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"Global Stock Index Performance Analysis Using Time Series Modeling"

Abstract

This paper investigates the performance and volatility changes of some major global stock indices using time series modelling techniques, particularly the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. In our analysis, we specifically focus on six prominent global indices: S&P500 (USA), FTSE100 (UK), DAX (Germany), Hang Seng (Hong Kong), BVSP (Brazil), and Nikkei 225 (Japan). By modelling conditional volatility, we provide insights into volatility persistence, market responsiveness to market shocks, and comparing these markets' behaviors. Our findings confirm strong volatility persistence in all indices and regional variations in the magnitude of short-term volatility.

Introduction

Global financial markets experience periods of stability and changes, often driven by macroeconomic shocks, political developments, or systematic risk events. Volatility modeling provides tools to understand and forecast such market dynamics. This philosophy also applies to GARCH (1,1) model to historical daily return data of six key global indices to assess and compare their volatility behavior.

The main objectives of this project are:

- To compute and model the return and volatility of selected stock indices.
- To evaluate the persistence of volatility using the GARCH models.
- To compare volatility characteristics across developing and developed markets.

GARCH models are widely used in both academic and applied finance due to their effectiveness in capturing volatility clustering. Studies such as Ugurlu et al. (2014) have demonstrated the usefulness of GARCH models in analyzing emerging markets, which further motivates our cross-market analysis.

Literature Review

Time series modeling has been a major field of study in financial econometrics for decades. Stock market volatility has especially been an area of importance. Volatility patterns such as clustering and persistence are crucial for understanding market behavior and risk management.

Engle (1982) introduced the Autoregressive Conditional Heteroskedasticity (ARCH) model, which provided a foundation for modeling time-varying volatility. Bollerslev (1986) extended this to the Generalized ARCH (GARCH) model, which allowed for both short-term stocks and longer-term volatility persistence to be captured. Garch remains the most widely applied model in empirical finance today because of its simplicity and effectiveness.

Numerous studies have confirmed that stock indices across both developed and emerging markets exhibit high volatility persistence. For example, Ugurlu, Thalassinos, and Muratoglu (2018) applied GARCH models to European emerging markets and Turkey, where they found consistent long-memory properties in volatility which are similar to those found in developed markets. This suggests that volatility modeling is relevant across different market contexts.

Research by Bekaert and Harvey (1997) found that emerging markets often show greater volatility and longer adjustment periods aftershocks compared to developed markets. This difference supports comparing indices like the S&P 500 and DAX to BVSP (Brazil) and Hang Seng (Hong Kong) as our project does. While the standard GARCH model captures volatility clustering, recent articles also emphasize asymmetric models like EGARCH or GJR-GARCH to capture leverage effects – where negative shocks tend to increase volatility more than positive shocks (Nelson, 1991). However, for a first level cross country analysis GARCH (1,1) provides a solid starting point.

In light of this literature, applying the GARCH (1,1) model to compare global stocks indices from both developed and emerging economies is appropriate. Our study builds upon this beginning work and updates it with more recent data (2000-2024) which offers insights into volatility dynamics across major markets.

Methodology

I. Data

We collected the daily closing price data from Yahoo Finance for the following indices:

- S&P 500 (^GSPC) - USA
- FTSE 100 (^FTSE) - UK
- DAX (^GDAXI) - Germany
- Hang Seng (^HSI)- Hong Kong
- Nikkei 225 (^N225) - Japan

- Bovespa (^BVSP) - Brazil

The time range spans from January 2000 to December 2024.

