Assignment 4 Lunar Lander Solution

1. You have completed the lab component of this Unit of Study. However, we would like you to undertake a final group task and design of a control system as a team. An early example of a computer based control system was on the Apollo lunar lander used to land the first astronauts on the moon. The control system for the full 3DOF platform took a team well over a year to design, largely due to the fact that many of the optimal control techniques required for the design had not yet been invented. We would like you to design a control system for a simplified 2D version of the lunar lander that will allow you to land the astronauts safely on the moon. You may choose to undertake this design using the classical, LTI methods we have studied by treating lateral and vertical control separately or you may wish to consider exploring the State Space techniques more fully, which will allow you to account for the coupled dynamics. Your task is to land the lunar module at a velocity of less than 1m/s at the origin of the landing site if the unit has initial conditions of position (0.5km, 160km) and an initial velocity of $\dot{y}(0)$=-0.7km/s. The gravitational constant on the moon is 1.6m/s^2. You have a limited fuel budget and the variable mass of the fuel will change the dynamics of the vehicle. Specifications of the vehicle are included in the following table. Begin by working out the dynamic models that govern the system and generating a Simulink model of the system. You may need to consider the control in stages to control the heading, lateral and vertical motion. Be sure to leave sufficient fuel (half the mass) remaining at the end of the landing to allow your astronauts to return from the surface of the moon. Submit a group report detailing your design and an analysis of its performance, clearly indicating the names and SIDs of the group members. [25 marks]

Table 1 - Lunar lander specifications

Specification	**Value**
Mass	15,000kg
Mass of Fuel	8,000kg
Moment of Inertia	100,000 kg m^2
Max. Ft propulsion	44kN
Max. Fl propulsion	0.5kN
Rocket Thruster Specific impulse	3.0kNs/kg
Gravitational Constant	1.6m/s^2

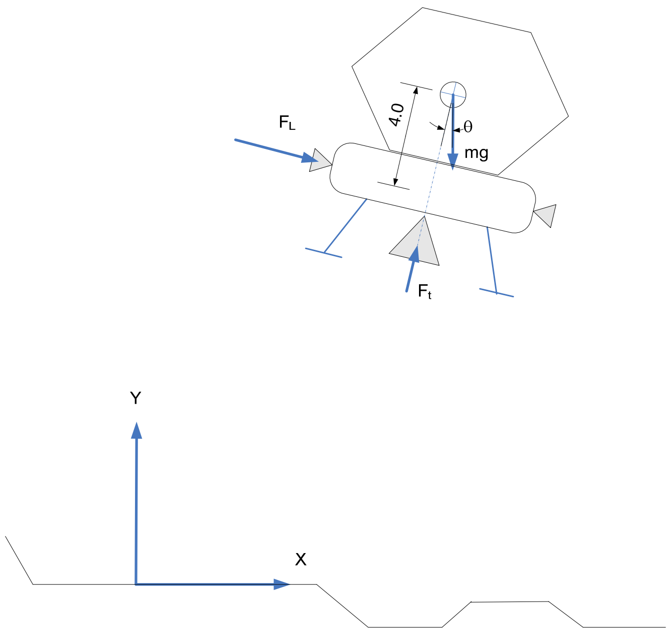

Figure 1 - Lunar lander schematic

A Possible Solution

The following presents one possible solution to this problem. There are many approaches that might be used. The first step in finding a solution for this problem is to model the dynamics of the vehicle. Clearly, the dynamics are coupled through the orientation and the location of the rockets. Using Newton's Second Law and the FBD shown above, we can derive the following equations of motion.

$$m\ddot{x} = F_l \cos\theta - F_t \sin\theta$$

$$m\ddot{y} = F_l \sin\theta + F_t \cos\theta - mg$$

$$J\ddot{\theta} = 4F_l$$

We can rearrange these equations to get them into a form that we could model with a block diagram. In this instance, we are interested in isolating the accelerations, as these will be used to generate the position and velocities of the body using integration blocks. Rearranging the equations yields

$$\ddot{x} = \frac{1}{m}\left(F_l \cos\theta - F_t \sin\theta\right)$$

$$\ddot{y} = \frac{1}{m}\left(F_l \sin\theta + F_t \cos\theta\right) - g$$

$$\ddot{\theta} = \frac{4F_l}{J}$$

We have to bear in mind that the mass is variable here given that we are burning fuel , that these equations are non-linear due to the sin and cos terms and that there is coupling between the three equations (i.e. the first two equations both depend on the orientation). This means that our classical LTI methods can't be used explicitly to design a controller for this system. However, we will use some of the insights gained in selecting our control strategy. We will begin by creating a block diagram for the equations of motion. Simulink provides us with some tools that will allow us to do this and the block diagram is shown in Figure 2.

