Mini Project - Linear Systems

Sem I AY2021/2022

Control System Design of a Continuous-flow Stirred Tank Reactor (CSTR)

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Abstract

The study of systems consists of four parts: modelling, setting up mathematical equations, analysis and design. A good knowledge on the modern control design techniques, which can bring the system under control, is desirable. Hence, in this report, there are design and analyse of various control systems. In this assignment, a typical third-order MIMO plant is provided. The duty is try to apply what have learnt from this module, like pole placement, LQR, decoupling, servo control and observer design. All the above investigations will be carried out or simulated by MATLAB and Simulink.

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Chapter 1 Introduction

In this project, the most important factors that we want to control is the component concentration of the reactant during reaction and the reaction temperature. The system is described by

$$\dot{x} = Ax + Bu + Bw$$
$$y = Cx$$

where

$$A = \begin{bmatrix} -1.7 & -0.25 & 0 \\ 23 & -30 & 20 \\ 0 & -200 - ab0 & -220 - ba0 \end{bmatrix} \tag{0.1}$$

$$B = \begin{bmatrix} 3 + a & 0 \\ -30 - dc & 0 \\ 0 & -420 - cd0 \end{bmatrix} \tag{0.2}$$

$$C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{0.3}$$

when a = 2, b = 8, c = 4, d = 1,

$$A = \begin{bmatrix} -1.7 & -0.25 & 0 \\ 23 & -30 & 20 \\ 0 & -480 & -1040 \end{bmatrix} \tag{0.4}$$

$$B = \begin{bmatrix} 5 & 0 \\ 44 & 0 \\ 0 & -830 \end{bmatrix} \tag{0.5}$$

$$C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{0.6}$$

$$x_0 = [1\ 100\ 200] \tag{0.7}$$

The aim of this project is to analyse the target plant by using the concepts learned in the class:

1. Applying Pole Placement control method to simulate the effects of the positions of poles on system performance.

2. Applying LQR control method to simulate the effects of weightings Q & R on system performance.

3. Designing a state observer using LQR method to monitor state estimation error and investigate effects of observer poles on state estimation error and closed-loop control performance.

4. Designing a decoupling controller with closed-loop stability to verify decoupling performance with stability.

5. Designing a servo controller such that the output error response goes to zero at the steady state.
6. Designing a controller to keep the state variables at steady state as close as possible to the set point.

Chapter 2 Controller Design Tasks

Section 1 Pole Placement Controller

From the equation for calculating overshoot and settling time,

$$M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} < 10\% \tag{1.1}$$

$$\Rightarrow \zeta > 0.5911$$

$$t_s = \frac{4}{\zeta\omega_n} < 40 \tag{1.2}$$

$$\Rightarrow \zeta\omega_n > 0.1$$

Select $\zeta = 0.8$, and $\omega_n = 0.25$, then

$$\lambda_1 = \zeta\omega_n + j\omega_n\sqrt{1-\zeta^2} = -0.2 + j0.1 \tag{1.3}$$

$$\lambda_2 = \zeta\omega_n - j\omega_n\sqrt{1-\zeta^2} = -0.2 - j0.1 \tag{1.4}$$

and,

$$\lambda_3 = -0.5 \tag{1.5}$$

Set unknown number $k_{11}, k_{12}, k_{13}, k_{21}, k_{22}, k_{23}$, let matrix

$$k^T = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix} \tag{1.6}$$

Controllability matrix

$$W = [B\ AB\ A^2B] = [b_1\ b_2\ Ab_1\ Ab_2\ A^2b_1\ A^2b_2] \tag{1.7}$$

$$C = [b_1\ Ab_1\ b_2] \tag{1.8}$$

$$C^{-1} = \begin{bmatrix} q_1^T \\ q_2^T \\ q_3^T \end{bmatrix} \tag{1.9}$$

$$T = \begin{bmatrix} q_2^T \\ q_2^T A \\ q_3^T \end{bmatrix} \tag{1.10}$$

Set,

$$\bar{K} = \begin{bmatrix} \bar{k}_{11} & \bar{k}_{12} & \bar{k}_{13} \\ \bar{k}_{21} & \bar{k}_{22} & \bar{k}_{23} \end{bmatrix} \tag{1.11}$$

$$\det(sI - A_d) = s^3 + \frac{9}{10} * s^2 + \frac{21}{80} * s + \frac{1}{32} \tag{1.12}$$

Then,

$$A_d = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{1}{32} & -\frac{21}{80} & -\frac{9}{10} \end{bmatrix} \tag{1.13}$$

$$K = \bar{K} * T = \begin{bmatrix} 2.52 & -5.65 & 3.57 \\ 0.28 & 0.03 & 1.60 \end{bmatrix} \tag{1.14}$$

Results:

The control system block diagram of section 1 is shown in *Figure 1-1*.

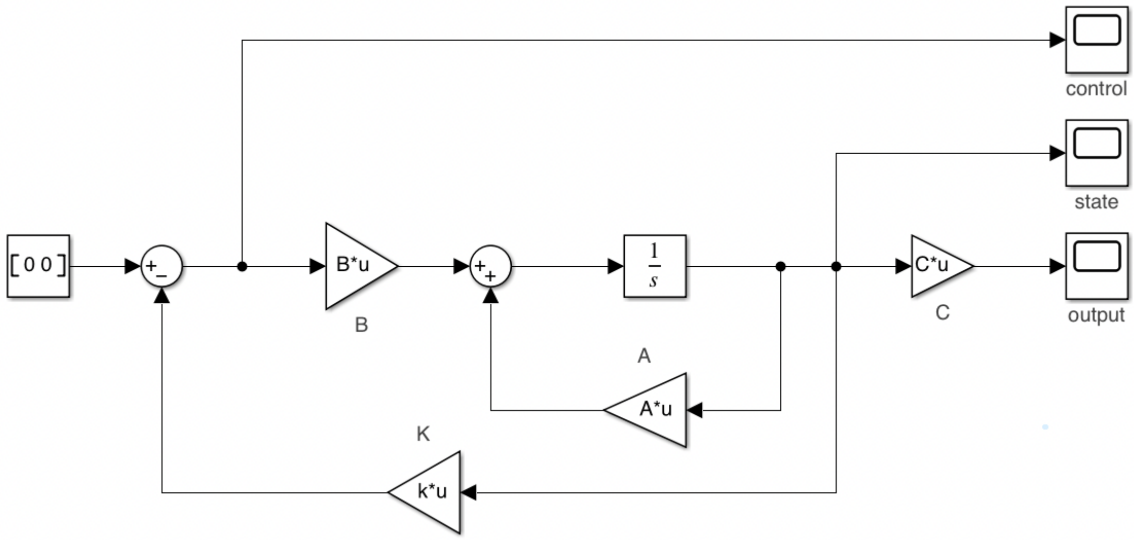

Figure 1-1 System Diagram for Section 1

The transient response by giving step reference signal for each input channel is shown in *Figure 1-2* and *Figure 1-3*.

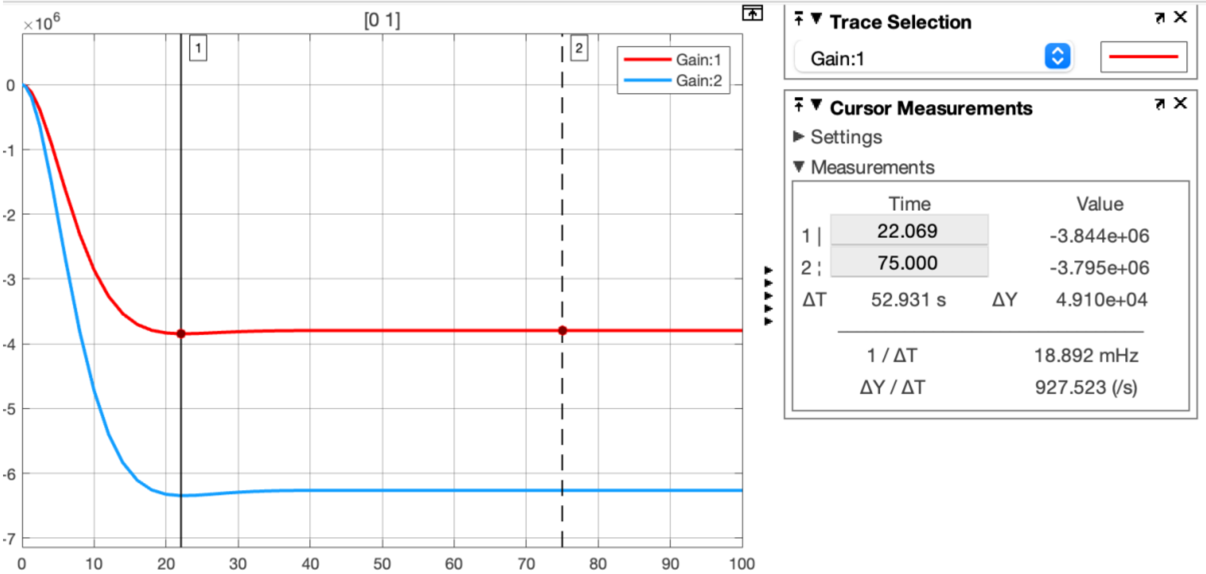

Figure 1-2 Transient Response by Giving Step Reference Signal (r = [0 1])

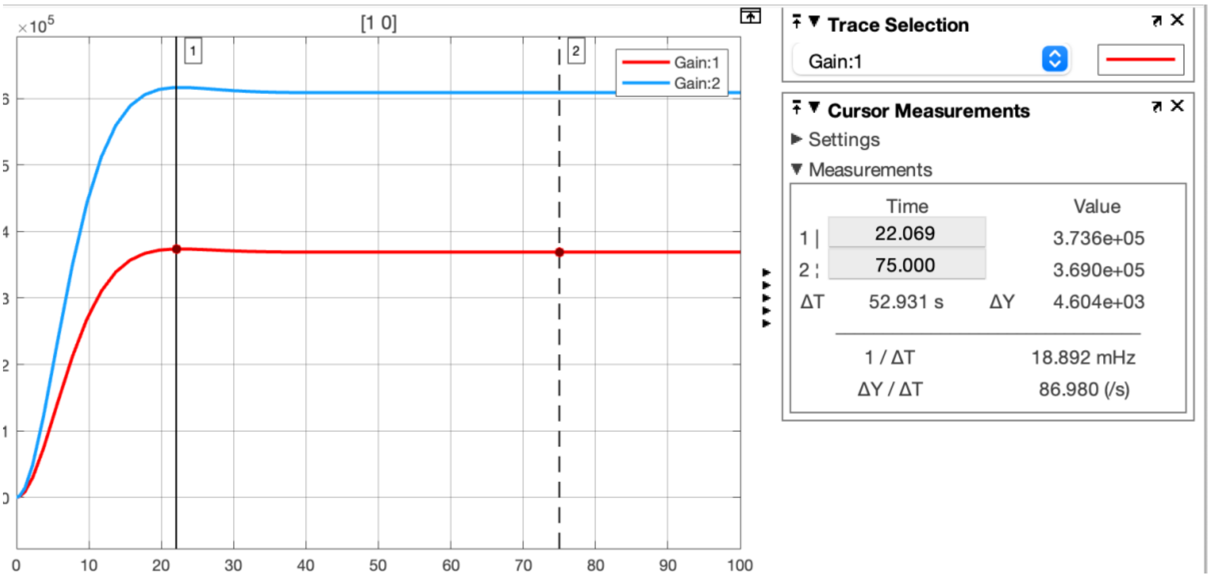

Figure 1-3 Transient Response by Giving Step Reference Signal (r = [1 0])

The overshoots of these transient response are all around 1%, and the 2% settling times are smaller than 20 seconds, which meet the requirements.

