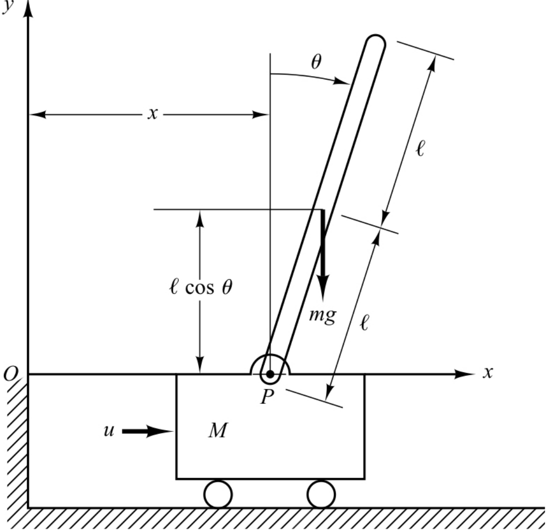

Carro

$$\left.\begin{aligned} P_{x1} &= x \longrightarrow V_{x1} = \dot{x} \\ P_{y1} &= 0 \longrightarrow V_{y1} = 0 \\ P_{z1} &= 0 \longrightarrow V_{z1} = 0 \end{aligned}\right\} M$$

Pendulo

$$\left.\begin{aligned} P_{x2} &= x + Ls\theta \longrightarrow V_{x2} = \dot{x} + Lc\theta\cdot\dot{\theta} \\ P_{y2} &= Lc\theta \longrightarrow V_{y2} = -Ls\theta\cdot\dot{\theta} \\ P_{z2} &= 0 \longrightarrow V_{z2} = 0 \\ \alpha_2 &= \frac{\pi}{2} - \theta \longrightarrow \omega_2 = -\dot{\theta} \end{aligned}\right\} \begin{aligned} & m \\ & I_{zz} = \\ & \tfrac{1}{12} mL^2 \end{aligned}$$

$$T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left[(\dot{x} + Lc\theta\cdot\dot{\theta})^2 + (-Ls\theta\cdot\dot{\theta})^2\right] + \frac{1}{2}I_{zz}\dot{\theta}^2$$

$$T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left[\dot{x}^2 + 2\dot{x}Lc\theta\cdot\dot{\theta} + L^2c^2\theta\cdot\dot{\theta}^2 + L^2s^2\theta\cdot\dot{\theta}^2\right] + \frac{1}{2}I_{zz}\dot{\theta}^2$$

$$T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left(\dot{x}^2 + 2\dot{x}Lc\theta\cdot\dot{\theta} + L^2\dot{\theta}^2\right) + \frac{1}{2}I_{zz}\dot{\theta}^2$$

$$U = mgLc\theta$$

$$L = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left(\dot{x}^2 + 2\dot{x}Lc\theta\cdot\dot{\theta} + L^2\dot{\theta}^2\right) + \frac{1}{2}I_{zz}\dot{\theta}^2 - mgLc\theta$$

$$\frac{\partial L}{\partial \dot{x}} = M\dot{x} + m(\dot{x} + Lc\theta\cdot\dot{\theta})$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = M\ddot{x} + m\left[\ddot{x} + \ddot{\theta}Lc\theta - \dot{\theta}^2Ls\theta\right]$$

$$\frac{\partial L}{\partial x} = 0$$

$$\boxed{F = (M+m)\ddot{x} + mL(\ddot{\theta}c\theta - \dot{\theta}^2s\theta)} \longrightarrow \left.\begin{aligned} s\theta &\cong \theta \\ c\theta &\cong 1 \\ \theta\dot{\theta}^2 &\cong 0 \end{aligned}\right\} \text{Angulos pequeños} \longrightarrow \boxed{F = (M+m)\ddot{x} + mL\ddot{\theta}}$$

*Modelo para controlar esta vueta

$$L = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left(\dot{x}^2 + 2\dot{x}Lc\theta\cdot\dot{\theta} + L^2\dot{\theta}^2\right) + \frac{1}{2}I_{zz}\dot{\theta}^2 - mgLc\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = m\dot{x}Lc\theta + mL^2\dot{\theta} + I_{zz}\dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = m\ddot{x}Lc\theta - m\dot{x}Ls\theta\cdot\dot{\theta} + mL^2\ddot{\theta} + I_{zz}\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -m\dot{x}Ls\theta\cdot\dot{\theta} + mgLs\theta$$

$$\tau = m\ddot{x}Lc\theta - \cancel{m\dot{x}Ls\theta\cdot\dot{\theta}} + mL^2\ddot{\theta} + I_{zz}\ddot{\theta} + \cancel{m\dot{x}Ls\theta\cdot\dot{\theta}} - mgLs\theta$$

$$\tau = m\ddot{x}Lc\theta + mL^2\ddot{\theta} + I_{zz}\ddot{\theta} - mgLs\theta \longrightarrow \left.\begin{matrix} s\theta \cong \theta \\ c\theta \cong 1 \\ \theta\dot{\theta}^2 \cong 0 \end{matrix}\right\} \text{Angulos pequeños}$$

$$\tau = m\ddot{x}L + mL^2\ddot{\theta} + I_{zz}\ddot{\theta} - mgL\theta$$

$$\boxed{\tau = (I_{zz} + mL^2)\ddot{\theta} + mL\ddot{x} - mgL\theta}$$

OJO con lo siguiente:

θ_1 NO es un grado de libertad que yo controle, es una variable del sistema por lo tanto $\underline{\underline{\tau = 0}}$

$$\boxed{(I_{zz} + mL^2)\ddot{\theta} + mL\ddot{x} = mgL\theta}$$ *Modelo importante

$$\boxed{F = (M+m)\ddot{x} + mL\ddot{\theta}}$$

* Estados del sistema

$$X = \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

$$\dot{x} = \dot{x}$$

$$\dot{\theta} = \dot{\theta}$$

$$\ddot{x} = -\frac{m^2L^2g}{I_{zz}(M+m) + MmL^2}\theta + \frac{I_{zz} + mL^2}{I_{zz}(M+m) + MmL^2}F$$

$$\ddot{\theta} = \frac{(M+m)mgL}{I_{zz}(M+m) + MmL^2}\theta - \frac{mL}{I_{zz}(M+m) + MmL^2}F$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\dfrac{m^2L^2g}{I_{zz}(M+m)+MmL^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{(M+m)mgL}{I_{zz}(M+m)+MmL^2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{I_{zz}+mL^2}{I_{zz}(M+m)+MmL^2} \\ 0 \\ \dfrac{-mL}{I_{zz}(M+m)+MmL^2} \end{bmatrix} F$$

$$\begin{bmatrix} \theta \\ x \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x & \dot{x} & \theta & \dot{\theta} \end{bmatrix}^T$$