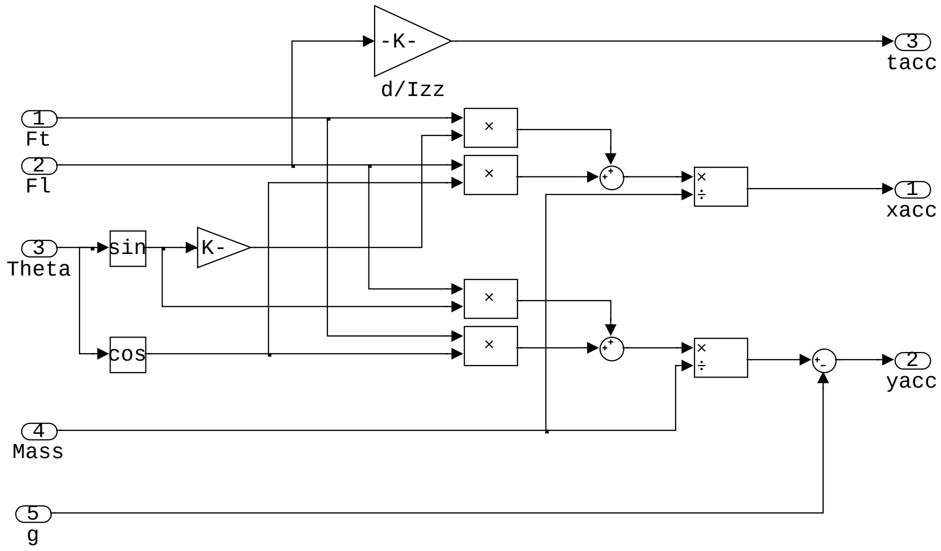

Figure 2 - Lunar lander equations of motion

The variables of interest, which appear on the right, are the x, y and theta accelerations respectively. These are a function of the applied forces, Ft and Fl, the orientation of the module, gravity and the mass of the vehicle. Remember that the mass is variable in this case. I have created a subsystem within my simulink model that takes the quantities on the left as inputs and has the three outputs shown on the right. The gain on the sin line is a -1 and the gain at the top is 4/100,000. This subsystem appears in the middle of the block diagram that describes the dynamics of the lander, shown in Figure 3.

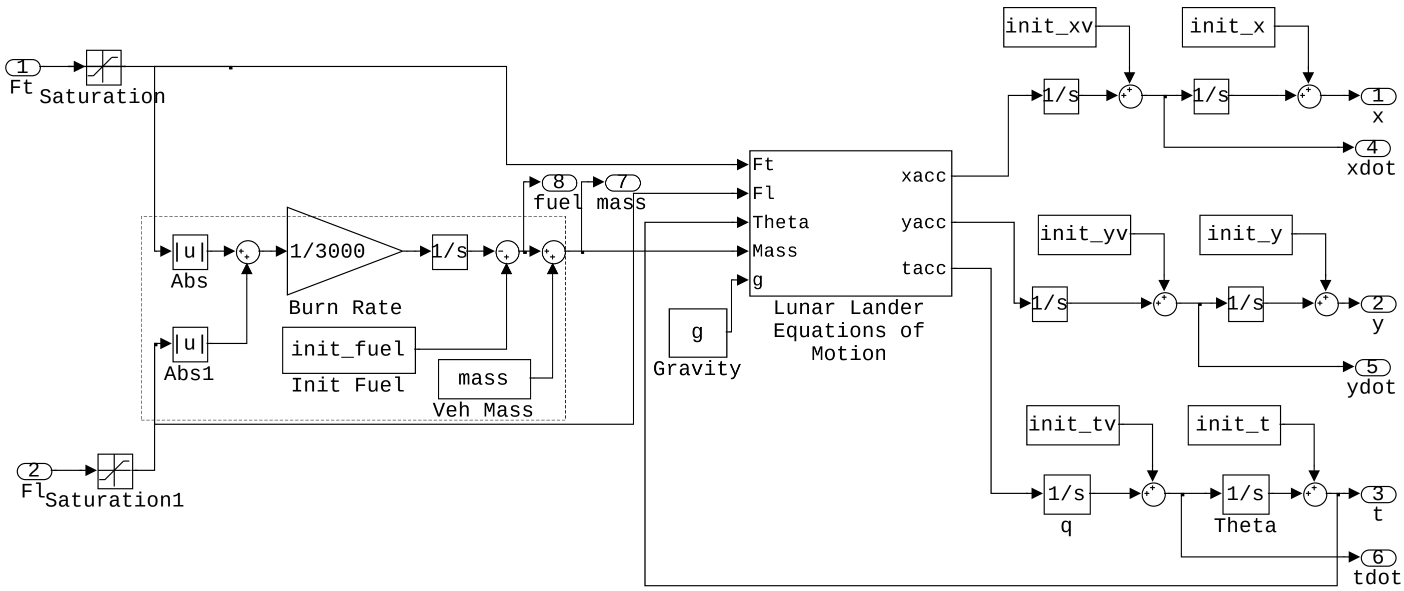

Figure 3 - Lunar lander dynamic model

Starting on the left of this diagram, there are the input forces used to control the lander. I have integrated saturation blocks which reflect the maximum forces of each of the thrusters (44kN and 0.5kN respectively). These forces are then fed into the block representing the equations of motion. The thrust values are also required to calculate the variable mass of the vehicle. This is shown in the dashed box above.

The absolute thrust values (you can't generate more fuel by running the thrusters in reverse) are summed and divided by the burn rate (3.0kNs/kg). The burn rate is then added to the initial mass of fuel (8,000 kg) and the mass of the vehicle (7,000kg). This yields the time changing mass of the vehicle which is fed to the dynamic model. I have also provided output blocks from this subsystem so I can monitor the mass of fuel remaining.

Ultimately, I want to compute the position and velocities of the vehicle and use these as the feedback for my controller. I take the output of the dynamic models (the accelerations) and integrate these twice to yield velocity and position. I have also allowed for initial conditions for each of these quantities. The outputs of this overall dynamic model are the positions and velocities of the vehicle and the mass of fuel remaining.

Finally, I am ready to start designing my controller. I am trying to control the x and y position of the lander using the two thrusters. However, I need to control the x position by manipulating the orientation of the lander. I'll start by tuning a controller about heading. In the figure below, the central block represents the subsystem shown in Figure 3.