With $\lambda_1 = -0.2 + j0.1$, $\lambda_2 = -0.2 - j0.1$, and $\lambda_3 = -0.5$, the three state variables and control signal are shown in *Figure 1-4* and *Figure 1-5*.

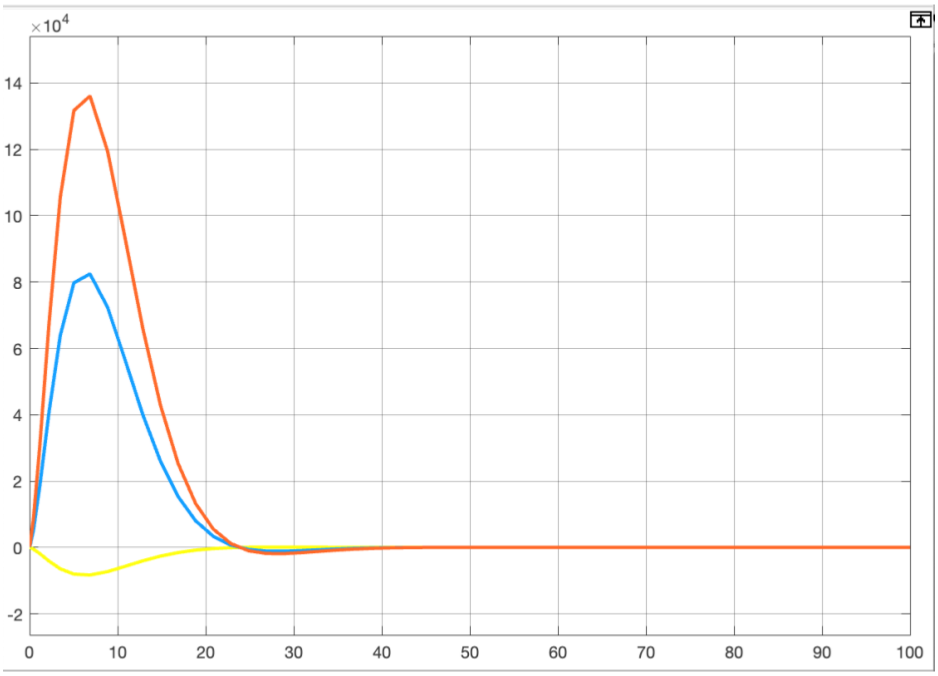

Figure 1-4 Three State Vatiables-1

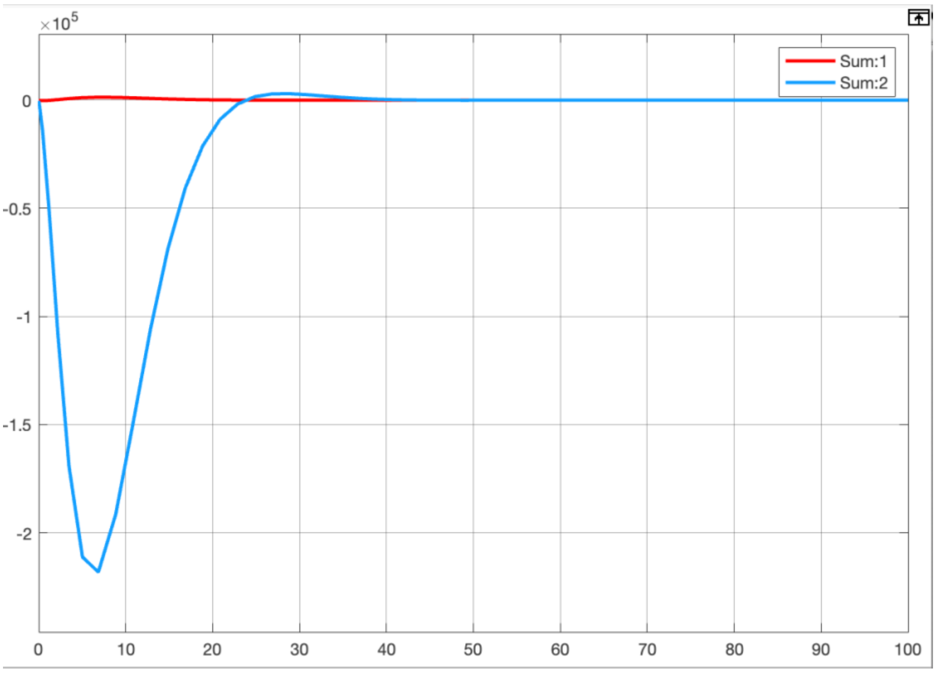

Figure 1-5 Control Signal-1

When the poles times 10, the state variables and the control signals are shown in *Figure 1-6* and *Figure 1-7*.

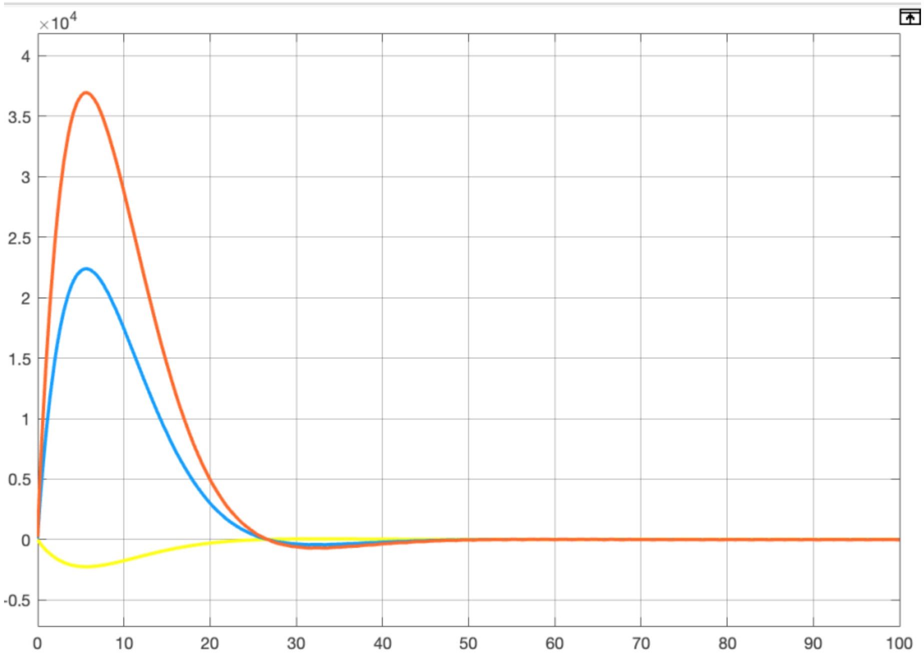

Figure 1-6 Three State Vatiables-2

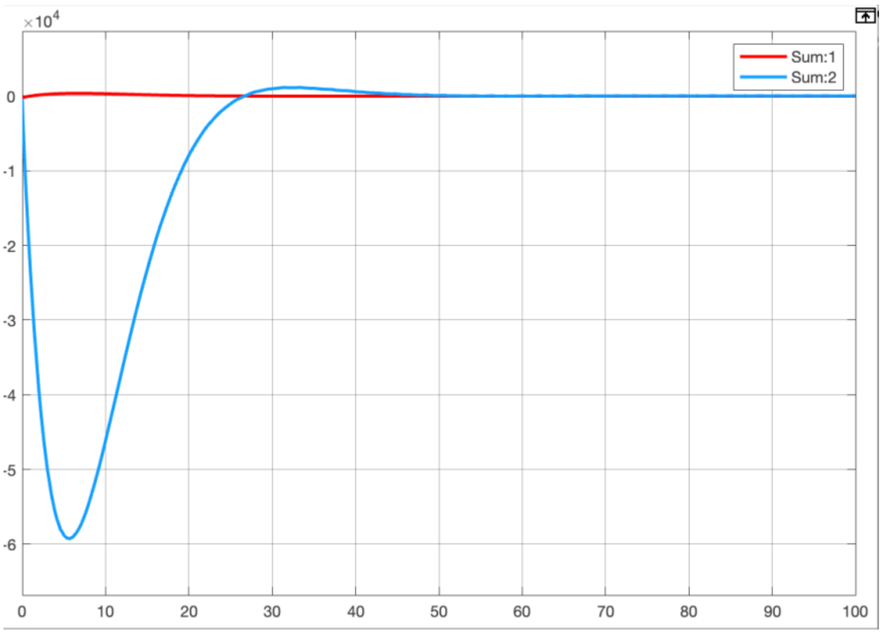

Figure 1-7 Control Signal-2

When the value of the pole changes to 10 times its original value, the system response speed does not change significantly, but all three state variables as well as the size of control signal become smaller.

Section 2 LQR Controller

Make

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix} \tag{2.1}$$

$$\mathrm{R} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{2.2}$$

and

$$\Gamma = \begin{pmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{pmatrix} \tag{2.3}$$

whose eigenvectors

$$\begin{pmatrix} v_i \\ \mu_i \end{pmatrix}, \tag{2.3}$$

where $i = 1, 3, 4$

$$P = [\mu_1, \mu_3, \mu_5][v_1, v_3, v_5]^{-1} = \begin{bmatrix} 0.37 & 0.02 & 0.0002 \\ 0.02 & 0.02 & -0.0003 \\ 0.0002 & -0.0003 & 0.003 \end{bmatrix} \tag{2.4}$$

$$K = R^{-1}B^T P = \begin{bmatrix} 0.87 & -0.56 & 0.015 \\ -0.12 & 0.27 & -2.15 \end{bmatrix} \tag{2.5}$$

Results:

The control system block diagram of section 2 is shown in *Figure 2-1*.

