Problem Set I – Introduction

(c) Chris Ormel

Hand in HW-problems before the announced date

Hand in before the announced deadline, either before the start of class or by uploading your solutions to the WebLearning system (pdf format only!). Write down your answers in English and state any additional assumptions that you have made, e.g., in case when you think that the phrasing is unclear. Collaboration is allowed, but you should hand in your own solution and be able to reproduce your answer independently if asked to (e.g, in class!). Students whom typeset their answers (with, e.g., LaTeX) or present an excellent layout will be awarded a 10% bonus, if all layout standards have been met. Late returns are penalized at a rate of 10% per day.

Exercise I.1 Sun's efficiency

(a) Compute the energy output (power) of the Sun per unit mass. Estimate the same quantity for a human being. Motivate your numbers (don't just look them up on baidu!)

Exercise I.2 Color-Magnitude diagram

In this exercise you will draw your own a color-magnitude diagram. You need basic computational skills to read in data files, manipulate, and plot the data. Nowadays, astronomers almost exclusively use `python` and `matplotlib` for such operations.

On our Tsinghua Cloud you will find two data sets:

1. `hipparcos-bright` – a sample of Hipparcos stars with V-magnitude brigher than 5.
2. `hipparcos-close` – the sample of Hipparcos stars with a parallax exceeding 50 mas (milli-arcsec).

(a) Choose one and then do the following:

- download and read in the data with your favorite plotting program (probably `python`)
- remove the entries where the type reads `UNIDENTIFIED`
- convert the entries in the `parallax` column to distance
- convert (apparent) magnitude (`vmag`) to absolute magnitude through the distance modulus
- Plot the V-magnitude (in the direction of increasing brightness) vs the B-V color. These diagrams are called color-magnitude diagrams.
- Highlight with a blue dot the star that has the greatest parallax and with a red dot the star in the sample that is the brightest in the sky.

(b) Why does the $B-V$ color index reach the same value, irrespective of whether magnitudes are given as apparent or absolute?

(c) Explain qualitatively that the difference between the magnitude in the "blue" band (B) and the magnitude in the "visual" (V) band reflect the stellar effective temperature, such that stars of higher B-V have lower $T_{\rm eff}$.

(d) On the plot, identify the following features (warning: depending on the sample you choose, they may not be there!):

- The Main Sequence
- RGB stars
- White Dwarfs

(e) Let's convert the color-magnitude diagram into a Hertzsprung-Russell diagram! To do so we must convert B-V color to effective temperature. These are often empirical relationships. Use:

$$B - V = \begin{cases} -3.684\log_{10}(T) + 14.551 & \text{if} \log_{10} T < 3.961 \\ 0.344[\log_{10}(T)]^2 - 3.402\log_{10} T + 8.037 & \text{if} \log_{10} T > 3.961 \end{cases} \tag{1}$$

(Reed 1998). In addition, you should convert V-magnitudes into bolometric velocities, the so-called bolometric correction BC, ($M_{\rm Bol} = V - BC$), see the book. The relation you could use is:

$$BC = -8.449x^4 + 13.421x^3 - 8.131x^2 - 3.901x - 0.438 \tag{2}$$

where $x = \log_{10} T_{\rm eff} - 4$ (this also follows Reed 1998 but note their typo). The bolometric magnitude of the Sun is +4.75.

Finally, plot the Hertsprung-Russell diagram (luminosity vs effective temperature) of your sample. Label lines of constant radius (0.1, 1.0, 10, $\ldots R_\odot$) and indicate the Sun.

Exercise I.3 Transits

(a) Show that the probability that a planet of radius R_p orbiting a star of radius $R_\star$ at a distance a will transit at some point in its orbit is

$$P_{\rm trans-orbit} = \frac{R_p + R_\star}{a} \tag{3}$$

independent of the distance to the observer. (You can assume a circular orbit)

(b) Also show that the probability that the planet is *currently* transiting is

$$P_{\rm transit-now} = \frac{1}{4}\left(\frac{R_p + R_\star}{a}\right)^2 \tag{4}$$

The *Kepler* mission's objective was to find an Earth-like planet orbiting a solar-type star at 1 au. To this end, *Kepler* was monitoring the brightness of about 100,000 stars.

(c) Assuming that all of the 100,000 stars harbor such an Earth-like planet, how many will (eventually) show a transit?

(d) What is the (maximum) percentage change in brightness of an Earth-like transit? What is the drop in magnitude? State the numerical values.

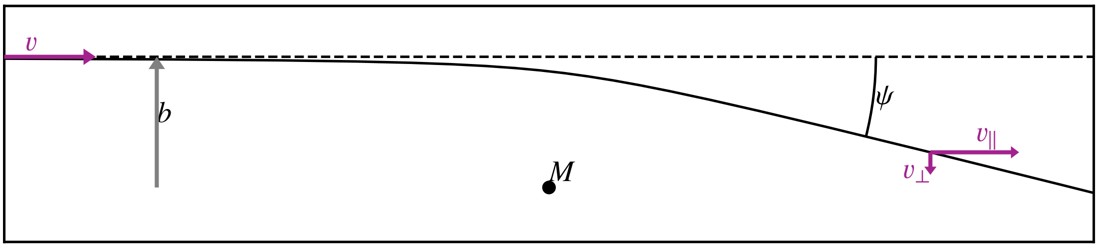

Figure 1: Scattering of a (massless) test particle by a massive body M.