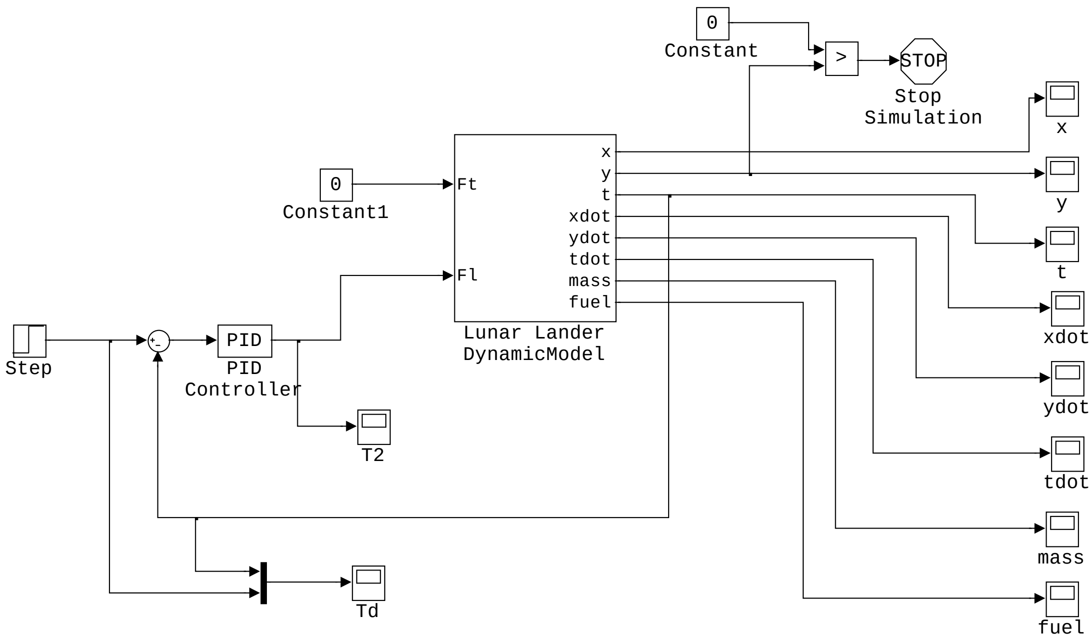

Figure 4 - Lunar lander heading control system

Given that the equations of motion of the heading include a double pole at the origin (i.e. there is only an s^2 in the transfer function for θ relative to the lateral thruster) I will probably need a PD controller to pull the roots away from origin and make the control of the orientation stable. This is shown in Figure 4 as a PID block. I will design the response to have a settling time of 40s and an overshoot of 16%. This is a second order system so my second order approximations should be reasonably good. To meet these specs, I need σ=0.1. For an overshoot of 16% I therefore require a root location at (-0.1±0.1sqrt(3)j).

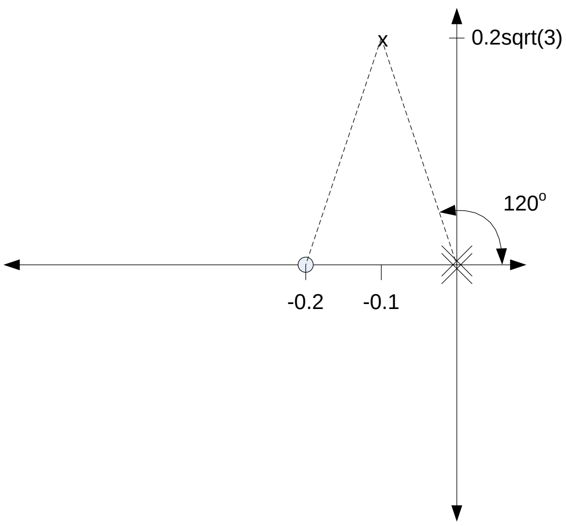

Figure 5 – Design point for heading control

Using the angle criteria, I can find the required zero location as follows.

$$\angle KG(s) = (2k+1)180$$
$$-120 - 120 + \theta_c = -180$$
$$\theta_c = 60$$
$$therefore\ z_c = 0.2$$

I also need to find the gain. This can be done using the magnitude criteria by substituting in the value of the root location (i.e. I am trying to find the value of K that will place the root at the design point).

$$|KG(s)| = 1$$
$$\left| K\left(s+0.2\right)\frac{4}{100,000s^2} \right| = 1$$
$$\left| K\left(-0.1+0.1\sqrt{3}j+0.2\right)\frac{4}{100,000\left(-0.1+0.1\sqrt{3}j\right)^2} \right| = 1$$
$$\left| \frac{K\left(0.1+0.1\sqrt{3}j\right)}{25,000\left(0.01-0.02\sqrt{3}j-0.03\right)} \right| = 1$$
$$K = 5000$$

The final orientation controller is therefore

$$D(s) = 5000\left(s+0.2\right)$$
$$= 5000s + 1000$$

The step response for the orientation controller is shown in Figure 6. This meets the design criteria in terms of overshoot and settling time.