II. Return Calculations

We calculated the log returns from adjusted daily closing prices using:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

III. Model Specification

The GARCH (1,1) model is specified as:

$\sigma_t^2 = w + \alpha \cdot \epsilon_{t-1}^2 + \beta \cdot \sigma_{t-1}^2$

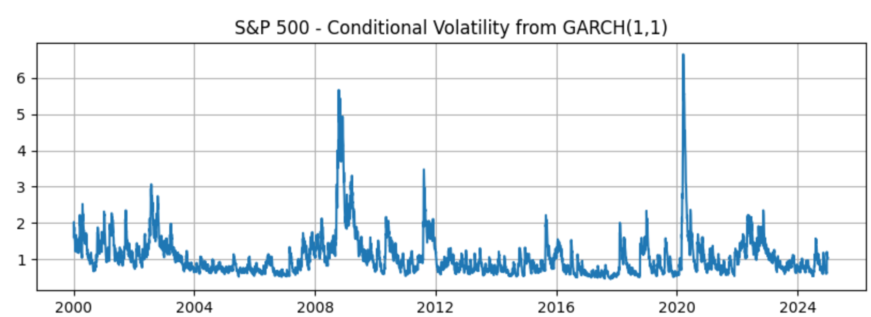
- w : constant
- α : impact from past shock (ARCH effect)
- β : volatility persistence (GARCH effect)

ADD- GARCH specifics

Results and Interpretation:

S&P 500 (USA)

- Mean return (μ) = 0.0655% per day
- Volatility persistence: $\alpha + \beta = 0.9828$

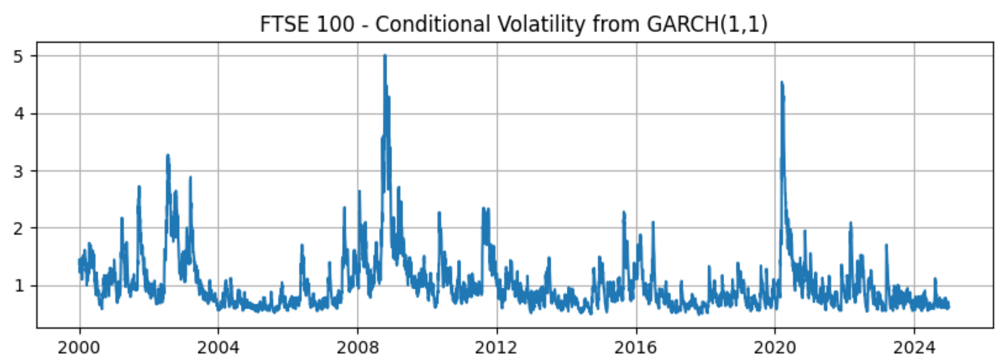


	Coefficient	P-value
μ	0.0655	1.67E-10
ω	0.0232	8.13E-07
α	0.1198	1.5E-22
β	0.863	0
α+β	0.9828	

- Interpretation: Very Strong volatility persistence. Market reacts significantly to past shocks and retains high volatility clustering

FTSE 100 (UK)

- Mean return = 0.0352%
- $\alpha + \beta = 0.9801$

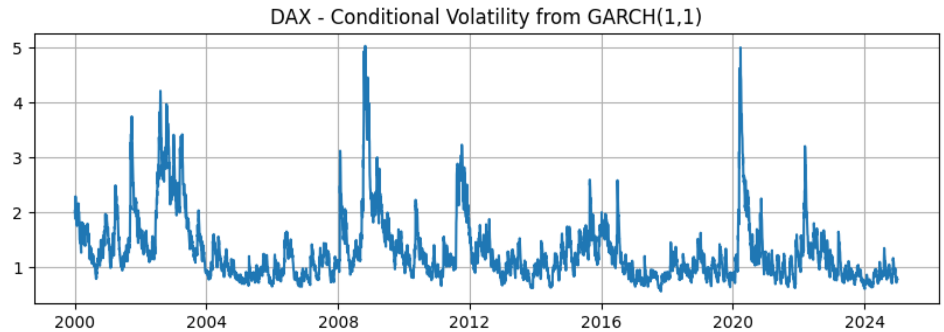


	Coefficient	P-value
μ	0.0352	0.000574
ω	0.0235	1.39E-05
α	0.1144	4.94E-14
β	0.8657	0
α+β	0.9801	

- Interpretation: Strong persistence with moderate mean return.

