. We measured the time of the first water-air reflection and then considered the sound velocity to be uniform and equals to the sound velocity at the surface "$c_0 = 1450 m/s$". Afterwards we multiplied this velocity by half the measured time and the calculated ocean depth was 3400.25m which is about 100m less than the actual depth or about 2.85% error.
Figure 7 demonstrates the simulation of the echosounder as explained above.

$$Time = \frac{Distance}{Velocity} \tag{6}$$

$$Round\,trip\,time = 2 \times \sum_{z=0}^{z=3500} \frac{dz}{C(z)} \tag{7}$$

$$Depth = C_0 \times t \tag{8}$$

$$Depth = \sum_{t=0}^{t=0.5Time} C(z) \times dt \tag{9}$$

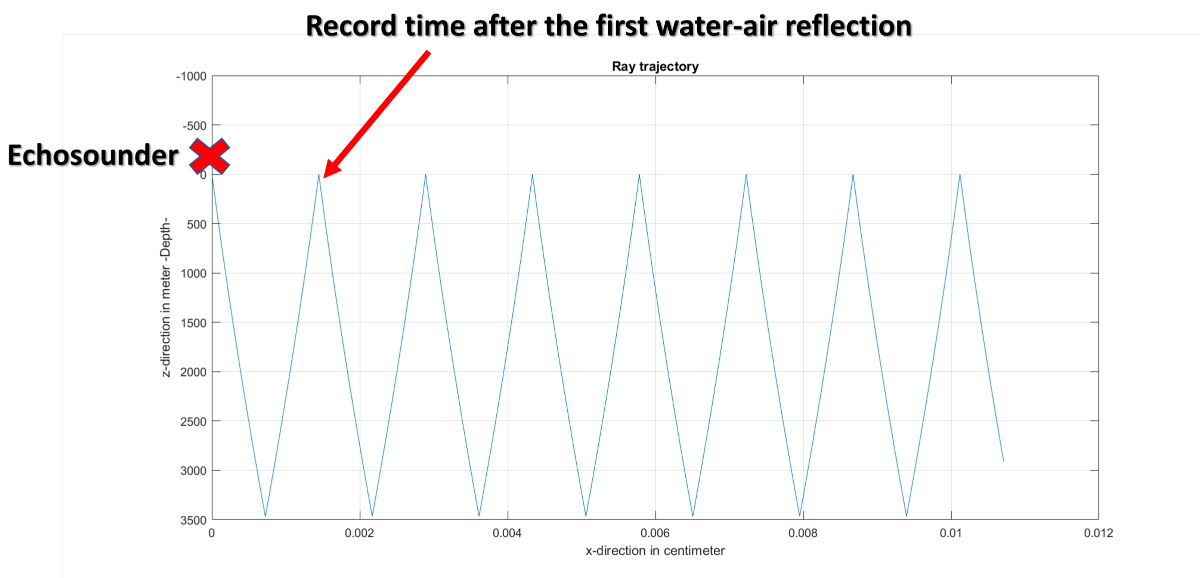

Figure 7: Simulating the echosounder

4 Mediterranean Sea

in this scenario, due to sound speed decreasing from above and below the source to some minimum we can observe Sound channels.

The depth of the minimum defines the sound channel axis as per figure 9 . The boundaries of the sound channel are sound speed maxima immediately above and below the sound channel axis. Within the sound channel, waves refract toward lower sound speeds, so sound rays above the minimum bend downward and sound rays below the minimum bend upward. Figure 8 illustrates the wave behaviour if the experiment was conducted with the previous scenario. It is clear the the sound wave has a very low amplitude which the physical translation of the previous explanation.

The likelihood of a sound ray remaining trapped depends on several factors:

The thickness of the sound channel (distance between the upper and lower boundaries).

Strength of the sound channel (the difference between the sound speeds at the axis and the boundary).

The sound frequency.

The propagation angle.