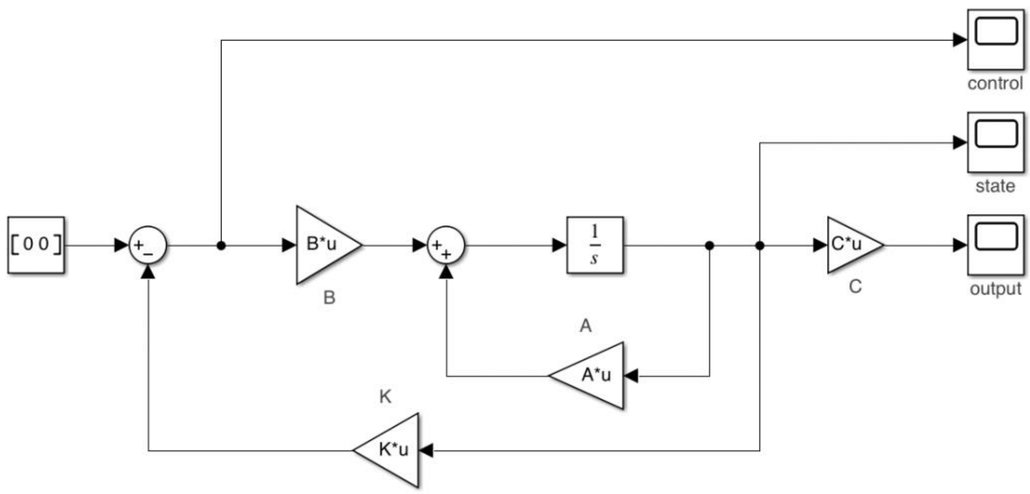

Figure 2-1 System Diagram for Section 2

The transient response by giving step reference signal for each input channel is shown in *Figure 2-2* and *Figure 2-3*.

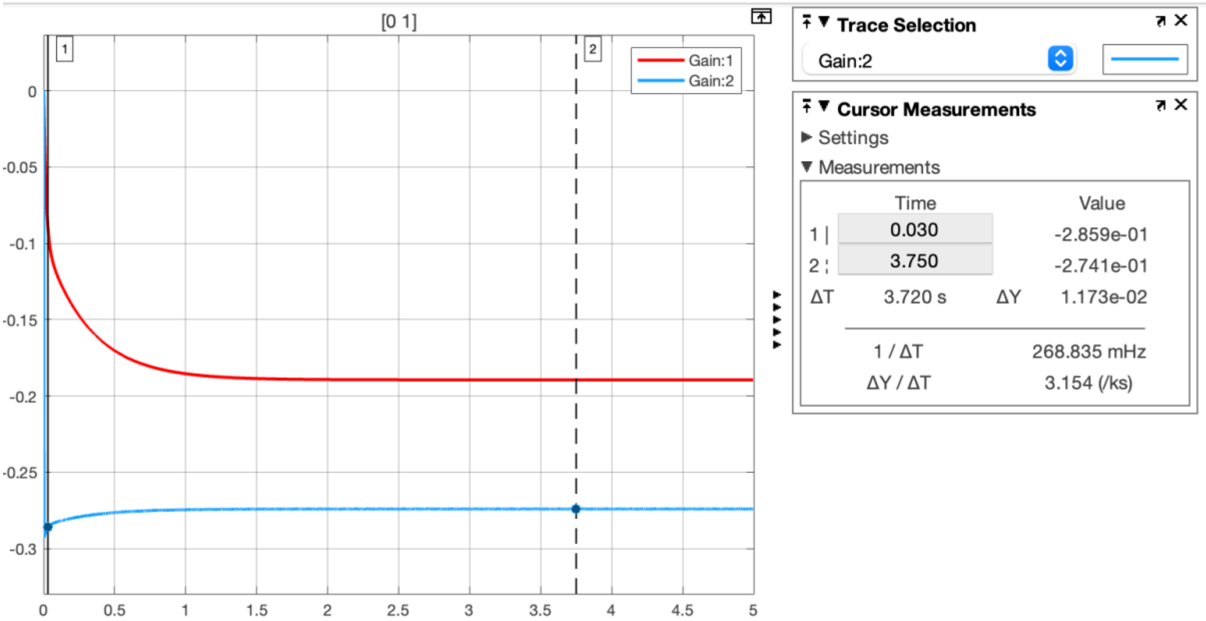

Figure 2-2 Transient Response by Giving Step Reference Signal (r = [0 1])

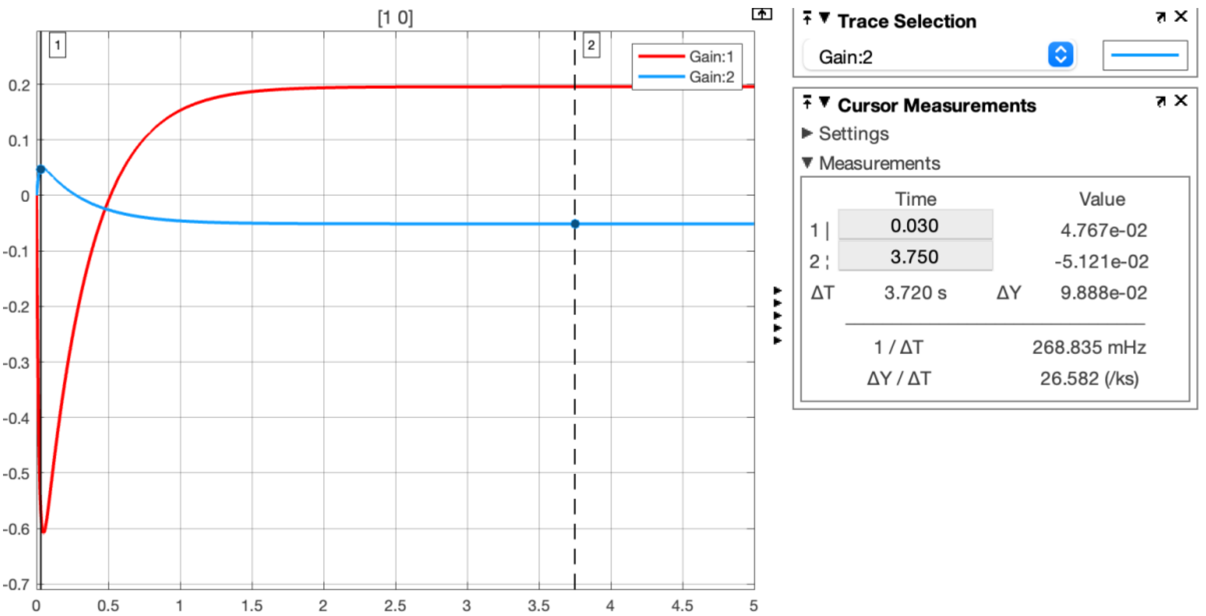

Figure 2-3 Transient Response by Giving Step Reference Signal (r = [1 0])

The overshoots of these transient response are all between 1% and 4%, in addition, the 2% settling times are obviously smaller than 20 seconds, which meet the requirements.

With the original Q and R, the state variables and the control signals are shown in *Figure 2-4* and *Figure 2-5*.

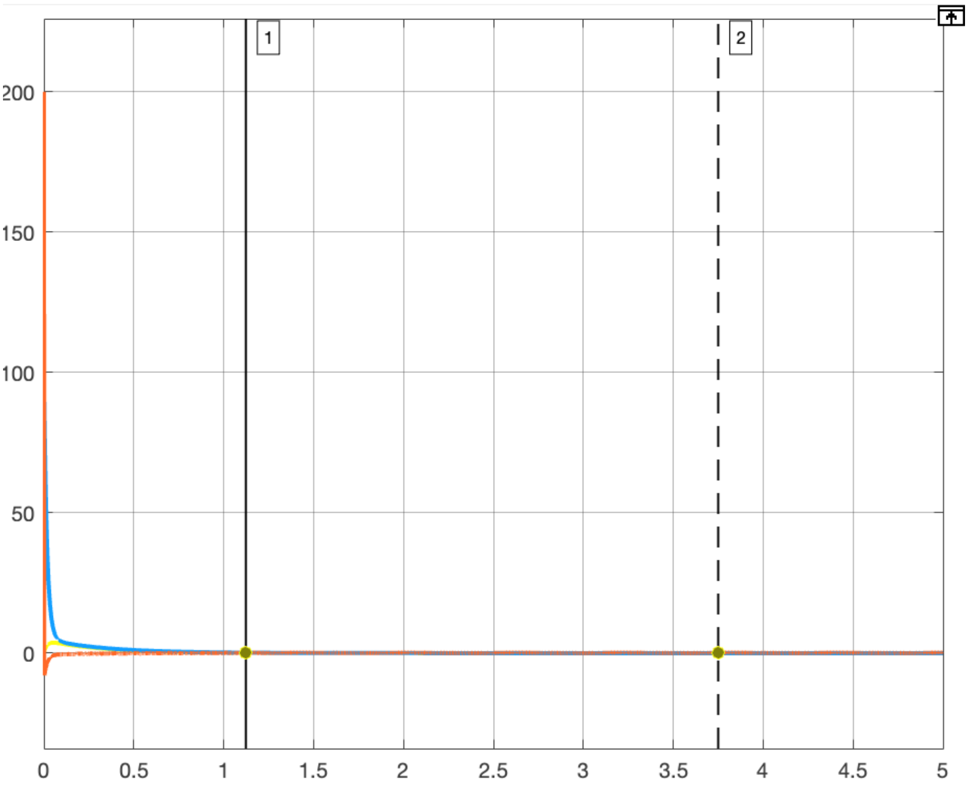

Figure 2-4 States Variables-1

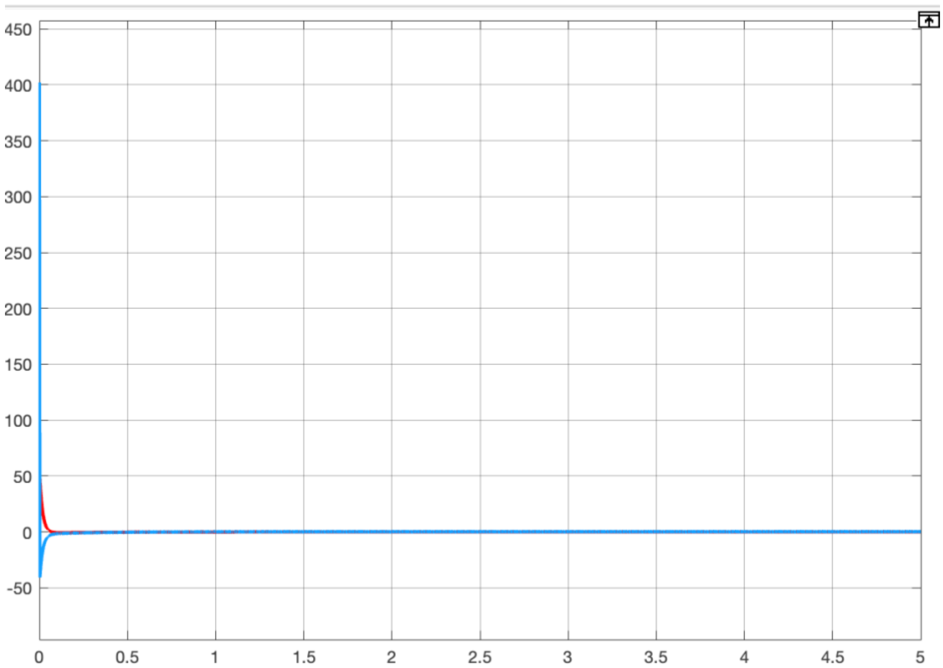

Figure 2-5 Control Signals-1

When matrix R times 10 or matrix Q times 10, there are no significant changes in response time, but the size of control signals become smaller when R become bigger and when size of control signals increase when Q become bigger, as shown in Figure 2-6 and Figure 2-7.