DAX (^GDAXI) - Germany

- Mean return= 0.0715%
- $\alpha + \beta = 0.9855$

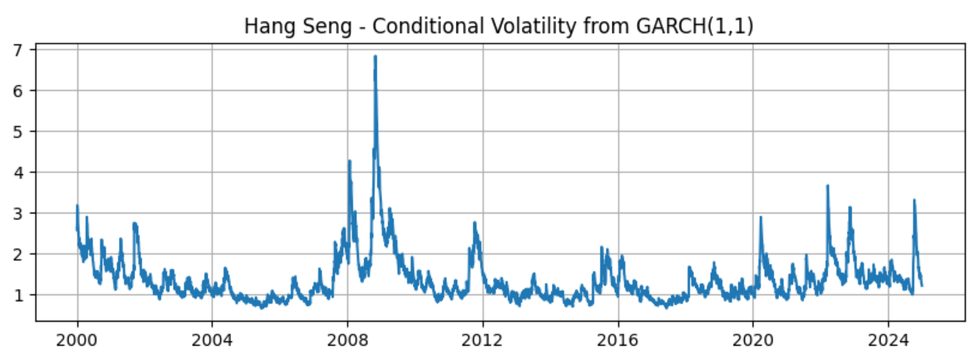


	Coefficient	P-value
μ	0.0715	1.08E-07
ω	0.0272	1.36E-05
α	0.0944	9.4E-15
β	0.8911	0
α+β	0.9855	

- Interpretation: Highest persistence, suggesting long-lasting market reactions to volatility shocks.

Hang Seng (^HSI)- Hong Kong

- Mean return = 0.0443%
- $\alpha + \beta = 0.9903$

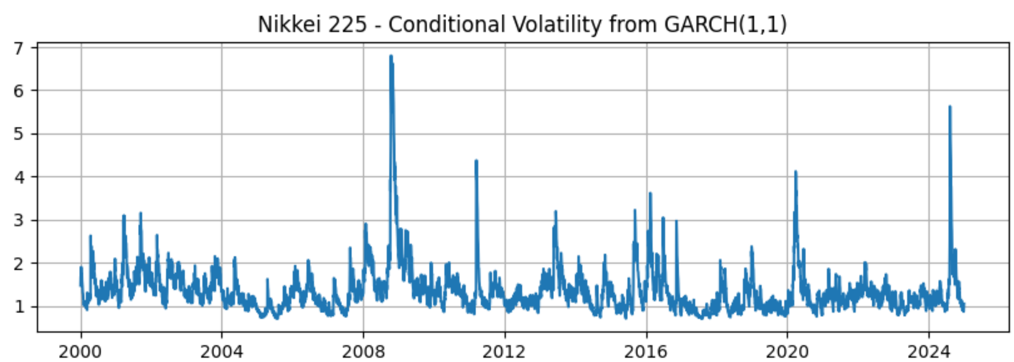


	Coefficient	P-value
μ	0.0443	0.002727
ω	0.0196	7.3E-05
α	0.0655	2.62E-18
β	0.9248	0
α+β	0.9903	

- Interpretation: Extremely persistent volatility. Likely reflects structural and systemic risk features in the Hong Kong market.