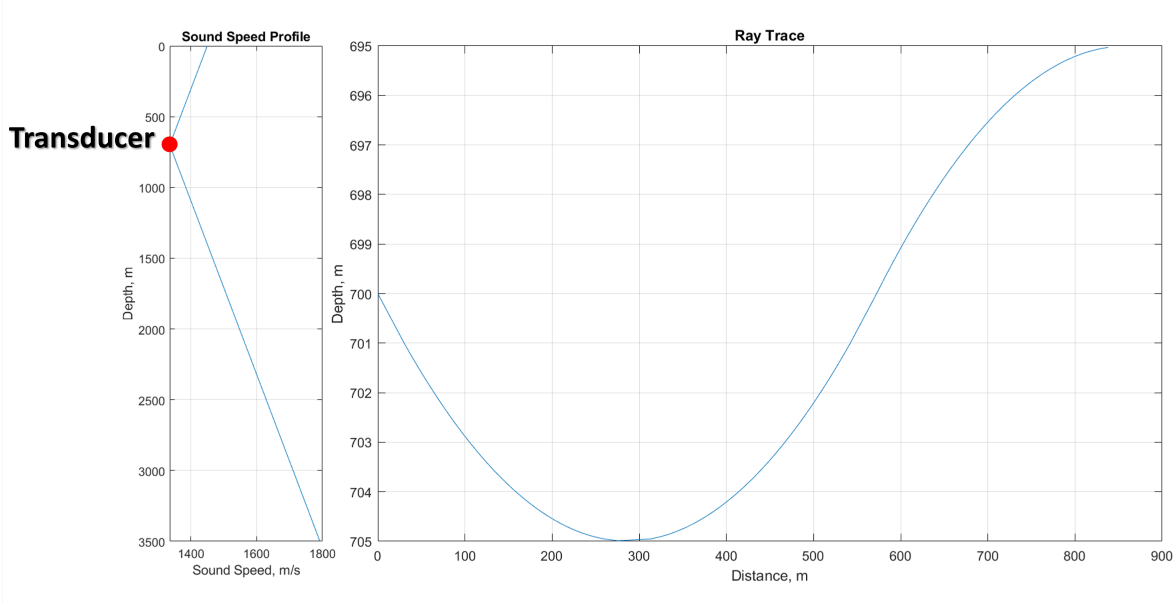

Figure 8: Wave behaviour if started at depth = 700m

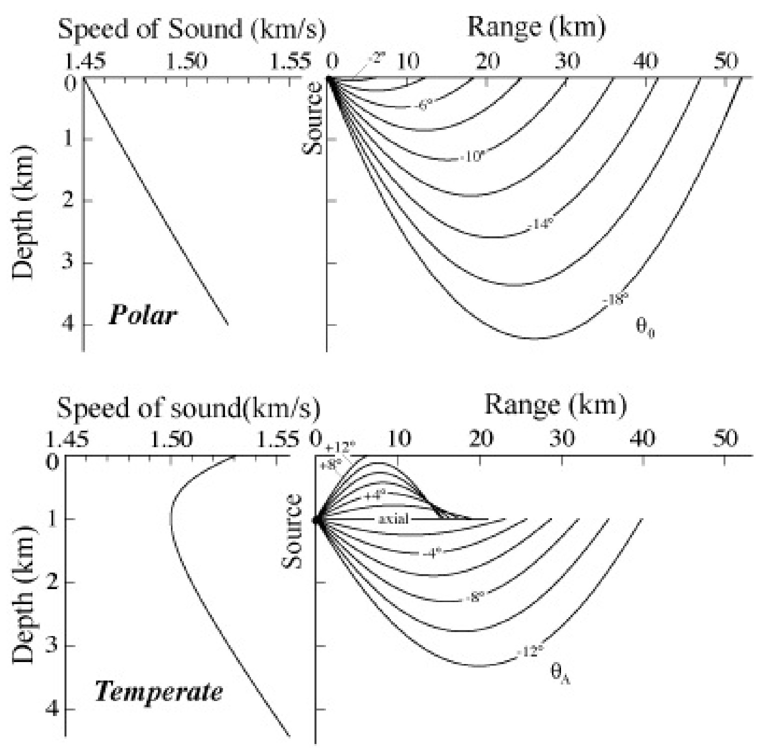

Figure 9: sound speed and path of travel through the water column. (Munk et al., 1995)