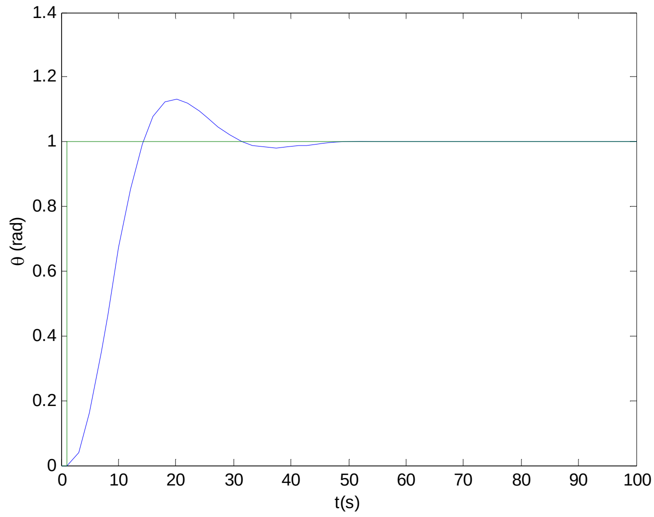

Figure 6 - Lunar lander tuned heading versus time

Now I can start thinking about control of the position of the lander. My objective is to bring the vehicle to the origin defining the landing site with a speed that isn't too high. I'll start with the x position. By looking at the FBD and the equations of motion, it's clear that I'll need to use the lateral thruster to change the orientation. This will then provide a component of the main thrust in the x direction. If I make the orientation proportional to the x position, the controller will tend to turn the lander towards the x=0 line and will push the vehicle towards that line. I will also want to make the heading proportional to the rate of change of x (so I don't overshoot too much – remember the dynamics of this system are relatively slow and non-linear). These two values, with associated gains, are now used to provide the heading reference to the system. This is shown in Figure 7. Here I have added a constant high thrust to the main thruster – otherwise the lander would not move in x.

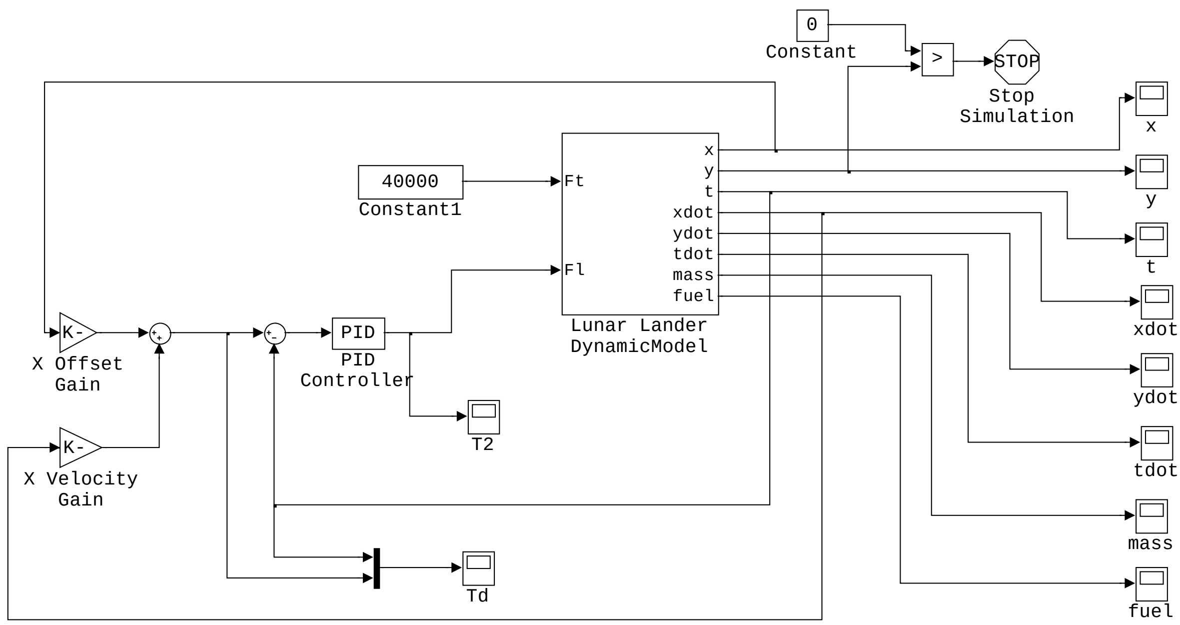

Figure 7 - Lunar lander x control system

This control strategy is effectively nesting the heading controller within another PD controller that uses the X position and velocity as feedback to provide a desired heading to the lander control system. Using values of gain for the X offset and X velocity of 0.0001 and 0.01 respectively yields a controlled change in X shown in Figure 8. The corresponding orientation is shown in Figure 9.