Nikkei 225 (^N225) - Japan

- Mean return = 0.0637%
- $\alpha + \beta = 0.9775$

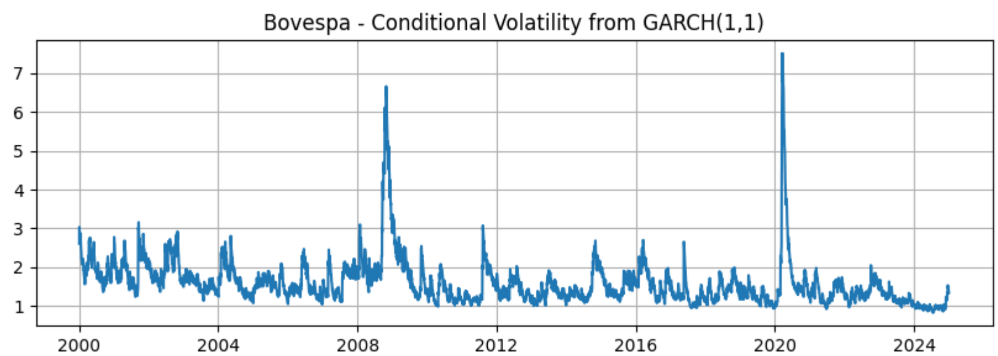


	Coefficient	P-value
μ	0.0637	1.76E-05
ω	0.0509	2.92E-06
α	0.113	1.86E-16
β	0.8645	0
α+β	0.9775	

- Interpretation: High but slightly lower persistence than Hang Seng. The market shows reactive volatility but recovers faster.

Bovespa (^BVSP) - Brazil

- Mean return = 0.0649%
- $\alpha + \beta = 0.9855$



	Coefficient	P-value
μ	0.0649	0.000287
ω	0.0404	3.23E-05
α	0.0715	5.88E-18
β	0.914	0
α+β	0.9855	

- Interpretation: Highest volatility persistence among emerging markets. Suggests prolonged volatility cycles and sensitivity to external factors.

Comparative Analysis

Index	α	β	$\alpha+\beta$
S&P 500	0.1198	0.8630	0.9828
FTSE 100	0.1144	0.8657	0.9801
DAX	0.0944	0.8911	0.9855
Hang Seng	0.0655	0.9248	0.9903

Nikkei 225	0.1130	0.8645	0.9775
Bovespa	0.0715	0.9140	0.9855

The updated results with extended data show even stronger persistence across all markets, especially Hang Seng and Bovespa. The DAX and Nikkei follow closely, indicating stable but prolonged volatility behavior in both developed and emerging economies.

Discussion

With extended historical data from 2000 onwards, the GARCH (1,1) model continues to confirm strong volatility clustering across all stock indices. The Hong Kong (Hang Seng) and Brazilian (Bovespa) indices exhibit the most persistent volatility. This is characteristic of markets with greater exposure to external shocks or structural inefficiencies.

DAX and S&P 500 show consistent patterns with slightly less persistence but still notable long-term volatility. Developed markets overall demonstrate stability with quicker return to equilibrium, while emerging markets linger in volatility cycles longer.

Future Extensions

This study solely focuses on modeling equity index volatility using the GARCH (1,1) framework across six global markets. However, future work can further broaden this scope in various ways:

- "Cross-Asset Volatility Analysis: Using key commodities such as crude oil and gold as comparative benchmarks can provide more comprehensive insights into how equity volatility co-moves with the basic commodity markets. These assets could potentially help us analyze how different markets respond differently to macroeconomic shocks."
- "Sector-level analysis: Analyzing volatility at the sectoral level, different industries such as technology, biotech, energy companies, may help to analyze unique patterns in aggregated index-level data."
- "Macroeconomic Variable Analysis: Adding more macroeconomic indicators such as interest rate, inflation, VIX (volatility index) to analyze more broader economic state."

Conclusion

Applying the GARCH (1,1) model on extended data enhances the validity of our findings on volatility dynamics. All indices reflect high persistence, with variations across regions. Understanding these differences is critical for global investors managing diversified portfolios and for policymakers observing systemic risk signals.

Future directions may include:

- Introducing asymmetric GARCH models (EGARCH, GJR-GARCH)
- Sectoral comparison (e.g., tech vs. energy indices)
- Incorporating macroeconomic or sentiment variables as exogenous inputs

Challenges and Limitations

This study encounters several challenges such as handling the technical aspects of data collection and modelling. Using Python and external libraries such as yfinance and arch, we encountered issues related to API rate limits, missing data, and model fitting errors. For example, Yahoo Finance often restricts repeated data and requests, which caused delays and required us to implement time delays and retry logics.

Reference

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