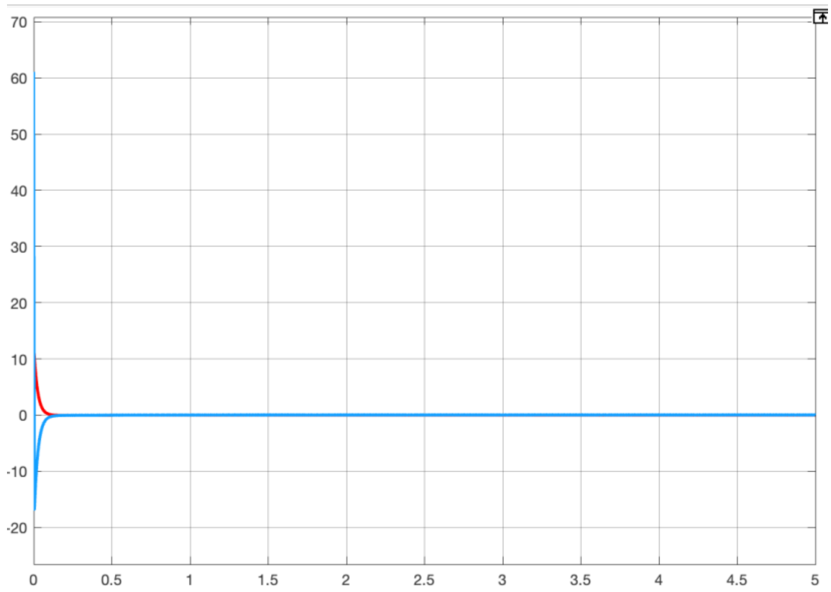

*Figure 2-6 Control Signals-R*10*

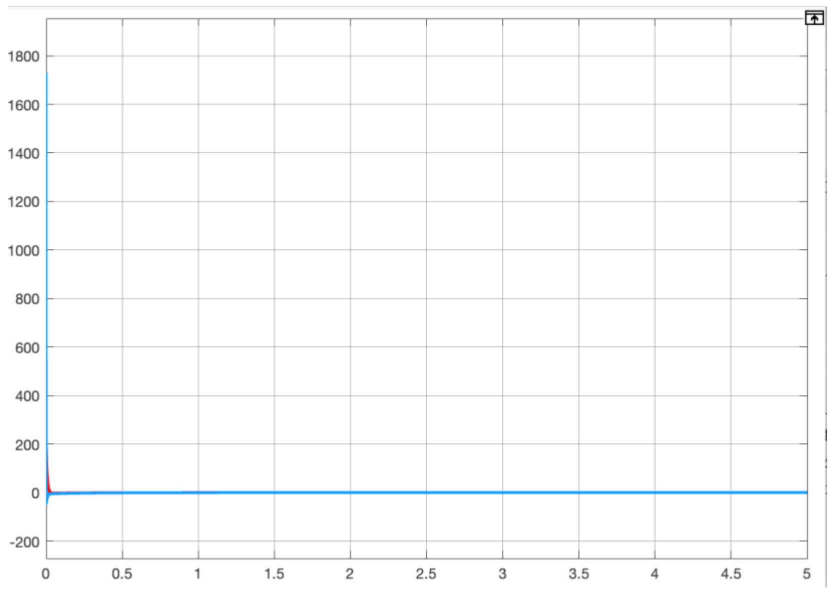

*Figure 2-7 Control Signals-Q*10*

Section 3 State Observer Using LQR Method

Similar as Section 2, we can get

$$K = \begin{bmatrix} 0.87 & -0.56 & 0.015 \\ -0.12 & 0.27 & -2.15 \end{bmatrix} \tag{3.1}$$

Then

$$\bar{A} = A^T, \bar{C} = C^T \tag{3.2}$$

$$W_c = [\,\bar{A} \quad \bar{A}\bar{C} \quad \bar{A}^2\,\bar{C}] \tag{3.3}$$

Choose three independent vectors from matrix W_c in the order from left to right as the matrix C.

$$C^{-1} = \begin{bmatrix} q_1^T \\ q_2^T \\ q_3^T \end{bmatrix} \tag{3.4}$$

$$d_1 = 2, d_2 = 1 \tag{3.5}$$

$$T = \begin{bmatrix} q_2^T \\ q_2^T A \\ q_3^T \end{bmatrix} \tag{3.6}$$

Use eigenvalues of system $[A - BK]$, to determine the poles of the observer and thus calculate the matrix L.

$$\bar{K} = \begin{bmatrix} \bar{k}_{11} & \bar{k}_{12} & \bar{k}_{13} \\ \bar{k}_{21} & \bar{k}_{22} & \bar{k}_{23} \end{bmatrix} \tag{3.7}$$

$$A_d = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1558 & -424 & -37 \end{bmatrix} \tag{3.8}$$

$$\det(sI - A_d) = s^3 + 37 * s^2 + 424 * s + 1558 \tag{3.9}$$

$$L = \bar{K}T = \begin{bmatrix} 0.74 & -31.7 & -481 \\ 5.7 & 424 & -1003 \end{bmatrix} \tag{3.10}$$

Results:

The control system block diagram of section 3 is shown in Figure 3-1.

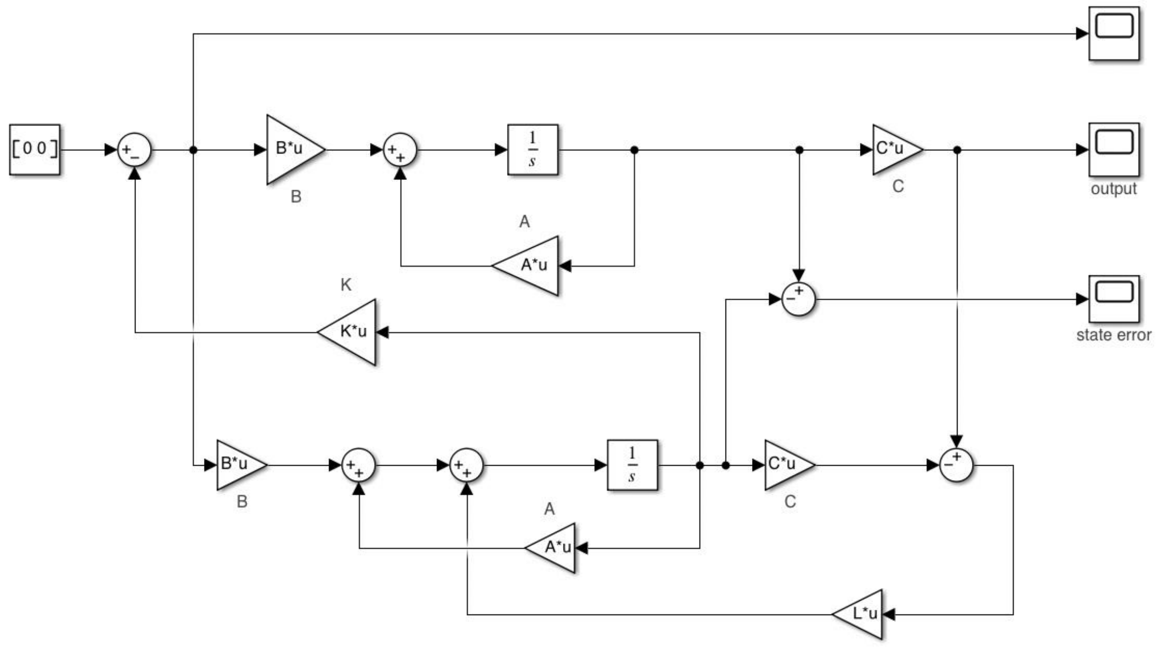

Figure 3-1 System Diagram for Section 3

The transient response by giving step reference signal for each input channel is shown in Figure 3-2 and Figure 3-3.

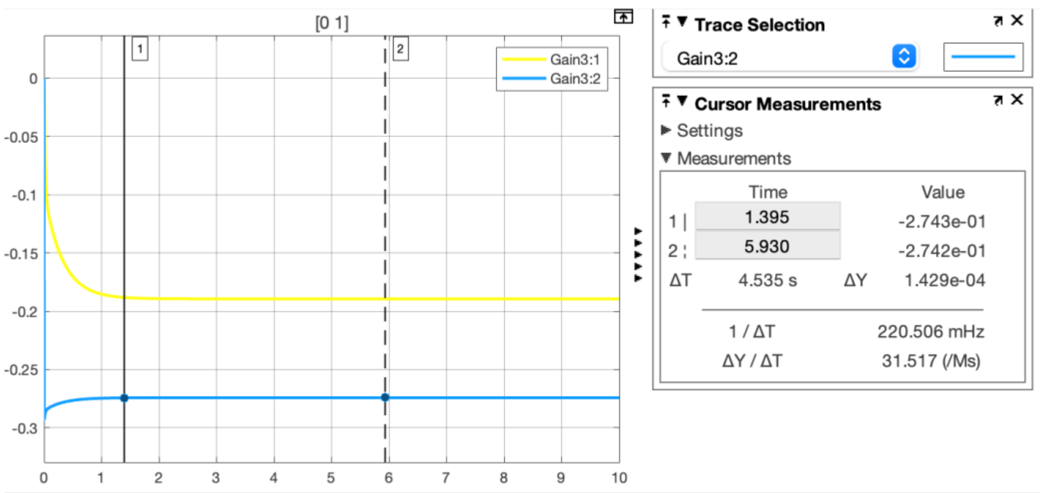

Figure 3-3 Transient Response by Giving Step Reference Signal (r = [0 1])

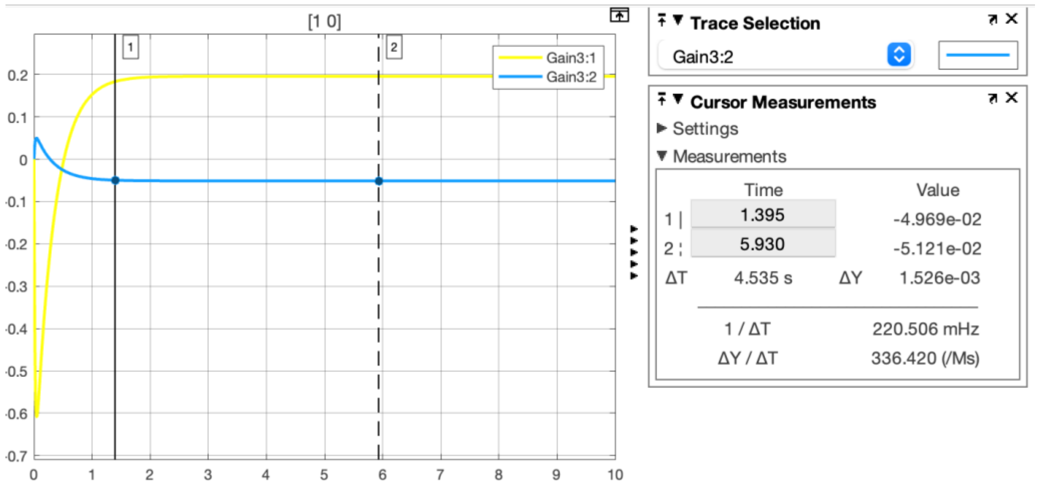

Figure 3-3 Transient Response by Giving Step Reference Signal (r = [1 0])

As shown in the scope, the overshoots of these transient response are all between 1% and 4%, in addition, the 2% settling times are obviously smaller than 20 seconds, which meet the requirements.