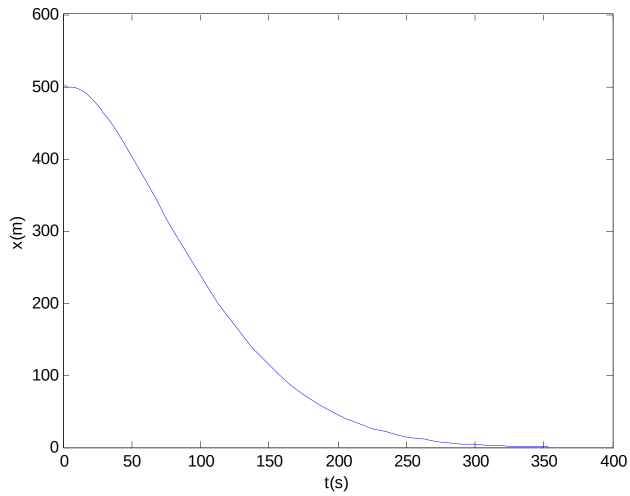

Figure 8 - Lunar lander x position

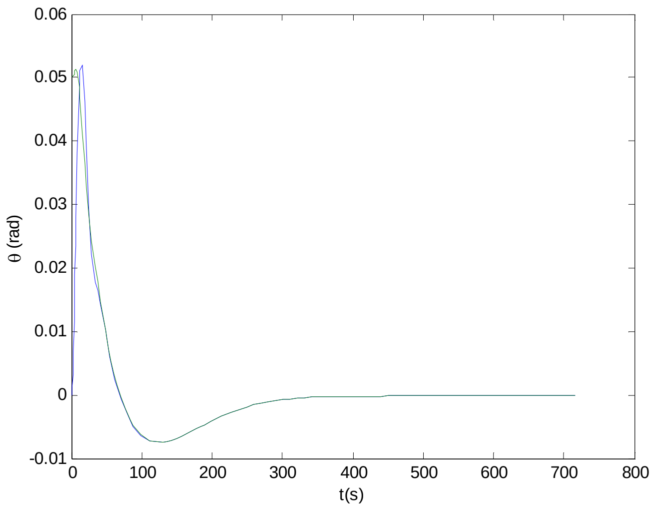

Figure 9 - Lunar lander orientation during x control

Now, I'm ready to think about y control. I have a very similar situation to the x position in that I'm trying to bring the craft down at the origin with a small velocity. In this case, however, I'm mostly interested in arresting the speed of the vehicle. There will be a small disturbance introduced by the fact that I'm simultaneously trying to control the x position, however as seen above the maximum heading deviation is approximately 2.86 degrees. This will make very little difference to the y control and the x position transient will have been brought to zero early in the descent. I will therefore use feedback from my y position and velocity to regulate the main thruster.

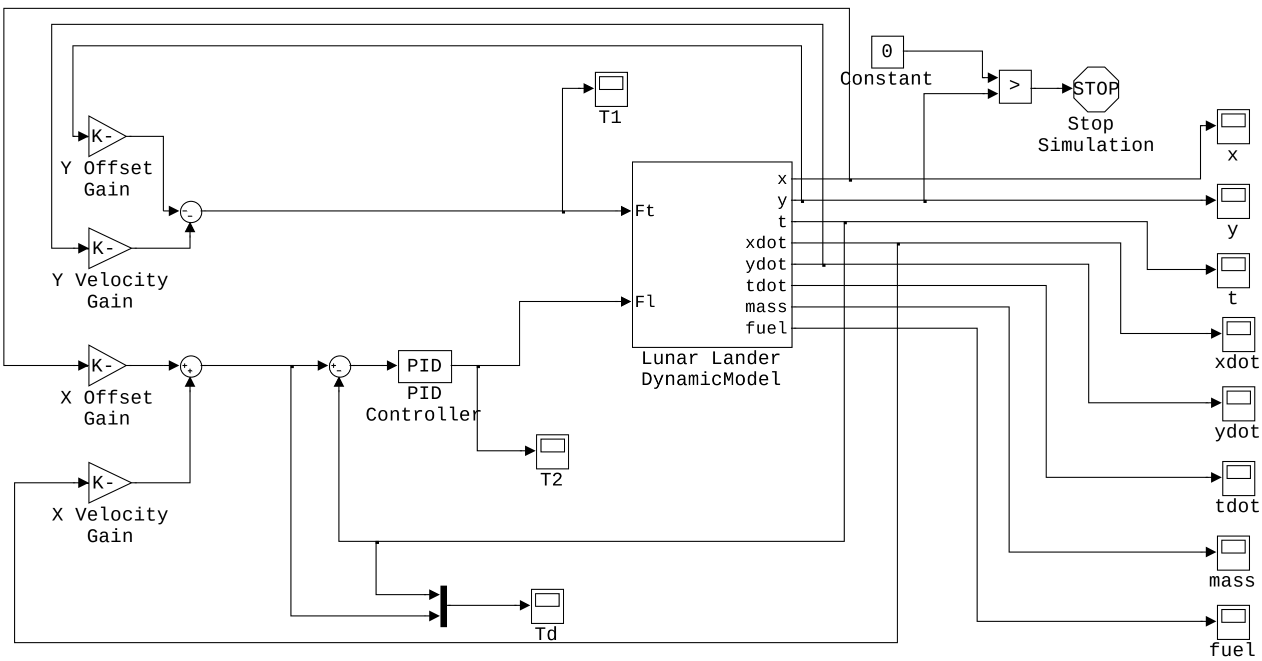

Figure 10 - Lunar lander control system

With Y offset and velocity gains of 11 and 2350 respectively, we achieve pretty respectable performance for both the position and velocity of the lander with respect to time. These are shown in Figure 11 and Figure 12.