White dwarfs are stellar objects – with the mass of the Sun – that have the size of the Earth. They can be mistaken for a planet.

(e) Angie argues that those White Dwarfs that are young and as bright as their host star (same effective temperature) do not show a transit signal. Bernie thinks they will. Who is right and why?

While Angie and Bernie are arguing, Charlie measures the radial velocity of the host star.

(f) Calculate the radial-velocity amplitude generated by (i) the Earth-like planet; (ii) the White Dwarf. Still assume a distance of 1 au and a solar-mass host star.

(g) Is an RV-measurement useful to rule out that the transiting body is a White dwarf?

(h) Is an RV-measurement useful to ensure that the transiting body is an Earth-mass planet?

Exercise I.4 Microlensing

Consider a gravitational encounter between a (massless) test particle of mass m and a massive body of mass M. We want to calculate the scattering angle ψ, as function of impact parameter b and velocity v, see Figure 1. Consider a scattering event at impact parameter b as illustrated in the figure.

(a) In the small-angle approximation ($\psi \ll 1$), we can use a perturbation approach. That is, we calculate the change in the velocity in the perpendicular direction $v_\perp$ based on the unperturbed motion (the straight line). Therefore assume

$$x = x_0 = vt; \qquad y = y_0 = b \tag{5}$$

and integrate the equation of motion over time to find an expression for $v_\perp$.

(b) From this, find the scattering angle ψ.

(c) Find the Einstein angle θ_E as defined in the class. Note: your expression will be off by a numerical factor. (Hint: you obviously need to substitute the speed of light for v and something else for the impact parameter b. The reason why the expression is wrong is that our derivation ignores relativistic effects.)

(d) If we assume that $d_s = 2d_l = 8\,\mathrm{kpc}$ and $M = 0.5\,M_\odot$ (very typical values for stars in the Milky Way bulge), what is the size of the Einstein ring? Give your answer both in angular units of milli-arcseconds (1 mas = 10^{-3} arcsec = $(\frac{10^{-3}}{3600})$ degree) as well as in Astronomical Units (au).

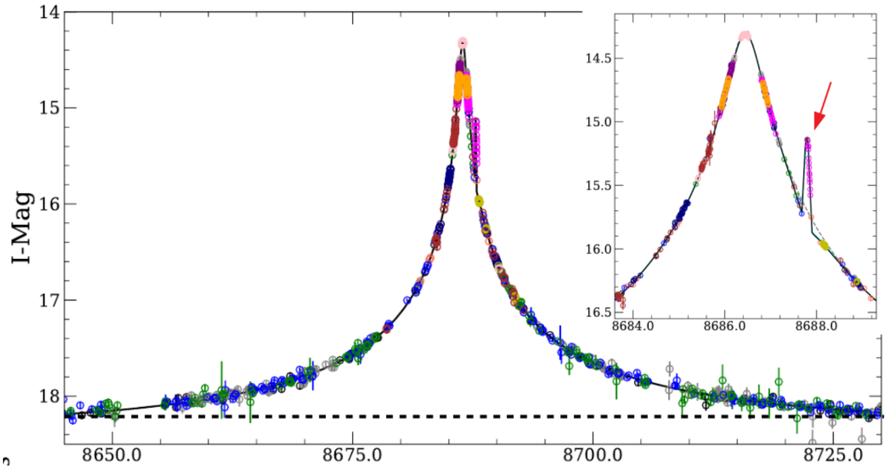

Figure 2: Example of microlensing event. X-axis is Julian date (units are days). The y-axis is brightness in magnitude units. The dashed line indicates the brightness of the star without lensing. After Yee et al. (2021). *Figure available on TsinghuaCloud* (`YeeZangEtalfig1-modified.png`).

In Figure 2 we see an example of a microlensing event, observed by an infamous Tsinghua graduate student. It should be clear that for these high magnification events, $\min(\beta)/\theta_E \ll 1$. In other words, the background star perfectly intersects the Einstein ring, entering at right angles and passing (almost) through the center.

(e) What is the magnification when the background source is located at $\beta = \pm\theta_E$? What is the increase in brightness due to the lensing at this position. Give your answer in magnitude units?

(f) Using your result from the previous question, obtain from Figure 2 the *duration* of the Einstein ring crossing, $t_{\mathrm{E,cross,star}}$.

In Figure 2 the deviation of the lightcurve indicated by the red arrow, is due to the planet. You can verify that the duration of *its* Einstein ring crossing (the Einstein ring of the planet) is about $t_{\mathrm{E,cross,planet}} = 0.25$ days.

(g) Give an expression for the planet-to-star mass ratio $m_p/m_\star$ in terms of the Einstein crossing times. What is the planet mass (in units of Earth) if the star is 0.5 $M_\odot$?

References

Reed, B. C. 1998, Journal of the Royal Astronomical Society of Canada, 92, 36

Yee, J. C., Zang, W., Udalski, A., et al. 2021, AJ, 162, 180