The state estimation errors with different observer poles are shown below. N is multiplier of the multiplier of the poles.

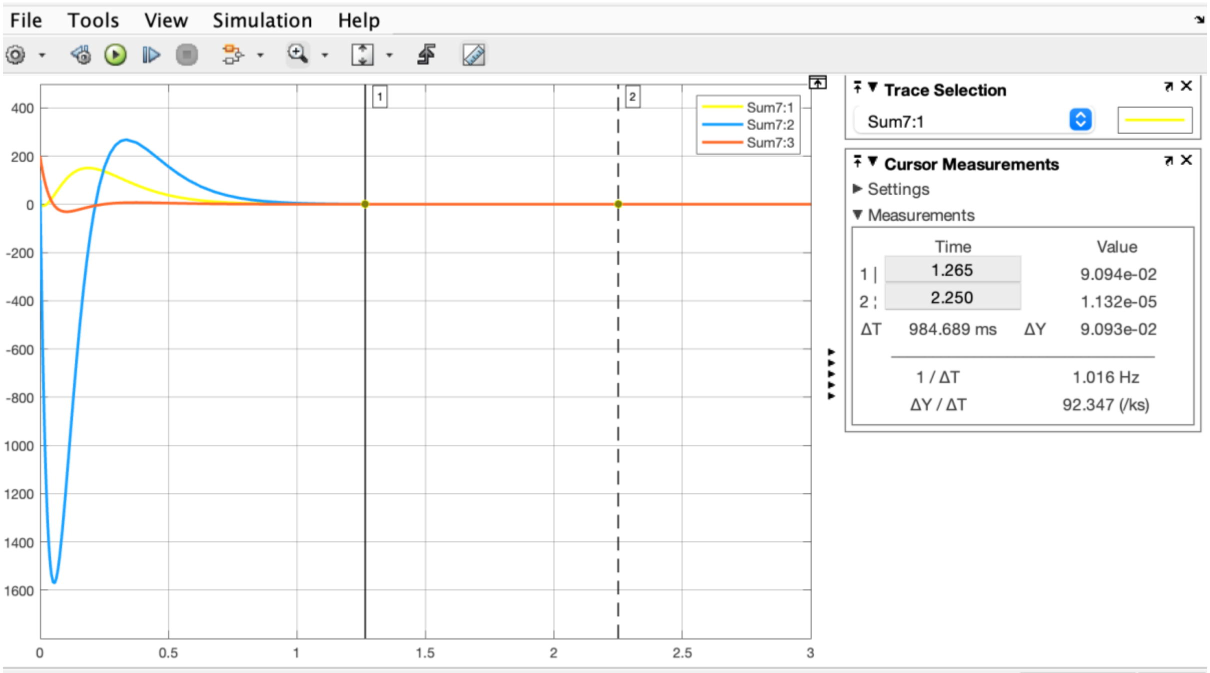

Figure 3-5 States Variables (N=3)

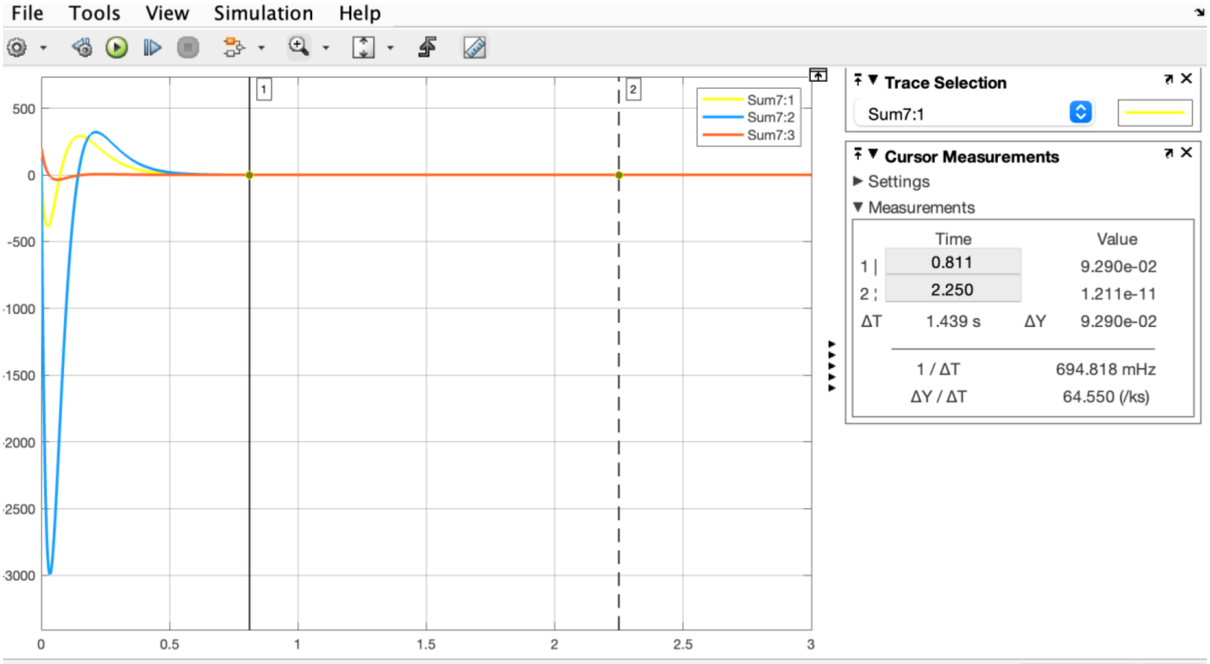

Figure 3-6 States Variables (N=5)

As we can see, when the absolute value of the real part of the poles increases, the state estimate error goes faster to zero. Besides, the faster state estimate error goes to zero, we assume the better close-loop control performance is. Therefore, the increase of the absolute value of the real part of the poles will improve the close-loop control performance.

Section 4 Decoupling Controller

$$C = \begin{pmatrix} c_1^T \\ \vdots \\ c_m^T \end{pmatrix} \tag{4.1}$$

$$G(s) = C(sI - A)^{-1}B \tag{4.2}$$

By calculation we get $\sigma_1 = 1$ and $\sigma_2 = 1$.

$$B^* = \begin{bmatrix} c_1^T A^{\sigma_1 - 1} B \\ c_2^T A^{\sigma_2 - 1} B \end{bmatrix} = \begin{bmatrix} c_1^T B \\ c_2^T B \end{bmatrix} = \begin{bmatrix} -44 & 0 \\ 0 & -830 \end{bmatrix} \tag{4.3}$$

$$H(s) = diag[(s+10)^{-1}, (s+10)^{-1}] \tag{4.4}$$

$$\phi_{f_1}(s) = s + 10, \tag{4.5}$$

$$\phi_{f_2}(s) = s + 10, \tag{4.6}$$

$$\phi_{f_1}(A) = A + 10I, \tag{4.7}$$

$$\phi_{f_2}(A) = A + 10I, \tag{4.8}$$

$$C^{**} = \begin{bmatrix} c_1^T \phi_{f_1}(A) \\ c_2^T \phi_{f_2}(A) \end{bmatrix} = \begin{bmatrix} c_1^T(A+I) \\ c_2^T(A+2I) \end{bmatrix} = \begin{bmatrix} 23 & -20 & 20 \\ 0 & -480 & -1030 \end{bmatrix}, \tag{4.9}$$

$$F = B^{*-1} = \begin{bmatrix} -0.0227 & 0 \\ 0 & -0.0012 \end{bmatrix}, \tag{4.10}$$

$$K = B^{*-1}C^{**} = \begin{bmatrix} -0.52 & 0.45 & -0.45 \\ 0 & 0.58 & 1.24 \end{bmatrix}, \tag{4.11}$$

Results:

The control system block diagram of section 4 is shown in Figure 4-1.

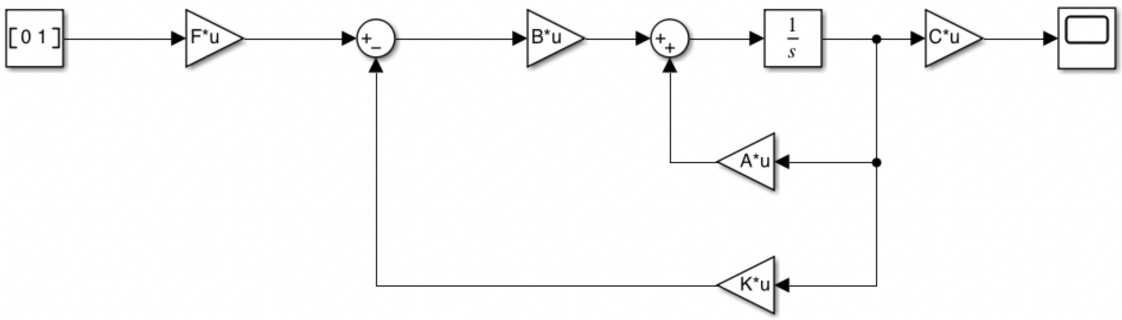

Figure 4-1 System Diagram for Section 4

The transient response with zero initial states by giving step reference signal for each input channel is shown in Figure 4-2 and Figure 4-3.