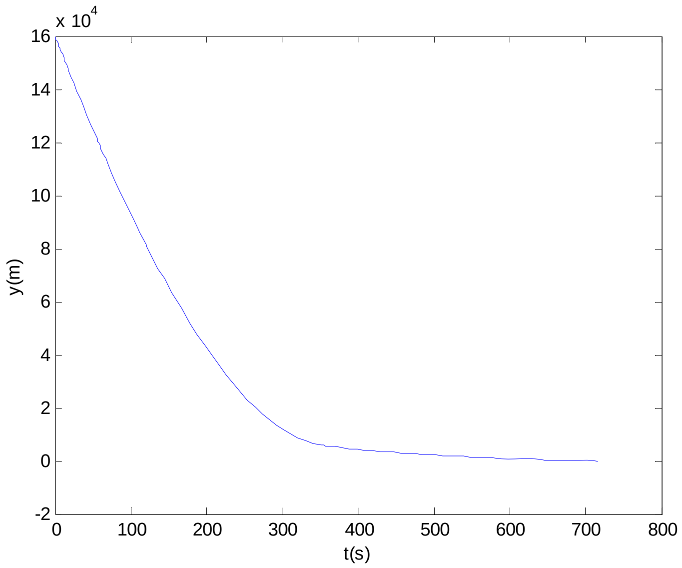

Figure 11 - Lunar lander y position

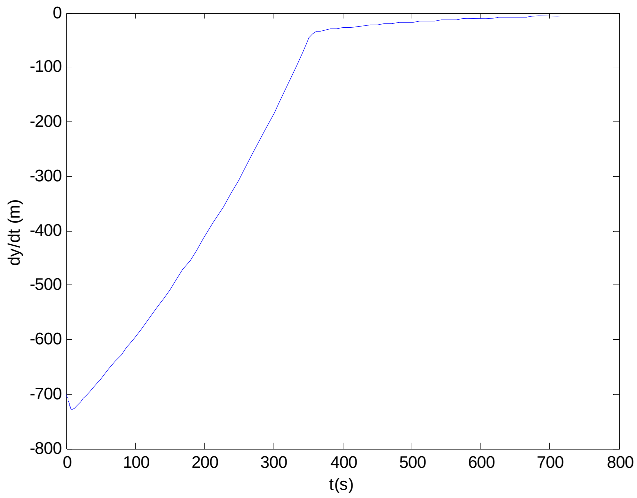

Figure 12 - Lunar lander y velocity

The velocity appears to be approaching zero as the lander touches down. However, zooming in I can see that the lander is still moving at approximately -5.5m/s on touchdown, as shown in Figure 13. The design criteria stipulates a required landing speed of less than -1m/s.

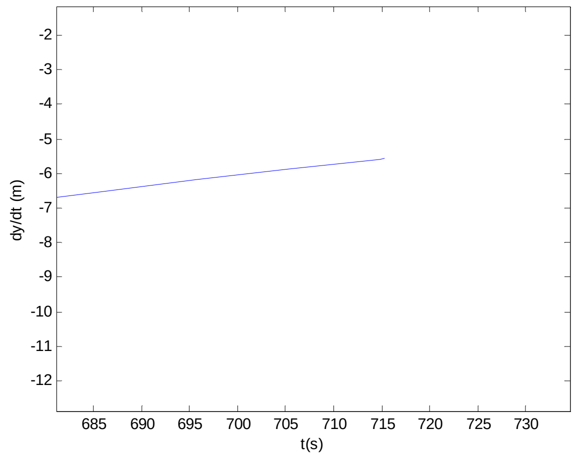

Figure 13 - Lunar lander y velocity zoom at touch down

In order to achieve this landing it would make sense to fire the rockets at the last minute to bring down the landing speed. An additional block is added in Figure 14 that adds a constant thrust of 12.5kN in the last 20m of the descent. This brings the velocity down to -0.5m/s on touchdown as shown in Figure 15.

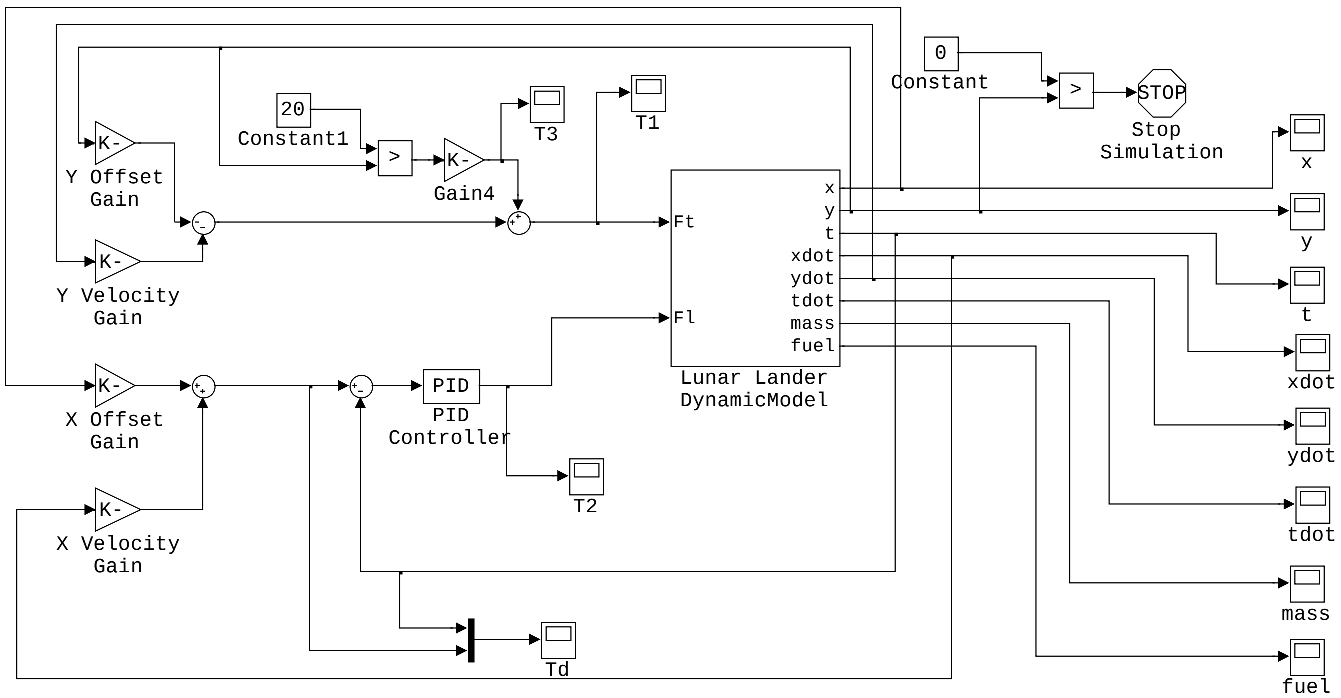

Figure 14 - Lunar lander control system

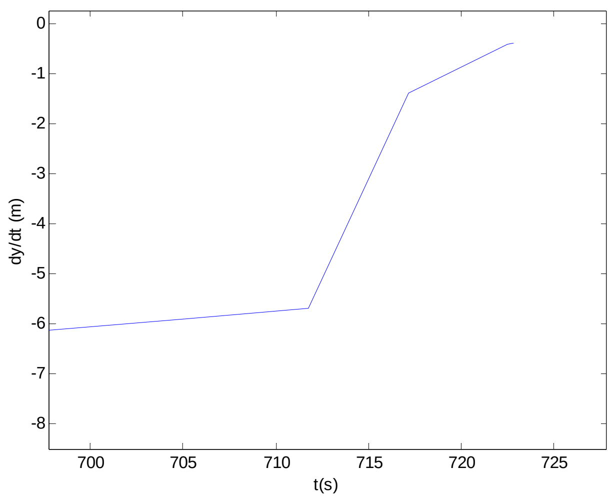

Figure 15 - Lunar lander y velocity on landing with additional thrust.

The resulting trajectory for this descent is shown in Figure 16. Notice that the scales on this figure are not the same.

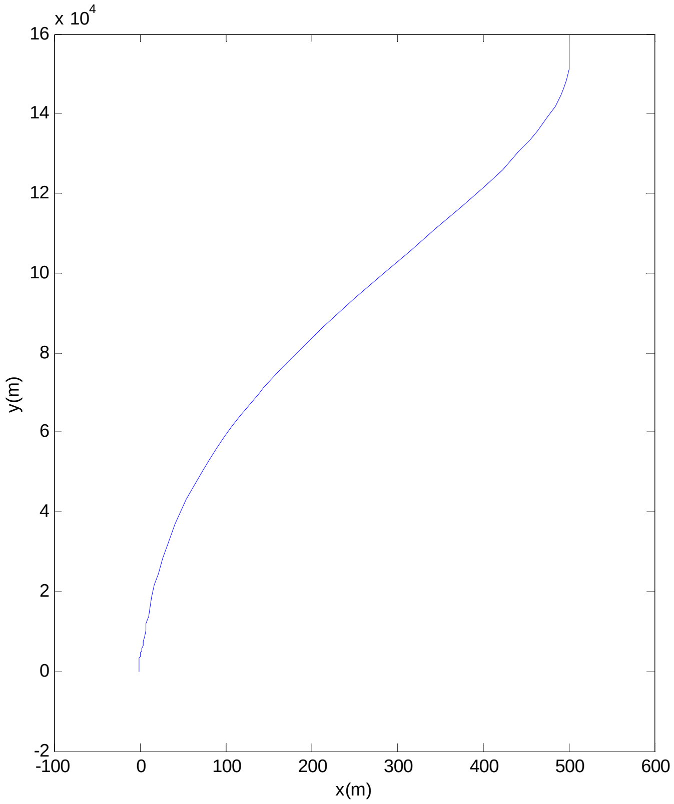

Figure 16 - Lunar lander trajectory

Alright, we have succeeded in landing the craft at the desired location and with the desired speed. Now the final design criteria is that I should have sufficient fuel left to get my astronauts back off the moon. Figure 17 shows the mass of fuel of the lander during the descent. The initial mass of fuel is 8000kg and on touchdown the mass remaining is approximately 1000kg. This doesn't meet the suggested requirement for half the fuel remaining.

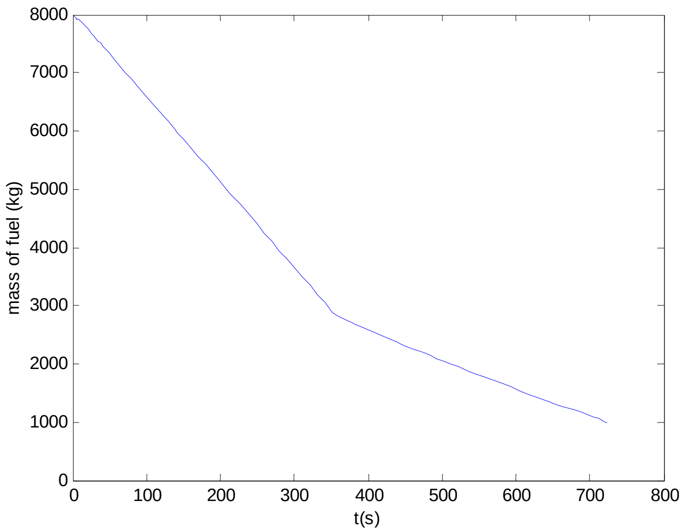

Figure 17 - Lunar lander mass of fuel during descent

In order to check if we can get the crew back, I will fire the main thrusters until I burn through the fuel. The escape velocity for the moon is approximately 2.4km/s. This means that I need to achieve this speed in order to escape the gravity pulling me back to the surface. I have modelled this on the left as a switch which turns off the thrust when there is no fuel remaining.