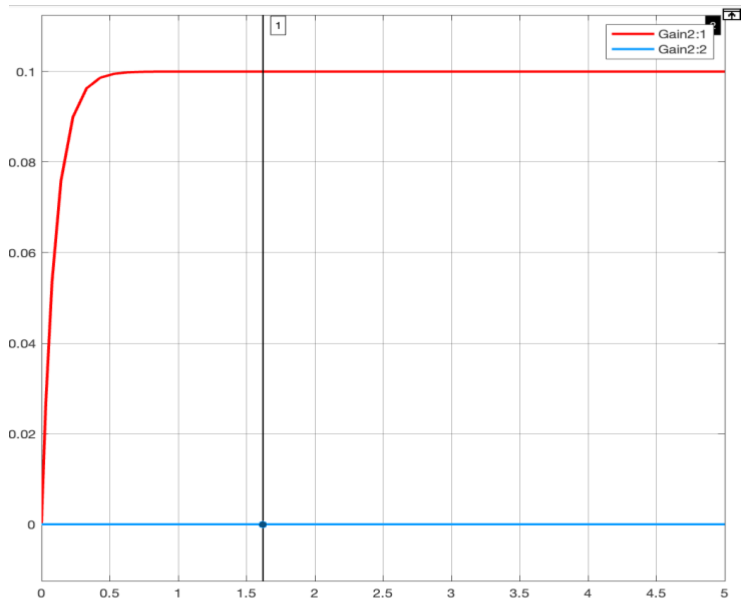

Figure 4-2 Transient Response by Giving Step Reference Signal (r = [1 0])

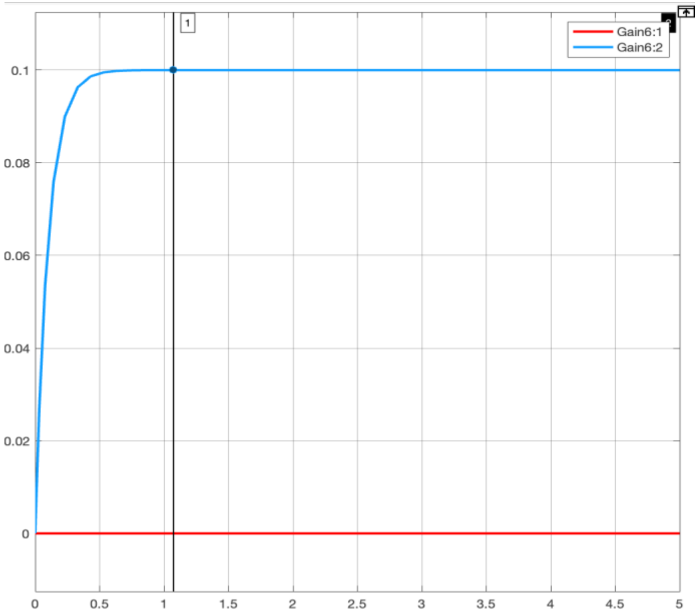

Figure 4-3 Transient Response by Giving Step Reference Signal (r = [1 0])

As shown in the scope, the overshoots of these transient response are all about 1%, in addition, the 2% settling times are obviously smaller than 20 seconds, which meet the requirements.

From Figure 4-2 and Figure 4-3, we can find that each input of the system now controls only one corresponding output, and each output is subject to the action of only one control. Hence, the system is decoupled successfully.

The initial response with respect to x_0 (due to equation 0.7) of the system is shown in Figure 4-4.

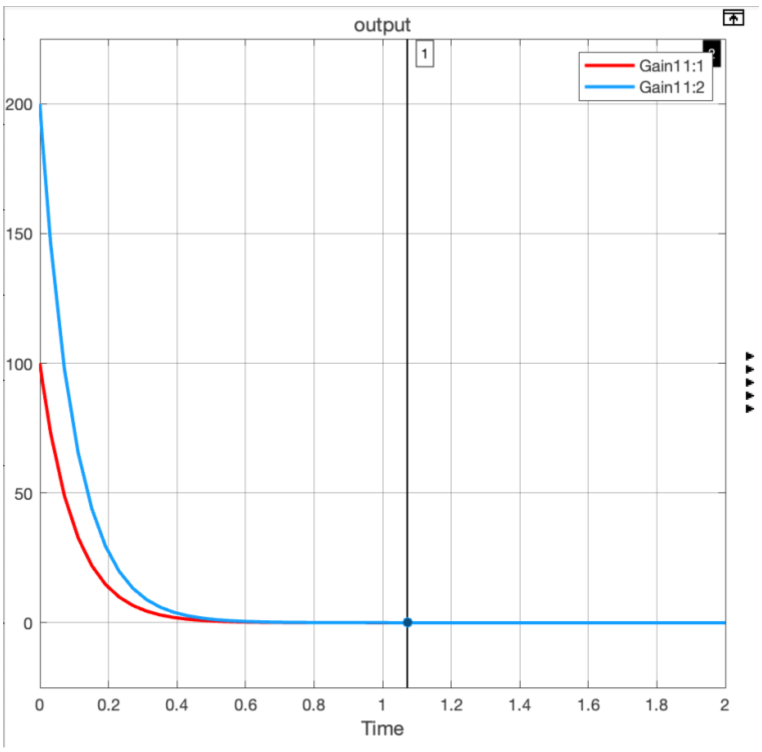

Figure 4-4 Initial Response with Respect to x_0

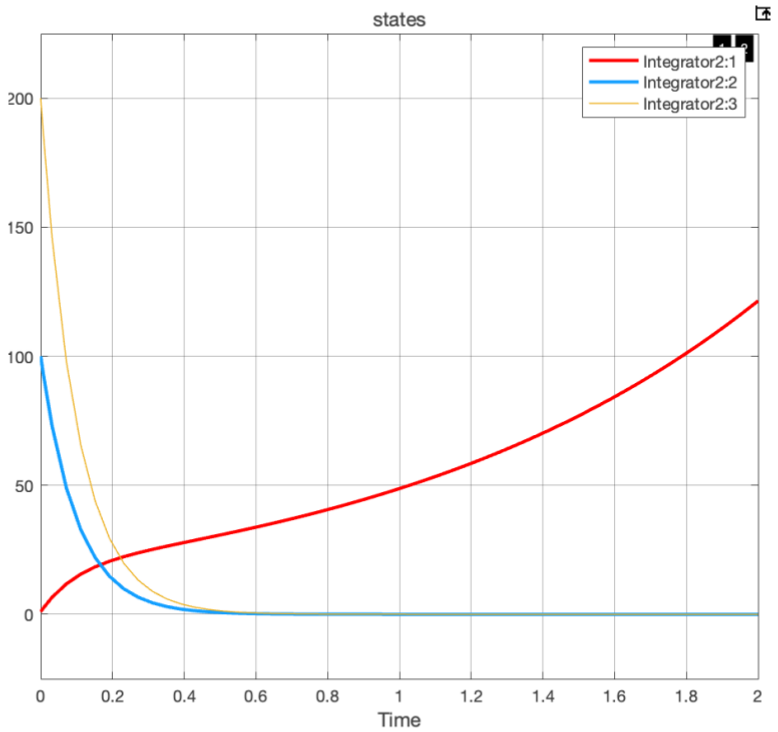

Figure 4-5 States Variables

From the Figure 4-5, with non-zero initial state, the first state variable is unbounded, so the system is not internal stable. Also, check the eigenvalues of $[\mathrm{A}-\mathrm{BK}]$, the eigenvalues are [0.9136 -10.0000 -10.0000] there are one positive eigenvalue, so it is the reason for internal instability.

Section 5 Servo Controller

$$\mathrm{y}_{sp} = r = \begin{bmatrix}100 & 150\end{bmatrix}^T \tag{5.1}$$

$$\mathrm{w} = \begin{bmatrix}-2 & 5\end{bmatrix}^T \tag{5.2}$$

$$\dot{v} = e(t) = r - y \tag{5.3}$$

$$\bar{x} = \begin{pmatrix} x \\ v \end{pmatrix} \tag{5.4}$$

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u + \bar{B}_w w + \bar{B}_r r \tag{5.5}$$

where

$$\bar{A} = \begin{bmatrix} A & O \\ -C & O \end{bmatrix}, \tag{5.6}$$

$$\bar{B} = \begin{pmatrix} B \\ O \end{pmatrix}, \tag{5.7}$$

$$\bar{B}_w = \begin{pmatrix} \bar{B}_w \\ O \end{pmatrix}, \tag{5.8}$$

$$\bar{B}_r = \begin{pmatrix} O \\ I \end{pmatrix}. \tag{5.9}$$

Similar to the Section 3, using LQR method to calculate the feedback gain matrix, $K = [K_1, K_2]$, and the observer feedback gain matrix L.

$$K_1 = \begin{bmatrix} 1.12 & -0.44 & -0.0003 \\ -0.25 & -0.04 & -0.35 \end{bmatrix} \tag{5.10}$$

$$K_2 = \begin{bmatrix} -0.77 & 0.63 \\ 0.63 & 0.77 \end{bmatrix} \tag{5.11}$$

$$L = \begin{bmatrix} 0.74 & 0.1 \\ -31.70 & 0.00 \\ -481.00 & -984.00 \end{bmatrix} \tag{5.12}$$

Results:

The control system block diagram of section 5 is shown in Figure 5-1.

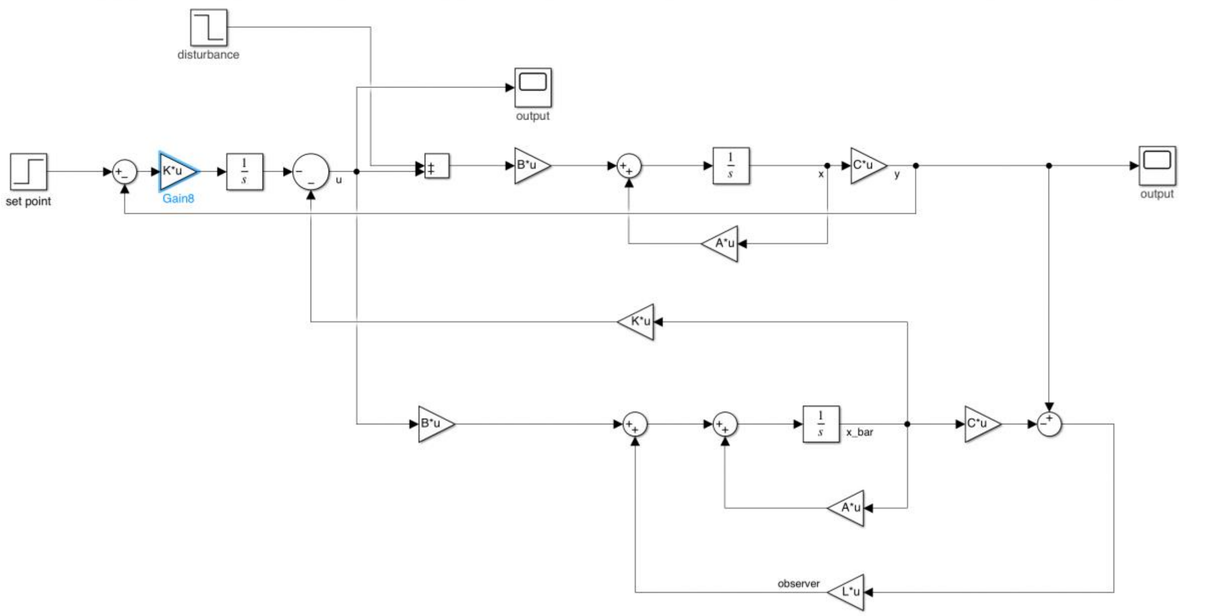

Figure 5-1 System Diagram for Section 5

The transient response by giving step reference signal for each input channel is shown in Figure 5-2 and Figure 5-3.