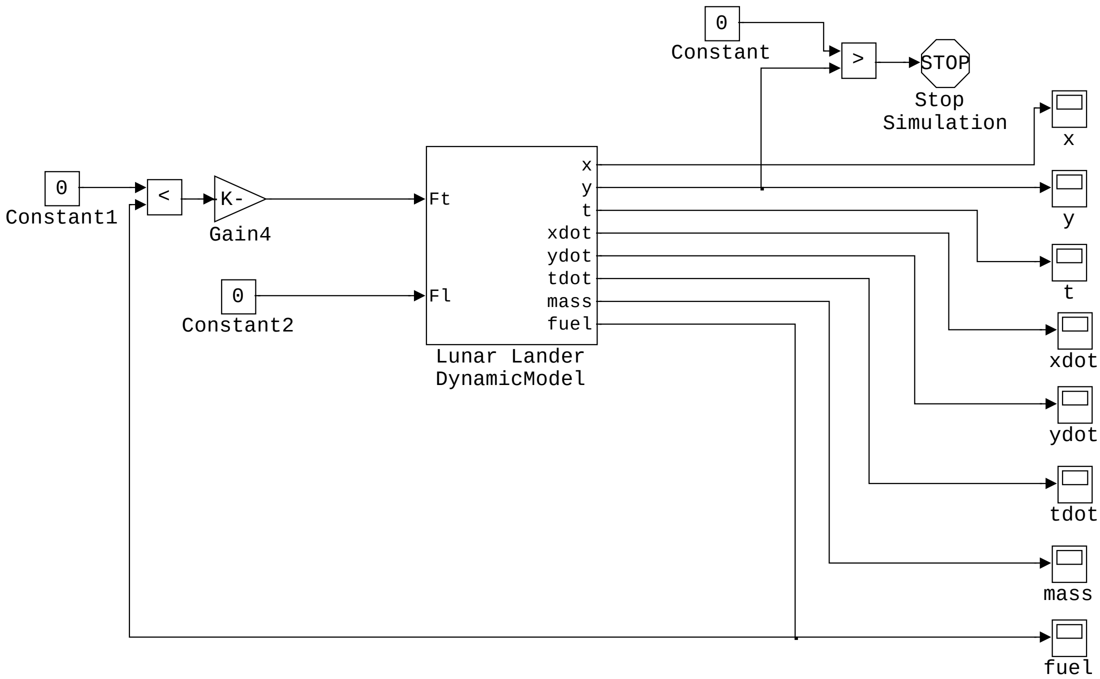

Figure 18 - Lunar lander control system ascent verification

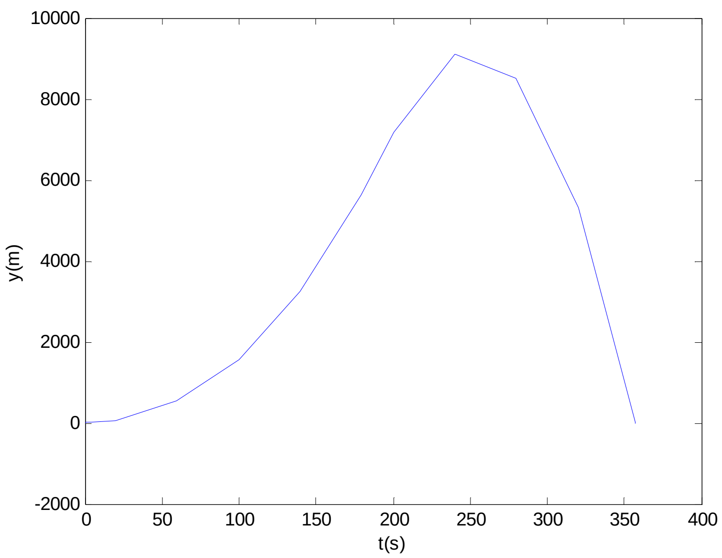

Figure 19 - Lunar lander y position ascent

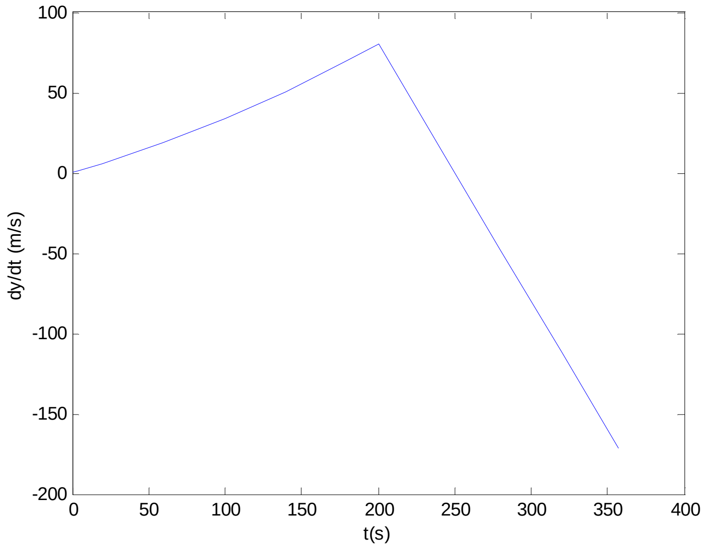

Figure 20 - Lunar lander ascent y velocity

Houston, we have a problem! The remaining fuel is not sufficient to return the craft from the surface of the moon. In the actual Apollo missions, the lander was comprised of two modules. The descent module and a lighter ascent module that was used to return the astronauts to the orbiting support spacecraft.