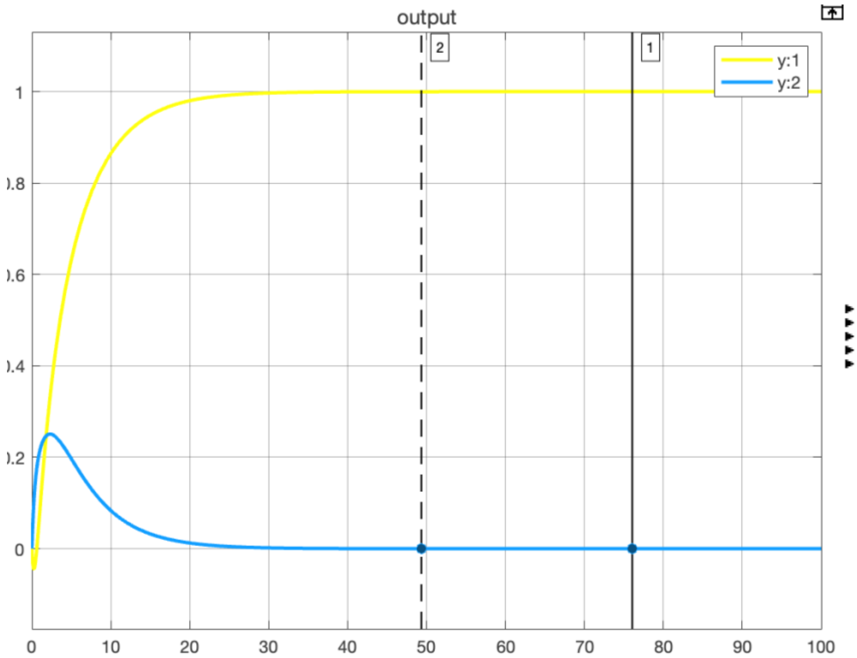

Figure 5-2 Transient Response by Giving Step Reference Signal (r = [1 0])

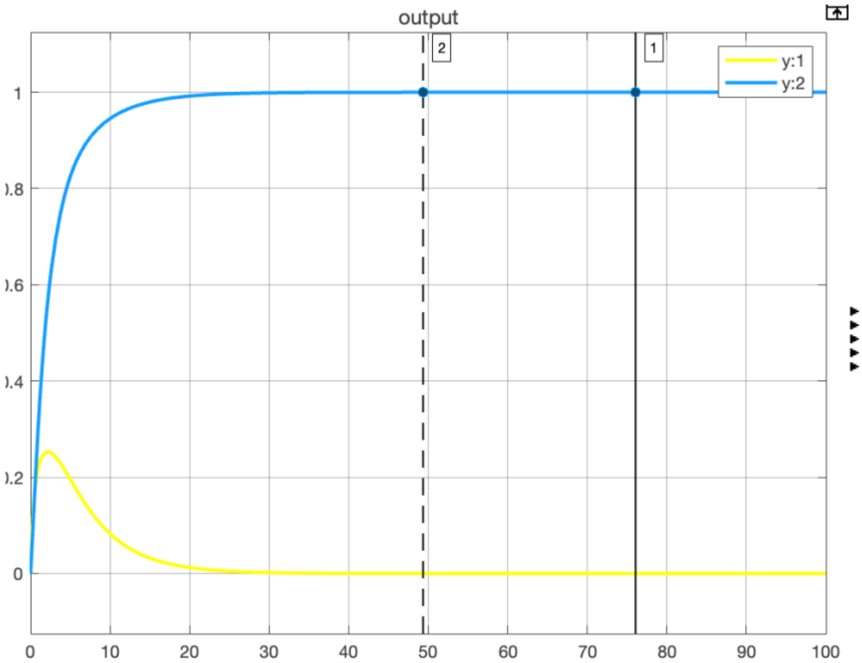

Figure 5-3 Transient Response by Giving Step Reference Signal (r = [1 0])

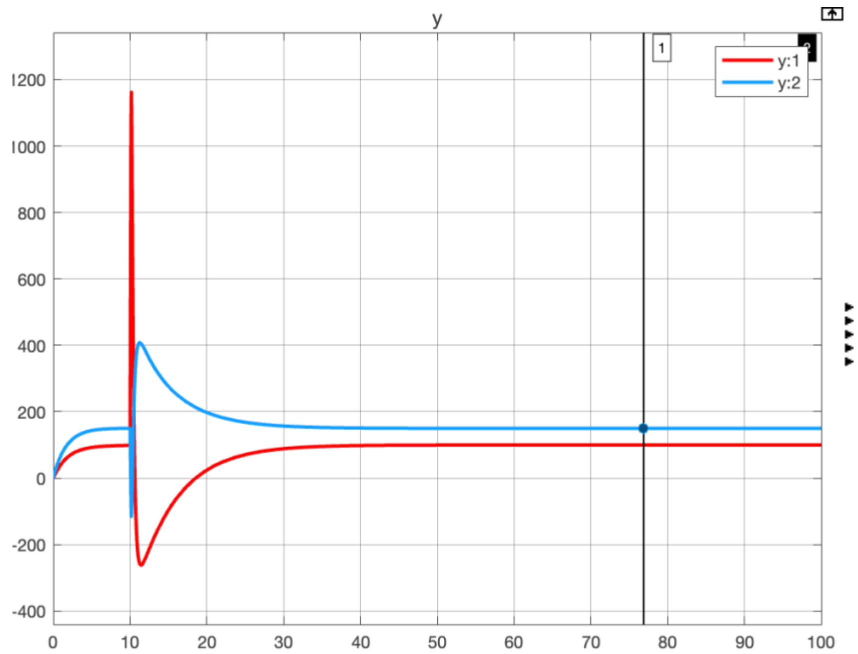

Figure 5-4 Output Signals

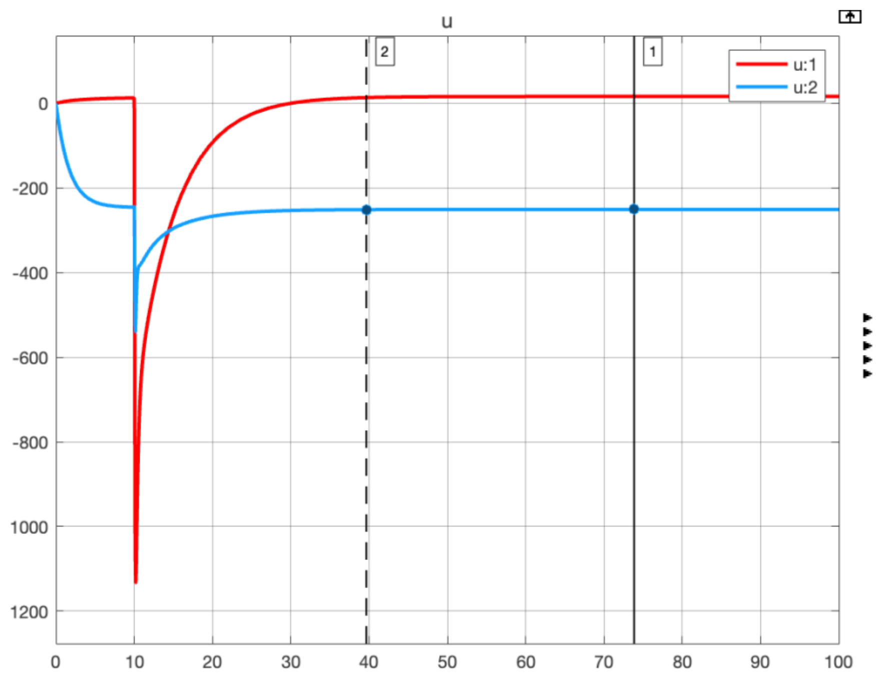

Figure 5-5 Control Signals

From Figure 5-4 and Figure 5-5, which plots the output signals of the servo system, we can find that as the time increases, the output signal gradually converges to the set points $y_{sp} = [100 \quad 150]^T$. Besides, since a disturbance is set at 10 seconds, both the output signal and the control signal have a large shock at 10 seconds due to the combined effect of the observer (good tracking ability).

Section 6 Manipulate Three State Variables

$$x_{sp} = [5 \quad 250 \quad 300]^T \tag{6.1}$$

Consider the system described by

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

At steady state,

$$\dot{x} = Ax + Bu = 0 \tag{6.2}$$

Since A and B are known, we can derive:

$$x = -A^{-1}Bu \tag{6.3}$$

$$\begin{cases} \dfrac{10772 * u_1}{3767} + \dfrac{415 * u_2}{7534} = 5 \\ \dfrac{10452 * u_1}{18835} - \dfrac{1411 * u_2}{3767} = 250 \\ \dfrac{-4824 * u_1}{18835} - \dfrac{18841 * u_2}{30136} = 300 \end{cases} \tag{6.4}$$

$$\begin{aligned} W &= diag(a + b + 1, c + 4, d + 5) \\ &= diag(11, 8, 6) \end{aligned} \tag{6.5}$$

$$\begin{aligned} J(x_s) &= \frac{1}{2}(x_s - x_{sp})^T W (x_s - x_{sp}) \\ &= \frac{1}{2}(-A^{-1}Bu - x_{sp})^T W (-A^{-1}Bu - x_{sp}) \end{aligned} \tag{6.6}$$

From

$$\frac{\partial J(x_s)}{\partial u_1} = 0 \tag{6.7}$$

$$\frac{\partial J(x_s)}{\partial u_2} = 0 \tag{6.8}$$

we can get

$$U = [u_1, u_2]^T = [14.6699, -538.865]^T, \tag{6.9}$$

The final states achieved

$$x = [12 \quad 210 \quad 333]^T \tag{6.10}$$

and the lost function

$$J = 9984.6$$

It is impossible to maintain the states x, around this set point. Because, there are only two inputs but three separate states needed to manipulate.

The control system block diagram of section 6 is shown in Figure 6-1.

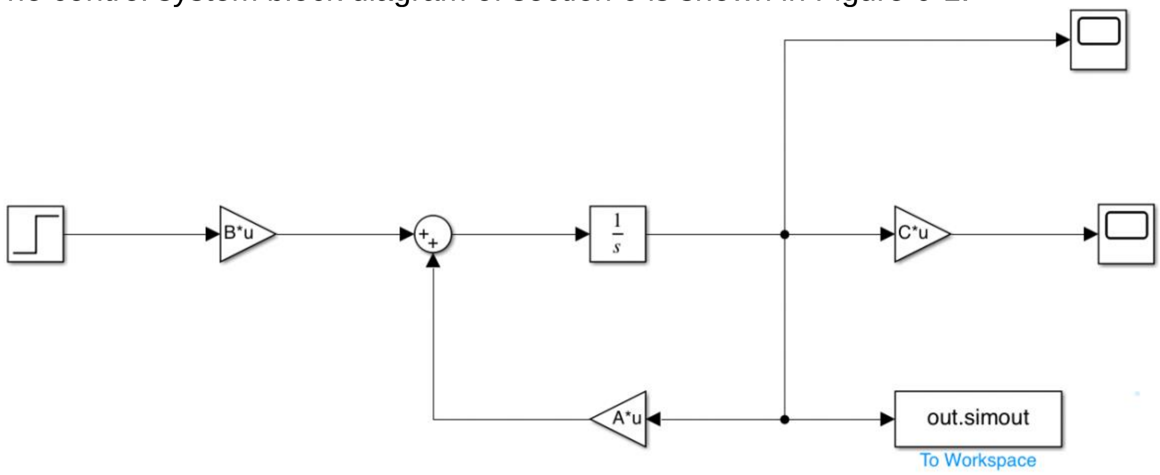

Figure 6-1 System Diagram for Section 6

Chapter 3 Conclusion

All the question in this assignment, is about control systems design of a continuous-flow stirred tank reactor(CSTR).
The assignment consists most of the chapter that I have learnt from this module, such like how to apply pole placement, how to design LQR controller, how to decouple a system, how to apply servo control and observer, etc.. During the process of doing this assignment and discussing with my classmate, I not only be familiar with how to use the Simulink in MATLAB to simulate the system, but also gave me a deeper understanding about this module.

Appendix

Code for Section 1

```
% a = 2, b = 8, c = 4, d = 1
A = [-1.7 -0.25 0; 23 -30 20; 0 -480 -1040];
B = [5 0; -44 0; 0 -830];
C = [0 1 0; 0 0 1];
M = [B A*B A*A*B]; % {B AB AAB}={b1 b2 Ab1 Ab2 AAb1 AAb2}
% r_M = rank(M); % 3
%%
```

```
l = 0.8; % l>0.5911 0.8
wn = 0.25; % l*wn>=0.1 0.25
j = [-l*wn + wn*1i*(1-l*l)^0.5, -l*wn - wn*1i*(1-l*l)^0.5, -0.5]; %0„ÄÇ5
k1 = place(A, B, j);
%% find k manually
% W = rref(M)
cc = inv([M(:,1) M(:,3) M(:,2)]); % d1=2 d2=1
Tq = [cc(2,:); cc(2,:)*A; cc(3,:)];
B_b = Tq * B;
A_b = Tq * A * pinv(Tq); % Matrix is close to singular or badly scaled.
Results may be inaccurate.
syms s k11 k12 k13 k21 k22 k23;
A_n = collect((s - (-l*wn + wn*1i*(1-l*l)^0.5))*(s - (-l*wn - wn*1i*(1-
l*l)^0.5))*(s + 0.5));
% s^3 + (9*s^2)/10 + (21*s)/80 + 1/32
A_d = [0 1 0; 0 0 1;(-1/32) (-21/80) (-0.9)];
k_b = [k11 k12 k13; k21 k22 k23];
comp = A_b - B_b*k_b;
[k11,k12,k13,k21,k22,k23] = solve(comp(2,1)==A_d(2,1),
comp(2,2)==A_d(2,2),comp(2,3)==A_d(2,3),comp(3,1)==A_d(3,1),
comp(3,2)==A_d(3,2),comp(3,3)==A_d(3,3));
k_b = [k11 k12 k13; k21 k22 k23];
k = k_b * Tq;
k = double(k);
```

Code for Section 2

```
clc
clear
% a = 2, b = 8, c = 4, d = 1
A = [-1.7 -0.25 0; 23 -30 20; 0 -480 -1040];
B = [5 0; -44 0; 0 -830];
C = [0 1 0; 0 0 1];
sys_ss = ss(A, B, C, 0);

% LQR
% Q = C'*C;
% Q(1,1) = 1;
Q = [1 0 0
    0 1 0
    0 0 10]*10;
R = [1 0
    0 1];

r11 = A;
r12 = -1 * B * pinv(R) * B';
r21 = -1 * Q;
r22 = -1 * A';
gamma = [r11 r12; r21 r22];
[vector, z] = eig(gamma);
eigvalue_minus = [];
for i=1:6
    if z(i,i)<0
        eigvalue_minus = [eigvalue_minus,i];
    end
end
v_minus = vector(:,eigvalue_minus);
v = v_minus(1:3,:);
```

```
m = v_minus(4:6,:);
P = m * pinv(v);
K = pinv(R) * B' * P;
```

Code for Section 3

```
clc
clear
% a = 2, b = 8, c = 4, d = 1
A = [-1.7 -0.25 0; 23 -30 20; 0 -480 -1040];
B = [5 0; -44 0; 0 -830];
C = [0 1 0; 0 0 1];

M = [B A*B A*A*B]; % {B AB AAB}={b1 b2 Ab1 Ab2 AAb1 AAb2}
sys_ss = ss(A, B, C, 0);

%% LQR
% Q = C'*C;
% Q(1,1) = 1;
Q = [1 0 0
    0 1 0
    0 0 10];
R = [1 0
    0 1];

r11 = A;
r12 = -1 * B * pinv(R) * B';
r21 = -1 * Q;
r22 = -1 * A';
G = [r11 r12; r21 r22];
[vector, z] = eig(G);
eigvalue_minus = [];
for i=1:6
    if z(i,i)<0
        eigvalue_minus = [eigvalue_minus,i];
    end
end
v_minus = vector(:,eigvalue_minus);
% v_minus = [vector(:,1) vector(:,3) vector(:,4)];
v = v_minus(1:3,:);
m = v_minus(4:6,:);
P = m * pinv(v);
K = pinv(R) * B' * P;

[~,g1] = eig(A-B*K);

%% obsever pole
n = 5;
eig_observer=round([n*g1(1) + 1i,n*g1(1) - 1i,2*n*g1(1)]);

lambda1=eig_observer(1);
lambda2=eig_observer(2);
lambda3=eig_observer(3);

M = [C' A'*C' A'^2*C'];
% rank(M);
% R1 = rref(M);
```

```
Rc = [M(:,1) M(:,3) M(:,2)];
P1 = pinv(Rc);
P = [P1(2,:)
    P1(2,:)*A'
    P1(3,:)];
A_b = P * A' * pinv(P);
C_b = P * C';

syms s k11 k12 k13 k21 k22 k23
coefficient = poly([lambda1, lambda2, lambda3]);
A_d=[0,1,0
    0,0,1
    -coefficient(4), -coefficient(3), -coefficient(2)];

syms s k11 k12 k13 k21 k22 k23;
k_b=[k11 k12 k13;
    k21 k22 k23];
comp = A_b - C_b * k_b;
[k11,k12,k13,k21,k22,k23]=solve(comp(2,1)==A_d(2,1),
comp(2,2)==A_d(2,2),comp(2,3)==A_d(2,3),comp(3,1)==A_d(3,1),
comp(3,2)==A_d(3,2),comp(3,3)==A_d(3,3));
k = [k11,k12,k13;k21,k22,k23]*P;
L=double(k)';
```

Code for Section 4

```
clc;
clear;
% a = 2, b = 8, c = 4, d = 1
A = [-1.7 -0.25 0; 23 -30 20; 0 -480 -1040];
B = [5 0; -44 0; 0 -830];
C = [0 1 0; 0 0 1];
D = [0 0; 0 0];
x0 = [1 100 200];
%%
syms s
for i=1:3
    g = C(1,:)*A^(i-1)*B;
    if g(1)~=0||g(2)~=0
       sigmoid1 = i;
       break
    end
end

for i=1:3
    g = C(2,:)*A^(i-1)*B;
    if g(1)~=0||g(2)~=0
       sigmoid2 = i;
       break
    end
end

C_star=[C(1,:)*A^sigmoid1
       C(2,:)*A^sigmoid2];
B_star=[C(1,:)*A^(sigmoid1-1)*B
       C(2,:)*A^(sigmoid2-1)*B];
F = inv(B_star);
```

```
fA1 = A + 10*eye(3);
fA2 = A + 10*eye(3);
K = F * [C(1,:)*fA1; C(2,:)*fA2];
eig(A - B*K)
```

Code for Section 5

```
clc;
clear;
% a = 2, b = 8, c = 4, d = 1
A = [-1.7 -0.25 0; 23 -30 20; 0 -480 -1040];
B = [5 0; -44 0; 0 -830];
C = [0 1 0; 0 0 1];

y_sp = [100; 150];
w = [-2; 5]; % disturbance
x0 = [1, 100, 200];
% rank([A B;C zeros(2,2)]);
A_b = [A zeros(3,2);-C zeros(2,2)];
B_b = [B;zeros(2,2)];
B_w_b = [B;zeros(2,2)];
B_r_b = [zeros(3,2)
        eye(2)];
% C_b = [C zeros(2,2)];
%% LQR
Q=[1 0 0 0 0;
   0 1 0 0 0;
   0 0 1 0 0;
   0 0 0 1 0;
   0 0 0 0 1];
R=[1 0
   0 1];
gamma=[A_b -B_b/pinv(R)*B_b'
    -Q -A_b'];
[vector,value]=eig(gamma);
value = sum(value);
v = vector(:, find(real(value)<0));
P=v(6:10,:)/v(1:5,:);

%% K = pinv(R)*B_b'*P
K = real(pinv(R)*B_b'*P);
K1 = K(:,1:3);
K2 = K(:,4:5);
%% obsever pole
n = 3;
[~,gamma1] = eig(A-B*K1);
eig_observer=round([n*gamma1(1) + 1i,n*gamma1(1) - 1i,2*n*gamma1(1)]);

lambda1=eig_observer(1);
lambda2=eig_observer(2);
lambda3=eig_observer(3);

M = [C' A'*C' A'^2*C'];
% rank(M);
% R1 = rref(M);
```

```
Rc = [M(:,1) M(:,3) M(:,2)];
P1 = pinv(Rc);
P = [P1(2,:)
    P1(2,:)*A'
    P1(3,:)];
A_b = P * A' * pinv(P);
C_b = P * C';

coefficient = poly([lambda1, lambda2, lambda3]);
A_d=[0,1,0
    0,0,1
    -coefficient(4), -coefficient(3), -coefficient(2)];

syms s k11 k12 k13 k21 k22 k23;
k_b=[k11 k12 k13
    k21 k22 k23];
comp = A_b - C_b * k_b;
[k11,k12,k13,k21,k22,k23]=solve(comp(2,1)==A_d(2,1),
comp(2,2)==A_d(2,2),comp(2,3)==A_d(2,3),comp(3,1)==A_d(3,1),
comp(3,2)==A_d(3,2),comp(3,3)==A_d(3,3));
k = [k11,k12,k13;k21,k22,k23]*P;
L=double(k)';
```

Code for Section 6

```
a = 2;
b = 8;
c = 4;
d = 1;
A = [-1.7 -0.25 0; 23 -30 20; 0 -480 -1040];
B = [5 0; -44 0; 0 -830];
C = [0 1 0; 0 0 1];
x_sp = [5; 250; 300];
w = [a+b+1 0 0
    0 c+4 0
    0 0 d+5];
syms s u1 u2
xs = pinv(s*eye(3)-A)*B*[u1;u2];
xs2 = subs(xs,s,0);
J = 1/2*(xs2 - x_sp)' * w * (xs2 - x_sp);
jacob = jacobian(J, [u1 u2]);
uu = solve(jacob==0);
u1 = double(uu.u1);
u2 = double(uu.u2);
U = [u1;u2];
x = out.simout(1000,:);
J_x = double(1/2*(x'-x_sp)'* w *(x'-x_sp));
```